# Considerations for teachers using manipulative materials

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# Considerations for teachers using manipulative materials

Classroom teachers of mathematics are witnessing an unprecedented period of proliferation in manipulative materials. Commercial catalogs list a great variety of available materials; professional journals carry many advertisements claiming that this device or that aid will provide a panacea for learning a certain mathematics topic; and professional meetings are frequently inundated with exhibits displaying new manipulative materials. This influx of newly available materials has precipitated many problems. The wide range of quality found among various materials has made the problem of selection much more difficult. It has made it impossible to list all available materials and discuss the merit - or lack of merit - of each. It has created doubts in some teachers' minds about the educational value of the materials. It has raised additional teacher-oriented questions such as, "What are some guidelines for selecting manipulative materials?", "What materials should be used?", "What are some dos and don'ts of using them?"

During the decade of the '60s, several fine articles appeared discussing considerations in the selection of learning materials (Berger and Johnson, 1959; Bernstein, 1963; Davidson, 1968; Hamilton, 1966; Spross, 1964). The present article is limited to a discussion of manipulative materials as opposed to other teaching aids. Furthermore, it is addressed specifically to classroom teachers in an effort to provide some current rationale, as well as guidelines, for the selection and use of manipulative materials.

## WHAT ARE MANIPULATIVE MATERIALS?

The use of the term *manipulative materials* raises one fundamental question, namely, "Just what are manipulative materials?" In this context, manipulative materials are objects or things that the pupil is "able to feel, touch, handle, and move. They may be real objects which have social application in our everyday affairs, or they may be objects which are used to represent an idea" (Grossnickle, Junge, and Metzner, 1951, p.162). Hence, not all teaching aids or instructional materials are manipulative materials. Suffice it to say here that manipulative materials appeal to several senses and are characterized by a physical involvement of pupils in an active learning situation.

## RATIONALE

In teaching mathematics, we are primarily concerned with concept formation as opposed to the memorization of facts. The mental processes involved in concept formation are much more complex than those associated with the memorization of a mass of isolated details. There is little disagreement among contemporary psychologists regarding the role of concept formation in the learning of mathematics. However, there are several existing theories about how to best foster proper concept formation. The results of recent psychological investigations into the ways children learn mathematics by men such as Jerome Bruner, Zolton Dienes, Robert Gagné,

Jean Piaget, and Richard Skemp are beginning to have an influence on mathematical pedagogy. In short, more is known today about the way children learn mathematics, and the general nature of the mathematics they are capable of learning at various stages, than has previously been known. Ironically, it is still not known precisely how children learn, but the efforts of researchers are continually providing new evidence to suggest (and oftentimes refute) various learning theories. Since learning is an individual matter and invariably dependent on numerous factors, some of which are quite elusive, it is highly unlikely that a comprehensive learning theory that is completely satisfactory to all people will evolve in the foreseeable future.

A comparison of prominent learning theories will not be made here, but it seems appropriate to identify the following statements, subscribed to by most learning psychologists:

1. Concept formation is the essence of learning mathematics.
2. Learning is based on experience.
3. Sensory learning is the foundation of all experience and thus the heart of learning.
4. Learning is a growth process and is developmental in nature.
5. Learning is characterized by distinct, developmental stages.
6. Learning is enhanced by motivation.
7. Learning proceeds from the concrete to the abstract.
8. Learning requires active participation by the learner.
9. Formulation of a mathematical abstraction is a long process.

This list is not exhaustive, nor are the statements independent. In fact, they are closely interrelated. Suffice it to say that the above statements the basic foundation underlying the rationale for using manipulative materials in learning mathematics.

Many prominent mathematics educators have strongly urged greater use of manipulative materials in teaching mathematics. The rationale for this emphasis seems educationally sound. Unfortunately, research studies in this area have "not been conclusive in either supporting or refuting the value of manipulative aids" (Beougher, 1967, p.31). Most of the questions cited by Brown and Abell (1965, p.548), such as, "Are there certain manipulative devices that lend themselves better to different methods of instruction?" and "Will a device help one child and hinder another?" are yet to be answered. One can only hope that quality research focused on manipulative materials and mathematics learning will provide some objective evidence relevant to the issues. In the meantime, classroom teachers are still faced with the problem of selecting and using manipulative materials in their classroom.

## *SELECTION CRITERIA*

The rapid increase in available commercial materials has made the job of selection not only difficult but also more crucial as the market is flooded with products. There are many criteria to consider in developing and procuring manipulative materials. In order to simplify this discussion, only important criteria in two basic categories, namely, pedagogical and physical, will be

considered. The proposed criteria are not exhaustive, nor is any hierarchy of importance suggested by the order in which they are discussed. Although some considerations are more significant than others, the relative importance attached to each criterion should be determined by the teacher. Any final evaluation of manipulative materials should weigh strengths and weaknesses against the educational potential.

Pedagogically there are many criteria to consider in selecting manipulative materials. One of the most important considerations is whether or not the materials serve the purpose for which they are intended. Furthermore, do these materials do something that could not be done as well or better without them? Since mathematics is mental, do the materials develop the desired mental imagery?

The following criteria should be included in any list purporting to identify pedagogical considerations in the selection of manipulative materials:

1. *The materials should provide a true embodiment of the mathematical concept or ideas being explored.* The materials are intended to provide concrete representations of mathematical principles. Therefore it is important that, above all else, the materials be mathematically appropriate.

2. *The materials should clearly represent the mathematical concept.* Concepts are embedded so deeply in some materials that few, if any, pupils extract relevant ideas from their experience with the materials. This problem is further compounded by materials that have extraneous distractors, such as bright colors, which actually serve as a hindrance to concept formation. These experiences result in an "I can't see the forest for the trees" complex. This is, of course, not all bad, as it requires pupils to cull out extraneous data, yet in many cases such materials serve more as a deterrent to correct concept formation than as an aid.

3. *The materials should be motivating.* There are many factors that ultimately contribute to motivation. Several of these, such as attractiveness and simplicity, will be discussed later. Materials with favorable physical characteristics will frequently stimulate the pupil's imagination and interest.

4. *The materials should be multipurpose if possible.* That is, they should be appropriate for use in several grade levels as well as for different levels of concept formation. Ideally, the materials should also be useful in developing more than a single concept. Such wide applicability is frequently achieved by using a portion or subset of materials. For example, logic or attribute blocks have much multiapplicability through the careful selection and use of pieces.

   This requirement should not preclude the procurement and use of materials designed exclusively for embodying one concept. In fact, if use of certain materials results in concept formation that is otherwise impossible, then such items should be considered. In other disciplines, such as science and physical education, considerable funds are spent on devices that teach a single concept. Shouldn't mathematics teachers have a similar opportunity? Besides, using materials (even those designed for one specific function) often suggests additional topics or concepts that might be explored.

5. *The materials should provide a basis for abstraction.* This underscores the importance of the requirement that materials correctly embody the concept. In addition, caution should be exercised to ensure that the concept being developed is commensurate with the level of abstraction needed to form the mental image. Care must also be taken to ensure that the level of abstraction is commensurate with the ability of the student to abstract.

6. *The materials should provide for individual manipulation.* That is, each pupil should have ample opportunity to physically handle the materials. This may be done individually or within a group, as circumstances dictate. Such manipulation utilizes several senses, including visual, aural, tactile, and kinesthetic. In general, the materials should exploit as many senses as possible. Compliance with this generalization is particularly important with younger children.

Physical criteria are important, since many sources of information available to teachers, such as commercial catalogs and brochures, describe physical features of the materials. A careful scrutiny of physical criteria would be helpful in initially screening manipulative materials. Among the physical characteristics to consider in selecting manipulative materials are the following:

1. *Durability* - The device must be strong enough to withstand normal use and handling by children. If and when maintenance is needed, it should be readily available at a reasonable cost.

2. *Attractiveness* - The materials should appeal to the child's natural curiosity and his desire for action. Materials in themselves should not divert attention away from the central concepts being developed. Nevertheless there are certain qualities - such as aesthetically pleasing design; precision of construction; durable, smooth, and perhaps colorful finish - that are desirable. Nothing can be more distracting than pieces of a tangram puzzle that do not fit properly or a balance beam that doesn't quite balance.

3. *Simplicity* - The degree of complexity is of course a function of the concept being developed and of the children involved, but generally the materials should be simple to operate and manipulate. Although the materials may lend themselves to a host of complex and challenging ideas, for example, the attribute or logic blocks, they should be simple to use. In an effort to construct and use simple devices, there is the inherent danger of oversimplifying or misrepresenting the concept. In all cases, care must be taken to ensure that the device properly embodies the mathematical concept. In addition, the design of materials should not require time-consuming, mundane chores such as distributing, collecting, and keeping an extensive inventory record of a large number of items.

4. *Size* - The materials should be designed to accommodate children's physical competencies and thus be easily manipulated. Storage is an important consideration directly related to size - no device should take up more than a reasonable amount of storage space. Suitability of size is also important in preventing misconceptions or distorted mental images within the child's mind.

5. *Cost* - The index used to assess the worth of materials must ultimately weigh their use against cost. In this context, cost is used in a broad sense. Thus cost must include the initial expenditure and maintenance and replacement charges based on the life expectancy of materials under normal classroom use. The teacher-related cost, a function of the time required to learn how to use the materials effectively, is an item of utmost importance. It is not uncommon for someone other than a classroom teacher to order mathematics materials; but without proper planning, orientation, and preparation, it is ludicrous to expect teachers to use new materials effectively with their pupils. Therefore, *any purchase of new materials should be accompanied by a planned program designed to familiarize the teacher with these materials.* As a result, any cost estimate for manipulative materials should reflect the teacher-education phase as well as the expenditure for materials.

Teachers are often confronted with the dilemma of whether to use commercial or homemade manipulative materials. Many manipulative materials are relatively easy to make and can often be produced by the teacher and/or pupils. There are many priceless, intangible by-products, such as additional mathematical insight and increased motivation, that result directly from classroom projects. Nevertheless, one should weigh production costs for homemade materials, including labor, materials, and so on, against the cost of similar commercial products. Quality, of course, is another consideration. Frequently there is a marked difference in quality between homemade and commercially produced materials.

It would be ideal if manipulative materials could meet all the aforementioned criteria. Finding such materials would be tantamount to finding a "fish that runs fast and flies high". Consequently the search continues. It is hoped, however, that these criteria will provide teachers with some guidelines for both the selection and the use of manipulative materials.

## *USING MANIPULATIVE MATERIALS*

There have been several fine lists summarizing uses and functions of teaching aids. Many such lists apply specifically to manipulative materials. Among the most common uses of manipulative materials are the following:

1. To vary instructional activities,
2. To provide experiences in actual problem-solving situations,
3. To provide concrete representations of abstract ideas,
4. To provide a basis for analyzing sensory data, so necessary in concept formation,
5. To provide an opportunity for students to discover relationships and formulate generalizations,
6. To provide active participation by pupils,
7. To provide for individual differences,
8. To increase motivation related, not to a single mathematics topic, but to learning in general.

From this list, it should be evident that manipulative materials may be used in a variety of ways. It should also be noted that the mere use of manipulative materials does not ensure that they are being used properly. Manipulative

materials must be used at the right time and in the right way if they are to be effective. Failure to select appropriate manipulative materials and failure to use them properly can destroy their effectiveness. Some specific dos and don'ts for teachers who plan to use manipulative materials follow:

1. *Do consider pedagogical and physical criteria in selecting manipulative materials.* A prerequisite for effective use of manipulative materials is their appropriateness. The physical criteria for manipulative materials as well as the pedagogical considerations should not be taken lightly.

2. *Do construct activities that provide multiple embodiment of the concept.* It is difficult, and often foolhardy, to abstract or generalize from a single experience. Thus the pupil should be presented with different situations manifesting the concept or structure to be learned. For example, in developing the concept of three, children might examine sets with three elements for one activity. The number line, balance beam, and Minnebars might also be used to provide different embodiments for the same concept. The case for multiple embodiment has been ably presented by Dienes. Although the idea is pedagogically sound, it has yet to receive widespread use by classroom teachers.

3. *Do prepare in advance for the activity.* Be sure you, as the teacher, use the manipulative materials in the complete activity before they are used by pupils. As you make this trial run, you should consider questions such as: What prerequisite skills are needed before these manipulative materials are introduced? Are the directions clear, and can they be easily followed? Are there an adequate number of leading questions? Are the manipulative materials commensurate with the level of the pupils and appropriate for the mathematical concept? What are some potential problem areas, and how might they be alleviated?

4. *Do prepare the pupils.* The type of preparation depends on both the manipulative materials being used and the age of the pupils. Above all else, be sure the pupils are ready to profit from experience with the materials. Care should be taken to provide the necessary directions for beginning the activity. One must guard against telling pupils precisely what to do with the materials, as this might sterilize the learning experience. On the other hand, sufficient direction should be provided to prevent mass confusion, which may quickly lead to discipline problems.

5. *Do prepare the classroom.* Check to ensure that all required materials are on hand. Also be sure they are operative, accessible, and available in sufficient quantity. The arrangement of the classroom furniture should be examined to ensure that it is suitable for the planned activities.

6. *Do encourage pupils to think for themselves.* The use of manipulative materials in an informal situation provides an ideal climate for creativity, imagination, and individual exploration. This atmosphere encourages pupils to think for themselves. However, in order to get students to begin and then continue to think for themselves, it is imperative that the teacher provide encouragement of and show respect for pupils' ideas. A teacher's dismissal of a student's idea as being trivial, incorrect, worthless, and so on, will repress future ideas.

7. *Do encourage group interaction.* Discussion within, as well as among, groups can be intellectually stimulating. Encourage students to communicate with their peers and teacher. The importance of having this opportunity to tell one's thoughts, observations, and ideas cannot be overestimated. As pupils grow older, this freedom to express personal ideas is accompanied by a responsibility to defend or at least support a position, should the need arise. Some teachers fear that one student will dominate a group of peers. This may sometimes happen; however, the careful selection of group membership can keep this problem at a minimum.

8. *Do ask pupils questions.* It is often essential that certain points be called to the pupils' attention. Sometimes big ideas are missed completely. Other times one child may divert group attention to some minor or obscure point. In either case, you, as the teacher, must be prepared to ask pertinent leading questions.

9. *Do allow children to make errors.* Some may view this as heresy. However, children must have an opportunity to be wrong or to make a mistake. Often greater learning and more lively discussion follow an incorrect answer than a correct one. Besides, the natural learning process is characterized by much trial-and-error learning. To do otherwise, that is, to attempt to eliminate incorrect answers or faulty speculation, is to create a highly artificial learning situation.

10. *Do provide follow-up activities.* Discussion, correlated readings, reports, and projects, as well as replications of activities, enhance the prospects of learning. Searching questions forcing pupils to further analyze and synthesize their results can be very productive, as they encourage students to "pull together the loose ends". They might be followed by additional questions that require extrapolation from these activities and encourage speculation on the outcome of other related events.

11. *Do evaluate the effectiveness of materials after using them.* Immediately upon the completion of an activity, it can be very helpful to note particular problem areas, strengths, weaknesses, and suggestions and to define areas of needed improvement as well as possible areas of modification. A continuous reevaluation of manipulative materials ultimately results in better materials as well as more effective use of them.

12. *Do exchange ideas with colleagues.* Many new functions of manipulative materials result from actual classroom use. Sometimes pupils either consciously or unconsciously propose additional uses. At times, informal exploration with manipulative materials by either teacher or children suggests new activities, which adds to the reservoir of potential uses for this set of manipulative materials. A mutual exchange of ideas among teachers allows each to profit from the experience of the others. Perhaps you remember the fable: If I have a dollar and you have a dollar and we exchange dollars, we both still have a dollar. However, if I have an idea and you have an idea and we exchange ideas we both now have two ideas.

Now for some teacher don'ts!

1. *Don't use manipulative materials indiscriminately.* Care must be taken to ensure that these materials properly embody the mathematical concept being developed. Be sure the materials and concept are commensurate with your objectives and the pupils' level of development.

2. *Don't make excessive use of manipulative materials.* They should be used only when they represent an integral part of the instructional program and when the program could not be achieved better without the materials. One exception to this might be manipulative materials that are directed more toward recreation. There are instances where the traditional curriculum fails to reach many pupils. Often the recreational aspect of manipulative materials has attracted the attention of these youngsters and eventually paved the way to more academically-oriented activities. Some teachers fear that excessive use of manipulative materials will lead to overdependence on physical representations. There are cases where the manipulative materials are used as "crutches". However, most pupils will gradually stop using the materials when they have reached a higher level of development. Signs of boredom from the children may indicate excessive use of manipulative materials, or may suggest the need for raising additional questions or extending the concepts being explored with the manipulative materials.

3. *Don't hurry the activity.* Once the concept has been developed, most children are eager to explore other ideas. However, every pupil should have ample opportunity to use the manipulative materials, thereby convincing himself of the principle or formulating the concept. Hurrying through the activity may impose unnecessary pressure on some pupils as well as creating a very artificial learning situation. Few individuals learn well when they are rushed. Some children may formulate the concept within minutes, whereas other children may require several days or perhaps months. Rushing children as they use manipulative materials does not solve the problem but rather compounds it.

4. *Don't rush from the concrete to the abstract level.* This is a sequel to the previous suggestion. Perhaps the most frequent error in using manipulative materials is the speed at which children are rushed from the concrete stage to the symbolic level. There seems to be some myth that you can't learn mathematics unless you are actually writing something, that is, working with symbols. This is, of course, nonsense! Most good mathematics at the primary level is done without symbolization. In fact, if serious consideration were given to Piaget's research, nearly all mathematics in the primary grades would be at the concrete stage. It must be noted that symbolization occurs quite late in concept formation. Symbols are reserved for describing or making a record of the concept or mathematical principle. Hence, they can only be properly used after the concept has been abstracted. Since the process of learning a mathematical abstraction is time-consuming, it is ludicrous (at least with most elementary children) to use manipulative materials for one or two days and then move directly to the symbolic level. The wrong kind of experience may result in the children's viewing manipulative materials as toys or entertainment, in no way related to mathematics.

5. *Don't provide all the answers.* In working with manipulative materials, pupils acquire experience in abstracting from a set of phenomena or a body of data.

As each child is actively involved in this process, conflicts frequently arise. One pupil has one answer, another child has a somewhat different result. Often the first reaction of the teacher is to settle the issue by providing the correct answer. It is difficult to resist the temptation to tell the correct answer, but resist the teacher must! To do otherwise is to discourage individual thought, squash natural curiosity to search for other solutions, promote dependence on the teacher rather than independence, and preclude further discussion of the problem, as everyone now knows the correct answer. On the other hand, you may decide to ask some leading questions; you may have the pupils explain their solution; you may wish to have them replicate the activity using the manipulative materials; or you may pursue some other alternative. Regardless of the option selected, the teacher must refrain from serving as the purveyor of truth and source of all knowledge. Remember that to children and adults alike, "The art of being a bore consists in telling everything."

## CONCLUSION

Perhaps the best one can do is identify those materials that best meet the criteria and then concentrate on developing effective ways of using them. This requires several steps. First, the desired learning must be clearly identified. Then manipulative materials that will aid in the learning process need to be selected. The third step requires that these materials be integrated into an organized learning sequence, so that pupils progress from the simple and concrete to the complex and abstract. Only in this way can manipulative materials be an integral part of the mathematics education program.

Remember that manipulative materials are not to be considered a substitute for teaching, - something one uses in lieu of teaching. There is not now, never has been, and, it is hoped, never will be a genuine substitute for a good teacher who knows how and what children need to learn and when they need to learn it!

## REFERENCES


Beougher, Elton E., *The review of the literature and research related to the use of manipulative aids in the teaching of mathematics.* Pontiac, Michigan: Special Publication of Division of Instruction, Oakland Schools, 1967.

Berger, Emil J. and Donovan A. Johnson, *A guide to the use and procurement of teaching aids for mathematics.* Washington, D.C.: National Council of Teachers of Mathematics, 1959.

Bernstein, Allen L., "Use of manipulative devices in teaching mathematics", *Arithmetic teacher 10*, May 1963, pp.280-83.

Brown, Kenneth E. and Theodore L. Abell, "Research in the teaching of elementary school mathematics", *Arithmetic teacher 12*, November 1965, pp.547-49.

Copeland, Richard W., *How children learn mathematics: teaching implications of Piaget's research.* New York: Macmillan Co., 1970.

Davidson, Patricia S., "An annotated bibliography of suggested manipulative devices", *Arithmetic teacher 15*, October 1968, pp.509-24.

Dienes, Zolton P., *Building up mathematics.* London: Hutchinson Education, 1960.

Hamilton, E.W., "Manipulative devices", *Arithmetic teacher 13*, October 1966, pp.461-67.

Grossnickle, Foster E., Charlotte Junge and William Metzner, "Instructional materials for teaching arithmetic", in *The teaching of arithmetic*, Fiftieth Yearbook of the National Society for the Study of Education, pt.2. Chicago: University of Chicago Press, 1951.

_____, *Multi-sensory aids in the teaching of mathematics.* Eighteenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1945.

Spross, Patricia, "Considerations in the selection of learning aids", *Arithmetic teacher 11,* May 1964, pp.350-53.

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