The use of attribute blocks: K - XII

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THE MATERIALS

Attribute blocks or logic blocks can be obtained commercially[1] or constructed from wood[2] by the teacher or school shop personnel. A set based on four attributes - shape (triangular, circular, square, and oblong), color (red, yellow, and blue), size (large and small), and thickness (thick and thin) - would consist of 48 blocks, one for each possible combination of the variables (Figure 1). Obviously, sets with more or fewer pieces could be easily formed. For instance, by eliminating thickness as an attribute, the set described above would be reduced to 24 blocks. Or, if another shape for instance, pentagon, were added, the set would consist of 60 blocks. For certain activities, a set for young children might contain only 6 blocks (based on 3 shapes and 2 colors).

GAMES AND ACTIVITIES

The attribute blocks can be used to provide a physical setting for a variety of experiences[3] which develop or illustrate principles of logical reasoning. Many of the suggested games can be played at various levels of sophistiction, and are therefore appropriate for learners ranging in ages from 5 to 16 (and up). Young children can discover and use intuitively certain valid modes of reasoning, while older students may be able to analyze strategies, consider all possibilities, and develop proofs using words and symbols.

The blocks can also serve as the universal set for a number of problems in basic counting and probability.

Sorting

Purpose: To acquaint students with the structure of the materials; to develop classification skills.

Instructions: The blocks are placed randomly on a table (or on the floor), and the students are asked to sort them, that is, to place together those which they think are alike in some way. The result, depending on the level of the students, might be several disjoint sets (such as red, blue, and yellow pieces) or a two or three dimensional matrix (as in Figure 1). Although young children generally require and enjoy a fair amount of free play with the materials before settling down to a directed task, they will often begin sorting naturally when confronted with the blocks.

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[1]Most suppliers of materials and aids for school mathematics and science handle attribute blocks. In Edmonton, Moyer-Vico Ltd. sells various sets of blocks along with guidebooks.

[2]For mimeographed instructions describing how to make your own attribute blocks out of wood, write to: Dr. Dossey, Mathematics Department, 313 Stevenson Hall, Illinois State University, Normal, Illinois 61761.

[3]Many of the activities dealing with logic are discussed in Z.P. Dienes and E.W. Golding, *Learning logic, logical games* (New York: Herder and Herder, 1966).

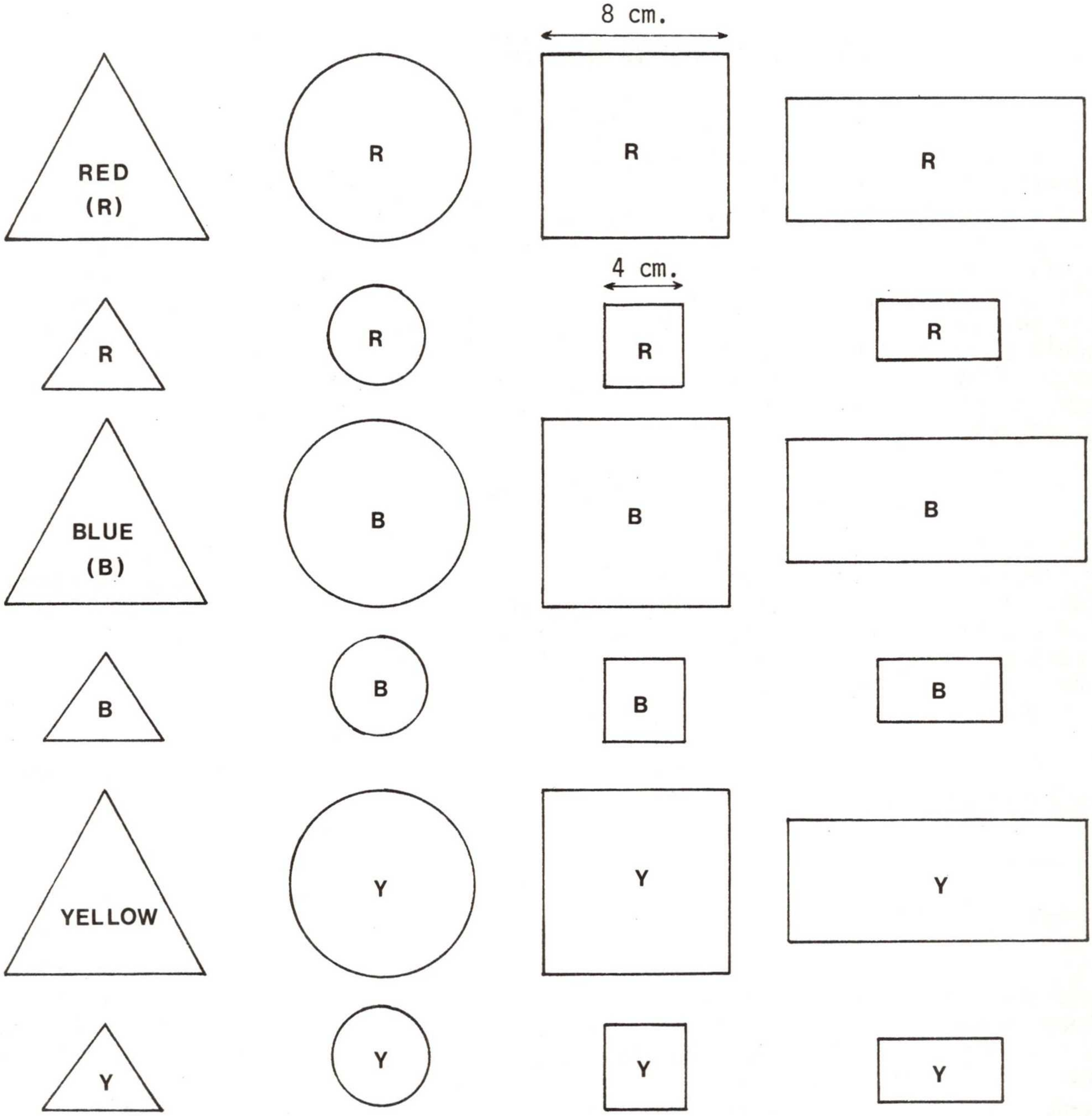


Figure 1. The Attribute Blocks: A set based on 4 shapes, 3 colors and 2 sizes. Two thicknesses of each of the above depicted blocks would result in a set of 48 pieces.

Difference games

Purpose: To further familiarize the learner with the relationships among the pieces; to focus on detecting similarities and differences; to develop strategies involving consideration of all possibilities.

Instructions: The one-difference game.

The first player chooses any block. The second player must select a block which differs from the first in exactly one way (shape, color, size or thickness). The next player then finds a block differing from the second in one way, and so on. For example: 1st player - small thick red triangle, 2nd player - small thick red circle, 3rd player - small thin red circle, and so on. Points are obtained by corectly choosing a block or by challenging a block incorrectly played by someone else.

The two-difference game.

Played as above except that a piece played must differ from the previous one in exctly 2 ways.

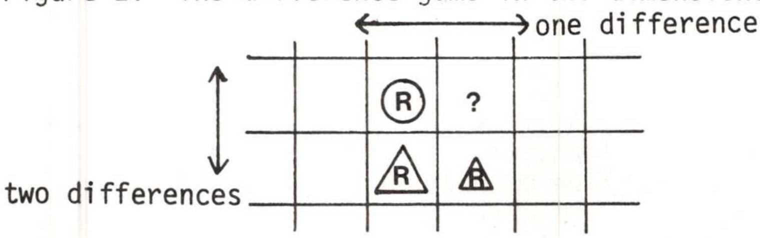
The three-difference game.

Pieces played must differ in exactly 3 ways. Students often discover that this can be viewed as a "one-same" game and that using this (logical) strategy simplifies the task.

Two dimensional difference game.

A grid is needed. One version of the game would require pieces to differ by one attribute in one direction and by two attributes in the other direction. In Figure 2, the square marked, ?, could be correctly filled in several ways (such as thin red large circle; thick red small circle). Players might wish to formulate a hypothesis concerning the relationships of correctly played diagonal pieces in this game. A scoring system might be instigated to encourage the filling of squares bordered by 2, 3 or 4 pieces. Suppose that, in attempting to fill such a square, a player decides that he is unable to find a block satisfying all required conditions. This may be either because all suitable pieces had been previously played or because the conditions make the existence of such a block logically impossible (see Figure 3). Determining (proving) that a space cannot be filled would be scored higher than simply correctly playing a block. Strategies of exhausting or considering all possible cases would be developed in such investigations.

Figure 2. The difference game in two dimensions. (The three blocks are all thin.)



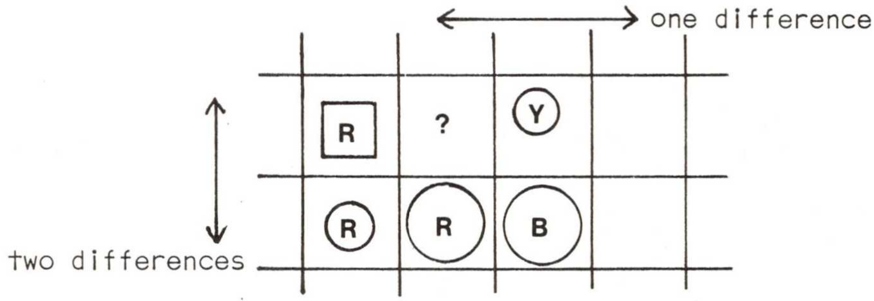


Figure 3. Filling the ? square is logically impossible. (All blocks depicted are thin.)

Guessing games

Purpose: To develop skills in asking relevant questions and in utilizing efficiently information obtained from the answers; to acquaint students with the use and power of the logical terms "not", "and", "or"; to illustrate a valid mode of logical reasoning.

Instructions: The teacher or game leader selects (in his mind) one of the 48 blocks. The players attempt to identify this block by asking questions which can be answered by "yes" or "no". For instance, if the answer to the question "Is it thin?" is "No", the players learn that they need not ask the question "Is it thick?" since this can be determined through logic. The situation can be analyzed as follows:

If thin or thick
and not thin
then thick

or symbolically

$$\begin{array}{l} p \vee q \\ \underline{\sim p} \\ \quad q \end{array}$$

An examination of the truth table of the related compound statement

$$[(p \vee q) \wedge \sim p] \rightarrow q$$

shows that the statement is a tautology and hence that this mode of reasoning is valid. Thus for the above argument, if the premises are true, the conclusion must always be true. To assist in the thinking process, a chart similar to the following might be kept by students as they play this game.

Answer	Deduction
not thick	thin
not red, not blue	yellow
square	square
...	

The meaning, advantages, and limitations of "and" and "or" questions may be brought out during the course of this activity. It should be discovered that questions such as "Is it the small, thin, red square?" or even "Is it large and blue?" are not generally productive or helpful. On the other hand, "or" questions provide the basis for more interesting discussions. One problem which can generate considerable interest is stated as follows: "Which is the better question to ask (would yield the most information)?" (1) Is it red?; or (2) Is it blue or yellow?" It is usually only after some debate that participants generally (not always unanimously) agree that the two questions are logically equivalent. Logically, if a block is either red or blue or yellow, then it is red or blue if and only if it is not yellow. Symbolically, if r, b, and y are mutually exclusive,

$$(r \vee b \vee y) \rightarrow [(r \vee b) \leftrightarrow \sim y)].$$

Now, what is the maximum number of questions required to identify the unknown block in the guessing game? The answer is 6 questions - one for size, one for thickness, 2 for color, and 2 for shape. Shape can be determined in two questions if the first one is an "or" question. For instance:

Question 1	Is it circular or triangular?	
Question 2	(Answer yes)	Is it circular?
	(Answer no)	Is it square?

In a far more difficult version of the guessing game, the objective is to determine a subset of the blocks defined as the conjunction of 2 attributes. Suppose, for example, that the game leader is thinking of the "thin circles". As the players point to various blocks one at a time, the leader must indicate whether each is or is not in the set he had in mind. A partitioning of the blocks in 2 classes is thus commenced. What strategy should be used to most efficiently identify the set? First, one block belonging to the set must be determined; suppose it is the *large red thin circle*. These attributes must then be varied one at a time while the other 3 are held constant. For example, the large red thin *square* might next be selected. Since this piece is *not* in the set, the conclusion would be that shape (circular) *is* a defining characteristic. On the other hand, since the *small* red thin circle *is* an exemplar of the set, size would *not* be a defining characteristic. Similarly, the relevance of color and of thickness could be tested.

Hoop activities

Purpose: To create Venn diagram-type illustrations of logical connectives and relations; to show the relation between sets and logic.

Instructions: Hoops (or rope or string) are required. Sample activities are as follows:

1. Place one hoop on the floor (or table). Put all the red blocks inside the hoop. The blocks outside the hoop then would be the "not-red" blocks. If the "not-red" blocks were placed inside a hoop, the blocks outside the hoop would be the "not (not-red)" blocks which are the red blocks, illustrating that $\sim(\sim r) \leftrightarrow r$.

2. To illustrate disjunction and conjunction, 2 (or more) hoops are required. The task might be to place the blue blocks in one hoop and the square blocks in another. To accomplish this, one must overlap the 2 hoops. The set of blocks is thus partitioned into 4 classes, each identified by the conjunction of 2 attributes as indicated in Figure 4. Such activities help to clarify the logical meanings of "and" and "(inclusive) or". One of DeMorgan's laws may

be illustrated as follows: Since the blocks inside either of the 2 hoops are "blue or square", those outside are "not (blue or square)". Hence not (blue or square) if and only if not blue and not square; or

$$\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q).$$

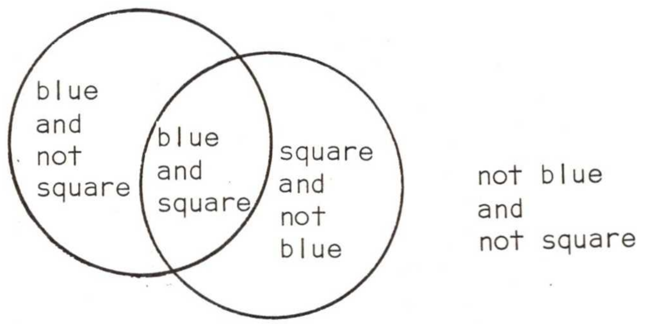


Figure 4. The conjunction of two attributes.

The number of elements in the intersection and union of two sets

Purpose: To allow students to discover methods for determining the number of elements in the intersection and union of 2 (non-disjoint) sets.

Instructions: Answers to the following questions can be obtained by counting. Students should be encouraged to discover methods (formulas) for finding answers to similar questions without counting. Answers obtained through using formulas arrived at inductively can then be checked by counting.

1. What fraction and how many of the blocks are

square?	Answer 1/4 (12)
red?	Answer 1/3 (16)
small?	Answer 1/2 (24)
thick?	Answer 1/2 (24)

2. How many blocks are red and square?
   Answer: By counting - 4.
   Methods:
   (a) 1/3 of 48, or 16, are red.
       1/4 of these, or 4, are square.
   (b) 1/3 are red and 1/4 are square.
       1/3 x 1/4 = 1/12 are red and square.
       1/12 x 48 = 4

3. How many blocks are either square or thick?
   Answer: By counting - 30.
   Method:
   # (square or thick) = # (square) + # (thick) - # (square and thick)
   = 12 + 24 - 6 = 30

Probability

Purpose: To determine empirically and theoretically the probabilities of simple events, independent events, and composite events (A or B, when A and B are not mutually exclusive).

Questions: If a block is selected at random, what is the probability that it is 1. blue? 2. red and square? 3. square or thick?

Instructions: To determine the probabilities empirically (to find the relative frequencies): For each of the 48 blocks, list the shape, color, size, and thickness on a separate slip of paper. Put the 48 slips in a box. Draw a slip at random; note the (1) color, (2) color and shape, (3) shape and thickness; and replace the slip. Repeat this procedure 100 times (or more).

The relative frequency is the number of times (1) blue, (2) red and square, (3) square or thick was noted divided by the total number of draws.

To determine the probability theoretically:

1. $\text{Prob (blue)} = \dfrac{\text{No. of blue blocks}}{\text{Total no. of blocks}} = \dfrac{16}{48} = \dfrac{1}{3}$

   Answers to the next 2 questions (and similar questions) can be found, first by counting or otherwise determining the number of blocks satisfying the given conditions. Formulas can then either be developed inductively or verified in the specific cases.

2. $\text{Prob (red and square)} = \dfrac{\text{No. of red and square blocks}}{48} = \dfrac{4}{48} = \dfrac{1}{12}$

   $= \text{Prob (red)} \times \text{Prob (square)} = \dfrac{1}{3} \times \dfrac{1}{4} = \dfrac{1}{12}$

3. $\text{Prob (square or thick)} = \dfrac{\text{No. of square or thick blocks}}{48} = \dfrac{30}{48} = \dfrac{5}{8}$

   $= \text{Prob (square)} + \text{Prob (thick)} - \text{Prob (square and thick)}$

   $= \dfrac{1}{4} + \dfrac{1}{2} - \dfrac{1}{8} = \dfrac{5}{8}$

Answers obtained to the above questions using empirical and theoretical procedures should be compared and any differences discussed.