Representing a reflection in the plane

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In reading mathematics education journals or attending conferences, one is struck by the increasing popularity of motion geometry as a way of bringing geometry back into the mathematical mainstream and as a natural way of reviving student interest in geometry.

There are many specific reasons for including motion geometry in the mathematics curriculum.[1] This author will not review these arguments for motion geometry but will pass directly to a review of some easy ways to model a line reflection in the plane. The line reflection, after all, is the basic building block of all motion geometry.

Mathematically, the line reflection in a plane has the following definition:

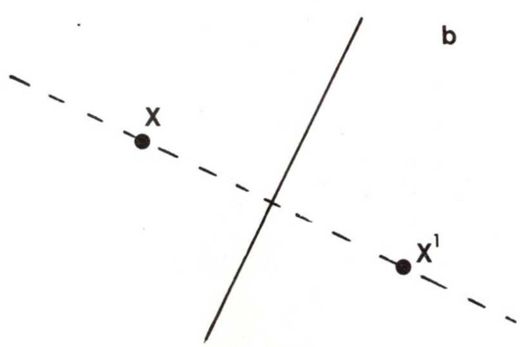
> Line Reflection of a Plane: Given a point x and a line b, x^1 is the reflection of x in line b if and only if b is the perpendicular bisector of the line segment determined by x and x^1.

In the remainder of this article, 5 easy ways of modelling a line reflection will be catalogued.

METHODS OF LINE REFLECTION

Construction

There is an obvious way of finding the reflection of a point in a line simply by constructing it with a compass and straightedge. In the diagram below, a line perpendicular to line b and passing through point x has to be constructed. Then x^1, the reflection of x in line b is constructed on this perpendicular so that it is in the opposite half-plane of b from x. Furthermore, x^1 must be the same distance from b as x is from b.



[1] *The mathematics teacher* since 1968 has had a number of articles which make a strong case for motion geometry.

Paper folding

In the diagram below, x is to be reflected in line b.

1. Mark the point x heavily with a soft-leaded pencil.
2. Fold the paper along line b, as in Figure 2.
3. After the folding, one should be able to see the markings for point x through the paper. On the backside of the paper, mark the position of point x with a pencil.
4. Open the paper and one will find that some of the marking from x will have been transferred to another point. This marks point x^1, the reflection of point x in line b.

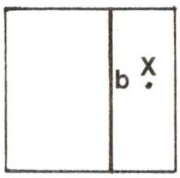


Figure 1

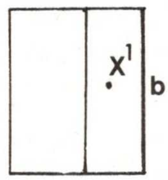
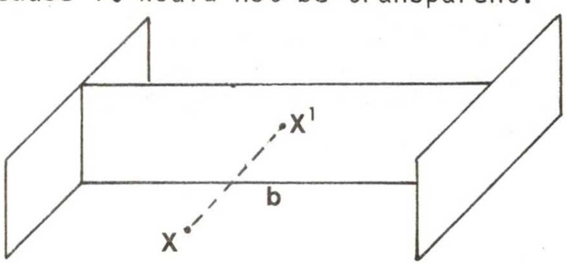


Figure 2

Mira

Mira[2] is a commercially available device that can be used to illustrate a line reflection. Incidentally, this device could be nicely used in a physics class when dealing with reflections there. The device is pictured below. It is made of some type of red plexiglass material. Again we have a point x which is to be reflected in line b. Now the bottom edge of the Mira is placed along the line b. By looking at the Mira one can see the reflection of point x. By reaching behind the Mira, one can mark the point x^1 in line with the reflection of point x. The point x^1 is the reflection of x in line b, as before. The plexi-glass material is both transparent and also reflective. A silvered mirror would not work because it would not be transparent.



Transposition

Again we have a point x which is to be reflected in line b (Figure 1).

1. Mark an arbitrary point z on line b (Figure 3).
2. Lay another sheet of paper down on top of the original sheet. Mark point x, point z and line b on this second sheet.
3. Turn the second sheet over using line b as an axis. Lay it down on top of the first sheet so that line b and point z in the two sheets correspond. Point x in the second sheet now corresponds to point x^1 in the first sheet. x^1 is the reflection image of x.

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[2]Mira is sold by Moyer-Vico Ltd., 10924 - 119 Street, Edmonton. The device sells for $2.95 individually or $2.36 each in classroom lots of 32 or more. A brochure is included which explains uses.

4. The position of x^1 can be marked by sticking a pin through point x in the second sheet. If a soft-leaded pencil is used on the second sheet then x^1 could also be determined in a manner similar to that of method two above.

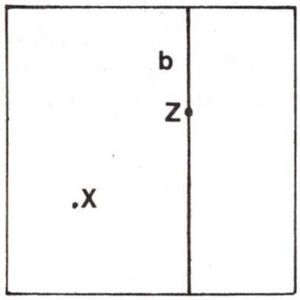

Figure 3

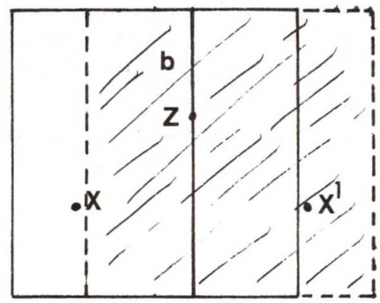

Figure 4

Line reflections in the coordinate plane

Performing line reflections in the coordinate plane is another easy and valuable experience in motion geometry. As an example, let the line of reflection be the graph of x = 4. Then the reflection of (7,3) will be (1,3). The points and lines of reflection can be varied to provide much valuable experience in motion geometry and also in the coordinate plane.

CONCLUDING REMARKS

In the examples given above, a single point was reflected in a line. Mathematically, a reflection in a line is defined point-to-point. However, in the classroom different kinds of geometric figures should be reflected in lines besides single points.

What conjectures can be made when a geometic figure is reflected in one line and that image is then reflected in a second line? This is an example of an open-ended question that would furnish a good introduction to motion geometry.

Reflections in a line lead naturally to symmetry conditions. Mathematically, symmetry can be defined only in terms of reflections.

Invariance is a key idea in motion geometry. What properties or characteristics of some geometric figure also hold for its reflection image? This is the essential question in experiments with invariance.

The 3 methods - paper folding, Mira, and transposition - mentioned above appear to be more valuable as mathematical experiences than do construction and line reflections, which both are less illustrative of the intrinsic properties of a line reflection. These judgments should be considered in choosing classroom models for a line reflection.

In working with the Mira method, does the relative position of the eye, with respect to point x and line b, make any difference? A good amount of mathematics and experimentation is involved with that question.

In this article, some models for a line reflection in the plane have been described along with some questions and ideas that lead on from such an introduction. The author hopes that classroom teachers will be able to try out some of the ideas described here. The author would enjoy hearing from anyone who has tried any of these methods or others.