# Chapter 2
# The Role of Communication in Mathematics

## Barbara J. Morrison

*This chapter focuses on the role of writing and classroom discourse in*

1. *confirming student understanding,*
2. *creating a risk-free environment, and*
3. *improving instruction.*

*Barbara J. Morrison provides a succinct summary of research supportive of such roles and outlines several areas in which writing can occur, including the following:*

1. *Short writing assignments*
2. *Class logs*
3. *Journals*
4. *Writing activities using the text*
5. *Writing on tests*
6. *Chapter summaries*
7. *Definition cards*
8. *Writing about problem solving*
9. *Personal math histories*

*A particular strength of this chapter is the way the author embeds her remarks in the literature about communication in mathematics. This not only provides references for further reading but also helps to integrate practice with theory.*

## Introduction

New knowledge about how students learn has had a tremendous impact on mathematics instruction and methodologies. The National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) presents standards to guide mathematics curriculum and pedagogy for the 1990s. The newly revised Alberta mathematics curriculum K–12 matches these directions in spirit and in philosophy. Alberta teachers have a unique opportunity to lead in improving instruction in mathematics teaching and learning. There is a tremendous research base and rationale for these changes and an abundance of supporting curricular documents developed by practising math specialists across North America.

Recent research on students as math learners supports students as active learners; learning as an active, dynamic and continuous process; and the learning theory that learners construct meaning by connecting new knowledge to concepts they already know. Research and practice that support these fundamental learning principles will guide our teaching into the next century. As we incorporate current research and methodologies from education and psychology, our teaching repertoires will include increased opportunities for collaboration, cooperative-learning techniques and full and active participation by all students.

The continued development and use of mathematical language and symbolism to communicate ideas and concepts form an integral part of our mandate to develop mathematical literacy/numeracy in our students. As with the nature of mathematics, communication in the later grades requires students to reason at more formal and symbolic levels. Alberta Education is increasing the emphasis on writing on the provincial Math 30 diploma examination, and "greater emphasis will be placed on the communication of students' understanding of mathematical concepts and procedures" (Alberta Education 1991).

This chapter explores issues and directions related to communication in mathematics. My comments focus on the writing aspects of communication and classroom discourse that are most useful in checking for student understanding, creating a risk-free environment and improving instruction in mathematics. The concrete teaching ideas and examples I use have been taken from many articles and conference presentations, as well as from practising teachers who shared some of the products of students' writing assignments.

## Why Change Our Practice?

"The traditional view has been that students learn to write in English classes and to compute in mathematics classes and 'never the twain shall meet' " (Davison and Pearce 1988, 42). Communication as it generally occurs in a teacher-directed, lecture-oriented lesson is most often students' responses to questions beginning "Does everyone understand. . . ?" or "Are there any questions about. . . ?" Usually only a few students nod their heads or respond, and the individual responses are short answers to lower-order questions which can hardly serve as reliable checks for all students' understanding. Often formal feedback on the teaching and learning process comes from students' results on tests or quizzes. In many cases after two or three weeks of instruction, we learn that half the students answer half the questions wrong!

The contemporary role of writing in math classes, as related to teaching and learning mathematics, has changed from transcribing information and ideas onto paper, to a writing-to-learn process. Writing as a process provides new experiences for students. It uses expressive and creative writing tasks (such as journals and personal notebooks), where students write without concern for the quality of writing per se.

## Advantages to Writing in Math Class

Richard Sagor (1991, 7) emphasizes the effect of this kind of reflection on achievement and describes a math writing project begun when a number of junior high teachers posed the question, "If writing is a window into thinking and if the act of writing helps improve comprehension, why not try it in our math classes?" The teachers used the following experimental design: They constructed and administered tests to divide four Grade 7 math classes into two groups according to achievement. Two of the classes were performing well and two performed below expectations. The lower-achieving classes were made the treatment group. Students were then given the opportunity to write about the math concepts they were learning the day before each math test. In every other respect, the groups were taught the same way. The teachers found that writing made a substantial difference in concept acquisition. The lower-achieving classes actually outperformed their classmates on every test.

Adele Le Gere (1991, 168) reports on research that found that

> students who write to learn actually do learn and retain concepts better than students who do not write as part of their course work.

A distinct advantage to frequent, short writing assignments is increased student participation. Every student has the opportunity for input as opposed to questions directed at an entire class which are answered by a select few. Support for writing in math classes is flooding the literature on learning mathematics (Nahrgang and Petersen 1986; Johnson 1983). Organizing thoughts on paper requires a higher-level mental thinking process. Through the process of synthesizing ideas, the students' understanding is clarified, the content is reinforced, and feedback is provided for the teacher on every student's level of understanding.

In-class writing or overnight homework assignments can also allow students to communicate in their own language and relate to their own real-world experiences. Writing assignments can force students to make connections from topics presented in class

to what they already know and to organize and synthesize ideas so that concepts become their own.

A third advantage comes from using individual journals in math class. Journal writing can lead to definite improvements in students' ability to organize their responses. Teachers can use writing assignments as diagnostic tools to reveal areas of confusion about and understanding of math concepts (Nahrgang and Petersen 1986).

In addition, brief and frequent informal writing assignments can be ideal opportunities for teachers to reflect on instruction by answering such questions as the following:

1. Do students understand?
2. What level of understanding are they at?
3. Is the main point of the lesson coming through?
4. Where are students having difficulty?
5. Where do I need to modify my instruction?

What students write about is the next important issue. Students are encouraged to write about such things as the mathematical content they are exploring, problem-solving experiences and progress, their own learning process—their attitudes, thoughts and feelings as they solve math problems—and the nature of math and its applications.

# Writing about Mathematical Content

## *Short Writing Assignments*

Short writing assignments have students writing regularly for a few minutes as they come into class, while they are settling in, at the end of class or to provide variety in their homework assignments. The following is a list of ideas for regular, short writing assignments (10 minutes twice a week):

1. State the mark you received on your last test and comment on how you feel you did.
2. How does . . . relate to. . . ? (Make connections from what they are studying to other topics in math or to those across disciplines.)
3. Where in your life have you recently used percent? Bring in clippings from the newspaper to show uses of percent.
4. Explain a rule or procedure for . . . and create your own example to show that you understand this procedure or rule.
5. Describe the graph of . . . by writing a story about what you see.

Short writing assignments might include statements such as "reflect on and clarify," "formulate," "compare and contrast" and "create a counter-example" or directions like the following:

1. Word Banks: Use the words *slope, intercept, coordinate, abscissae, ordinate, point, tangent, parallel, perpendicular* and *y-intercept* in a true sense to help you write a paragraph about . . . and. . . .
2. Write a question containing one or more of these words: *right angle, perpendicular, side, hypotenuse, parallel.*
3. Write a true sentence using the words *rectangle* and *square* but without using the word *always.*
4. Write a paragraph about pyramids and cones.
5. Write a crib sheet for a friend who has fallen behind in. . . .
6. Use the words *intercepted, centre* and *angle* to help you write a definition of a central angle.

Other suggestions to use as starting points for teachers who would like to ease into incorporating a variety of writing activities related to mathematical content are class logs and journal activities.

## *Class Logs*

Logs are written accounts, available during each class period (perhaps in a duotang), that provide information on previous math classes. They are detailed accounts that can be written by one student each day and include the date, homework, announcements and pertinent information about the class,

topic, activity or investigation. As a classroom routine, the log provides a written summary of previous lessons for students who are absent, as well as the opportunity for students to write actively about the lesson. Students can be assigned on a rotational basis and graded for their entries. Some teachers ask students to include a personal comment about the lesson.

## Journals

A diary-like math journal or file is useful for organizing students' regular writing assignments. It should be available when the students are working on math and allow for spontaneity as well as teacher-directed writing assignments. An expressive writing style, not to be confused with "free writing," should be encouraged. Teachers who use journals report that vague assignments produce vague responses. Teachers must consider the response they expect and clarify the assignment for students ahead of time. By presenting good prompts, teachers can force students to take a stand. Grading responses will depend on how well the students support their stand. Here are two examples of good prompts: "We learned two ways to graph quadratic functions before I used the graphing calculator. The easiest way for me to do it is . . . because. . . ." and "What I'm finding hardest right now is. . . ."

There are a variety of ways to approach journal assignments, including the following:

1. Completion type statements: Prompts can be used to focus on a small part of a bigger topic for summarizing, analyzing, comparing or expressing feelings. A good prompt
   (a) encourages students to take a stand,
   (b) focuses on the main idea,
   (c) is written in first person,
   (d) takes the form of a statement,
   (e) depends on the feedback desired, and
   (f) provides student choice.
2. Use of lead sentences: These have many of the characteristics of prompts but are usually more specific, for example:
   (a) The discriminant is useful in determining the kinds of roots of a quadratic equation. The discriminant. . . .
   (b) The factors of 18 are 1, 2, 3, 6, 9 and 18. Factors are numbers that. . . .
   (c) The only natural number that is neither prime nor composite is 1. A prime number is. . . .
3. Warm-ups: When teachers want students to put together several topics in order to see the larger picture, they can ask a simple question (not requiring much thought), followed by a second, more difficult question, as in the following example:
   (a) Complete the sentence: Lines that are perpendicular have slopes that. . . .
   (b) State the equation of a line that is perpendicular to $y = 3x + 2$.

## Writing Activities Using the Text

Senior high students need practice using texts. As students progress in their schooling, their learning experiences become increasingly self-directed and dependent on textual materials. Texts are valuable resources for them and ones they need to feel comfortable using when they are alone at home, as well as at school. Writing assignments can enhance students' comfort and familiarity with the text and lessen math anxiety. Consider the following ideas:

1. Familiarity with the "whole picture" is an important way for teachers to assist their students. Many texts provide an inventory of problem-solving strategies, a glossary, answers, historical notes, and technology and applications sections. Students can learn from paying attention to the layout of the text, the sequencing of topics and how each section relates to the topic being developed. Any limitations of the text should also be noted for students. Many students accept printed material as gospel truth. Teachers should reward students who find errors or ambiguities.
2. Have students examine topic development. Students can learn about mathematical

content and processes by studying and writing about worked-out examples. Texts that use bold type for key words or small print to highlight hints for the students also allow students to write about these terms or to discuss reasons why they think the authors inserted the tips.

3. Have students explain or justify each step in a solution presented on a text page, create another example or counter-example or rewrite the page the way they would prefer to have seen it written.
4. Try this one yourself to see the richness of the exercise!
   (a) Justify each of the following steps in multiplying $(x + 4)(x + 2)$.
   $$\begin{aligned}(x + 4)(x + 2) &= x(x+2) + 4(x + 2)\\ &= x^2 + 2x + 4x + 8\\ &= x^2 + (2 + 4)x + 8\\ &= x^2 + 6x + 8\end{aligned}$$
5. Have students practise analyzing the conditions of a problem: Can they change the given information and the question being asked to create a new problem? What are the restrictions? Have students select a problem that can be enriched, write a simpler problem or select a less interesting problem to explore or think about.
6. Have students reflect on text exercises. How are they alike? How do they differ? Do they move from simple to complex? How are they sequenced? An effective cooperative-learning strategy has students in pairs or groups select which exercises from each section they will do and then directs them to correct and discuss each other's work. This experience provides fresh insights for other students and fosters an appreciation for different ways of solving problems.
7. Have students list four topics from a chapter in the text and write a summary for one. Provide clear directions on what must be included in the summaries.

Students should appreciate that texts are written by ordinary people to help others understand and learn about mathematics rather than by people who are strange or removed from the real world. This may help demystify math for them.

### *Writing on Tests*

Some teachers have included short writing assignments on their tests. Such questions could require feedback on concepts or skills, as well as personal comments regarding students' learning experiences, for example:

1. Compare and contrast. . . .
2. Describe your understanding of the procedure for factoring the difference of two squares. Create an example of your own using the numbers . . . to verify your statements.
3. The most difficult thing I had to learn in this class was. . . .
4. Have students analyze one problem on the test that they found hardest to do, using correct vocabulary directed at students' thought processes and approach, any pattern they recognized and some knowledge of how they could proceed.
5. Use verbs such as the following to promote higher-level responses: justify, restate, summarize, support, read and explain, and hypothesize.

## Study Aids and Reviews

The changing nature of society and increased demands on students' time challenge teachers to evaluate the amount and kind of homework assignments given to students. Many students don't do, and don't see the importance of doing, regular math homework. Review is critical to learning, retaining concepts and being successful in mathematics. The following ongoing writing assignments involve students more actively in their own learning.

### *Chapter Summaries*

At the end of a unit or study topic, reflection is a valuable part of the problem-solving process. However, it is more than just looking back (from Polya's four-step problem

solving model). It also involves looking ahead and internalizing a broader picture of the concepts and skills being studied. Relating what has just been learned to what was previously known and looking for the bigger picture allow students to make critical connections between one chapter and the next, one topic and the next.

### *Definition Cards*

I recall a story about a senior high student whose second language was English. He was doing poorly in his high school class. When the teacher asked him how he had gotten that far, he responded, "In junior high, my teacher made me write out what each thing was, and I had to make an example too!" Definition cards can be used as an activity in which students write about math terms and give examples, work in pairs, revise and refine or give a specific example in their own words, and pose a problem to be solved. A group game can also be designed using sets of three cards—one with the definition written on it, another with an example to illustrate the term or concept and a third with a phrase to explain the meaning. The game can be played like Concentration, where students take turns exposing cards, three at a time, looking for a match.

## Writing About Problem Solving

Margaret Ford (1990, 35) reports on research by Donald Graves who found that

> simply asking students to write or giving them opportunities to write, does not produce better writers. . . . Guiding students through a writing process is the key to producing better writers and better thinkers. The task of developing problem-solving ability is similar because problem solving, like writing, is a process. Guiding students through the process of problem solving is essential if they are to become better problem solvers.

As a strategy for problem solving, Ford supports the writing process and advocates using writing to help students focus on the question being asked and look for essential information in the question.

Communication skills play a key role in teachers' assessment of student thinking and for monitoring and reporting on students' progress in problem solving. Students need to develop mathematical language facility to clearly convey their ideas and thought processes in solving problems and reflecting on mathematical experiences and relationships.

To construct meaning by connecting new ideas to concepts they already know, students will need experience in expressing their own generalizations discovered through investigations and to write convincing arguments to validate them.

As mathematics teachers, we can learn from our language arts colleagues who share "secrets" with their students to help them write. These secrets include strategies in prewriting, drafting, revising and publishing, with continuous sharing and conferencing.

In the prewriting stage, the teacher furnishes a stimulus or context, perhaps a real-world problem, which connects to what students already know. The context motivates and arouses interest; it is a springboard from which to introduce a new idea. In mathematics, we look for real ways to teach that thought-through problem solving is the first step in the process.

The writing stage provides time for students to explore, think about the problem and organize their thoughts and ideas, without the pressure of presenting a clean copy.

In revision, students focus and clarify their ideas, create their own examples and counter-examples, and add or delete information. In this stage, students present a draft copy for teacher or peer reaction.

The final phase in the writing process is the publication of a good copy and final form, putting the thoughts on paper.

Ongoing conferencing allows teachers or peers to listen to, read and give feedback by asking questions that focus on the content

of the writing. Discussion promotes the kind of verbal thinking that develops ideas. Writing places the thoughts on paper and acts as a lens through which to view, magnify and focus those thoughts (WIMP 1989–91).

# Writing about Attitudes, Thoughts and Feelings

## *Personal Math Histories*

One way to begin a new semester or math course is to find out where your students are coming from. Students' attitudes, backgrounds and personal math histories can be revealing and can assist teachers in getting to know their students and in planning for instruction. It may be more appropriate for young students to write about what they expect to achieve or learn in math this year.

## *Attitudes Toward Mathematics*

Mathematical dispositions, such as a student's self-confidence, willingness to solve problems and perseverence and interest in math, contribute to mathematical literacy and student achievement. Students' dispositions are part of the informal ongoing assessment which is monitored as students progress through high school math programs. One effective way of monitoring is to collect data continually through anecdotal records and teacher observations. Another way is to involve students in self-assessment and reflection.

Journal assignments can also provide teacher and student with insights into attitudes and feelings. Maintaining collections of individual student's responses to your prompts, over an entire course, is useful for determining changes and progress in mathematics. This data can be used in parent and student conferences. Teachers should allow students to change prompts so they feel comfortable, but the intent in any change should be consistent with the kind of information teacher wants to collect. Notice the "I" form for the following prompts:

1. As a problem solver, I have no problem doing . . . but . . . still bothers me. . . .
2. Yesterday I learned that. . . .
3. So far in this course, I. . . .

# Writing about Math and Its Applications

Opportunities for writing about issues, historical figures in math and applications will reinforce students' understanding of the connection between mathematics and society. Essay questions, projects and reports can be used as long-term assignments to develop awareness of the value of mathematics and appreciation for its use in the real world. The newer statistics and probability strands of the curriculum promote group work on real-world problems and meaningful investigations.

## *Research Topics*

Teachers can have students research topics from math history and/or the discoveries of famous mathematicians, write a major paper or develop a complex problem to be solved. Alberta Education's senior high school problem-solving monograph *Problem Solving in Mathematics: Focus for the Future* (1987) is a valuable resource that includes solutions to several interesting project problems.

## *Math Hunt*

One of the most successful research projects I have encountered was a math hunt conducted by Cynthia Ballheim at St. Mary's High School in Calgary. Students, working in groups, were asked to find 10 adults who used math in their careers. As a long-term assignment, the students compiled documentation from interviews with the adults and wrote a final report, which included a signature and the nature of the mathematics the adults used. The students received bonus points for getting a person to speak to the class. Project results were so diverse that the class concluded that everyone uses math. One girl remarked in

a surprised way, "Did you know? Even my mother uses math!" A second student told me that it wasn't until then that she really tried in math class because she realized that she would need to know more math when she graduated.

## Assessing Student Writing

Teachers must establish a purpose when selecting writing assignments and must then evaluate whether or not that purpose is met. Change happens slowly and gradually, so proceed slowly and begin with what is comfortable for you. This will take patience, but the rewards are worthwhile. Incorporating writing experiences takes little more time than repeated lecturing. By showing students' that their thoughts are valued, a spirit of openness and trust can be built to promote math teaching and learning. By writing back and responding to students' work, assigning grades and including writing assignments as part of the course mark, teachers can convince students of communication's importance in learning mathematics.

Teachers who have used writing-to-learn approaches in their classrooms have struggled with the time it takes. This time commitment depends on what the teacher decides to do with students' responses. In some cases, teachers read assignments on a "spot-check" basis. In others, students share their responses orally, exchange with peers or hand their papers in. In all cases, students should receive regular feedback from teachers. If students take time to write, teachers need to find the time to write back!

Mathematical errors should be a secondary consideration in some writing assignments, but they should always be brought to students' attention. In their evaluations, teachers should keep the primary focus on thoughts and ideas rather than on spelling and punctuation.

Students should see their errors as opportunities for learning. An understanding and acknowledgement that errors are a part of learning will help to establish a risk-free environment. Asking students to proofread their own as well as each other's work is effective for correcting errors and clarifying meanings. Having students write about their errors on tests or homework assignments is also an effective learning tool. When students are asked to write, they are really being asked to think. This will take perseverence from students and encouragement from teachers. The following questions should guide students in their efforts to analyze what they are doing and why:

1. What are you doing?
2. Why are you doing it?
3. How is it going to help you find an answer to this problem?

The aim of establishing a regular series of questions like this is to encourage students to reflect continually on their thinking, their use and choice of strategies, and their problem-solving procedures. By continually asking students to clarify, paraphrase and elaborate, to describe how they reach answers as well as the difficulties they encounter solving problems, teachers are providing them with valuable skills for the future.

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