Chapter 5

The Language of Mathematics Learning: Lexicon and Grammar

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Because Laurie Walker teaches education language arts at the University of Lethbridge, it might be assumed that he would approach the language of mathematics learning from a language-experience viewpoint. He does and, along the way, allows those of us who have a primary interest in language learning to bring our understanding of this domain more explicitly into our practice as mathematics teachers.

He describes a "grammar of mathematics" with illustrations from division and shows how differences in passive and active forms can lead to confusion. He also offers insight into errors commonly encountered or made by children.

He concludes by suggesting that

> *if there is a gap between everyday language and the language of mathematics found in the technical terms and in the grammar, a general proposal for teaching the subject may be useful: That children be encouraged and helped to learn math in their own language, at least initially.*

This is good advice, particularly when taken in context with the rest of the chapters in this monograph.

Introduction

A prison instructor was working with an inmate on math concepts. The student was having problems with percentage, apparently unable to grasp the idea and its use. Changing the subject, the instructor asked what the inmate's crime had been.

"Drug dealing," the inmate replied.

"When you were pushing drugs, how did you work out how much of each sale went to your supplier?"

"He just got his *end*."

"Well, an *end* is the same as percentage."

The instructor reported that this conversation was a breakthrough for both him and the student, who then went on to grasp the concept of percentage and use it successfully in problem solving.

This anecdote illustrates a general curriculum issue: the choice between starting with the general abstract concept or starting with the student's own experience. The issue has particular relevance to mathematics where there is a profusion of abstract terms and concepts, said by mathematicians to be more precise in meaning than terms used in everyday situations. If we accept a simple relationship between words and the world, one in which words simply label or represent objects and ideas in our experience, the issue of the curriculum starting point is not so serious; it is simply a matter of learning the right labels for things that we know. If, on the other hand, the relationship is more complex and if some ideas are unavailable until the words call them into being, teachers have to accept that their students may have difficulty forging links between the official labels for mathematics concepts and their own experience of the world.

For example, the medieval mind took a long time to see the connection between a brace of pheasants, a pair of stockings and a courting couple. These were experienced as concrete things, and the linking abstraction of "two-ness" was not available. Perhaps literacy reveals the abstract level at which experiences have common elements and qualities. In such a way, the concept of percentage may lie beyond the immediate and more concrete "end" of a drug dealer, the "half" of a fair sharing of a chocolate bar between two friends, or the "tip" to a restaurant server. In other words, the inmate's learning block arose because he saw percentage as an esoteric part of the world of school mathematics, an official discourse that was not supposed to have anything to do with real life. In his view, mathematics words are the currency of those who have been inducted into the privileged discourse community of mathematicians by gatekeeping teachers. To outsiders, excluded by tests and examinations, the language categories and rules are rather mysterious. Some try to gain late entry to the community through adult upgrading courses and struggle with exotic things such as proper and improper fractions, never having encountered them in their working lives. Two sources of exclusion from the effective learning of mathematics are related to the language subject: the lexicon and the grammar of math.

The Technical Language of Mathematics

The written texts of mathematics are replete with technical vocabulary. I counted 22 such words in one 36-page unit of a Grade 5 textbook (Kelly et al. 1987, 124–60). Students must read text material containing words such as "array," "pictograph," "digit," "quotient," "multiple," "addend," "short division" and "long division." There are three kinds of these technical words: some are words unique to math—"quotient," "divisor," and "addend"; others have precise mathematical uses similar to their everyday function—"remainder," "average" and "bring down"; a third group consists of words that have mathematical meanings different from their ordinary functions—"rounding," "even" and "root."

Words in the first group, those unique to mathematics, are difficult to connect to children's prior experience and need special treatment in class. Children need lots of correct and incorrect examples as they work toward definition and classification. They need opportunities to express definitions in their own words rather than in the forbidding technical language of the textbook. It is easier to grasp "the quotient is what you get when you divide one number into a larger number" than the textbook definition—"[t]he number (a ÷ b) or (b ÷ a) which results from applying the division operation to the numbers 'a' and 'b' " (Fleenor et al. 1974). After working through problems, children can be helped with these new technical words by a chart display on which the "New Words of the Week" are listed with definitions, examples and sample problems.

Words in the second category, having similar meanings in mathematics and everyday life, could be approached differently. The term *odd*, for example, in the mathematical sense of "odd number," could be introduced through the notion of oddness: someone who is odd, an odd story, the odd one out. The idea that *odd* means unusual, eccentric, different or left out could then be extended to the mathematical idea that *odd* refers to numbers that cannot be separated into pairs without a remainder. The etymology of the word might be interesting to some children. The Middle English word *odde* meant a point of land having the shape of a triangle and therefore possessing a third angle, the one left out when the other two were paired. It would help children to realize that certain math words are used with more precise versions of their everyday meanings. Experience in this case is a dependable starting point.

In the case of the third category, it is not. These words have math meanings that conflict with their everyday usage. In baseball, for example, one talks about a runner *rounding* third base; in carpentry, one thinks of *rounding* off a corner. Neither of these uses is particularly helpful in thinking about *rounding* off numbers in mathematics. In these cases, students need to be alerted to the different uses that mathematics makes of words.

The Grammar of Mathematics

In addition to the lexical aspects of the language of mathematics, an area of grammatical difficulty may be unique to the language in which mathematical concepts and operations are expressed. The proposal conveyed by the slogan Language Across the Curriculum, made popular by the British report *A Language for Life* (Department of Education and Science 1975), is that teaching and learning are closely linked to the particular ways in which thought is expressed in the different disciplines. There is a language of science, a language of history and a language of literature, for example, each representing special ways of understanding the world. An example of the uniqueness of mathematical language is the particular relationships between the expression of number relationships in mathematical symbols (called math sentences) and their expression in language (language sentences). These relationships have been studied in detail by Hull (1985) with respect to division.

He observed that just as language sentences can be expressed in the active voice (the chairperson opened the meeting) or the passive voice (the meeting was opened by the chairperson), math sentences vary in the same way. For example, one could show a division sentence actively:

1.0 $6\overline{)12}$ with quotient 2.

Reading from left to right, this math sentence could be expressed in language as

1.1 "Six goes into twelve twice" or "six divides into twelve two times."

The relationship between the mathematical voice and the language voice seems straightforward; the elements are processed from left to right in order of their appearance. Similarly with the passive voice, it is expressed mathematically as

1.2 $12 \div 6 = 2$

and in language as

1.3 "Twelve divided by six is two" or "twelve shared by six is two."

Again, expression and processing proceed from left to right in the order in which the elements appear.

However, it is possible to read an active math sentence as a passive language sentence so that 1.0 could be transformed to read passively as 1.3, and 1.2 could be transformed to read actively as 1.1. In each case, the grammatical transformation is accompanied by a transposition of elements so that the math symbols are no longer read from left to right. There are two learning issues here. First, research into children's language development shows that passive constructions are mastered later than active ones. Berko-Gleason (1985, 250) reported research showing that full comprehension of passive structures is not acquired until between 11 and 13 years of age. Thus, children in elementary grades may not be comfortable with passive math and language sentences.

Second, the inadvertent transformation of active and passive forms while translating between math and language sentences, without recognizing the potential processing complexity, may be a source of difficulty. Such transformation may occur in the course of oral instruction as teachers explain operations such as division. It may also appear in textbook presentations.

For example, in *Math Quest Five* (Kelly et al. 1987), the unit on division with one-digit divisors uses both active and passive forms. On page 124 under a picture of plants appears the language statement:

1.4 28 plants divided into 4 rows is 7 plants in each row.

Then appears the math sentence

1.5 $28 \div 4 = 7$.

These are both passive sentences. However, note the potential for misunderstanding in the preposition "into." In 1.4, the sense is clear enough—28 plants divided into 4 rows. In 1.5, if children carry over the "into" idea, they might read the math sentence as "28 divided into 4." This is ambiguous because it could mean either "how many 28s are there in 4?" or "how many 4s are there in 28?" It would be helpful to replace the preposition "into" in 1.4 with "by" in the math sentence 1.5.

The passive construction is maintained on page 126, except that "divided into" has become "shared by":

1.7 24 shared by 4 people gives 6 to each person.

On page 127, there is an unacknowledged switch from a passive language sentence

1.8 How many packages can be made from 50 peaches?

to an active math sentence

1.9 $8 \overline{\smash{)}50}$

This is followed by an exercise involving completing active math sentence problems. Then, on page 129, an explanation of how to carry out division operations on a calculator begins with an active math sentence:

1.10 To compute $7 \overline{\smash{)}59}$ on a calculator, press these keys.

To many math teachers, this grammatical difference must seem trivial. However, Hull (1985, 53) noted that the variety of language expressions for the mathematical operation of division was vast. He went on:

> If S is a subject who does the dividing, and O is an object (like a cake) or an amount, and D is the number representing the divisor, there are at least seven types that seem to be idiomatic—two commands and five statements:
>
> Divide O into D (e.g. divide the cake into two)
> Divide D into O (e.g. divide 2 into 6)
> S divides O into D
> S divides D into O
> O divides into D
> O is divided into D
> D is divided into O

Hull also noted that *divide* has a number of synonymous expressions: "share," "go into," "into," "how many . . . in" and "partition." However, these do not all behave in the same way grammatically. For example, "six goes into twelve" is not the same as "six divided into twelve" or "six divided by twelve." The prepositions "into" and "by" are implicated in these differences.

Hull (1985, 54) explored 100 11-year-olds' understanding of *divide.* First, by oral questioning, he established that they all thought they knew what the operation was and, that indeed, most of them could carry out simple division operations. However, when he asked them to translate math sentences into language sentences, their knowledge was less secure. For example, for the math sentence $2 \overset{2}{\overline{\smash{)}4}}$ their translations included

Two divided by four is two
Two divided into four is two
Two share four is two
Two shared into four is two
Two shared between four is two

Thirty-seven children made this kind of error involving transforming the active math sentence into passive language structures and thereby changing the relationships of the elements.

Their translations for the sentence $12 \div 6 = 2$ included

Six divided twelve is two
Six share twelve is two
Twelve shared by six is two
Six shared into twelve is two
Six shared by twelve is two
Six shared between twelve is two

Hull observed that many of their language translations were meaningless or wrong. There was a ritualistic quality to some of these language sentences, as though they were utterances that applied to the mystery of mathematics without any fidelity to the real-world use of language. Their language sentences were derived from math as an activity separate from the world in which they lived their normal lives.

Hull concluded that children's difficulties with division as a math operation arose from the complex translations between math sentences and language sentences, from the transformations between passive and active structures and from the transpositions of elements in these translations and transformations. The grammar of division is an example of the complexity of the language of mathematics, which may cause difficulty for learners. To be aware of the potential difficulty and to be able to observe carefully the language of math teaching and learning are starting points for teachers who wished to help their students achieve fluency and control.

Conclusion

If there is a gap between everyday language and the language of mathematics found in the technical terms and in the grammar, a general proposal for teaching the subject may be useful. This is that children be encouraged and helped to learn math in their own language, at least initially. Talking about math in his own language helped the prison inmate. Using exploratory talk in math classrooms helps bridge the gap between mathematics and everyday experience and its expression in everyday language. The precision of math terms could be approached from this starting point. Likewise, using exploratory writing in math writing journals might illuminate the link and indicate to teachers where links were not being made. It is interesting to note a shift of emphasis in the literature from a focus on how to teach children to read mathematical text to the use of writing in math lessons (Abel and Abel 1988; Davison and Pearce 1988; Johnson 1983; Miller 1989; Nahrgang and Petersen 1986). This may be part of a shift from a subject focus in the math curriculum to a learner focus. Teaching reading skills to math students implies that students have to be changed to fit the subject; using exploratory talk and writing implies that the subject has to be changed to fit the learner. However, before conservatives see this proposal as a loss of rigor, this latter change, as far as language is concerned, is strategic in that it permits more students to pass safely through the door to the discourse community of mathematicians where everyone is at home with technical terminology and grammatical complexity. Then no one need approach percentage through the experience of drug dealing.

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