Chapter 6
"Why Are We Doing This?": Reporting Back on Mathematical Investigations

David Pimm

This chapter demonstrates how research and practice can together offer teachers the best in guidance and understanding of what is happening when students learn mathematics. It also offers a glimpse of how distinctions made linguistically can lead to insights into what it means to know mathematics as well as to ways in which to enhance mathematical understanding in the classroom.

David Pimm considers the following a significant difficulty facing all mathematics teachers:

> *How to encourage movement in their students from sole use of the predominantly informal spoken language with which they are fluent . . . to a range of language modes and styles including the formal written language that is frequently perceived to be one hallmark of successful mathematical study.*

The movement from informal spoken language to the more formal written mode is detailed within the context of mathematical investigations by focusing on the process of reporting back to the whole class. David Pimm provides excerpts from lessons to illustrate his points and draws his conclusions within the context of the research literature surrounding his project.

This chapter will be of particular interest to those who attended his two sessions at the MCATA annual conference in Edmonton in 1991.

Many aspects of and relationships between mathematics and language can be highlighted as part of the mathematics education enterprise. In this article, I will explore just a couple of aspects related to the teaching and learning of mathematics, but it is crucial for all of us to find ways of talking about the varied components of mathematical activity itself, which is a complex phenomenon. One means for achieving this is to focus on particular features of doing mathematics, which might then afford teachers greater insight into what is happening in and between their students when in the mathematics classroom. This article and this monograph focus particularly on linguistic aspects of doing and teaching mathematics, in particular the social context and community that classrooms exemplify. (Aspects of some of the themes mentioned in passing in this article are examined in greater depth in Pimm 1987.)

Spoken and Written Language

As Douglas Barnes (1976) points out, communication is not the only function of language. For instance, externalizing thought through spoken or written language can provide greater access to one's own thoughts (for oneself as well as for others), thus aiding the reflection process, without which learning rarely takes place. In mathematics, language can also be used to conjure and control mental images (see, for example, some of the mental geometry activities in Beeney et al. 1982). Spoken and written language have many characteristics and functions, and it can be useful for mathematics teachers to be aware of them to encourage and foster appropriate growth in their students' language abilities in mathematics.

Written language externalizes thought in a relatively stable and permanent form, so it may be reflected on by the writer and provide access for others. Writing things down can be used to find out what one thinks, enable one to refer back to something later (serving as an external memory) or send a message to someone who is not present. As a consequence of the need to fulfill these functions, one common characteristic of written language is for it to be more self-contained and able to stand on its own, with far more of the referents internal to the formulation, than spoken language, which can be employed to communicate successfully despite being full of indefinite "theses," "its" and "over theres," from other factors present in the communicative situation.

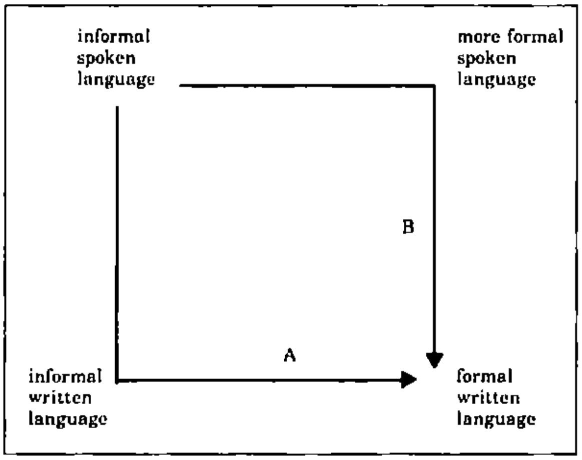
Speech frequently fulfills more direct communicative functions, but because speech is invisible, it does not persist except in memory. Its intangible quality renders it less permanent but more readily altered and revised. In addition, spoken language can be used flexibly to point, to focus attention, in situations where physical indication is not possible or prohibited for some reason.

Spoken and written language are rarely interchangeable, in that choice alone seldom determines the channel of communication. Other dimensions which frequently affect this decision include the weighing of private/public, informal/formal, interactive/one-way, face-to-face/disembodied, spontaneous/planned and context-bound/context-free. (See Rubin 1980 or Hudson 1984 for further exploration of these differences, as well as those of structural organization.)

One difficulty facing mathematics teachers is how to encourage movement in their students from sole use of the predominantly informal spoken language with which they are fluent (Brown 1982) to a range of language modes and styles including the formal written language frequently perceived to be one hallmark of successful mathematical activity.

It is a probable pedagogical error to assume continuous development *from* spoken *to* written language. It is not a linear move—students need to develop particular skills with both channels. Nonetheless, two basic ways are used to approach development of written mathematics moving out from a presumed strength of spoken skills and the context of the classroom. The first (and far more common) is to encourage students to write down their informal utterances and then work on making this written language more self-sufficient (route A, Diagram 1), for example, by using brackets and other written devices to convey similar information to that which is conveyed orally by stress or intonation.

Diagram 1



James and Mason (1982) discuss 10- and 11-year-old students moving back and forth between various spoken and written representations of combinations of Cuisenaire rods, focusing directly on questions of reaccessing the original situation from the linguistic representation to provide criteria for judging the adequacy of a given written expression. One example they discuss is how a spoken description of "pink and white [pause] four times" can be recorded in various mixtures of mathematical symbols and written words and whether other interpretations are possible. Written forms explored included

4 Pink and White
4 Pink and 4 White
Pink and White x 4
Pink + White x 4

Repeated access over time is another essential function of written records, as Martin Hughes' work (1986) with much younger children makes clear. Hughes invited preschool and early elementary age children to make marks on labels attached to the lids of tins, each of which contained a different number of cubes. The lids were replaced and the tins shuffled: a game was then proposed in which they were to guess how many cubes were in each (or a particular) tin. His categorization of responses into four types (idiosyncratic, pictographic, iconic and symbolic) provides a useful means of talking about many students' attempts at symbolization at different age levels.

Further research exploring the self-contained aspect of written messages includes the study by Balacheff (1988) of 13-year-old students' notions of proof. The activity was to

> write a message which will be given to other pupils of your own age which is to provide a means of calculating the number of diagonals of a polygon when you know the number of vertices it has.

Students in pairs wrote their claims (on a single sheet of paper) about this mathematical situation to communicate what they had found out. By providing them with plausible justification for writing a message, Balacheff was able to access their level of writing proficiency as well as to explore the various styles and conventions of justification employed (and, somewhat incidently, their understanding of polygons).

A second route to greater control over formal, written mathematical language (route B, Diagram 1) might be to work on the formality and self-sufficiency of the spoken language before it is written down. For this to be feasible, constraints must be placed on the communicative situation to remove features allowing spoken language to be merely one part of the communication, rather than the entirety of the message.

Such situations often have some of the attributes of a game, and provided students take on the proposed activity as worthy of engagement, then those students can rehearse more formal spoken language skills in a setting which requires them. One such scenario is provided in a lesson where the focus of mathematical attention was a complex geometric poster (Jaworski 1985). Students were invited to take the hot seat (a chair in front of the poster facing the rest of the class) and to say what they had seen to the rest of the class, without pointing or touching. These constraints focused the challenge onto the language being used to point at the picture.

The situation is an artificial one. In real life, one can often point which, together with spoken language, is completely adequate for effective communication. However, provided the artificiality is accepted (as the rules of the game), learning can take place that would otherwise not so readily have happened. There is an interesting paradox here in how quite artificial teaching can give rise to natural learning in certain circumstances. The point of being in mathematics classes is to provide students with experiences they would not get elsewhere. The mathematics classroom is not and should not be a natural setting, and attempts to make it so diminish its potential power.

A second instance of attempts to work on pupil speech directly, and the one I will focus on for the rest of this article, comes from the context of mathematical "investigations," when students are invited to report back to the class about what they have done and found out. As the background assumptions (explicit and tacit) of such activity may not be familiar, I first outline some of the traditions that have developed in England with respect to students' mathematical activity under the generic heading of "investigations."

Mathematical Investigations

Broadly speaking, one intent of the "new" English teaching in the 1960s was to move away from teaching language syntax and the study of acclaimed literary works toward greater attention to self-expression and creative writing. Students were supposed to formalize their informal writing gradually (in a way similar to route A); yet, among other things, this required students to reinvent precisely those conventions of written language that are essentially arbitrary.

While there was a similar "new" mathematics, it was less a comparable refocusing along the spectrum between individual and cultural expression and values than a replacement of one set of content aims by another. Closer attention to student creativity in mathematics, and an acceptance that teaching the grammar of mathematics together with a study of culturally-enshrined works or theorems was an impoverished view of potential student involvement in mathematical activity, was only widely accepted a decade or more later. The 1980s, not the 1960s, was NCTM's "decade of problem solving," and even then many of the examples of so-called problem solving did not allow students much scope for exploration or invention.

In England, *Mathematics Counts*, known as The Cockcroft Report, was a seminal national government document on teaching mathematics in elementary and secondary schools (Department of Education and Sciences 1982). One paragraph in particular (para. 243) stated:

> Mathematics teaching at all levels should include opportunities for:
>
> - exposition by the teacher;
> - discussion between teacher and students and between students themselves;
> - appropriate practical work;
> - consolidation and practice of fundamental skills and routines;
> - problem solving, including the application of mathematics to everyday situations;
> - investigational work.
>
> [my bullets]

Three of these modes (discussion, practical and investigational work) were relatively novel (particularly at the secondary level), and the report has led to the promotion of these activities in mathematics classrooms. The report also legitimized the actions of those teachers who had been moving to broaden the scope of appropriate mathematics classroom activities. Among others, increasing numbers of members of the Association of Teachers of Mathematics, frequently writing in the Association's *Mathematics Teaching* journal, had been describing and indirectly promoting offering students open-ended mathematical situations for exploration over the preceding 20 or more years.

Eric Love (1988, 249–51) has produced a thoughtful account of this period, analyzing the ways people try to describe mathematical activity beyond a purely content description and analyzing in particular the birth of the mathematical "investigation":

> In contrast to the tasks set by the teacher—doing exercises, learning definitions, following worked examples—in mathematical activity the thinking, decisions, projects undertaken were under the control of the learner. It was the learner's activity. . . .
>
> In the early writings on mathematical activity, there is no mention of "investigation" in the sense of "doing an investigation." It is interesting to see this construction develop from pupils "investigating such-and-such," or "carrying out an investigation into such-and-such," no doubt originally as a shorthand, but soon taking on a life of its own. The path to formalisation had begun.
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> In the last decade "investigations" have become institutionalised—as part of formal requirements for assessment of courses. . . . They also appear in the official recommendations of Cockcroft and HMI.*
>
> Such a development is a typical one in education—the often commented upon way

* Her Majesty's Inspectorate (HMI) work for the Department of Education and Science, and their primary brief is to inspect and write reports on the state of teaching in the public schools.

in which originally liberating ways of working become formalised and codified, losing their purpose as they become adapted for different ends or by those who have no personal commitment to the underlying intentions.

One example of such "institutionalization" has been that post-Cockcroft paragraph 243, there has been the emergence of the phrases "investigation lesson" or "discussion lesson" in the same way as the terms "algebra lesson" or "decimals lesson" would formerly have been used to describe the main focus. Such use is another example also of this process of action turning into noun, whereby everything ends up as content to be taught in a lesson. These expressions mistake the means for the end—the end is not just discussion per se. I have written elsewhere (Mason and Pimm 1986; Pimm 1987) on the naive assumption that student discussion is wholly and everywhere a good thing, rather than being a particular tool with strengths and weaknesses available for use in various situations.

The "investigation lesson" has its common structure of whole-group posing of the task, students then working together in small groups and then finally reporting back to the whole class. This format has become an example of the new orthodoxy and, unexamined, can be as harmful to the health of teacher-student communication as the old pattern of the teacher-led, blackboard-focused lesson. Nonetheless, the activity of reporting back raises some interesting linguistic questions, as I indicated earlier, in relation to student language and styles of speech in the mathematics classroom context.

Reporting Back to the Class

Because of the more formal nature of the language situation (particularly if rehearsal is encouraged), reporting back can lead to more formal, "public" language being used and to greater structured reflection on the task. Thus the demands of the situation alter the required language use. When preparing for course work write-ups, for example, a prior stage of oral reporting back can help with selection of material and the emphasis placed on various parts of it.

One key question is for whose benefit the reporting back is done, which may be particularly pertinent if you have observed sessions that were apparently painful for some participants. Requiring students to report back can also be intrusive and perhaps unhelpful if some groups are not at the stage of summarizing what they have done and are still engaged with the problem. Take some time to think about who benefits (if anyone) and why.

Three possible answers and corresponding sets of potential justifications to this question follow:

1. The student(s) doing the reporting: For them, plausible justifications include developing a range of communication skills, their use of language and social

confidence. A further important possibility is developing skill at reflecting on a mathematical experience and distilling it into forms whereby they and others may learn.

2. The other students listening: For them, potential benefits include hearing alternative approaches and that perhaps others besides themselves had difficulties, the chance to ask genuine questions of other students and to engage in trying to understand what others have done on a task that they also have worked on.
3. The teacher: Potential benefits include a range of opportunities to make contextually based meta-remarks about methods, results and processes (perhaps indicating the task is open and that a number of ways of proceeding are possible or different emphases can be placed), as well as to value students' work and broaden their experience (possibly by sharing an original idea with the rest of the class—"Did anyone else. . . ?").

   Reporting back can also provide the teacher with access to what the students think the task was about, as well as less tangible but nonetheless interesting information such as what they think is important (in terms of what they select to talk about) and their level and detail of oral expression of mathematical ideas. But it can also be *assumed* that reporting back *has* to happen, perhaps meeting the need (or expectation) for some sort of whole-group ending, "pulling the session together." If that is the case, then many of the above benefits may not accrue, because attention is not being paid to drawing them out.

Here are five further questions that form key issues involving mathematics education and language arising from the task of reporting back on mathematical investigations. Bear them in mind (and formulate more of your own) as you read through the classroom transcripts provided:

1. What sort of language (style, structure, organization, register, and so on) does the student reporter use?
2. How can the tension between wanting the student(s) to say themselves what they have done and wanting to use what they say to make general remarks about how to undertake investigative work be handled?
3. How can students develop the skills of selection of and reflection on what to report? How can they acquire a sense of audience?
4. To whom is the reporter talking?
5. What justifications does the teacher explicitly offer the class for having them report back on their work to the rest of the class?

To provide a clearer feel for what is involved in practice, I give below two excerpts from actual class lessons using reporting back. Then I explore some of the more general issues arising from inviting students to engage in such a task, organized around the five questions offered above.

The Lessons

Both lessons have been transcribed from videotape, and much of the relevant detail is nonverbal: who is looking at whom, where people are standing or sitting, what equipment is being pointed to and so on. As reader, you face the problem alluded to earlier of only having the written record to interrogate. Nonetheless, I hope that these accounts provide you with some actual detail of what might be said and how report-backs in classrooms might be carried out.

Lesson 1

This lesson (which takes place over three 50-minute classes) is with a mixed-ability class of 30 10- and 11-year-olds, and teacher Steve Wilson is working on a mathematical investigation involving movements on a square grid. Students have previous experience of a similarly investigative nature and are developing their ability to explore questions and apply mathematical processes such as predicting and testing, and convincing others. Recording ideas and results forms an important part of their work.

Steve starts the lesson with the whole group by having a 3 x 3 set of mats on the floor and inviting eight pupils to stand on them, leaving one corner mat free. The pupil (Robert) in the diagonally opposite corner from the empty mat is asked to wear a red cap, and the first task is to see if, by moving sideways or vertically only, Robert can be moved to the currently empty (target) square. It proves possible, moves are counted and further attempts have external leaders guiding the students. The class counts the number of moves so far aloud as a group. Steve then introduces the question of the minimum number of moves possible and alerts the class to the possibility of exploring different-sized grids with the same question.

In an interview after the lesson, Steve comments, "There will be sessions where I bring it together to compare and to sort of look at the work they have done so far."

The report-back phase of the second lesson went as follows:

Steve: There are a lot of good ideas that have come out of the various groups. I had a chance to see those ideas because I got to go around and see what you are doing. But it would be quite nice if you could also share those ideas with the rest of the class, so they can see the sort of things you've been doing. It may help to give some ideas to you as well.

We've got the overhead projector [OHP] if you want to write anything on the screen; we've got our equipment over there—our cubes, squares, the same sort of things we've been using before. If you want to demonstrate on the OHP, that's fine; if you want to use a table, that's fine; and again, if you want to just speak, that's fine.

One person from the group or two people can come up and explain—it's up to you, alright? Just to explain briefly what you've achieved over the last day or two, [pause] let's start with this group, shall we? Who's going to come up from this group?

Come and stand here. [near him]

Paul: [very hesitantly] We've been trying to see if there's eight between all the numbers and we found out there was—and the more squares there are, the harder it gets.

Steve: OK. The harder it gets—stay there. [near to group's table] Do you want to add anything to that?

[No response]

You found this difference of eight does carry on no matter how many squares you've got in there. But you're also saying the larger the size of square, the more difficult it is . . . to do what?

Paul: I'm getting a bit confused.

Steve: Is counting the problem or is moving the problem—are you happy where to move?

[Paul nods]

So it's keeping count. [turns to rest of class] Did anybody else find keeping count was a problem? . . .

Let's go onto this group—Diane, Kevin, Mary.

Mary: [standing at the OHP, reading from her book] We were checking out the pink [the target cube was pink in their representation of the problem using Unifix apparatus] and the way in which the blank moves. The thing about the eight—the eight does carry on; you keep adding eight every time, and the stair of the pink—we tried it on a 2 x 2 and a 3 x 3, 4 x 4 and I think we tried it on a 5 x 5, and everytime it went in a staircase and we had a bit of trouble with the blank, because it kept going back on itself, so we lost track of where it had got to.

Steve: Right. Following the path of the blank—let's just remind ourselves of that. Here's the cubes. Can you demonstrate what you did with a 3 x 3?

Mary: The blank goes from there to there, then it goes round to there, then we go back to that square, so it's retraced and gone back to there, so we've got two arrows going round in the square, so it goes back to there again, to the centre, so you've got another loop. Then it goes back to the top where you haven't had it before, then it goes back down again, so everytime the arrows are going back to the same place that they were before. So you've normally got about two or three arrows going to each square.

Steve: I think it's quite interesting. I don't know if you can see this [holds up her book to class], but they've actually traced the path of the gaps and also the target one as well, and you can see they use a system of arrows. We can actually trace this method of going round and round. It's quite interesting, isn't it? How we make a note of these things is quite difficult sometimes. Thank you very much.

Emma: We did a table [starts to draw table, with whole numbers along both sides]. This is the number of moves and this is the number of squares. Then we did the arrows.

Steve: [to rest of class] Do you fully understand what she is saying?

Class: No.

Steve: I'm not sure that I do. However, if I can hold this up again? You're trying to plot the moves, aren't you?

Emma: Yes.

Steve: If we can go back to your 3 x 3 grid. That's quite interesting. . . .

[In many of these interchanges, Steve is standing at the front near the student speaking, but to one side facing the rest of the class. He quite often repeats, rebroadcasts and reinterprets what the students are saying. This is particularly true of the next pair.]

John: We've not been taking 7 x 7 grids, but like 6 x 9 ones; then we added one onto the first side and took one off the other side until you got down to where you couldn't move any more.

6 x 9 = 33 moves
7 x 8 = 39 moves

Then we started predicting them as we came up with a pattern in the results.

8 x 7 = 41
9 x 6 = 45
10 x 5 = 51

Steve: Can you see what they are doing?

Class: No.

Steve: They're working on a different-sized grid. They're working on rectangular grids. . . .

Simon: To find any number, for example, 18 x 18, minus 2 to make 16 x 16, and then times 16 by 8 because the answers jump in 8, and each time then add on 5 from the first one, 2 by 2, because you don't have 8 there to add on, and you get the minimum moves for 18 x 18.

Steve: Understand that? [pause] A lot of things have happened, haven't they? It shows that if we take a particular problem, you can continue it in lots of different ways. Can you produce theories on how patterns developed? Can you notice the patterns? How do you recall what's going on? Are there different ways of recording and, again, are there any things to notice in the way you record things, are there patterns in those? Can you produce theories? Can you test them, and can you convince yourself that they are correct? If you're convinced, how can you convince other people? Can you produce some sort of a method, an equation, a function rule, to explain it to other people? What I think is also quite interesting is how we can change the problem and both of you [indicates two students] were working on a way of changing a problem.

Lesson 2

This is with an older group of 13- and 14-year-old students and their teacher Peter Gates. The problem was to find out how to make a race once round a track fair; in other words, what was the stagger necessary between each lane on a standard 400 m lap? They were working on scale drawings of the track and using a variety of materials including string. At the beginning of this (second) lesson, they are sitting in groups at tables.

Peter: Let's get started. Now yesterday, we umm, let's just try to cast our minds back to what we were doing toward the end. Remember it was a fairly packed session but you were working on this race track? Can we just try to get each of you to construct in your own mind what the problem was that I set you? OK. Anyone like to remind us what the problem was I set you? . . . Anyone?

John: It was all about the 400 m race track, with four, five, four lanes, and you had to make it fair for each person to run, where you were going to put the start.

Peter: So we're trying to make it fairer and that was basically the problem. Now I've spoken to most of you in your groups yesterday. What I want you to do now, in the next 10 minutes or so, is just to make sure among your group that you've had a fair crack at that problem and that you've got a reasonable solution to it, and then I'd like you to arrange for someone in your group to report to the whole class on how you solved that problem and on what the problem was. We'll then quickly go round each group to see how you each group tackled it individually. So you've got 5–10 minutes to tidy up loose ends. Make sure you know what you are going to say. And then we'll come back together as a group.

Can I remind you of the very last thing that you did last session. . . . What was that? Don't put your hands up. What was the very last thing you did? [pause] All got it? If I remember rightly, I asked you to jot down in your books where you were, to write down what you were doing, what you were thinking and so on. The reason for doing that was that you were going to come in today, a new day, the start of a new day and have forgotten what we'd done.

Can you look now, before we get started, at what you wrote yesterday? Read through it and see if it helps you get back into the problem. If it helps you, you wrote it well yesterday. If it doesn't help you much, you didn't do it well yesterday.

[They then work for 15 minutes in small groups.]

Can we come back together then? Right. Can you remember as we go round that part of the reason for doing this is giving you an idea of how other people tackled the problem? No group here knows what any other group did at the moment. Another part of the reason for doing this is so that we can share our ideas. Another reason, or another thing we can get out of this, another advantage, is that the person who's speaking is having to put their ideas into words. It helps them if you can remember that and try to be as encouraging as you can.

What I want [to know] is how you managed to tackle this problem and the solution if you have one. [pause] OK?

[cuing the first group] Robert.

Robert: To start with, we measured the diameter of each lane, we multiplied it by 3.14 to find the circumference, and we measured the straight which was 9.6 cm and we added it to each circumference to find out the length of each lane. We found out that lane 2 was 1.57 cm longer than lane 1, lane 3 was 1.57 cm longer than lane 2, and so on. So to make it fairer for the runners, we had to set a stagger of 1.57 cm longer for each lane.

Peter: Anyone like to ask any questions? Is that not clear to anyone?

Shital: Did you all get the same answers for the circumference and then the straight? Did you get the same measurements?

Robert: It was 1.57 cm each time.

Steve: We got the same measurement for the straights.

Peter: Is that what you mean?

Shital: Yes.

George: How did you measure the circumference of the track?

Robert: We took the diameter of the lanes and multiplied it by 3.14. I imagined it was a circle.

[silence in class]

Lucy: First, we measured all, each lane with a piece of string. First we measured the outside lane; we found that that was 50 cm. Then the third lane was 49 cm and so on. Each time it went down a centimetre. So we decided that we should stagger them out a centimetre behind each other.

Peter: So on your diagram they are a centimetre behind.

Lucy: Yes.

Peter: Can I just have a look at that? Right, have a look at this then [takes diagram and shows class]. That was Lucy's, Anita's *and Marsha's solution for the problem.* Anything you notice? Any comments? Any questions you want to ask? [pause; cues] Robert.

Robert: What's the rectangle for?

[The rectangle is drawn around the track, touching in four places symmetrically.]

Lucy: We just wanted to see if we had a rectangle, how big the whole thing would be—if it was like that. It was 62 cm, but we found it was nothing to do with the track.

Peter: So you drew a rectangle around it to see how big the whole thing was . . .

Lucy: Yes.

Peter: . . . but you found that had nothing to do with the problem. Good.

Sarah: How did you work out the stagger in each lane?

Lucy: We measured it with the string, and each time we measured it it was 1 cm apart for each lane. If we took it away, no added it, to each lane a centimetre, they'd all be the same.

Peter: What is the real stagger? [The drawing is 1 cm to 8 m.]

Lucy: Eight metres, that's what we thought.

[Peter cues Lewis]

Lewis: Well, first we decided to draw a rectangle around the outside of the track, and we tried to measure this as accurately as we could. And then we drew in diagonals to find the centre; then we drew in horizontal and vertical lines to split the track up into four sectors, 100 m in each sector. And then we measured the inside lane up to the first sector and multiplied by four. We kept on finding it wouldn't tally up, it wouldn't go up to 400 m, and after a lot of discussion, we found out that we had drawn the rectangle so it was practically a parallelogram—it wasn't a rectangle at all. The sides were all wrong. So we had to get a new piece of paper and set it all out and draw it again; we've just started to measure the inside lane, and it has just started to tally up now. That's about as far as we got.

Peter: Is everyone clear about what they think Lewis is saying? Notice how close that group's answer is to this group's. It's been interesting to see the different approaches, bits of string, pi and so on, and you've all come to similar conclusions.

Gordon: What's the triangle got to do with it?

Lewis: Triangle? Do you mean diagonal? It helps you find the centre of the track.

[Peter cues Shital]

Shital: We all measured the straight on our own pieces of paper and we all got different answers for that, some were 8.5, some were 9, 9.6 and like that. Everything we tried, we got different answers from everyone and every time we double-checked it, we got other different answers. So we didn't really get anything except we got one measurement and that was around 12 m between the two positions. That was about it.

Peter: Alright, 12 seems to be at least the right sort of order. It doesn't appear to be way out, but didn't you get anything at all out of the activity? Anyone in the group can answer that one. Did you get anything at all about doing that activity? [Students look at one another.] No? Anyone else care to say something about what you can learn or what they might learn from that experience? [pause] I'll give you my story. My impression of it is that when I was talking to those students one thing they said to me before is how they didn't seem to be working well together. They were all doing it on their own, all doing their own individual problem, and I've got the feeling that at the end of the day, or the end of that problem, they were trying to solve it themselves and were getting frustrated they were all getting different answers. I think we can see that each group has got slightly different answers from other groups, but it's the way you tackled the problem that's been nice and interesting and similar. . . .

Questioning the Lessons

1. *What sort of language (style, structure, organization, register, and so on) does the student reporter use?*

   How can the teacher help resolve the tension between the fact that increased preparation time can improve performance but can also emphasize the public-speaking aspect of the event, with its attendant social pressures on performance? Spontaneous requests, however, without much time for reflection or organization, though offering a potentially less formal context for speaking, can merely result in recollections of the last thing students were doing.

   Steve's class was still using informal, context-specific spoken language. This may have been due to the nature of what they were trying to say but also may involve the OHP as a means of amplifying a local conversational setting (where such informality might be more appropriate) into a global one.

2. *How can the tension between wanting the students to say themselves what they have done and wanting to use what they say to make general remarks about how to undertake investigative work be handled?*

   This tension can be particularly strongly felt in cases where teachers have seen something that they value as a higher-order process (specializing systematically, developing notation, coping with getting stuck or whatever) while circulating around the small groups. The process may not have been a salient one for students (unless the teacher made a big point of it at the time), so they probably have little idea either of what to emphasize or why this particular incident is being focused on. If the teacher has made the point to them, why are they being asked to repeat it?

   Teachers are reluctant to say for the students what one of the students has done, instead of prompting with something like "Why don't you say it, because you did it?" A truthful response from the student might be "Because it is *your* anecdote. I don't know what significance it has for you." Students do not have the teacher's reasons for highlighting particular parts of what they have done. It is far better to say something like "I want the rest of the class to know about something related to what you did. At the moment, I am the only one who knows what that is. What I saw you doing was . . . and it struck me because. . . ."

Peter Gates made a particular point of this in his last interchange where he told the group what *he* thought they might have gotten from the activity. Steve Wilson had a number of examples where he made general points out of the particular report-backs he had heard, and he, on occasion, asked the general audience whether they had experienced something similar to what had just been reported.

3. *How can students develop the skills of selection of and reflection on what to report? How can they acquire a sense of audience?*

Obviously, this is contingent on the perceived audience and purpose behind carrying out the reporting back. Who knows what might be worth telling about? The teacher has no control over what comes out. He or she can only work on it after it has emerged from the student, though it is likely that what they (student and teacher) then do with the response will influence later reporters.

One important skill is the ability of students to disembed their discourse from the knowledge of the group who saw the work developed, so that someone who was not there can follow what is being described. The tendency is for the reporter to assume that everyone will know what he or she is talking about.

One difference between Peter's and Steve's classes is the students' willingness to ask each other questions after a report-back. Steve, by rebroadcasting, has already done it for them.

4. *To whom is the reporter talking?*

Students often address themselves to the teacher, the person who, besides their own group, probably knows most about what they have done—and the students know this. How might the teacher deflect the reporter's attention to the rest of the class?

In Barbara Jaworski's account (1985) of a poster lesson she comments:

> Most pupils, on taking the hot seat, started off addressing their comments to Irene [the teacher], stopping now and then in what they were saying to allow her to comment, or to solicit her comments. In some cases she replied to them or prompted them directly, which then encouraged a two-way exchange with the rest of the class as audience. In order to get the rest of the class to participate she then had to overtly invite comments from them. What in fact started to encourage more general discussion was Irene's deflecting of the invitation to comment to others in the group.

If the teacher reinterprets what the student says for the rest of the class (possibly by playing a role, which in television interviews is known as "audience's friend"), what effects might this have on the reporter? By playing "audience's friend," the teacher can ease the strain on reporters, by taking the focus off them, possibly by reinterpreting or expanding for the audience and then moving into more general questioning of the class. This role was particularly apparent in the pattern of interchange in Steve Wilson's class. But it also may result in the teacher being looked to as broadcaster and interpreter of the person reporting back and thus acting as an intermediary between reporter and audience which may get in the way. After all, if reporting back were an effective technique in and of itself, there would be little need for the teacher to intervene—the reporting back would do its own work.

If the teacher has adopted this role, then he or she may be asking questions on behalf of the audience or more directly explaining the student's words to the others. Whether the teacher says "tell us" or "tell them" may be an important difference in cuing reporters as to whom they should be facing and speaking.

Where is the teacher standing and where is the control? If there is a silence, whose responsibility is it to fill it? Where is the audience's attention and who are they asking questions of? If it is predominantly a conversation between the reporter and the teacher, what is the intended role for the students who are being invited to listen?

What are the students supposed to be doing while the report-back is being given? What would you (as their teacher) like to be happening in their heads and what could you do to help bring it about? What techniques do you employ for deflecting or involving the audience? Are students too concerned about the fact that their turn is coming up (and are perhaps rehearsing what they are going to say) to attend to what the current reporter is saying? How can they be encouraged to be active listeners?

5. *What justifications does the teacher explicitly offer the class for reporting back on their work to the rest of the class?*

In the two excerpts, the teachers were trying to say why they value reporting back. In particular, look back to what Steve said at the beginning to set up the reporting back. How does what the students did relate to this? How does what he said differ from the guidance Peter gave to his older students?

What covert justifications does the teacher have? What are various student views about why they have been asked to engage in this activity? Do they mirror what the teacher has said?

There can be some difficulty conveying to students what it is they are being asked to do: If the activity is too vague, the students do not know why they are being asked to carry it out and can flounder; if too precise, students will tend to do exactly that, thereby constraining what might happen. Overspecification also retains the teacher's control and initiative rather than handing this over (at least in part) to the students.

This is a familiar tension which lies at the heart of teaching. One general formulation [referred to there as the didactic tension] runs as follows (Mason 1988, 33):

> The *more* explicit I am about the behaviour I wish my pupils to display, the more likely it is that they will display that behaviour without recourse to the understanding which the behaviour is meant to indicate; that is, the more they will take the *form* for the substance.
>
> The less explicit I am about my aims and expectations about the behaviour I wish my pupils to display, the less likely they are to notice what is (or might be) going on, the less likely they are to see the point, to encounter what was intended, or to realise what it was all about.

Conclusion

Madeleine Goutard (1968) claimed that one role of the teacher when working with students on their own mathematical activity was to "help the students follow their own intentions through, strengthen their own intuitions and carry their own creations to a higher level." Reporting back can assist in these aims, but considerable care needs to be taken to ensure that this is achieved. Not least, this requires students to take on the activity of reporting back as one *they* choose to undertake, rather than merely another task that they are doing "for the teacher."

Reporting back can place some quite sophisticated linguistic demands on students in terms of communicative competence—that is, knowing how to use language to communicate in certain circumstances. Here it includes how to choose what to say, taking into account what you know and what you believe your audience knows. Educational linguist Michael Stubbs (1980, 115) claims, "A general principle in teaching any kind of communicative competence, spoken or written, is that the speaking, listening, writing or reading should have some genuine communicative purpose." Yet this is at odds with my earlier comment about the classroom being an avowedly unnatural, artificial setting, being precisely the place where all the necessary learning that does not take place naturally and spontaneously has to be confronted. Nonetheless, provided the pupils are willing to take on whatever constraints the activities entail, then there is the chance of their learning. It is still an open question as to whether or not working directly at increasing the formality of spoken mathematical language, prior to working on improving written control of language, will assist written fluency. I feel it is a worthwhile prospect that deserves further exploration.

Students learning mathematics are acquiring communicative competence in mathematical language, and classroom activities can be examined from this perspective to assess the opportunities they offer for learning. Teachers cannot make students learn—at best, teachers can provide well-thought-out situations that allow students to engage in mathematical ideas and develop skills that use spoken and written language to that end.

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