Chapter 1
Writing to Communicate Mathematics

Yvonne Pothier

The strength of this chapter is in the richness and power of the example episodes Yvonne Pothier presents as concrete examples of the value of writing in mathematics learning.

She has grouped the writing examples under six categories:

1. *Writing about a concept's meaning*
2. *Describing a process*
3. *Responding to a question*
4. *Reporting on an activity*
5. *Writing problems*
6. *Writing solutions to problems*

Dr. Pothier concludes her chapter by stating that many benefits can be derived from these writing activities. I would go further and suggest that one of the hidden benefits of writing to learn mathematics is the potential of the activity for both formative and summative evaluation, particularly when a portfolio is used. The writing illustrated here provides information expressing children's understanding of mathematics and would provide opportunities for others (such as teachers or parents) to gain insight into children's grasp of mathematics.

Introduction

Growing attention is being given to communicating mathematics in elementary classrooms (Baker and Baker 1990; Edmunds and Stoessiger 1990; NCTM 1989; Wilde 1991). This is a marked deviation from tradition and, increasingly, elementary teachers perceive the benefits to be gained.

Communicating mathematics can take on many forms. In an elementary classroom, it can happen in children talking while engaged in a task; reporting to others; using illustrations, diagrams and symbols to communicate their messages; writing about what they are doing; and discussing and reacting to other children's communications (Baker and Baker 1990).

This chapter highlights writing as an effective mode of communicating mathematics for elementary school children. The writing can be preceded or followed by class dialogue or discussion and can be accompanied by diagrams and symbolic representations of the ideas being communicated. Students can be asked to write reports on explorations, solution processes, explanations and problems or to respond to mathematical questions. During the writing activity, it is important to allow students the freedom to decide how best to place on paper whatever they want to present.

The Value of Communicating Mathematics

Communicating forces one to consider meaning. Students, knowing that they are expected to communicate orally or in writing on the mathematical activity, will engage wholeheartedly in the task to derive as much meaning as possible. A student may self-question: What is the meaning I want to convey? How can I best communicate the meaning that emerges from the task for me? This attention to meaning can only enhance the child's understanding of mathematics.

Writing about Mathematics

A teacher who wishes to have students write about mathematics may have to

change his or her instruction methods. Following a typical textbook page by page will not generate an animated class discussion nor move a child to write about the activity. Varied mathematical activities that have proven motivational for writing are presented below, together with samples of students' written communications.

Writing about a Concept's Meaning

Mathematical explorations using concrete materials provide excellent contexts for students to write about the mathematics they are doing. For example, after they have worked with physical materials to develop a concept, children could be asked to write about the concept's meaning for them.

The following children's writing samples were composed after several experiences partitioning shapes and discrete sets in fractional parts. Two example partitioning tasks are presented first:

Sharing a Cake

Each child was given a large colored square and narrow strips of white bristol board. The children were directed to pretend they had to cut the "cake" in fair shares (equal parts) to serve at a birthday party. The narrow strips were to be used to show how they would cut the cake to get fair shares. The children experimented with the number of equal parts they were able to attain on the square shape. Their partitionings were recorded on paper.

Subsequent tasks included partitioning a rectangle, a parallelogram, a circle and other regular shapes.

Sharing a Dozen

The children were each given an egg carton and a dozen small objects to serve as "eggs." They were to remove a number of eggs from the carton and to tell what part of the dozen had been "eaten." The challenge question was to try to find different ways to tell how many eggs had been eaten. (For example, if four eggs had been eaten, one could say that four-twelfths or one-third had been eaten.)

A Grade 3 class was invited to write a story about the fractions one-half or one-third; some students included diagrams or drawings. The following are some of their written responses:

> My mom made 6 cookies. My friend and I ate 1 and a half cookie each so three cookies were gone so we had a half left.
>
> —*Vicki*

> My cat had 9 kittens. I gave away 6 of the little kittens. Now I have 3 little kittens or ONE THIRD of the kittens.
>
> —*Roger*

> One day I went to the "It Store" and I bought 14 scratch 'n sniff sticers. The next day I went to my friends house and we traded sticers. We traded and I gave her 7 sticers or *half* of the sticers. [The 14 stickers were drawn with 7 crossed out.]
>
> P.S. half means you have two equal parts and you take one away. Then you have half.
>
> —*Jack*

> My mommy got a pizza for me and my brother. My mommy cut it in eight pices. I had 2 pices and my brother 2 pices of pizza. All together we eat half of the pizza.
>
> —*Barb*

> One day I went to the candy store. I bought a chocolate bar while I was there. After I bought the chocolate bar I cut it in thirds—in other words I cut it into three equal parts. Then I ate one third of the chocolate bar.
>
> —*Don*

An important aspect of the fraction concept is the idea of equality. Yet, children often use fractional names to describe uneven parts of a whole. Through children's writings, teachers learn whether or not students are thinking about equality when determining fractional parts. For instance, in the responses above, Don is careful to state that he cut the bar "into three equal parts," whereas Barb simply states that the pizza was cut in "eight pices." A teacher

might ask Barb about the pieces of pizza: Was there anything special about the eight pieces of pizza? The idea of equality is generally not problematic when partitioning discrete sets.

Describing a Process

Writing about a just-completed process causes one to reflect on the process. This reflection can provoke insights into the process, thus yielding more meaning from the activity.

A Grade 2 class had been doing two-digit number work, such as grouping by tens to model numbers, counting objects and writing number sequences. One day, each student was given an empty egg carton and a bag of small objects (buttons, craft sticks, short lengths of plastic straws or plastic discs). Each bag contained 350–500 objects. The students were directed to find out how many objects were in the bags by using only 10 of the eggcups in their cartons to group the objects. The egg cartons could be emptied, and the process of making groups continued. The students chose when to write about the activity; that is, they could write as they were counting or after they had found the total. Examples of students' writings follow:

> I put 10 groups of 10 in 10 hols and that makes 100. I put 10 groups of 10 in 10 hols and that makes 200. I put 4 groups of 10 and 3 laft over and that makes 342 and I'm going to cep on going.
>
> —*Shane*

> I am counting blocks. Now I have 100. I got 100 from grouping tens. Now I have 200. I got 200 from grouping more tens. I now have 300. I got 300 from still grouping tens. I have ended at 350. I have grouped all these numbers and got 350.
>
> —*Jenny*

> I am counting blocks. I have ten sets of ten so I have 100. I have another ten sets of ten so I have 200. I have 240 blocks. I have another ten sets of ten so I have 300 blocks. I have another ten sets of ten so I have 400. I have 460!
>
> —*Michael*

> I put 10 sticks in each whole. I have 100. There are 10 wholes. I put 10 more sticks in each whole. Theres 10 wholes. I have 200. I put 10 sticks in 10 wholes now I have 300. I put 10 more sticks in 10 more wholes and found four more. All together I have 404 sticks.
>
> —*Kate*

From this writing activity, teachers learn about the children's understanding of *hundred*. These four children seem to know the meaning of *hundred*, but they differ in their ability to define it. Shane and Michael provide rather mature definitions of *hundred* as "ten groups of ten" and "ten sets of ten." Jenny writes about grouping tens without stating how many groups of ten she counted to make a hundred. Kate's account is the most immature, as Kate seems unable to think of a group as an entity but focuses on the parts (single sticks and single holes) rather than the whole (10 sticks make one ten; 10 tens make one hundred). Children like Kate would probably benefit from a session at which they read and discussed their written accounts of the counting process.

Responding to a Question

Teachers can have students respond in writing to questions about the mathematics they are learning. Important information about students' thinking can be taken from such responses.

During a unit on two-digit numbers, a Grade 2 class was asked to respond to the question, "What do you know about 50?" Sample responses were

> The number is half way to one hundred. There are no ones. 50 is an even number.
>
> —*Anne*

> 50 is halfway to one hundred. It is made out of 5 tens. 25 + 25 = 50.
>
> —*Jane*

> It's 50 + 50 − 50 + 50 − 50 = 50.
>
> —*Derek*

Another question posed to the class was "35 + 35 + 35 = ___? Is the answer more

than 100? Write about how you found out." Three sample responses are

> First I remembered that 3 3's equaled 9. So I knew that three 30's equaled 90, so I added 15, and it left me with 105!
>
> —*Ann*

> I knew that 25 + 25 + 25 = 75. I also knew that 35 is 10 more than 25 so 35 + 35 + 35 = 105.
>
> —*Dave*

> I found out because 30 + 30 is 60 and 5 + 5 is 10 so another 10 is 70 so another 30 is 100 and another 5 is 105.
>
> —*Lorne*

A Grade 6 class was asked to respond to the question "½ + ⅛ = 1. Is this correct? How do you know?" and to justify their answer in writing. Three student responses are presented below:

> The answer is wrong. You have half so you have to add another half to get 1. ⅛ is not equivalent to one half so you'll end up with less than 1.
>
> —*Andy*

> No, because one half plus one half equals one whole. One eighth does not equal one half.
>
> —*Kim*

> ½ + ⅛ are not correct because you have to make the bottom the same so if you x the 2 by 8 and the 8 by 2 you'd get 16 so it would be 2/16.
>
> —*Joshua*

Another question presented to the same class was "Everybody gets three-fourths of a pizza. How might this happen?" Sample student responses are

> This can happen by having four people and 3 pizzas. So each person gets 3 fourths of the pizzas. Or you can dubble it. I like the kind of problems that take major thinking!
>
> —*Carl*

> There is four people and 3 pizzas. 3 of the people receive ¾ of one pizza. The remaining person gets ¼ from each pizza.
>
> —*John*

Children's written responses to mathematical questions can be revealing for the mathematical terminology used and the conceptual understanding demonstrated. Teachers could commend students for using proper terminology (even number, equivalent, equaled) and could encourage them to use mathematical terms in their writing.

The responses to the question about the sum of 35 + 35 + 35 reveal some thinking strategies in mental computation. Ann, Dave and Lorne used different but appropriate strategies. Students also reveal their conceptual understanding of fractions in written responses to simple open-ended questions. In the responses to the second question, all students but Joshua demonstrate an understanding of the value of the fractions under consideration; Joshua's thinking appears to be rule-governed.

Reporting on an Activity

Given open-ended activities, children can be asked to write about what they did and what they discovered. The activities can be structured and narrow in focus. For example, a class could be provided with a collection of small objects and asked to make different-sized groups and to write about what they did. The open-endedness of the activity allows students to decide on a group size that interests them. The following examples were written by Grade 1 students:

> I have 35 buttons. I have made seventine groups of two and 1 left over. I have made 7 groups of five and 0 left over. I have made three groups of ten and five left over.
>
> —*Carol*

> I have 32 buttons. I made 6 groups of 5 and had 2 left over. I made 4 groups of 8 and had 0 left. I made 32 groups of one and had 0 left.
>
> —*Tanya*

> I have 28 buttons. I made 5 groups of 5 and had 3 left. I made 7 groups of 4 and had 0 left. I made 8 groups of 0 and had 28 left.
>
> —*Joshua*

The accounts of groupings made by Tanya and Joshua are particularly interesting. The teacher was surprised to observe that a student who knew how many counters she had would choose to make groups of one and, as the event occurred, would *re-count* the groups of one before she wrote about her findings. Joshua's behavior is interesting from the point of view of his exploration of zero. He had been given eight small plates to use to make groups. Having chosen zero as the group size, Joshua spread the plates out and then counted his buttons to find out how many were left. Written records of children's work allow teachers to observe, in this case, what numbers children select to make groups. An interesting observation is that 10 was not a frequent number selected.

Writing Problems

In the past, if writing was ever used in math classes, it was usually done to have students write word problems. This exercise can be valuable for distinguishing between addition and subtraction or multiplication and division situations. Today, teachers would more likely move beyond this activity to encourage students to compose different types of problem situations. For example, students could be asked to write comparison subtraction problems, measurement division problems, cross-product multiplication problems or two-step problems.

Writing story problems can be done as early as Grade 1 as the following examples demonstrate:

Wos a pon a tim I wat to the bith I so three ros on the sad and tan I sa five and all totr tr was 8.
(Once upon a time I went to the beach. I saw three rocks on the sand and then I saw five and altogether there were eight.)

—*Ann*, Grade 1

There was 4 doks in the sea and 4 more kam alag. haw mane do I hav all togithr? I got 8.
(There were four ducks in the sea and four more came along. How many do I have altogether? I have eight.)

—*Tom*, Grade 1

There was 23 people ridding some whale. Then 63 more people came to ride some whales. Then 15 more people came to ride some whales. How many people came to ride some whales?

```
 23
 63
+15
---
101
```

There are 101 people ridding whales.

—*Rick*, Grade 2

The zoo ceper had 20 sels. he nedit 17 more because he codi't put on shwos. haw many sels dose he have now he has 37.

—*Meg*, Grade 2

Jennifer is going to have a birthday party 4 kids were there 6 kids were missing. How many are all together?

```
  4
add
  6
---
 10
```

—*Clara*, Grade 2

In reading problems composed by students, teachers can note the types of problem written, which operation is most frequently used and whether or not the questions posed suit the information provided.

Writing Solutions to Problems

Problem solving has become an important mathematical activity in elementary classrooms. This activity has been expanded in scope and substance to include multistep and nonroutine problems. Students are encouraged to discuss strategies and solutions, ask questions and reflect on their solution processes (NCTM 1989). The aim is to develop confidence in their ability to use mathematics.

Given interesting problems to solve, children enjoy devising solution processes and writing about them. They come up with ingenious ways of setting up tables, drawing diagrams and developing strategies for problem solving (Pothier and Sawada 1990).

If students are expected to develop a solution process in detail, they must have sufficient time to do so. The feeling of being rushed may discourage a sincere search for a solution or careful recording of the solution process used.

Examples of solutions to three types of problems are presented below:

Problem 1

The school library has acquired some new books. The four books about computers are one-third of the nonfiction books. How many nonfiction books were bought?

Solutions:

IIII	IIII	IIII	= 12
⅓	⅓	⅓	

The library bought 12 nonfiction books. I know this because, if 4 books are ⅓ then ⅔ is 12 books.

—*Jane*, Grade 6

12 books

□□□□	□□□□	□□□□	
⅓	⅓	⅓	= ⅔

I added ⅓ + ⅓ + ⅓ which equals 3/3. Then I added the amounts of the books which equals 12 books. So 12 non-fiction books were bought.

—*Clara*, Grade 6

□ = 1 book

□□□□	□□□□	□□□□
⅓	⅓	⅓

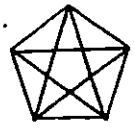
12 non-fiction books were bought. If 4 computer books are one-third of all the non-fiction books I need two more thirds. A third = 4 so 3 x 4 = 12.

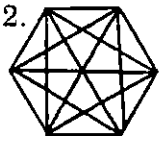
—*Mark*, Grade 6

Problem 2

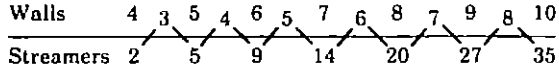
Pretend you have been hired to decorate a room. You need to attach colored streamers across the ceiling connecting opposite corners of the room. How many streamers would be needed for a 10-sided room?

Solution:

1. 
   - 5 streamers
   - walls form a pentagon
   - we made a pentagon (used strips of cloth to act it out)

2. 
   - 9 streamers
   - walls form a hexagon
   - we made a hexagon with the streamers

Pattern:



I feel proud of myself for having discovered the pattern.

—*Sonia*, Grade 6

Problem 3

Jagan has three pairs of pants, four sweaters and two pairs of shoes. From how many different pant/sweater/shoes combinations can Jagan choose what to wear to school on Monday?

Solution:

Pants	Sweaters	Shoes	No. of Ways
1	1	1	1
1	2	1	2
1	3	1	3
1	4	1	4
1	1	2	5
1	2	2	6
1	3	2	7
1	4	2	8

Think: Since there were 3 pants and each had 8 different ways you get 24 different ways to dress.

—*Carl*, Grade 5

When students are solving problems, teachers should encourage them not only to answer the questions asked but also to record their solution processes (including abandoned procedures) and to write about their thinking or feelings as they worked through the problems. This information can assist teachers in assessing students' problem-solving abilities.

Summary

Many benefits can be derived from the written activities described here. The inherent reflective activity in writing can be productive to mathematics learning. Students can come to see relationships or connections between different modes of representation (concrete materials, diagrams, pictures) and mathematical ideas. Formal mathematical terms will become more meaningful to students if used in expressing their own mathematical thinking.

Having students write in mathematics class can also aid teachers. Misconceptions will be revealed, as will insightful thinking. Such information can help teachers make curricular decisions for classes or individual students.

A few years ago, I suggested to a Grade 4 teacher that she have her students write about the mathematics they were doing. The teacher, who enjoyed planning different writing activities for her class as part of the language arts program, was surprised at my suggestion and responded, "It never dawned on me to have my students write during mathematics class."

In the past, writing has been too infrequently used in mathematics classes. Let us look toward the day when students and teachers view writing as an integral part of doing and learning mathematics.

References

Baker, A., and J. Baker. *Mathematics in Process.* Portsmouth, N.H.: Heinemann, 1990.

Edmunds, J., and R. Stoessiger. "Using Natural Learning Processes in Mathematics." *Mathematics in School* 19 (May 1990): 30–33.

NCTM (National Council of Teachers of Mathematics). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

Pothier, Y., and D. Sawada. "Students Value Time and a Patient Teacher." *Mathematics in School* 19, no. 13 (1990): 38–39.

Wilde, S. "Learning to Write About Mathematics." *Arithmetic Teacher* 38, no. 6 (February 1991): 38–43.

Further Reading

Burns, M. "Beyond 'The Right Answer'. . . Helping Your Students Make Sense Out of Math." *Learning* 88 (January 1988): 31–36.

Katterns, B., and C. Carr. "Talking with Children about Multiplication." *Arithmetic Teacher* 33 (April 1986): 18–21.