MATH Monograph No. 10

Editor: Daiyo Sawada

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Communication Communication Communication Communication in the MATHEMATICS CLASSROOM

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David Pimm is a senior lecturer in mathematics education at the Open University in Great Britain. He prepares written, video and computer-based materials for teachers at all levels. His research interests are in the interrelations between language and mathematics and analyzing classroom discourse in particular, and he wrote *Speaking Mathematically: Communication in Mathematics Classrooms* (London: Routledge, 1987) on the topic. During the 1991–92 academic year, David was on leave at Simon Fraser University.

Yvonne Pothier is associate professor of mathematics education at Mount Saint Vincent University in Halifax. A former elementary and secondary school teacher, Yvonne teaches professional development courses for teachers, including graduate education. As part of her research, she has conducted clinical interviews with several hundred elementary school children and, in recent years, has taught Grades 1–6 in short-term collaborative teaching projects.

Daiyo Sawada is a professor in the University of Alberta's Elementary Education department where, among other things, he teaches ED EL 396 "Communication Through Mathematics." He is a member of the current MCATA executive and was program director for the 1991 MCATA annual conference in Edmonton. Daiyo is also principal author for *MathWorlds*, a new mathematics program for Alberta's elementary schools being developed by Reidmore Books. His most recent interest has been in a literary approach to mathematics.

Laurie Walker has taught and studied English language arts for many years and is an education professor and associate vice-president (academic) at the University of Lethbridge. Language-Across-the-Curriculum took him to the language of mathematics as an issue in children's successful acquisition of numeracy.

Foreword

This monograph has been two years in the making. When it was first suggested in Spring 1990, the NCTM publication *Curriculum and Evaluation Standards for School Mathematics* had just been released. In that document, "Mathematics as Communication" was listed as the second of four standards to serve from K–12. At the "Thinker's Conference" in Red Deer, the MCATA executive approved a monograph focusing on this standard.

The monograph brings together two approaches to communication in the classroom which can be identified in the following questions:

1. What can I *do* in my classroom?
2. How can I *understand* what I'm doing?

These two questions are drawn together in a final section called:

3. What kind of teaching can I *practise*?

If enhancing mathematics learning through communication sounds promising, I am sure you will find many excellent suggestions from Yvonne Pothier, Barbara J. Morrison, Susan Burgoyne and Marilyn Burgoyne in the first three chapters.

If you also hope to further your own understanding of the role of communication in the learning of mathematics, then I am sure the works of Laurie Walker and David Pimm in chapters 5 and 6 will trigger many thought-provoking ideas. I have included my thoughts and reflections in chapters 4 and 7.

I have prefaced each chapter with a short overview/commentary to provide further insight and/or background information. The overviews are set apart in italics.

Happy reading.

Daiyo Sawada

Part I

What Can I Do in My Classroom?

Chapter 1
Writing to Communicate Mathematics

Yvonne Pothier

The strength of this chapter is in the richness and power of the example episodes Yvonne Pothier presents as concrete examples of the value of writing in mathematics learning.

She has grouped the writing examples under six categories:

1. *Writing about a concept's meaning*
2. *Describing a process*
3. *Responding to a question*
4. *Reporting on an activity*
5. *Writing problems*
6. *Writing solutions to problems*

Dr. Pothier concludes her chapter by stating that many benefits can be derived from these writing activities. I would go further and suggest that one of the hidden benefits of writing to learn mathematics is the potential of the activity for both formative and summative evaluation, particularly when a portfolio is used. The writing illustrated here provides information expressing children's understanding of mathematics and would provide opportunities for others (such as teachers or parents) to gain insight into children's grasp of mathematics.

Introduction

Growing attention is being given to communicating mathematics in elementary classrooms (Baker and Baker 1990; Edmunds and Stoessiger 1990; NCTM 1989; Wilde 1991). This is a marked deviation from tradition and, increasingly, elementary teachers perceive the benefits to be gained.

Communicating mathematics can take on many forms. In an elementary classroom, it can happen in children talking while engaged in a task; reporting to others; using illustrations, diagrams and symbols to communicate their messages; writing about what they are doing; and discussing and reacting to other children's communications (Baker and Baker 1990).

This chapter highlights writing as an effective mode of communicating mathematics for elementary school children. The writing can be preceded or followed by class dialogue or discussion and can be accompanied by diagrams and symbolic representations of the ideas being communicated. Students can be asked to write reports on explorations, solution processes, explanations and problems or to respond to mathematical questions. During the writing activity, it is important to allow students the freedom to decide how best to place on paper whatever they want to present.

The Value of Communicating Mathematics

Communicating forces one to consider meaning. Students, knowing that they are expected to communicate orally or in writing on the mathematical activity, will engage wholeheartedly in the task to derive as much meaning as possible. A student may self-question: What is the meaning I want to convey? How can I best communicate the meaning that emerges from the task for me? This attention to meaning can only enhance the child's understanding of mathematics.

Writing about Mathematics

A teacher who wishes to have students write about mathematics may have to change his or her instruction methods. Following a typical textbook page by page will not generate an animated class discussion nor move a child to write about the activity. Varied mathematical activities that have proven motivational for writing are presented below, together with samples of students' written communications.

Writing about a Concept's Meaning

Mathematical explorations using concrete materials provide excellent contexts for students to write about the mathematics they are doing. For example, after they have worked with physical materials to develop a concept, children could be asked to write about the concept's meaning for them.

The following children's writing samples were composed after several experiences partitioning shapes and discrete sets in fractional parts. Two example partitioning tasks are presented first:

Sharing a Cake

Each child was given a large colored square and narrow strips of white bristol board. The children were directed to pretend they had to cut the "cake" in fair shares (equal parts) to serve at a birthday party. The narrow strips were to be used to show how they would cut the cake to get fair shares. The children experimented with the number of equal parts they were able to attain on the square shape. Their partitionings were recorded on paper.

Subsequent tasks included partitioning a rectangle, a parallelogram, a circle and other regular shapes.

Sharing a Dozen

The children were each given an egg carton and a dozen small objects to serve as "eggs." They were to remove a number of eggs from the carton and to tell what part of the dozen had been "eaten." The challenge question was to try to find different ways to tell how many eggs had been eaten. (For example, if four eggs had been eaten, one could say that four-twelfths or one-third had been eaten.)

A Grade 3 class was invited to write a story about the fractions one-half or one-third; some students included diagrams or drawings. The following are some of their written responses:

My mom made 6 cookies. My friend and I ate 1 and a half cookie each so three cookies were gone so we had a half left.

—Vicki

My cat had 9 kittens. I gave away 6 of the little kittens. Now I have 3 little kittens or ONE THIRD of the kittens.

—Roger

One day I went to the "It Store" and I bought 14 scratch 'n sniff sticers. The next day I went to my friends house and we traded sticers. We traded and I gave her 7 sticers or *half* of the sticers. [The 14 stickers were drawn with 7 crossed out.]

P.S. half means you have two equal parts and you take one away. Then you have half.

—Jack

My mommy got a pizza for me and my brother. My mommy cut it in eight pices. I had 2 pices and my brother 2 pices of pizza. All together we eat half of the pizza.

—Barb

One day I went to the candy store. I bought a chocolate bar while I was there. After I bought the chocolate bar I cut it in thirds—in other words I cut it into three equal parts. Then I ate one third of the chocolate bar.

—Don

An important aspect of the fraction concept is the idea of equality. Yet, children often use fractional names to describe uneven parts of a whole. Through children's writings, teachers learn whether or not students are thinking about equality when determining fractional parts. For instance, in the responses above, Don is careful to state that he cut the bar "into three equal parts," whereas Barb simply states that the pizza was cut in "eight pices." A teacher might ask Barb about the pieces of pizza: Was there anything special about the eight pieces of pizza? The idea of equality is generally not problematic when partitioning discrete sets.

Describing a Process

Writing about a just-completed process causes one to reflect on the process. This reflection can provoke insights into the process, thus yielding more meaning from the activity.

A Grade 2 class had been doing two-digit number work, such as grouping by tens to model numbers, counting objects and writing number sequences. One day, each student was given an empty egg carton and a bag of small objects (buttons, craft sticks, short lengths of plastic straws or plastic discs). Each bag contained 350–500 objects. The students were directed to find out how many objects were in the bags by using only 10 of the eggcups in their cartons to group the objects. The egg cartons could be emptied, and the process of making groups continued. The students chose when to write about the activity; that is, they could write as they were counting or after they had found the total. Examples of students' writings follow:

> I put 10 groups of 10 in 10 hols and that makes 100. I put 10 groups of 10 in 10 hols and that makes 200. I put 4 groups of 10 and 3 laft over and that makes 342 and I'm going to cep on going.
>
> *—Shane*

> I am counting blocks. Now I have 100. I got 100 from grouping tens. Now I have 200. I got 200 from grouping more tens. I now have 300. I got 300 from still grouping tens. I have ended at 350. I have grouped all these numbers and got 350.
>
> *—Jenny*

> I am counting blocks. I have ten sets of ten so I have 100. I have another ten sets of ten so I have 200. I have 240 blocks. I have another ten sets of ten so I have 300 blocks. I have another ten sets of ten so I have 400. I have 460!
>
> *—Michael*

> I put 10 sticks in each whole. I have 100. There are 10 wholes. I put 10 more sticks in each whole. Theres 10 wholes. I have 200. I put 10 sticks in 10 wholes now I have 300. I put 10 more sticks in 10 more wholes and found four more. All together I have 404 sticks.
>
> *—Kate*

From this writing activity, teachers learn about the children's understanding of *hundred.* These four children seem to know the meaning of *hundred,* but they differ in their ability to define it. Shane and Michael provide rather mature definitions of *hundred* as "ten groups of ten" and "ten sets of ten." Jenny writes about grouping tens without stating how many groups of ten she counted to make a hundred. Kate's account is the most immature, as Kate seems unable to think of a group as an entity but focuses on the parts (single sticks and single holes) rather than the whole (10 sticks make one ten; 10 tens make one hundred). Children like Kate would probably benefit from a session at which they read and discussed their written accounts of the counting process.

Responding to a Question

Teachers can have students respond in writing to questions about the mathematics they are learning. Important information about students' thinking can be taken from such responses.

During a unit on two-digit numbers, a Grade 2 class was asked to respond to the question, "What do you know about 50?" Sample responses were

> The number is half way to one hundred. There are no ones. 50 is an even number.
>
> *—Anne*

> 50 is halfway to one hundred. It is made out of 5 tens. 25 + 25 = 50.
>
> *—Jane*

> It's 50 + 50 − 50 + 50 − 50 = 50.
>
> *—Derek*

Another question posed to the class was "35 + 35 + 35 = ___? Is the answer more than 100? Write about how you found out." Three sample responses are

> First I remembered that 3 3's equaled 9. So I knew that three 30's equaled 90, so I added 15, and it left me with 105!
>
> —*Ann*

> I knew that 25 + 25 + 25 = 75. I also knew that 35 is 10 more than 25 so 35 + 35 + 35 = 105.
>
> —*Dave*

> I found out because 30 + 30 is 60 and 5 + 5 is 10 so another 10 is 70 so another 30 is 100 and another 5 is 105.
>
> —*Lorne*

A Grade 6 class was asked to respond to the question "½ + ⅛ = 1. Is this correct? How do you know?" and to justify their answer in writing. Three student responses are presented below:

> The answer is wrong. You have half so you have to add another half to get 1. ⅛ is not equivalent to one half so you'll end up with less than 1.
>
> —*Andy*

> No, because one half plus one half equals one whole. One eighth does not equal one half.
>
> —*Kim*

> ½ + ⅛ are not correct because you have to make the bottom the same so if you x the 2 by 8 and the 8 by 2 you'd get 16 so it would be 2/16.
>
> —*Joshua*

Another question presented to the same class was "Everybody gets three-fourths of a pizza. How might this happen?" Sample student responses are

> This can happen by having four people and 3 pizzas. So each person gets 3 fourths of the pizzas. Or you can dubble it. I like the kind of problems that take major thinking!
>
> —*Carl*

> There is four people and 3 pizzas. 3 of the people receive ¾ of one pizza. The remaining person gets ¼ from each pizza.
>
> —*John*

Children's written responses to mathematical questions can be revealing for the mathematical terminology used and the conceptual understanding demonstrated. Teachers could commend students for using proper terminology (even number, equivalent, equaled) and could encourage them to use mathematical terms in their writing.

The responses to the question about the sum of 35 + 35 + 35 reveal some thinking strategies in mental computation. Ann, Dave and Lorne used different but appropriate strategies. Students also reveal their conceptual understanding of fractions in written responses to simple open-ended questions. In the responses to the second question, all students but Joshua demonstrate an understanding of the value of the fractions under consideration; Joshua's thinking appears to be rule-governed.

Reporting on an Activity

Given open-ended activities, children can be asked to write about what they did and what they discovered. The activities can be structured and narrow in focus. For example, a class could be provided with a collection of small objects and asked to make different-sized groups and to write about what they did. The open-endedness of the activity allows students to decide on a group size that interests them. The following examples were written by Grade 1 students:

> I have 35 buttons. I have made seventine groups of two and 1 left over. I have made 7 groups of five and 0 left over. I have made three groups of ten and five left over.
>
> —*Carol*

> I have 32 buttons. I made 6 groups of 5 and had 2 left over. I made 4 groups of 8 and had 0 left. I made 32 groups of one and had 0 left.
>
> —*Tanya*

> I have 28 buttons. I made 5 groups of 5 and had 3 left. I made 7 groups of 4 and had 0 left. I made 8 groups of 0 and had 28 left.
>
> —*Joshua*

The accounts of groupings made by Tanya and Joshua are particularly interesting. The teacher was surprised to observe that a student who knew how many counters she had would choose to make groups of one and, as the event occurred, would *re-count* the groups of one before she wrote about her findings. Joshua's behavior is interesting from the point of view of his exploration of zero. He had been given eight small plates to use to make groups. Having chosen zero as the group size, Joshua spread the plates out and then counted his buttons to find out how many were left. Written records of children's work allow teachers to observe, in this case, what numbers children select to make groups. An interesting observation is that 10 was not a frequent number selected.

Writing Problems

In the past, if writing was ever used in math classes, it was usually done to have students write word problems. This exercise can be valuable for distinguishing between addition and subtraction or multiplication and division situations. Today, teachers would more likely move beyond this activity to encourage students to compose different types of problem situations. For example, students could be asked to write comparison subtraction problems, measurement division problems, cross-product multiplication problems or two-step problems.

Writing story problems can be done as early as Grade 1 as the following examples demonstrate:

Wos a pon a tim I wat to the bith I so three ros on the sad and tan I sa five and all totr tr was 8.
(Once upon a time I went to the beach. I saw three rocks on the sand and then I saw five and altogether there were eight.)

—*Ann*, Grade 1

There was 4 doks in the sea and 4 more kam alag. haw mane do I hav all togithr? I got 8.
(There were four ducks in the sea and four more came along. How many do I have altogether? I have eight.)

—*Tom*, Grade 1

There was 23 people ridding some whale. Then 63 more people came to ride some whales. Then 15 more people came to ride some whales. How many people came to ride some whales?

$$\begin{array}{r} 23 \\ 63 \\ +15 \\ \hline 101 \end{array}$$

There are 101 people ridding whales.

—*Rick*, Grade 2

The zoo ceper had 20 sels. he nedit 17 more because he codi't put on shwos. haw many sels dose he have now he has 37.

—*Meg*, Grade 2

Jennifer is going to have a birthday party 4 kids were there 6 kids were missing. How many are all together?

$$\begin{array}{r} 4 \\ \text{add} \\ 6 \\ \hline 10 \end{array}$$

—*Clara*, Grade 2

In reading problems composed by students, teachers can note the types of problem written, which operation is most frequently used and whether or not the questions posed suit the information provided.

Writing Solutions to Problems

Problem solving has become an important mathematical activity in elementary classrooms. This activity has been expanded in scope and substance to include multistep and nonroutine problems. Students are encouraged to discuss strategies and solutions, ask questions and reflect on their solution processes (NCTM 1989). The aim is to develop confidence in their ability to use mathematics.

Given interesting problems to solve, children enjoy devising solution processes and writing about them. They come up with ingenious ways of setting up tables, drawing diagrams and developing strategies for problem solving (Pothier and Sawada 1990).

If students are expected to develop a solution process in detail, they must have sufficient time to do so. The feeling of being rushed may discourage a sincere search for a solution or careful recording of the solution process used.

Examples of solutions to three types of problems are presented below:

Problem 1

The school library has acquired some new books. The four books about computers are one-third of the nonfiction books. How many nonfiction books were bought?

Solutions:

|||| |||| |||| = 12

1/3 1/3 1/3

The library bought 12 nonfiction books. I know this because, if 4 books are 1/3 then 3/3 is 12 books.

—*Jane*, Grade 6

12 books

□□□□ □□□□ □□□□

1/3 1/3 1/3 = 3/3

I added 1/3 + 1/3 + 1/3 which equals 3/3. Then I added the amounts of the books which equals 12 books. So 12 non-fiction books were bought.

—*Clara*, Grade 6

□ = 1 book

□□□□ □□□□ □□□□

1/3 1/3 1/3

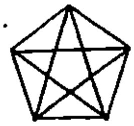
12 non-fiction books were bought. If 4 computer books are one-third of all the nonfiction books I need two more thirds. A third = 4 so 3 x 4 = 12.

—*Mark*, Grade 6

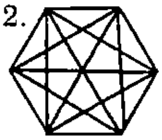
Problem 2

Pretend you have been hired to decorate a room. You need to attach colored streamers across the ceiling connecting opposite corners of the room. How many streamers would be needed for a 10-sided room?

Solution:

1. 

- 5 streamers
- walls form a pentagon
- we made a pentagon (used strips of cloth to act it out)

2. 

- 9 streamers
- walls form a hexagon
- we made a hexagon with the streamers

Pattern:

Walls	4	3	5	4	6	5	7	6	8	7	9	8	10
Streamers	2		5		9		14		20		27		35

I feel proud of myself for having discovered the pattern.

—*Sonia*, Grade 6

Problem 3

Jagan has three pairs of pants, four sweaters and two pairs of shoes. From how many different pant/sweater/shoes combinations can Jagan choose what to wear to school on Monday?

Solution:

Pants	Sweaters	Shoes	No. of Ways
1	1	1	1
1	2	1	2
1	3	1	3
1	4	1	4
1	1	2	5
1	2	2	6
1	3	2	7
1	4	2	8

Think: Since there were 3 pants and each had 8 different ways you get 24 different ways to dress.

—*Carl*, Grade 5

When students are solving problems, teachers should encourage them not only to answer the questions asked but also to record their solution processes (including abandoned procedures) and to write about their thinking or feelings as they worked through the problems. This information can assist teachers in assessing students' problem-solving abilities.

Summary

Many benefits can be derived from the written activities described here. The inherent reflective activity in writing can be productive to mathematics learning. Students can come to see relationships or connections between different modes of representation (concrete materials, diagrams, pictures) and mathematical ideas. Formal mathematical terms will become more meaningful to students if used in expressing their own mathematical thinking.

Having students write in mathematics class can also aid teachers. Misconceptions will be revealed, as will insightful thinking. Such information can help teachers make curricular decisions for classes or individual students.

A few years ago, I suggested to a Grade 4 teacher that she have her students write about the mathematics they were doing. The teacher, who enjoyed planning different writing activities for her class as part of the language arts program, was surprised at my suggestion and responded, "It never dawned on me to have my students write during mathematics class."

In the past, writing has been too infrequently used in mathematics classes. Let us look toward the day when students and teachers view writing as an integral part of doing and learning mathematics.

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Chapter 2
The Role of Communication in Mathematics

Barbara J. Morrison

This chapter focuses on the role of writing and classroom discourse in

1. *confirming student understanding,*
2. *creating a risk-free environment, and*
3. *improving instruction.*

Barbara J. Morrison provides a succinct summary of research supportive of such roles and outlines several areas in which writing can occur, including the following:

1. *Short writing assignments*
2. *Class logs*
3. *Journals*
4. *Writing activities using the text*
5. *Writing on tests*
6. *Chapter summaries*
7. *Definition cards*
8. *Writing about problem solving*
9. *Personal math histories*

A particular strength of this chapter is the way the author embeds her remarks in the literature about communication in mathematics. This not only provides references for further reading but also helps to integrate practice with theory.

Introduction

New knowledge about how students learn has had a tremendous impact on mathematics instruction and methodologies. The National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) presents standards to guide mathematics curriculum and pedagogy for the 1990s. The newly revised Alberta mathematics curriculum K–12 matches these directions in spirit and in philosophy. Alberta teachers have a unique opportunity to lead in improving instruction in mathematics teaching and learning. There is a tremendous research base and rationale for these changes and an abundance of supporting curricular documents developed by practising math specialists across North America.

Recent research on students as math learners supports students as active learners; learning as an active, dynamic and continuous process; and the learning theory that learners construct meaning by connecting new knowledge to concepts they already know. Research and practice that support these fundamental learning principles will guide our teaching into the next century. As we incorporate current research and methodologies from education and psychology, our teaching repertoires will include increased opportunities for collaboration, cooperative-learning techniques and full and active participation by all students.

The continued development and use of mathematical language and symbolism to communicate ideas and concepts form an integral part of our mandate to develop mathematical literacy/numeracy in our students. As with the nature of mathematics, communication in the later grades requires students to reason at more formal and symbolic levels. Alberta Education is increasing the emphasis on writing on the provincial Math 30 diploma examination, and "greater emphasis will be placed on the communication of students' understanding of mathematical concepts and procedures" (Alberta Education 1991).

This chapter explores issues and directions related to communication in mathematics. My comments focus on the writing aspects of communication and classroom discourse that are most useful in checking for student understanding, creating a risk-free environment and improving instruction in mathematics. The concrete teaching ideas and examples I use have been taken from many articles and conference presentations, as well as from practising teachers who shared some of the products of students' writing assignments.

Why Change Our Practice?

"The traditional view has been that students learn to write in English classes and to compute in mathematics classes and 'never the twain shall meet' " (Davison and Pearce 1988, 42). Communication as it generally occurs in a teacher-directed, lecture-oriented lesson is most often students' responses to questions beginning "Does everyone understand. . . ?" or "Are there any questions about. . . ?" Usually only a few students nod their heads or respond, and the individual responses are short answers to lower-order questions which can hardly serve as reliable checks for all students' understanding. Often formal feedback on the teaching and learning process comes from students' results on tests or quizzes. In many cases after two or three weeks of instruction, we learn that half the students answer half the questions wrong!

The contemporary role of writing in math classes, as related to teaching and learning mathematics, has changed from transcribing information and ideas onto paper, to a writing-to-learn process. Writing as a process provides new experiences for students. It uses expressive and creative writing tasks (such as journals and personal notebooks), where students write without concern for the quality of writing per se.

Advantages to Writing in Math Class

Richard Sagor (1991, 7) emphasizes the effect of this kind of reflection on achievement and describes a math writing project begun when a number of junior high teachers posed the question, "If writing is a window into thinking and if the act of writing helps improve comprehension, why not try it in our math classes?" The teachers used the following experimental design: They constructed and administered tests to divide four Grade 7 math classes into two groups according to achievement. Two of the classes were performing well and two performed below expectations. The lower-achieving classes were made the treatment group. Students were then given the opportunity to write about the math concepts they were learning the day before each math test. In every other respect, the groups were taught the same way. The teachers found that writing made a substantial difference in concept acquisition. The lower-achieving classes actually outperformed their classmates on every test.

Adele Le Gere (1991, 168) reports on research that found that

> students who write to learn actually do learn and retain concepts better than students who do not write as part of their course work.

A distinct advantage to frequent, short writing assignments is increased student participation. Every student has the opportunity for input as opposed to questions directed at an entire class which are answered by a select few. Support for writing in math classes is flooding the literature on learning mathematics (Nahrgang and Petersen 1986; Johnson 1983). Organizing thoughts on paper requires a higher-level mental thinking process. Through the process of synthesizing ideas, the students' understanding is clarified, the content is reinforced, and feedback is provided for the teacher on every student's level of understanding.

In-class writing or overnight homework assignments can also allow students to communicate in their own language and relate to their own real-world experiences. Writing assignments can force students to make connections from topics presented in class to what they already know and to organize and synthesize ideas so that concepts become their own.

A third advantage comes from using individual journals in math class. Journal writing can lead to definite improvements in students' ability to organize their responses. Teachers can use writing assignments as diagnostic tools to reveal areas of confusion about and understanding of math concepts (Nahrgang and Petersen 1986).

In addition, brief and frequent informal writing assignments can be ideal opportunities for teachers to reflect on instruction by answering such questions as the following:

1. Do students understand?
2. What level of understanding are they at?
3. Is the main point of the lesson coming through?
4. Where are students having difficulty?
5. Where do I need to modify my instruction?

What students write about is the next important issue. Students are encouraged to write about such things as the mathematical content they are exploring, problem-solving experiences and progress, their own learning process—their attitudes, thoughts and feelings as they solve math problems—and the nature of math and its applications.

Writing about Mathematical Content

Short Writing Assignments

Short writing assignments have students writing regularly for a few minutes as they come into class, while they are settling in, at the end of class or to provide variety in their homework assignments. The following is a list of ideas for regular, short writing assignments (10 minutes twice a week):

1. State the mark you received on your last test and comment on how you feel you did.
2. How does . . . relate to. . . ? (Make connections from what they are studying to other topics in math or to those across disciplines.)
3. Where in your life have you recently used percent? Bring in clippings from the newspaper to show uses of percent.
4. Explain a rule or procedure for . . . and create your own example to show that you understand this procedure or rule.
5. Describe the graph of . . . by writing a story about what you see.

Short writing assignments might include statements such as "reflect on and clarify," "formulate," "compare and contrast" and "create a counter-example" or directions like the following:

1. Word Banks: Use the words *slope, intercept, coordinate, abscissae, ordinate, point, tangent, parallel, perpendicular* and *y-intercept* in a true sense to help you write a paragraph about . . . and. . . .
2. Write a question containing one or more of these words: *right angle, perpendicular, side, hypotenuse, parallel.*
3. Write a true sentence using the words *rectangle* and *square* but without using the word *always.*
4. Write a paragraph about pyramids and cones.
5. Write a crib sheet for a friend who has fallen behind in. . . .
6. Use the words *intercepted, centre* and *angle* to help you write a definition of a central angle.

Other suggestions to use as starting points for teachers who would like to ease into incorporating a variety of writing activities related to mathematical content are class logs and journal activities.

Class Logs

Logs are written accounts, available during each class period (perhaps in a duotang), that provide information on previous math classes. They are detailed accounts that can be written by one student each day and include the date, homework, announcements and pertinent information about the class, topic, activity or investigation. As a classroom routine, the log provides a written summary of previous lessons for students who are absent, as well as the opportunity for students to write actively about the lesson. Students can be assigned on a rotational basis and graded for their entries. Some teachers ask students to include a personal comment about the lesson.

Journals

A diary-like math journal or file is useful for organizing students' regular writing assignments. It should be available when the students are working on math and allow for spontaneity as well as teacher-directed writing assignments. An expressive writing style, not to be confused with "free writing," should be encouraged. Teachers who use journals report that vague assignments produce vague responses. Teachers must consider the response they expect and clarify the assignment for students ahead of time. By presenting good prompts, teachers can force students to take a stand. Grading responses will depend on how well the students support their stand. Here are two examples of good prompts: "We learned two ways to graph quadratic functions before I used the graphing calculator. The easiest way for me to do it is . . . because. . . ." and "What I'm finding hardest right now is. . . ."

There are a variety of ways to approach journal assignments, including the following:

1. Completion type statements: Prompts can be used to focus on a small part of a bigger topic for summarizing, analyzing, comparing or expressing feelings. A good prompt
   (a) encourages students to take a stand,
   (b) focuses on the main idea,
   (c) is written in first person,
   (d) takes the form of a statement,
   (e) depends on the feedback desired, and
   (f) provides student choice.
2. Use of lead sentences: These have many of the characteristics of prompts but are usually more specific, for example:
   (a) The discriminant is useful in determining the kinds of roots of a quadratic equation. The discriminant. . . .
   (b) The factors of 18 are 1, 2, 3, 6, 9 and 18. Factors are numbers that. . . .
   (c) The only natural number that is neither prime nor composite is 1. A prime number is. . . .
3. Warm-ups: When teachers want students to put together several topics in order to see the larger picture, they can ask a simple question (not requiring much thought), followed by a second, more difficult question, as in the following example:
   (a) Complete the sentence: Lines that are perpendicular have slopes that. . . .
   (b) State the equation of a line that is perpendicular to $y = 3x + 2$.

Writing Activities Using the Text

Senior high students need practice using texts. As students progress in their schooling, their learning experiences become increasingly self-directed and dependent on textual materials. Texts are valuable resources for them and ones they need to feel comfortable using when they are alone at home, as well as at school. Writing assignments can enhance students' comfort and familiarity with the text and lessen math anxiety. Consider the following ideas:

1. Familiarity with the "whole picture" is an important way for teachers to assist their students. Many texts provide an inventory of problem-solving strategies, a glossary, answers, historical notes, and technology and applications sections. Students can learn from paying attention to the layout of the text, the sequencing of topics and how each section relates to the topic being developed. Any limitations of the text should also be noted for students. Many students accept printed material as gospel truth. Teachers should reward students who find errors or ambiguities.
2. Have students examine topic development. Students can learn about mathematical

content and processes by studying and writing about worked-out examples. Texts that use bold type for key words or small print to highlight hints for the students also allow students to write about these terms or to discuss reasons why they think the authors inserted the tips.

3. Have students explain or justify each step in a solution presented on a text page, create another example or counterexample or rewrite the page the way they would prefer to have seen it written.
4. Try this one yourself to see the richness of the exercise!
   (a) Justify each of the following steps in multiplying $(x + 4)(x + 2)$.
   $$\begin{aligned}(x + 4)(x + 2) &= x(x+2) + 4(x + 2)\\ &= x^2 + 2x + 4x + 8\\ &= x^2 + (2 + 4)x + 8\\ &= x^2 + 6x + 8\end{aligned}$$
5. Have students practise analyzing the conditions of a problem: Can they change the given information and the question being asked to create a new problem? What are the restrictions? Have students select a problem that can be enriched, write a simpler problem or select a less interesting problem to explore or think about.
6. Have students reflect on text exercises. How are they alike? How do they differ? Do they move from simple to complex? How are they sequenced? An effective cooperative-learning strategy has students in pairs or groups select which exercises from each section they will do and then directs them to correct and discuss each other's work. This experience provides fresh insights for other students and fosters an appreciation for different ways of solving problems.
7. Have students list four topics from a chapter in the text and write a summary for one. Provide clear directions on what must be included in the summaries.

Students should appreciate that texts are written by ordinary people to help others understand and learn about mathematics rather than by people who are strange or removed from the real world. This may help demystify math for them.

Writing on Tests

Some teachers have included short writing assignments on their tests. Such questions could require feedback on concepts or skills, as well as personal comments regarding students' learning experiences, for example:

1. Compare and contrast. . . .
2. Describe your understanding of the procedure for factoring the difference of two squares. Create an example of your own using the numbers . . . to verify your statements.
3. The most difficult thing I had to learn in this class was. . . .
4. Have students analyze one problem on the test that they found hardest to do, using correct vocabulary directed at students' thought processes and approach, any pattern they recognized and some knowledge of how they could proceed.
5. Use verbs such as the following to promote higher-level responses: justify, restate, summarize, support, read and explain, and hypothesize.

Study Aids and Reviews

The changing nature of society and increased demands on students' time challenge teachers to evaluate the amount and kind of homework assignments given to students. Many students don't do, and don't see the importance of doing, regular math homework. Review is critical to learning, retaining concepts and being successful in mathematics. The following ongoing writing assignments involve students more actively in their own learning.

Chapter Summaries

At the end of a unit or study topic, reflection is a valuable part of the problem-solving process. However, it is more than just looking back (from Polya's four-step problem solving model). It also involves looking ahead and internalizing a broader picture of the concepts and skills being studied. Relating what has just been learned to what was previously known and looking for the bigger picture allow students to make critical connections between one chapter and the next, one topic and the next.

Definition Cards

I recall a story about a senior high student whose second language was English. He was doing poorly in his high school class. When the teacher asked him how he had gotten that far, he responded, "In junior high, my teacher made me write out what each thing was, and I had to make an example too!" Definition cards can be used as an activity in which students write about math terms and give examples, work in pairs, revise and refine or give a specific example in their own words, and pose a problem to be solved. A group game can also be designed using sets of three cards—one with the definition written on it, another with an example to illustrate the term or concept and a third with a phrase to explain the meaning. The game can be played like Concentration, where students take turns exposing cards, three at a time, looking for a match.

Writing About Problem Solving

Margaret Ford (1990, 35) reports on research by Donald Graves who found that

> simply asking students to write or giving them opportunities to write, does not produce better writers. . . . Guiding students through a writing process is the key to producing better writers and better thinkers. The task of developing problem-solving ability is similar because problem solving, like writing, is a process. Guiding students through the process of problem solving is essential if they are to become better problem solvers.

As a strategy for problem solving, Ford supports the writing process and advocates using writing to help students focus on the question being asked and look for essential information in the question.

Communication skills play a key role in teachers' assessment of student thinking and for monitoring and reporting on students' progress in problem solving. Students need to develop mathematical language facility to clearly convey their ideas and thought processes in solving problems and reflecting on mathematical experiences and relationships.

To construct meaning by connecting new ideas to concepts they already know, students will need experience in expressing their own generalizations discovered through investigations and to write convincing arguments to validate them.

As mathematics teachers, we can learn from our language arts colleagues who share "secrets" with their students to help them write. These secrets include strategies in prewriting, drafting, revising and publishing, with continuous sharing and conferencing.

In the prewriting stage, the teacher furnishes a stimulus or context, perhaps a real-world problem, which connects to what students already know. The context motivates and arouses interest; it is a springboard from which to introduce a new idea. In mathematics, we look for real ways to teach that thought-through problem solving is the first step in the process.

The writing stage provides time for students to explore, think about the problem and organize their thoughts and ideas, without the pressure of presenting a clean copy.

In revision, students focus and clarify their ideas, create their own examples and counter-examples, and add or delete information. In this stage, students present a draft copy for teacher or peer reaction.

The final phase in the writing process is the publication of a good copy and final form, putting the thoughts on paper.

Ongoing conferencing allows teachers or peers to listen to, read and give feedback by asking questions that focus on the content of the writing. Discussion promotes the kind of verbal thinking that develops ideas. Writing places the thoughts on paper and acts as a lens through which to view, magnify and focus those thoughts (WIMP 1989–91).

Writing about Attitudes, Thoughts and Feelings

Personal Math Histories

One way to begin a new semester or math course is to find out where your students are coming from. Students' attitudes, backgrounds and personal math histories can be revealing and can assist teachers in getting to know their students and in planning for instruction. It may be more appropriate for young students to write about what they expect to achieve or learn in math this year.

Attitudes Toward Mathematics

Mathematical dispositions, such as a student's self-confidence, willingness to solve problems and perseverence and interest in math, contribute to mathematical literacy and student achievement. Students' dispositions are part of the informal ongoing assessment which is monitored as students progress through high school math programs. One effective way of monitoring is to collect data continually through anecdotal records and teacher observations. Another way is to involve students in self-assessment and reflection.

Journal assignments can also provide teacher and student with insights into attitudes and feelings. Maintaining collections of individual student's responses to your prompts, over an entire course, is useful for determining changes and progress in mathematics. This data can be used in parent and student conferences. Teachers should allow students to change prompts so they feel comfortable, but the intent in any change should be consistent with the kind of information teacher wants to collect. Notice the "I" form for the following prompts:

1. As a problem solver, I have no problem doing . . . but . . . still bothers me. . . .
2. Yesterday I learned that. . . .
3. So far in this course, I. . . .

Writing about Math and Its Applications

Opportunities for writing about issues, historical figures in math and applications will reinforce students' understanding of the connection between mathematics and society. Essay questions, projects and reports can be used as long-term assignments to develop awareness of the value of mathematics and appreciation for its use in the real world. The newer statistics and probability strands of the curriculum promote group work on real-world problems and meaningful investigations.

Research Topics

Teachers can have students research topics from math history and/or the discoveries of famous mathematicians, write a major paper or develop a complex problem to be solved. Alberta Education's senior high school problem-solving monograph *Problem Solving in Mathematics: Focus for the Future* (1987) is a valuable resource that includes solutions to several interesting project problems.

Math Hunt

One of the most successful research projects I have encountered was a math hunt conducted by Cynthia Ballheim at St. Mary's High School in Calgary. Students, working in groups, were asked to find 10 adults who used math in their careers. As a long-term assignment, the students compiled documentation from interviews with the adults and wrote a final report, which included a signature and the nature of the mathematics the adults used. The students received bonus points for getting a person to speak to the class. Project results were so diverse that the class concluded that everyone uses math. One girl remarked in a surprised way, "Did you know? Even my mother uses math!" A second student told me that it wasn't until then that she really tried in math class because she realized that she would need to know more math when she graduated.

Assessing Student Writing

Teachers must establish a purpose when selecting writing assignments and must then evaluate whether or not that purpose is met. Change happens slowly and gradually, so proceed slowly and begin with what is comfortable for you. This will take patience, but the rewards are worthwhile. Incorporating writing experiences takes little more time than repeated lecturing. By showing students' that their thoughts are valued, a spirit of openness and trust can be built to promote math teaching and learning. By writing back and responding to students' work, assigning grades and including writing assignments as part of the course mark, teachers can convince students of communication's importance in learning mathematics.

Teachers who have used writing-to-learn approaches in their classrooms have struggled with the time it takes. This time commitment depends on what the teacher decides to do with students' responses. In some cases, teachers read assignments on a "spot-check" basis. In others, students share their responses orally, exchange with peers or hand their papers in. In all cases, students should receive regular feedback from teachers. If students take time to write, teachers need to find the time to write back!

Mathematical errors should be a secondary consideration in some writing assignments, but they should always be brought to students' attention. In their evaluations, teachers should keep the primary focus on thoughts and ideas rather than on spelling and punctuation.

Students should see their errors as opportunities for learning. An understanding and acknowledgement that errors are a part of learning will help to establish a risk-free environment. Asking students to proofread their own as well as each other's work is effective for correcting errors and clarifying meanings. Having students write about their errors on tests or homework assignments is also an effective learning tool. When students are asked to write, they are really being asked to think. This will take perseverence from students and encouragement from teachers. The following questions should guide students in their efforts to analyze what they are doing and why:

1. What are you doing?
2. Why are you doing it?
3. How is it going to help you find an answer to this problem?

The aim of establishing a regular series of questions like this is to encourage students to reflect continually on their thinking, their use and choice of strategies, and their problem-solving procedures. By continually asking students to clarify, paraphrase and elaborate, to describe how they reach answers as well as the difficulties they encounter solving problems, teachers are providing them with valuable skills for the future.

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Chapter 3
Communicating in Mathematics Class

Susan Burgoyne and Marilyn Burgoyne

Thirty students to one teacher? With this ratio in mind, the teacher needs to become the facilitator in the learning environment. The students need to become active participants in order to construct mathematics concepts. A facilitator and participants function well in a cooperative group-learning situation.

These statements express well this chapter's focus on

1. *learning environments,*
2. *the teacher as facilitator,*
3. *cooperative-learning groups,*
4. *problem solving and the writing process,*
5. *writing implementation in class,*
6. *learning logs, and*
7. *math storybooks.*

These foci are coherently and insightfully related to NCTM's Curriculum and Evaluations Standards for School Mathematics.

The chapter also includes an annotated bibliography.

Communicating in Math Class?

Keith bent his head in frustration. He glanced at his math textbook and stared at the almost blank page in his scribbler. He looked at his calculator. Mrs. Lewis, his Grade 7 math teacher, stopped to see if he needed help solving word problems. He shook his head to indicate that he did not. How could he possibly admit to Mrs. Lewis that he could not read or understand many of the word problems?

Jenny glanced across the aisle to see how Keith was doing. He looked as puzzled as she felt. She had followed all of Mrs. Lewis's directions. She had underlined key words and data. She had an idea of how to set up an equation to solve the problem. However, she was confused and did not know how to continue. Mrs. Lewis was too busy to help her. If only she could ask David, who sat behind her, to give her a hint. Jenny was sure that with a little help she could continue solving the problems.

David wondered why Jenny kept turning to look at his answer page. Did she need some help? He would help her if he could, but Mrs. Lewis did not permit talking between students. He continued working at the problems—they were easy! He wanted to be the best math student in the class.

Mrs. Lewis surveyed the 30 Grade 7 students in her classroom. She had been teaching for 10 years but felt that she needed to learn and to grow in order to help her students. If she could be more aware of their individual math concerns and questions, she felt that she could tailor her lessons to meet their specific needs. She could teach the problem-solving process more effectively by allowing her students to use a variety of mathematical skills and concepts. How could she accomplish this with so many students and such limited time?

The dilemma of Mrs. Lewis and her math class is common. Many students have difficulty in reading and then solving word problems from textbooks. As a result, the levels of frustration for student and teacher are high. What can be done to teach problem solving effectively in the classroom?

To assist teachers and students, the National Council of Teachers of Mathematics (NCTM) has developed the *Curriculum and*

Evaluation Standards for School Mathematics (1989). This document explores the curricular goals of the mathematics program and suggests methods by which progress toward these goals may be assessed. This chapter considers the social and technological changes occurring in society and how these changes have affected mathematics teaching and learning in schools.

Thirty students to one teacher? With this ratio in mind, the teacher needs to become the facilitator in the learning environment. The students need to become active participants to construct mathematics concepts. A facilitator and participants function well in a cooperative group-learning situation. The teacher models the problem-solving process desired, and the students gain practice by solving problems in groups. Students will learn to deal with new or unknown vocabulary. They will learn to test strategies and validate solutions and will develop mathematical power. The teacher as facilitator has created a learning environment that allows students to develop mathematical reasoning, understanding and communication.

Students need to be able to explain their strategies and reasoning in writing as well as speech. Writing enables the students to

1. reflect on personal thinking (it is now visible on a page),
2. express understanding in a private way (a student may be uncomfortable in an oral situation),
3. formulate ideas and organize thoughts,
4. make abstract ideas more accessible,
5. connect math to the real world,
6. work out a problem individually (to have a debate with oneself), and
7. expose learning problems, concerns, conjectures and questions.

This writing process can be linked directly to the problem-solving process in math as illustrated in the following chart:

Writing Process	*Problem-Solving Process*
1. Prewriting	1. What is being asked? Underline the key words, key data.
Exploratory Talk	
2. Writing	2. Decide on a plan of action. Solve the problem. (Calculate)
3. Revising, editing	3. Check back to see if your answer makes sense.

By analyzing students' written work, the teacher will understand how well students are coping in the problem-solving situation. Some groups and individuals may require additional instruction.

New approaches to teaching mathematics call for new methods of evaluation as well. Observations, interviews and student learning logs (journals) are encouraged to promote communication in mathematics. Different views of student understanding are presented.

A specific assessment tool is the student learning log, outlined as follows:

1. The student keeps an ongoing record of learning in a specific notebook or journal. This writing is informal unless a preset goal has been established.
2. The student may respond to specific process questions by writing in the learning log.
3. The student may record thoughts, feelings, questions and observations as he or she progresses through a problem.
4. The student can organize and test ideas through writing in the learning log. Ongoing dialogue with the teacher is created.
5. The student may be introduced to—or reminded of—a topic in math through

using the learning log as a diagnostic tool (for example, ratio and proportion, area, perimeter).

By having students respond through learning logs, the teacher is better able to see what students are learning and to understand student concerns and problems. Enrichment and remedial strategies may be used as required to meet individual student needs.

Mathematics is now seen as a problem-solving process. It is something that students do. Students explore, examine, interpret and use data. Students use problem-solving strategies independently or in small cooperative-learning groups. Students communicate through talking and writing about mathematical reasoning to form and evaluate conjectures and arguments. Students construct personal meaning from mathematics experiences. The teacher assists students in the mathematical studies process (refer to Appendix 1).

Teachers such as Mrs. Lewis want to give students mathematical power. They are willing to act as small-group facilitators and to encourage communication in mathematics. Their classroom learning environments will become interactive and noisy. When evaluating writing, teachers will have to ignore mechanics and grammar and concentrate on content. They will have to write themselves to show their students that writing is important. They will have to relax and have fun with their attempts and their students' attempts to communicate about mathematics in writing.

Mrs. Lewis will learn more about why Keith cannot read and so cannot solve math word problems from the text. She will learn that Jenny needs a few hints now and then. She will learn that David is an excellent problem solver who is willing to help others. She will learn about herself and the needs of her students. After making changes, Mrs. Lewis's mathematics class will become student-centred.

Writing in Mathematics Class!

"Writing in Math!? Who ever heard of that? It is probably just another one of those fads." As a math consultant and classroom teacher, I often hear such comments when I discuss using writing in mathematics class. Students and adults find this approach to learning math concepts new and different. After discussing the writing process and its benefits to understanding mathematical concepts, educators and learners accept this valuable learning strategy.

Why Use Writing in Math?

Writing in math provides an opportunity for students to become active participants in acquiring knowledge. If children write about their experiences with math and reflect on the learning process that has occurred, then they are able to clarify their thinking and incorporate the ideas and concepts into their own belief systems. Through the writing process, the learner is incorporating mathematical terms into personal everyday language. In addition, writing is a vehicle for learners to use if they are uncomfortable in oral situations; the written environment is less threatening for some learners.

As a classroom teacher, I observed that learning and understanding took place when a student was able to explain and write to another student what was done to solve a problem. By writing like this to classmates, students are reflecting on their own thought processes as well as communicating information and strategies to others. Through writing in math, we are helping learners develop the belief that they have the power to do mathematics and that they have control over their own success or failure.

Learning specific to mathematics needs to make sense and be logical and enjoyable, not simply memorizing rules and procedures. Richard Skemp (1989) believes that there are two types of understanding: instrumental understanding (rules without reasons) and relational understanding (knowing what to do and why). Unfortunately, instrumental understanding is more popular because it seems easier to achieve. However, in the long run, relational understanding has the power we are looking for in creating mathematical knowledge. For example, through relational-type activities, learners are able to adapt easily when presented with new situations. Also, the information remains with learners longer; thus mathematical reasoning can occur during a lifetime. The use of writing in math encourages relational understanding to occur.

Through the writing process, learners are able to organize, internalize, clarify and reflect on the knowledge that they have created, experienced and explored. In other words, learners are experiencing relational understanding. By observing my students participating in writing, I found that they tended to become more confident in their ability to reason and communicate mathematically. It also became evident that they were more prepared to discuss and justify their thinking.

Many articles have been written dealing specifically with why we would use writing in mathematics class. However, the question still remains, "How are teachers going to implement the writing process in class?"

Implementing Writing in Mathematics Class

Learning Logs

Meaningful learning in math class can be facilitated through writing. One method I found extremely helpful for enhancing communication was using learning logs. A learning log is excellent for keeping an ongoing record of learning as it occurs. There are several approaches to using learning logs. One is to have the students respond to process questions. For example, in my mathematics class, students would write responses to process questions such as

1. What did you understand in class today?
2. What didn't you understand?
3. At what point in the lesson did you get confused?
4. What activity did you like doing?

It is important to note that the writing in the learning log is informal and is not being grammatically evaluated by the teacher, unless a preset goal has been established.

For young students who are unable to write, I have found it useful for them to draw pictures or representations in their learning logs about their thoughts.

By having students respond to this type of questioning, teachers are able to see what students are learning as well as better understand student concerns and problems. In effect, teachers are able to provide for individual differences in how the students can be helped or how the program can be enriched.

Another approach to learning logs is to have students jot down predictions or expectations of a new topic. In this way, the teacher is looking for the background knowledge of the learner to use in individualizing lesson planning.

I have also used one learning log for a whole class. Each day a different student writes down or draws a representation, or both, about the math activities that took place that day. The student assigned to do the learning log a particular day must observe class activities and be able to put forth his or her interpretation of the day's happenings.

Problem Solving

Problem solving is an essential component of communication in math. Through problem solving, the learner discusses, reads, writes and listens to mathematical ideas and constructs knowledge. Through the problem-solving vehicle, the student becomes mathematically literate.

To focus specifically on the writing process in problem solving, I have found it beneficial for students to write their own problems. Students are able to incorporate the concepts and ideas being covered in class in original problems. This also helps students become more aware of the information essential to solving problems.

Letter Writing

Yes, you can write a letter in math. Have your students write a letter to a classmate telling him or her about something learned in math class. Through this form of communication, learners reflect on and organize their thoughts, while the receivers gain knowledge from their peers.

Storybooks

An exciting way to incorporate writing in math is to have children create math storybooks. The books can range from number books to stories about shapes and sizes. If you let the imagination of the learners take over, you will often be amazed at the writing that will take place with math as the overall theme. We have "young writers" in language arts, why not in mathematics?

Conclusion

The previously mentioned strategies are but a few of the ways that we as educators can incorporate writing into mathematics classes. A major goal of mathematics instruction is to help students develop the belief that they have the power to do mathematics and that they have control over their own success or failure. By using writing in our math classes, we provide opportunities for students to gain relational understanding and the power to do mathematics. Students will achieve increased knowledge about writing and become confident and knowledgeable math participants.

Appendix 1

Reference articles in the *Arithmetic Teacher* and the *Mathematics Teacher* journals that support the NCTM *Standards* document and its objectives for mathematics education are outlined here.

1. Wilde, S. "Learning to Write About Mathematics." *Arithmetic Teacher* 38, no. 6 (1991): 38–43.

This article explores using writing as a regular part of the mathematics curriculum. Creating word problems, using writing as a diagnostic tool and reflecting in math journals are discussed.

2. Davison, D. M., and D. L. Pearce. "Using Writing Activities to Reinforce Mathematics Instruction." *Arithmetic Teacher* 35, no. 8 (1988): 42–45.

This article features students engaged in writing activities in mathematics: direct use of language, linguistic translation, summarizing, applied use of language and creative use of language.

3. Mumme, J., and N. Shepherd. "Implementing the Standards: Communication in Mathematics." *Arithmetic Teacher* 38, no. 1 (1990): 18–22.

This article looks at communication in the mathematics classroom. Examples of student writing are given.

4. Thompson, C. "Number Sense and Numeration in Grades K–8." *Arithmetic Teacher* 37, no. 1 (1989): 22–24.

This article examines the development sequence of number meanings through students' experiences linked to physical materials. The use of estimation is also examined in terms of the reasonableness of a proposed solution to a problem-solving situation. Activities dealing with whole numbers and decimals are provided.

5. Lindquist, M. "The Measurement Standards." *Arithmetic Teacher* 37, no. 2 (1989): 22–26.

This article examines the measurement process. The activities outlined use pattern blocks to explore the concept of area. Students examine geometrical shapes, such as the rectangle, parallelogram, hexagon, trapezoid, triangle and square. A tile floor is designed, and the angles are examined. Charts are made, and students draw conclusions from the data collected.

6. Howden, H. "Patterns, Relationships, and Functions." *Arithmetic Teacher* 37, no. 3 (1989): 18–24.

This article looks at explorations of patterns and functions. The student activities encourage describing, analyzing and creating patterns. Tables and graphs are interpreted. Functions and patterns are used to represent and solve problems.

7. Rowan, T. "The Geometry Standards in K–8 Mathematics." *Arithmetic Teacher* 37, no. 6 (1990): 24–28.

This article looks at geometry and spatial sense for students in the classroom. The students construct geometric shapes and three-dimensional models (tetrahedron, cube). Basic shapes, such as the rectangle, parallelogram and triangle, are used to discuss the concept of area.

8. Burrill, G. "Statistics and Probability." *Mathematics Teacher* 83, no. 2 (1990): 113–18.

This article looks at statistics and the collection, organization and interpretation of data. It offers teacher guidelines for lesson preparations.

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Part II

How Can I Understand What I'm Doing?

Chapter 4
Languaging Numbers in a Multicultural Setting

Daiyo Sawada

In recent years, the business community in Alberta and Alberta Education have taken keen interest in mathematics education in China, Korea, Taiwan and Japan. The Alberta Chamber of Resources and Alberta Education have jointly commissioned an indepth study of Asian mathematics education and have derived many implications for Alberta schools. Their report International Comparisons in Education—Curriculum, Values and Lessons *is available from*

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If you have already examined this report, you will find it quite different from the interpretations of this chapter. This does not necessarily mean that one is wrong and the other right. There are many ways to interpret different cultural practices.

Of Japanese heritage, Daiyo Sawada has tried to interpret Japanese educational practice within a perspective that does not violate its culture. He has related his remarks to a multicultural setting so that applied language differences and multilingual counting can be appreciated as ways to improve mathematics and *cultural understanding.*

To begin, I would like to present two class lists. One is from my son's Grade 6 class, and the other is from my own Grade 6 class in the mid-1950s. Can you identify which class is my son's and which is mine?

Class A	*Class B*
Arno	George
Vivek	Gail
Mark	Judy
Jayan	Herman
Emily	Elaine
Kamil	Shirley
Jaime	Lawrence
Kwang	Ken
Philina	Carol
Nana	Bobby
Hamish	Ruby
Rishi	Don
Saul	Jim
Brandon	Lynn
Mia	Lloanne
Mintike	Noel

I'm sure you had little difficulty identifying Class A as my son's and Class B as mine [some names given are pseudonyms]. I grew up in Pincher Creek and my son in Edmonton, but the difference in class makeup is not simply accounted for by the rural/urban difference. The multicultural composition of my son's class is more an indication of the cultural diversity that exists in Canada: My son's class looks like a mini-United Nations. The cultural composition of today's classrooms offers a unique opportunity to enrich mathematics in ways which were formerly unavailable. However, to seize this new opportunity, we teachers may need to adopt a different perspective on our practice. In this chapter, I would like to indicate what this new perspective might be and how it might be brought forth in the classroom through language.

A Cultural/Lingual Perspective

Look again at the two class lists. In both classes, each child is unique. In Class B (my class), Herman had failed a grade despite his reasonably high IQ. He had a bad attitude and even lipped off now and again. He was a slow learner, didn't pay much attention to anything and rarely came to school with his homework done. Herman needed to be motivated because his attention span was so short. Lawrence, on the other hand, was a fluent reader. Teachers often called on him for answers and, on the rare occasion that he was unable to give the desired answer, the question would be dismissed as inappropriate. Lawrence was always prompt with his assignments and often went beyond what was required. He was self-motivated and enjoyed coming to school. He had a high IQ. As a generalization, I think it safe to say that teachers would try to understand the differences between Herman and Lawrence in terms of psychology and to try to accommodate these differences through individualized instruction. In this response, differences are considered to be handled through *instruction.*

In Class A (my son's class), Vivek was from India. He would often speak English with a strong Hindi accent although he could also speak English with only a Canadian accent. He would adopt the Hindi way of speaking to be entertaining and appreciated by his buddies. He knew a great deal about India and often shared this knowledge in class. In contrast, Mintike came recently from Ethiopia. He could tell story after story about the atrocities that he had experienced firsthand. Accounts in textbooks or even in newspapers seemed thinly one-dimensional compared to the stories Mintike could relate. While teachers may try to understand the differences between Vivek and Mintike in terms of psychology and to deal with the two youngsters using different instructional processes, the differences between Vivek and Mintike can be understood psychologically only with great distortion. These differences are cultural and cannot be legitimately understood using only psychology. Moreover, neither Vivek's nor Mintike's first language was English, and it is worthwhile noting that language and culture have a close relation.

My point can be expressed in the following:

1. Individual differences in Class B were largely understood as being psychological in nature and were tolerated as inevitable. Teaching consisted largely of overcoming these differences using instruction methods that either worked regardless of these differences or catered differentially to these differences (individualized instruction).
2. Individual differences in Class A are strongly cultural as well as psychological. While these differences could also be overcome with instructional methods, such an approach would sacrifice the rich cultural content latent in the classroom. Instead, it may be wiser to consider these differences as *sources of curriculum* rather than to deal with them as instructional problems to overcome (Sawada 1989). Considered as curricular sources, these differences offer new learning opportunites not present in Class B.

If we consider the cultural differences of our students as sources of curriculum rather than as individual aberations or ethic distractions, then we can tap a rich resource which normally is overlooked or deliberately avoided in many classrooms. While it may be clearer how different cultural backgrounds can operate as curricular sources for social studies or langugage arts, their significance in mathematics learning may be less obvious (Sawada 1990). This chapter suggests ways in which this curricular resource of culture, so plentiful in many Canadian classrooms, can enrich mathematics classes.

Cultural Differences in Mathematics Achievement

Cross-cultural comparisons in mathematics achievement have given rise to many controversial claims. High scores achieved by Asian (Korean, Chinese, Japanese) students have often been dismissed as the results of slave-like adherence to excessive perseverance and rote approaches to learning. Recent and methodologically rigorous research reveals that the substantially higher mathematical achievement attained by Asian students from K–12 is valid at all levels of cognitive complexity, from simple verbal knowledge through to open-ended creative problem solving, including the ability to carry out complex visual transformations mentally. A comprehensive and readable study of this kind has been published recently by the National Council of Teachers of Mathematics (NCTM) (Stevenson et al. 1990; Stigler, Lee and Stevenson 1990). Stigler and Baranes (1988, 294) come to the following conclusion:

> In summary, the Asian advantage in mathematics, at least at the elementary school level, is not restricted to narrow domains of computation, but rather pervades all aspects of mathematical reasoning. This result has also been obtained in studies of Korean children. . . . Taken together, these findings should provide ample motivation for examining cultural differences in the way mathematics is taught in Japanese, Chinese, and American classrooms.

One of the striking findings of this research has been the fascinating relationships between the higher achievement in number work by Asian students and the particular kind of number language used. Strong evidence now shows that in addition to cultural differences, differences in language, particularly in spoken language, contribute to the superior performance of Asian students. I will examine these relationships to find ways of enriching Canadian use of number language to understand number concepts. I will do so at the early childhood level so that whatever benefits can be derived can be made available during the early grades.

Language and Number—The Asian Case

In this section, I do three things. First, I display the counting words Japanese children use (the Chinese and Korean number names follow exactly the same structure) and invite readers to contemplate how the structure of these number names might make early number competence easier to acquire. Second, I indicate that the superior performance of Asian children on number tasks can indeed be traced to, if not completely explained by, the structure of the number language they use. And third, I indicate how we can make such language structures available to Canadian children so that they too may benefit from this particular way of languaging numbers.

Japanese Counting

To indicate the structure of the number words Japanese children use, I list the first 12 number names and invite the reader to fill in the rest up to 100. To guide this activity, I have inserted a few number names along the way as readers' aids.

1. ichi	11. juichi	21. nijuinchi	31.	41.
2. ni	12. juni	22.	32.	42.
3. san	13.	23.	33.	43.
4. shi	14.	24.	34.	44.
5. go	15.	25. nijugo	35.	45.
6. roku	16.	26.	36.	46.
7. shichi	17.	27.	37.	47.
8. hachi	18.	28.	38. sanjuhachi	48.
9. ku	19. juku	29.	39.	49.
10. ju	20. niju	30.	40.	50. goju

51.	61.	71.	81.	91.
52.	62.	72.	82.	92.
53.	63.	73.	83.	93.
54.	64.	74.	84.	94.
55. gojugo	65.	75.	85.	95.
56.	66.	76.	86.	96.
57.	67.	77.	87.	97.
58.	68.	78.	88.	98.
59.	69.	79.	89.	99. kujuku
60.	70.	80.	90.	100.

In case you are wondering, 100 isn't *kuku*, although of course such a name makes perfect sense—just as "ten ten" would make good sense in English. The Japanese word for 100 is *hyaku* and 101 would be *hyaku ichi* and 512 would be *gohyaku juni.*

How long do you think it would take you to master these counting words up to 100 once you had learned the names for single-digit numbers?

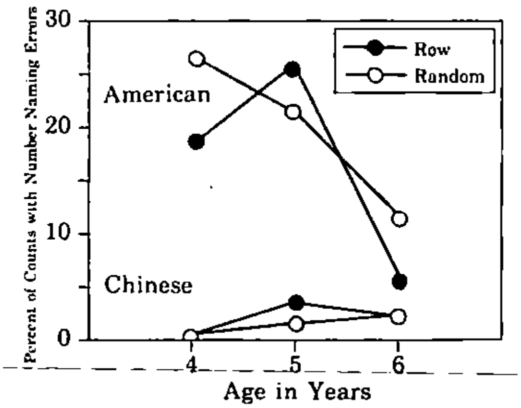
If one compares these words with the English equivalents, one is struck by how logically consistent one set is and how seemingly arbitrary the other. In particular,

1. the structure of the spoken symbols in Japanese matches the written numeral structures quite closely. The base-ten structure is embodied in both written and spoken forms. This congruence between spoken and written forms is violated in upredicatable ways in English;
2. while the written and spoken symbols possess base-ten structure in Japanese, the spoken words in English often do not. Even worse is that for the first few occurrences of the base-ten structure in the written form (that is 10, 11, 12) there is not even a hint of the base-ten structure in the spoken form: The spoken words *eleven* and *twelve* sound just like names for single-digit numbers. This is unfortunate because Canadian children do not get even a hint that there is a pattern underlying our counting words—the spoken words mask the pattern entirely.

To establish that the superior number competence of Asian children is due in large measure to language differences rather than overall cognitive competence, I show two graphs comparing American and Chinese youngsters aged four to six on counting tasks. The Chinese and Japanese oral systems are similar in adhering to the base-ten structure. The first graph shows the percentage of counting performances containing number-naming errors, such as skipping a number, saying a number word out of sequence or saying a nonstandard name. As can be seen, the Chinese performance is nearly error-free at age four, while American children are still making mistakes at age six.

Figure 1

Percentages of Counting Performances Containing Number-Naming Errors: American and Chinese Comparisons



Note: Number-naming errors include mistakes in enunciating the conventional counting sequence such as skipping a number or producing a nonstandard name. (Figure modified from Stigler and Baranes 1988)

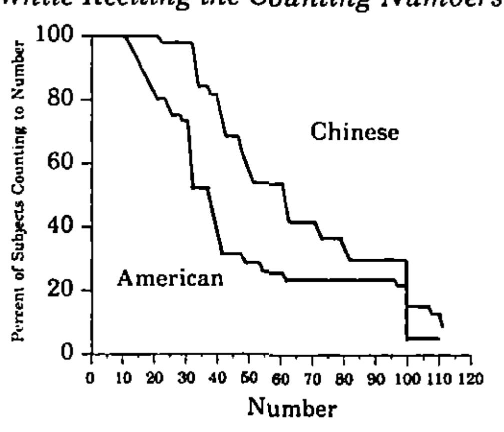
From this graph it is clear that Chinese youngsters are much better counters. However, this superior performance may not be due simply to the structure of the spoken number words; it could also be due to increased parental pressure to do counting tasks. Evidence such as the following helps to eliminate this alternative hypothesis:

1. If other counting errors are considered, such as counting the same object twice or skipping over an object while counting, then the Chinese youngsters make as many errors as the Americans.
2. If children are simply asked to count to the highest number they can (oral counting), Chinese children again outperform American children. More significant, however, is the point in the counting sequence at which the difference in performance occurs. If the structure of the oral symbols makes any difference, then the point in the counting sequence at which it should have an effect is precisely at the shifts in place value. Figure 2 confirms this.

These two graphs convincingly demonstrate that the congruence between the written and oral symbols used in Chinese language gives Chinese children a definite head start, a head start they never relinquish. These findings for Chinese children have been replicated for Japanese and Korean children.

Figure 2

Percentage of Chinese and American Children Reaching a Given Number While Reciting the Counting Numbers



Note: The data are based on the highest number reached in abstract counting, permitting one omission of a single number. The rapid decline in performance by American children occurs when place value became important. (Figure modified from Stigler and Baranes 1988)

Classroom Suggestions

Is there anything we can do in Canadian classrooms that will enable our children to benefit from these research findings? A great deal can be done in terms of attitude and learning activities.

Attitude

In Canadian classrooms, many children approach mathematics as a repetitive activity that rarely makes sense; it just goes on and on year after year. Many give up trying to make sense of it and resort to the frustratingly painful strategy of brute-force memorization. Contrast this with the Japanese (or Chinese or Korean) child; recall how easy it was for you to learn the Japanese counting words up to 100. My interpretation is this: Japanese children find out early that counting makes a great deal of sense. The number name that comes after "ten" is not just another nonsense syllable (such as *eleven*) but begins instead to suggest a pattern ("ten-one" followed by "ten-two" and so on). In other words, the Japanese child is immersed in a setting that makes sense. Can you imagine such a child saying to herself, "Hey, I can figure this out!"? I submit that Canadian children never get to say this at age four, and as a result their attitude toward mathematics is born within an attitude of arbitrary roteness. For many, this attitude may last a lifetime.

Learning Activities

Multicultural Counting Festival

Most Alberta communities have a Heritage Festival of some sort. I suggest that your classroom have a Multicultural Counting Festival for a week. Presuming that your students come from a variety of cultural backgrounds, the idea is to have children, perhaps working in groups, show others how to count (perhaps to 100) in another language. (If your classroom has a dearth of different languages, community members could be invited to act as group leaders.)

If children in your class speak six languages, for example, then six groups could be formed, each around a native speaker. Group members would first learn the counting system from their "resident" expert, and then the group could devise a method for teaching their newly learned counting system to the rest of the class.

For the sharing exercise, counting booths could be set up in the classroom, and children could go to other booths to learn to count in another language. Perhaps other classes in the school would be interested in attending the festival too.

Inventing an Alternative Counting System

The most fundamental question in all of elementary school mathematics is never encountered by elementary school children, the question being "Can you make up a set of counting words that would work better than our English system?" This question is appropriate at any grade level and, if students haven't as yet encountered it in their mathematical careers, they should work on it regardless of their age. This question is also an excellent way of reviewing the many counting systems encountered at the counting festival.

You might approach this with your class by saying, "We have seen how people in different cultures have solved the problem of counting. Each of the solutions is different. Which did you find was the easiest to learn? The hardest? The simplest? The most complicated? Which did you like best? What other questions can we ask?" The idea is to generate a discussion and critique of the counting solutions that have come down through the ages.

You might also ask, "Now that we know how others have solved (or messed up!) the counting problem, do you think we could create a better solution?"

Students could profitably spend a whole period (and more) on this problem as a counting festival follow-up. You as teacher could also solve the problem. The students are bound to come up with a multitude of solutions. Everyone could share solutions, perhaps using the booths again. I find the following solution attractive for many reasons, and I share it with preservice teachers every year:

1. one	11. ty-one	21. twoty-one	31.	41.
2. two	12. ty-two	22.	32.	42.
3. three	13. ty-three	23.	33.	43.
4. four	14.	24. twoty-four	34.	44.
5. five	15.	25.	35.	45.
6. six	16.	26.	36.	46.
7. seven	17.	27.	37.	47.
8. eight	18.	28.	38.	48. fourty-eight
9. nine	19.	29.	39.	49.
10. ty	20. twoty	30. threety	40. fourty	50.
51.	61.	71.	81.	91.
52.	62.	72.	82.	92.
53.	63.	73.	83.	93.
54.	64.	74.	84.	94.
55.	65.	75.	85.	95.
56.	66. sixty-six	76.	86.	96.
57.	67.	77.	87.	97.
58.	68.	78.	88.	98.
59.	69.	79.	89.	99. ninety-nine
60. sixty	70.	80. eighty	90.	100. hundred

Conclusion

We Westerners can respond in many ways to the realization that Asian Pacific Rim countries are doing something in mathematics education that enables their children to do very well. Rather than shouting that our children need higher standards or need to strive to be first in the world, why not encourage languaging numbers in a multicultural setting?

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Chapter 5
The Language of Mathematics Learning: Lexicon and Grammar

Laurie Walker

Because Laurie Walker teaches education language arts at the University of Lethbridge, it might be assumed that he would approach the language of mathematics learning from a language-experience viewpoint. He does and, along the way, allows those of us who have a primary interest in language learning to bring our understanding of this domain more explicitly into our practice as mathematics teachers.

He describes a "grammar of mathematics" with illustrations from division and shows how differences in passive and active forms can lead to confusion. He also offers insight into errors commonly encountered or made by children.

He concludes by suggesting that

> *if there is a gap between everyday language and the language of mathematics found in the technical terms and in the grammar, a general proposal for teaching the subject may be useful: That children be encouraged and helped to learn math in their own language, at least initially.*

This is good advice, particularly when taken in context with the rest of the chapters in this monograph.

Introduction

A prison instructor was working with an inmate on math concepts. The student was having problems with percentage, apparently unable to grasp the idea and its use. Changing the subject, the instructor asked what the inmate's crime had been.

"Drug dealing," the inmate replied.

"When you were pushing drugs, how did you work out how much of each sale went to your supplier?"

"He just got his *end*."

"Well, an *end* is the same as percentage."

The instructor reported that this conversation was a breakthrough for both him and the student, who then went on to grasp the concept of percentage and use it successfully in problem solving.

This anecdote illustrates a general curriculum issue: the choice between starting with the general abstract concept or starting with the student's own experience. The issue has particular relevance to mathematics where there is a profusion of abstract terms and concepts, said by mathematicians to be more precise in meaning than terms used in everyday situations. If we accept a simple relationship between words and the world, one in which words simply label or represent objects and ideas in our experience, the issue of the curriculum starting point is not so serious; it is simply a matter of learning the right labels for things that we know. If, on the other hand, the relationship is more complex and if some ideas are unavailable until the words call them into being, teachers have to accept that their students may have difficulty forging links between the official labels for mathematics concepts and their own experience of the world.

For example, the medieval mind took a long time to see the connection between a brace of pheasants, a pair of stockings and a courting couple. These were experienced as concrete things, and the linking abstraction of "two-ness" was not available. Perhaps literacy reveals the abstract level at which experiences have common elements and qualities. In such a way, the concept of percentage may lie beyond the immediate and more concrete "end" of a drug dealer, the "half" of a fair sharing of a chocolate bar between two friends, or the "tip" to a restaurant server. In other words, the inmate's learning block arose because he saw percentage as an esoteric part of the world of school mathematics, an official discourse that was not supposed to have anything to do with real life. In his view, mathematics words are the currency of those who have been inducted into the privileged discourse community of mathematicians by gatekeeping teachers. To outsiders, excluded by tests and examinations, the language categories and rules are rather mysterious. Some try to gain late entry to the community through adult upgrading courses and struggle with exotic things such as proper and improper fractions, never having encountered them in their working lives. Two sources of exclusion from the effective learning of mathematics are related to the language subject: the lexicon and the grammar of math.

The Technical Language of Mathematics

The written texts of mathematics are replete with technical vocabulary. I counted 22 such words in one 36-page unit of a Grade 5 textbook (Kelly et al. 1987, 124–60). Students must read text material containing words such as "array," "pictograph," "digit," "quotient," "multiple," "addend," "short division" and "long division." There are three kinds of these technical words: some are words unique to math—"quotient," "divisor," and "addend"; others have precise mathematical uses similar to their everyday function—"remainder," "average" and "bring down"; a third group consists of words that have mathematical meanings different from their ordinary functions—"rounding," "even" and "root."

Words in the first group, those unique to mathematics, are difficult to connect to children's prior experience and need special treatment in class. Children need lots of correct and incorrect examples as they work toward definition and classification. They need opportunities to express definitions in their own words rather than in the forbidding technical language of the textbook. It is easier to grasp "the quotient is what you get when you divide one number into a larger number" than the textbook definition—"[t]he number (a ÷ b) or (b ÷ a) which results from applying the division operation to the numbers 'a' and 'b' " (Fleenor et al. 1974). After working through problems, children can be helped with these new technical words by a chart display on which the "New Words of the Week" are listed with definitions, examples and sample problems.

Words in the second category, having similar meanings in mathematics and everyday life, could be approached differently. The term *odd*, for example, in the mathematical sense of "odd number," could be introduced through the notion of oddness: someone who is odd, an odd story, the odd one out. The idea that *odd* means unusual, eccentric, different or left out could then be extended to the mathematical idea that *odd* refers to numbers that cannot be separated into pairs without a remainder. The etymology of the word might be interesting to some children. The Middle English word *odde* meant a point of land having the shape of a triangle and therefore possessing a third angle, the one left out when the other two were paired. It would help children to realize that certain math words are used with more precise versions of their everyday meanings. Experience in this case is a dependable starting point.

In the case of the third category, it is not. These words have math meanings that conflict with their everyday usage. In baseball, for example, one talks about a runner *rounding* third base; in carpentry, one thinks of *rounding* off a corner. Neither of these uses is particularly helpful in thinking about *rounding* off numbers in mathematics. In these cases, students need to be alerted to the different uses that mathematics makes of words.

The Grammar of Mathematics

In addition to the lexical aspects of the language of mathematics, an area of grammatical difficulty may be unique to the language in which mathematical concepts and operations are expressed. The proposal conveyed by the slogan Language Across the Curriculum, made popular by the British report *A Language for Life* (Department of Education and Science 1975), is that teaching and learning are closely linked to the particular ways in which thought is expressed in the different disciplines. There is a language of science, a language of history and a language of literature, for example, each representing special ways of understanding the world. An example of the uniqueness of mathematical language is the particular relationships between the expression of number relationships in mathematical symbols (called math sentences) and their expression in language (language sentences). These relationships have been studied in detail by Hull (1985) with respect to division.

He observed that just as language sentences can be expressed in the active voice (the chairperson opened the meeting) or the passive voice (the meeting was opened by the chairperson), math sentences vary in the same way. For example, one could show a division sentence actively:

1.0 $6 \overline{)12}$ with quotient 2 ($\begin{array}{r} 2 \\ 6\overline{)12} \end{array}$)

Reading from left to right, this math sentence could be expressed in language as

1.1 "Six goes into twelve twice" or "six divides into twelve two times."

The relationship between the mathematical voice and the language voice seems straightforward; the elements are processed from left to right in order of their appearance.

Similarly with the passive voice, it is expressed mathematically as

1.2 $12 \div 6 = 2$

and in language as

1.3 "Twelve divided by six is two" or "twelve shared by six is two."

Again, expression and processing proceed from left to right in the order in which the elements appear.

However, it is possible to read an active math sentence as a passive language sentence so that 1.0 could be transformed to read passively as 1.3, and 1.2 could be transformed to read actively as 1.1. In each case, the grammatical transformation is accompanied by a transposition of elements so that the math symbols are no longer read from left to right. There are two learning issues here. First, research into children's language development shows that passive constructions are mastered later than active ones. Berko-Gleason (1985, 250) reported research showing that full comprehension of passive structures is not acquired until between 11 and 13 years of age. Thus, children in elementary grades may not be comfortable with passive math and language sentences.

Second, the inadvertent transformation of active and passive forms while translating between math and language sentences, without recognizing the potential processing complexity, may be a source of difficulty. Such transformation may occur in the course of oral instruction as teachers explain operations such as division. It may also appear in textbook presentations.

For example, in *Math Quest Five* (Kelly et al. 1987), the unit on division with one-digit divisors uses both active and passive forms. On page 124 under a picture of plants appears the language statement:

1.4 28 plants divided into 4 rows is 7 plants in each row.

Then appears the math sentence

1.5 $28 \div 4 = 7$.

These are both passive sentences. However, note the potential for misunderstanding in the preposition "into." In 1.4, the sense is clear enough—28 plants divided into 4 rows. In 1.5, if children carry over the "into" idea, they might read the math sentence as "28 divided into 4." This is ambiguous because it could mean either "how many 28s are there in 4?" or "how many 4s are there in 28?" It would be helpful to replace the preposition "into" in 1.4 with "by" in the math sentence 1.5.

The passive construction is maintained on page 126, except that "divided into" has become "shared by":

1.7 24 shared by 4 people gives 6 to each person.

On page 127, there is an unacknowledged switch from a passive language sentence

1.8 How many packages can be made from 50 peaches?

to an active math sentence

1.9 $8\overline{)50}$

This is followed by an exercise involving completing active math sentence problems. Then, on page 129, an explanation of how to carry out division operations on a calculator begins with an active math sentence:

1.10 To compute $7\overline{)59}$ on a calculator, press these keys.

To many math teachers, this grammatical difference must seem trivial. However, Hull (1985, 53) noted that the variety of language expressions for the mathematical operation of division was vast. He went on:

> If S is a subject who does the dividing, and O is an object (like a cake) or an amount, and D is the number representing the divisor, there are at least seven types that seem to be idiomatic—two commands and five statements:
>
> Divide O into D (e.g. divide the cake into two)
> Divide D into O (e.g. divide 2 into 6)
> S divides O into D
> S divides D into O
> O divides into D
> O is divided into D
> D is divided into O

Hull also noted that *divide* has a number of synonymous expressions: "share," "go into," "into," "how many . . . in" and "partition." However, these do not all behave in the same way grammatically. For example, "six goes into twelve" is not the same as "six divided into twelve" or "six divided by twelve." The prepositions "into" and "by" are implicated in these differences.

Hull (1985, 54) explored 100 11-year-olds' understanding of *divide*. First, by oral questioning, he established that they all thought they knew what the operation was and, that indeed, most of them could carry out simple division operations. However, when he asked them to translate math sentences into language sentences, their knowledge was less secure. For example, for the math sentence $\begin{array}{r}2\\2\overline{)4}\end{array}$ their translations included

Two divided by four is two
Two divided into four is two
Two share four is two
Two shared into four is two
Two shared between four is two

Thirty-seven children made this kind of error involving transforming the active math sentence into passive language structures and thereby changing the relationships of the elements.

Their translations for the sentence $12 \div 6 = 2$ included

Six divided twelve is two
Six share twelve is two
Twelve shared by six is two
Six shared into twelve is two
Six shared by twelve is two
Six shared between twelve is two

Hull observed that many of their language translations were meaningless or wrong. There was a ritualistic quality to some of these language sentences, as though they were utterances that applied to the mystery of mathematics without any fidelity to the real-world use of language. Their language sentences were derived from math as an activity separate from the world in which they lived their normal lives.

Hull concluded that children's difficulties with division as a math operation arose from the complex translations between math sentences and language sentences, from the transformations between passive and active structures and from the transpositions of elements in these translations and transformations. The grammar of division is an example of the complexity of the language of mathematics, which may cause difficulty for learners. To be aware of the potential difficulty and to be able to observe carefully the language of math teaching and learning are starting points for teachers who wished to help their students achieve fluency and control.

Conclusion

If there is a gap between everyday language and the language of mathematics found in the technical terms and in the grammar, a general proposal for teaching the subject may be useful. This is that children be encouraged and helped to learn math in their own language, at least initially. Talking about math in his own language helped the prison inmate. Using exploratory talk in math classrooms helps bridge the gap between mathematics and everyday experience and its expression in everyday language. The precision of math terms could be approached from this starting point. Likewise, using exploratory writing in math writing journals might illuminate the link and indicate to teachers where links were not being made. It is interesting to note a shift of emphasis in the literature from a focus on how to teach children to read mathematical text to the use of writing in math lessons (Abel and Abel 1988; Davison and Pearce 1988; Johnson 1983; Miller 1989; Nahrgang and Petersen 1986). This may be part of a shift from a subject focus in the math curriculum to a learner focus. Teaching reading skills to math students implies that students have to be changed to fit the subject; using exploratory talk and writing implies that the subject has to be changed to fit the learner. However, before conservatives see this proposal as a loss of rigor, this latter change, as far as language is concerned, is strategic in that it permits more students to pass safely through the door to the discourse community of mathematicians where everyone is at home with technical terminology and grammatical complexity. Then no one need approach percentage through the experience of drug dealing.

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Chapter 6
"Why Are We Doing This?": Reporting Back on Mathematical Investigations

David Pimm

This chapter demonstrates how research and practice can together offer teachers the best in guidance and understanding of what is happening when students learn mathematics. It also offers a glimpse of how distinctions made linguistically can lead to insights into what it means to know mathematics as well as to ways in which to enhance mathematical understanding in the classroom.

David Pimm considers the following a significant difficulty facing all mathematics teachers:

> *How to encourage movement in their students from sole use of the predominantly informal spoken language with which they are fluent . . . to a range of language modes and styles including the formal written language that is frequently perceived to be one hallmark of successful mathematical study.*

The movement from informal spoken language to the more formal written mode is detailed within the context of mathematical investigations by focusing on the process of reporting back to the whole class. David Pimm provides excerpts from lessons to illustrate his points and draws his conclusions within the context of the research literature surrounding his project.

This chapter will be of particular interest to those who attended his two sessions at the MCATA annual conference in Edmonton in 1991.

Many aspects of and relationships between mathematics and language can be highlighted as part of the mathematics education enterprise. In this article, I will explore just a couple of aspects related to the teaching and learning of mathematics, but it is crucial for all of us to find ways of talking about the varied components of mathematical activity itself, which is a complex phenomenon. One means for achieving this is to focus on particular features of doing mathematics, which might then afford teachers greater insight into what is happening in and between their students when in the mathematics classroom. This article and this monograph focus particularly on linguistic aspects of doing and teaching mathematics, in particular the social context and community that classrooms exemplify. (Aspects of some of the themes mentioned in passing in this article are examined in greater depth in Pimm 1987.)

Spoken and Written Language

As Douglas Barnes (1976) points out, communication is not the only function of language. For instance, externalizing thought through spoken or written language can provide greater access to one's own thoughts (for oneself as well as for others), thus aiding the reflection process, without which learning rarely takes place. In mathematics, language can also be used to conjure and control mental images (see, for example, some of the mental geometry activities in Beeney et al. 1982). Spoken and written language have many characteristics and functions, and it can be useful for mathematics teachers to be aware of them to encourage and foster appropriate growth in their students' language abilities in mathematics.

Written language externalizes thought in a relatively stable and permanent form, so it may be reflected on by the writer and provide access for others. Writing things down can be used to find out what one thinks, enable one to refer back to something later (serving as an external memory) or send a message to someone who is not present. As a consequence of the need to fulfill these functions, one common characteristic of written language is for it to be more self-contained and able to stand on its own, with far more of the referents internal to the formulation, than spoken language, which can be employed to communicate successfully despite being full of indefinite "theses," "its" and "over theres," from other factors present in the communicative situation.

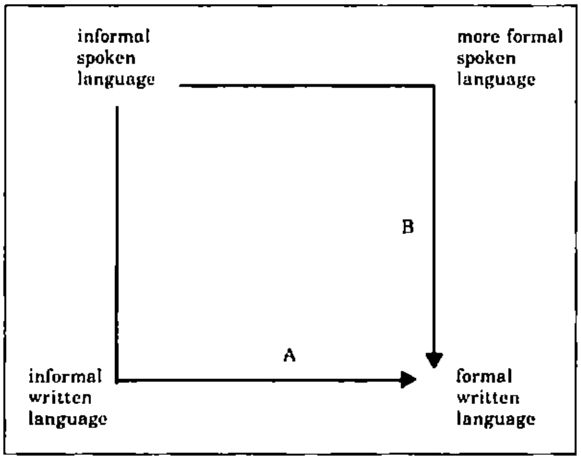
Speech frequently fulfills more direct communicative functions, but because speech is invisible, it does not persist except in memory. Its intangible quality renders it less permanent but more readily altered and revised. In addition, spoken language can be used flexibly to point, to focus attention, in situations where physical indication is not possible or prohibited for some reason.

Spoken and written language are rarely interchangeable, in that choice alone seldom determines the channel of communication. Other dimensions which frequently affect this decision include the weighing of private/public, informal/formal, interactive/one-way, face-to-face/disembodied, spontaneous/planned and context-bound/context-free. (See Rubin 1980 or Hudson 1984 for further exploration of these differences, as well as those of structural organization.)

One difficulty facing mathematics teachers is how to encourage movement in their students from sole use of the predominantly informal spoken language with which they are fluent (Brown 1982) to a range of language modes and styles including the formal written language frequently perceived to be one hallmark of successful mathematical activity.

It is a probable pedagogical error to assume continuous development *from* spoken *to* written language. It is not a linear move—students need to develop particular skills with both channels. Nonetheless, two basic ways are used to approach development of written mathematics moving out from a presumed strength of spoken skills and the context of the classroom. The first (and far more common) is to encourage students to write down their informal utterances and then work on making this written language more self-sufficient (route A, Diagram 1), for example, by using brackets and other written devices to convey similar information to that which is conveyed orally by stress or intonation.

Diagram 1



James and Mason (1982) discuss 10- and 11-year-old students moving back and forth between various spoken and written representations of combinations of Cuisenaire rods, focusing directly on questions of reaccessing the original situation from the linguistic representation to provide criteria for judging the adequacy of a given written expression. One example they discuss is how a spoken description of "pink and white [pause] four times" can be recorded in various mixtures of mathematical symbols and written words and whether other interpretations are possible. Written forms explored included

4 Pink and White
4 Pink and 4 White
Pink and White x 4
Pink + White x 4

Repeated access over time is another essential function of written records, as Martin Hughes' work (1986) with much younger children makes clear. Hughes invited preschool and early elementary age children to make marks on labels attached to the lids of tins, each of which contained a different number of cubes. The lids were replaced and the tins shuffled: a game was then proposed in which they were to guess how many cubes were in each (or a particular) tin. His categorization of responses into four types (idiosyncratic, pictographic, iconic and symbolic) provides a useful means of talking about many students' attempts at symbolization at different age levels.

Further research exploring the self-contained aspect of written messages includes the study by Balacheff (1988) of 13-year-old students' notions of proof. The activity was to

> write a message which will be given to other pupils of your own age which is to provide a means of calculating the number of diagonals of a polygon when you know the number of vertices it has.

Students in pairs wrote their claims (on a single sheet of paper) about this mathematical situation to communicate what they had found out. By providing them with plausible justification for writing a message, Balacheff was able to access their level of writing proficiency as well as to explore the various styles and conventions of justification employed (and, somewhat incidently, their understanding of polygons).

A second route to greater control over formal, written mathematical language (route B, Diagram 1) might be to work on the formality and self-sufficiency of the spoken language before it is written down. For this to be feasible, constraints must be placed on the communicative situation to remove features allowing spoken language to be merely one part of the communication, rather than the entirety of the message.

Such situations often have some of the attributes of a game, and provided students take on the proposed activity as worthy of engagement, then those students can rehearse more formal spoken language skills in a setting which requires them. One such scenario is provided in a lesson where the focus of mathematical attention was a complex geometric poster (Jaworski 1985). Students were invited to take the hot seat (a chair in front of the poster facing the rest of the class) and to say what they had seen to the rest of the class, without pointing or touching. These constraints focused the challenge onto the language being used to point at the picture.

The situation is an artificial one. In real life, one can often point which, together with spoken language, is completely adequate for effective communication. However, provided the artificiality is accepted (as the rules of the game), learning can take place that would otherwise not so readily have happened. There is an interesting paradox here in how quite artificial teaching can give rise to natural learning in certain circumstances. The point of being in mathematics classes is to provide students with experiences they would not get elsewhere. The mathematics classroom is not and should not be a natural setting, and attempts to make it so diminish its potential power.

A second instance of attempts to work on pupil speech directly, and the one I will focus on for the rest of this article, comes from the context of mathematical "investigations," when students are invited to report back to the class about what they have done and found out. As the background assumptions (explicit and tacit) of such activity may not be familiar, I first outline some of the traditions that have developed in England with respect to students' mathematical activity under the generic heading of "investigations."

Mathematical Investigations

Broadly speaking, one intent of the "new" English teaching in the 1960s was to move away from teaching language syntax and the study of acclaimed literary works toward greater attention to self-expression and creative writing. Students were supposed to formalize their informal writing gradually (in a way similar to route A); yet, among other things, this required students to reinvent precisely those conventions of written language that are essentially arbitrary.

While there was a similar "new" mathematics, it was less a comparable refocusing along the spectrum between individual and cultural expression and values than a replacement of one set of content aims by another. Closer attention to student creativity in mathematics, and an acceptance that teaching the grammar of mathematics together with a study of culturally-enshrined works or theorems was an impoverished view of potential student involvement in mathematical activity, was only widely accepted a decade or more later. The 1980s, not the 1960s, was NCTM's "decade of problem solving," and even then many of the examples of so-called problem solving did not allow students much scope for exploration or invention.

In England, *Mathematics Counts*, known as The Cockcroft Report, was a seminal national government document on teaching mathematics in elementary and secondary schools (Department of Education and Sciences 1982). One paragraph in particular (para. 243) stated:

> Mathematics teaching at all levels should include opportunities for:
> - exposition by the teacher;
> - discussion between teacher and students and between students themselves;
> - appropriate practical work;
> - consolidation and practice of fundamental skills and routines;
> - problem solving, including the application of mathematics to everyday situations;
> - investigational work.
>
> [my bullets]

Three of these modes (discussion, practical and investigational work) were relatively novel (particularly at the secondary level), and the report has led to the promotion of these activities in mathematics classrooms. The report also legitimized the actions of those teachers who had been moving to broaden the scope of appropriate mathematics classroom activities. Among others, increasing numbers of members of the Association of Teachers of Mathematics, frequently writing in the Association's *Mathematics Teaching* journal, had been describing and indirectly promoting offering students open-ended mathematical situations for exploration over the preceding 20 or more years.

Eric Love (1988, 249–51) has produced a thoughtful account of this period, analyzing the ways people try to describe mathematical activity beyond a purely content description and analyzing in particular the birth of the mathematical "investigation":

> In contrast to the tasks set by the teacher—doing exercises, learning definitions, following worked examples—in mathematical activity the thinking, decisions, projects undertaken were under the control of the learner. It was the learner's activity. . . .
>
> In the early writings on mathematical activity, there is no mention of "investigation" in the sense of "doing an investigation." It is interesting to see this construction develop from pupils "investigating such-and-such," or "carrying out an investigation into such-and-such," no doubt originally as a shorthand, but soon taking on a life of its own. The path to formalisation had begun.
>
> In the last decade "investigations" have become institutionalised—as part of formal requirements for assessment of courses. . . . They also appear in the official recommendations of Cockcroft and HMI.*
>
> Such a development is a typical one in education—the often commented upon way

* Her Majesty's Inspectorate (HMI) work for the Department of Education and Science, and their primary brief is to inspect and write reports on the state of teaching in the public schools.

in which originally liberating ways of working become formalised and codified, losing their purpose as they become adapted for different ends or by those who have no personal commitment to the underlying intentions.

One example of such "institutionalization" has been that post-Cockcroft paragraph 243, there has been the emergence of the phrases "investigation lesson" or "discussion lesson" in the same way as the terms "algebra lesson" or "decimals lesson" would formerly have been used to describe the main focus. Such use is another example also of this process of action turning into noun, whereby everything ends up as content to be taught in a lesson. These expressions mistake the means for the end—the end is not just discussion per se. I have written elsewhere (Mason and Pimm 1986; Pimm 1987) on the naive assumption that student discussion is wholly and everywhere a good thing, rather than being a particular tool with strengths and weaknesses available for use in various situations.

The "investigation lesson" has its common structure of whole-group posing of the task, students then working together in small groups and then finally reporting back to the whole class. This format has become an example of the new orthodoxy and, unexamined, can be as harmful to the health of teacher-student communication as the old pattern of the teacher-led, blackboard-focused lesson. Nonetheless, the activity of reporting back raises some interesting linguistic questions, as I indicated earlier, in relation to student language and styles of speech in the mathematics classroom context.

Reporting Back to the Class

Because of the more formal nature of the language situation (particularly if rehearsal is encouraged), reporting back can lead to more formal, "public" language being used and to greater structured reflection on the task. Thus the demands of the situation alter the required language use. When preparing for course work write-ups, for example, a prior stage of oral reporting back can help with selection of material and the emphasis placed on various parts of it.

One key question is for whose benefit the reporting back is done, which may be particularly pertinent if you have observed sessions that were apparently painful for some participants. Requiring students to report back can also be intrusive and perhaps unhelpful if some groups are not at the stage of summarizing what they have done and are still engaged with the problem. Take some time to think about who benefits (if anyone) and why.

Three possible answers and corresponding sets of potential justifications to this question follow:

1. The student(s) doing the reporting: For them, plausible justifications include developing a range of communication skills, their use of language and social

confidence. A further important possibility is developing skill at reflecting on a mathematical experience and distilling it into forms whereby they and others may learn.

2. The other students listening: For them, potential benefits include hearing alternative approaches and that perhaps others besides themselves had difficulties, the chance to ask genuine questions of other students and to engage in trying to understand what others have done on a task that they also have worked on.
3. The teacher: Potential benefits include a range of opportunities to make contextually based meta-remarks about methods, results and processes (perhaps indicating the task is open and that a number of ways of proceeding are possible or different emphases can be placed), as well as to value students' work and broaden their experience (possibly by sharing an original idea with the rest of the class—"Did anyone else. . . ?").

   Reporting back can also provide the teacher with access to what the students think the task was about, as well as less tangible but nonetheless interesting information such as what they think is important (in terms of what they select to talk about) and their level and detail of oral expression of mathematical ideas. But it can also be *assumed* that reporting back *has* to happen, perhaps meeting the need (or expectation) for some sort of whole-group ending, "pulling the session together." If that is the case, then many of the above benefits may not accrue, because attention is not being paid to drawing them out.

Here are five further questions that form key issues involving mathematics education and language arising from the task of reporting back on mathematical investigations. Bear them in mind (and formulate more of your own) as you read through the classroom transcripts provided:

1. What sort of language (style, structure, organization, register, and so on) does the student reporter use?
2. How can the tension between wanting the student(s) to say themselves what they have done and wanting to use what they say to make general remarks about how to undertake investigative work be handled?
3. How can students develop the skills of selection of and reflection on what to report? How can they acquire a sense of audience?
4. To whom is the reporter talking?
5. What justifications does the teacher explicitly offer the class for having them report back on their work to the rest of the class?

To provide a clearer feel for what is involved in practice, I give below two excerpts from actual class lessons using reporting back. Then I explore some of the more general issues arising from inviting students to engage in such a task, organized around the five questions offered above.

The Lessons

Both lessons have been transcribed from videotape, and much of the relevant detail is nonverbal: who is looking at whom, where people are standing or sitting, what equipment is being pointed to and so on. As reader, you face the problem alluded to earlier of only having the written record to interrogate. Nonetheless, I hope that these accounts provide you with some actual detail of what might be said and how report-backs in classrooms might be carried out.

Lesson 1

This lesson (which takes place over three 50-minute classes) is with a mixed-ability class of 30 10- and 11-year-olds, and teacher Steve Wilson is working on a mathematical investigation involving movements on a square grid. Students have previous experience of a similarly investigative nature and are developing their ability to explore questions and apply mathematical processes such as predicting and testing, and convincing others. Recording ideas and results forms an important part of their work.

Steve starts the lesson with the whole group by having a 3 x 3 set of mats on the floor and inviting eight pupils to stand on them, leaving one corner mat free. The pupil (Robert) in the diagonally opposite corner from the empty mat is asked to wear a red cap, and the first task is to see if, by moving sideways or vertically only, Robert can be moved to the currently empty (target) square. It proves possible, moves are counted and further attempts have external leaders guiding the students. The class counts the number of moves so far aloud as a group. Steve then introduces the question of the minimum number of moves possible and alerts the class to the possibility of exploring different-sized grids with the same question.

In an interview after the lesson, Steve comments, "There will be sessions where I bring it together to compare and to sort of look at the work they have done so far."

The report-back phase of the second lesson went as follows:

Steve: There are a lot of good ideas that have come out of the various groups. I had a chance to see those ideas because I got to go around and see what you are doing. But it would be quite nice if you could also share those ideas with the rest of the class, so they can see the sort of things you've been doing. It may help to give some ideas to you as well.

We've got the overhead projector [OHP] if you want to write anything on the screen; we've got our equipment over there—our cubes, squares, the same sort of things we've been using before. If you want to demonstrate on the OHP, that's fine; if you want to use a table, that's fine; and again, if you want to just speak, that's fine.

One person from the group or two people can come up and explain—it's up to you, alright? Just to explain briefly what you've achieved over the last day or two, [pause] let's start with this group, shall we? Who's going to come up from this group?

Come and stand here. [near him]

Paul: [very hesitantly] We've been trying to see if there's eight between all the numbers and we found out there was—and the more squares there are, the harder it gets.

Steve: OK. The harder it gets—stay there. [near to group's table] Do you want to add anything to that?

[No response]

You found this difference of eight does carry on no matter how many squares you've got in there. But you're also saying the larger the size of square, the more difficult it is . . . to do what?

Paul: I'm getting a bit confused.

Steve: Is counting the problem or is moving the problem—are you happy where to move?

[Paul nods]

So it's keeping count. [turns to rest of class] Did anybody else find keeping count was a problem? . . .

Let's go onto this group—Diane, Kevin, Mary.

Mary: [standing at the OHP, reading from her book] We were checking out the pink [the target cube was pink in their representation of the problem using Unifix apparatus] and the way in which the blank moves. The thing about the eight—the eight does carry on; you keep adding eight every time, and the stair of the pink—we tried it on a 2 x 2 and a 3 x 3, 4 x 4 and I think we tried it on a 5 x 5, and everytime it went in a staircase and we had a bit of trouble with the blank, because it kept going back on itself, so we lost track of where it had got to.

Steve: Right. Following the path of the blank—let's just remind ourselves of that. Here's the cubes. Can you demonstrate what you did with a 3 x 3?

Mary: The blank goes from there to there, then it goes round to there, then we go back to that square, so it's retraced and gone back to there, so we've got two arrows going round in the square, so it goes back to there again, to the centre, so you've got another loop. Then it goes back to the top where you haven't had it before, then it goes back down again, so everytime the arrows are going back to the same place that they were before. So you've normally got about two or three arrows going to each square.

Steve: I think it's quite interesting. I don't know if you can see this [holds up her book to class], but they've actually traced the path of the gaps and also the target one as well, and you can see they use a system of arrows. We can actually trace this method of going round and round. It's quite interesting, isn't it? How we make a note of these things is quite difficult sometimes. Thank you very much.

Emma: We did a table [starts to draw table, with whole numbers along both sides]. This is the number of moves and this is the number of squares. Then we did the arrows.

Steve: [to rest of class] Do you fully understand what she is saying?

Class: No.

Steve: I'm not sure that I do. However, if I can hold this up again? You're trying to plot the moves, aren't you?

Emma: Yes.

Steve: If we can go back to your 3 x 3 grid. That's quite interesting. . . .

[In many of these interchanges, Steve is standing at the front near the student speaking, but to one side facing the rest of the class. He quite often repeats, rebroadcasts and reinterprets what the students are saying. This is particularly true of the next pair.]

John: We've not been taking 7 x 7 grids, but like 6 x 9 ones; then we added one onto the first side and took one off the other side until you got down to where you couldn't move any more.

6 x 9 = 33 moves
7 x 8 = 39 moves

Then we started predicting them as we came up with a pattern in the results.

8 x 7 = 41
9 x 6 = 45
10 x 5 = 51

Steve: Can you see what they are doing?

Class: No.

Steve: They're working on a different-sized grid. They're working on rectangular grids. . . .

Simon: To find any number, for example, 18 x 18, minus 2 to make 16 x 16, and then times 16 by 8 because the answers jump in 8, and each time then add on 5 from the first one, 2 by 2, because you don't have 8 there to add on, and you get the minimum moves for 18 x 18.

Steve: Understand that? [pause] A lot of things have happened, haven't they? It shows that if we take a particular problem, you can continue it in lots of different ways. Can you produce theories on how patterns developed? Can you notice the patterns? How do you recall what's going on? Are there different ways of recording and, again, are there any things to notice in the way you record things, are there patterns in those? Can you produce theories? Can you test them, and can you convince yourself that they are correct? If you're convinced, how can you convince other people? Can you produce some sort of a method, an equation, a function rule, to explain it to other people? What I think is also quite interesting is how we can change the problem and both of you [indicates two students] were working on a way of changing a problem.

Lesson 2

This is with an older group of 13- and 14-year-old students and their teacher Peter Gates. The problem was to find out how to make a race once round a track fair; in other words, what was the stagger necessary between each lane on a standard 400 m lap? They were working on scale drawings of the track and using a variety of materials including string. At the beginning of this (second) lesson, they are sitting in groups at tables.

Peter: Let's get started. Now yesterday, we umm, let's just try to cast our minds back to what we were doing toward the end. Remember it was a fairly packed session but you were working on this race track? Can we just try to get each of you to construct in your own mind what the problem was that I set you? OK. Anyone like to remind us what the problem was I set you? . . . Anyone?

John: It was all about the 400 m race track, with four, five, four lanes, and you had to make it fair for each person to run, where you were going to put the start.

Peter: So we're trying to make it fairer and that was basically the problem. Now I've spoken to most of you in your groups yesterday. What I want you to do now, in the next 10 minutes or so, is just to make sure among your group that you've had a fair crack at that problem and that you've got a reasonable solution to it, and then I'd like you to arrange for someone in your group to report to the whole class on how you solved that problem and on what the problem was. We'll then quickly go round each group to see how you each group tackled it individually. So you've got 5–10 minutes to tidy up loose ends. Make sure you know what you are going to say. And then we'll come back together as a group.

Can I remind you of the very last thing that you did last session. . . . What was that? Don't put your hands up. What was the very last thing you did? [pause] All got it? If I remember rightly, I asked you to jot down in your books where you were, to write down what you were doing, what you were thinking and so on. The reason for doing that was that you were going to come in today, a new day, the start of a new day and have forgotten what we'd done.

Can you look now, before we get started, at what you wrote yesterday? Read through it and see if it helps you get back into the problem. If it helps you, you wrote it well yesterday. If it doesn't help you much, you didn't do it well yesterday.

[They then work for 15 minutes in small groups.]

Can we come back together then? Right. Can you remember as we go round that part of the reason for doing this is giving you an idea of how other people tackled the problem? No group here knows what any other group did at the moment. Another part of the reason for doing this is so that we can share our ideas. Another reason, or another thing we can get out of this, another advantage, is that the person who's speaking is having to put their ideas into words. It helps them if you can remember that and try to be as encouraging as you can.

What I want [to know] is how you managed to tackle this problem and the solution if you have one. [pause] OK?

[cuing the first group] Robert.

Robert: To start with, we measured the diameter of each lane, we multiplied it by 3.14 to find the circumference, and we measured the straight which was 9.6 cm and we added it to each circumference to find out the length of each lane. We found out that lane 2 was 1.57 cm longer than lane 1, lane 3 was 1.57 cm longer than lane 2, and so on. So to make it fairer for the runners, we had to set a stagger of 1.57 cm longer for each lane.

Peter: Anyone like to ask any questions? Is that not clear to anyone?

Shital: Did you all get the same answers for the circumference and then the straight? Did you get the same measurements?

Robert: It was 1.57 cm each time.

Steve: We got the same measurement for the straights.

Peter: Is that what you mean?

Shital: Yes.

George: How did you measure the circumference of the track?

Robert: We took the diameter of the lanes and multiplied it by 3.14. I imagined it was a circle.

[silence in class]

Lucy: First, we measured all, each lane with a piece of string. First we measured the outside lane; we found that that was 50 cm. Then the third lane was 49 cm and so on. Each time it went down a centimetre. So we decided that we should stagger them out a centimetre behind each other.

Peter: So on your diagram they are a centimetre behind.

Lucy: Yes.

Peter: Can I just have a look at that? Right, have a look at this then [takes diagram and shows class]. That was Lucy's, Anita's *and Marsha's solution for the problem.* Anything you notice? Any comments? Any questions you want to ask? [pause; cues] Robert.

Robert: What's the rectangle for?

[The rectangle is drawn around the track, touching in four places symmetrically.]

Lucy: We just wanted to see if we had a rectangle, how big the whole thing would be—if it was like that. It was 62 cm, but we found it was nothing to do with the track.

Peter: So you drew a rectangle around it to see how big the whole thing was . . .

Lucy: Yes.

Peter: . . . but you found that had nothing to do with the problem. Good.

Sarah: How did you work out the stagger in each lane?

Lucy: We measured it with the string, and each time we measured it it was 1 cm apart for each lane. If we took it away, no added it, to each lane a centimetre, they'd all be the same.

Peter: What is the real stagger? [The drawing is 1 cm to 8 m.]

Lucy: Eight metres, that's what we thought.

[Peter cues Lewis]

Lewis: Well, first we decided to draw a rectangle around the outside of the track, and we tried to measure this as accurately as we could. And then we drew in diagonals to find the centre; then we drew in horizontal and vertical lines to split the track up into four sectors, 100 m in each sector. And then we measured the inside lane up to the first sector and multiplied by four. We kept on finding it wouldn't tally up, it wouldn't go up to 400 m, and after a lot of discussion, we found out that we had drawn the rectangle so it was practically a parallelogram—it wasn't a rectangle at all. The sides were all wrong. So we had to get a new piece of paper and set it all out and draw it again; we've just started to measure the inside lane, and it has just started to tally up now. That's about as far as we got.

Peter: Is everyone clear about what they think Lewis is saying? Notice how close that group's answer is to this group's. It's been interesting to see the different approaches, bits of string, pi and so on, and you've all come to similar conclusions.

Gordon: What's the triangle got to do with it?

Lewis: Triangle? Do you mean diagonal? It helps you find the centre of the track.

[Peter cues Shital]

Shital: We all measured the straight on our own pieces of paper and we all got different answers for that, some were 8.5, some were 9, 9.6 and like that. Everything we tried, we got different answers from everyone and every time we double-checked it, we got other different answers. So we didn't really get anything except we got one measurement and that was around 12 m between the two positions. That was about it.

Peter: Alright, 12 seems to be at least the right sort of order. It doesn't appear to be way out, but didn't you get anything at all out of the activity? Anyone in the group can answer that one. Did you get anything at all about doing that activity? [Students look at one another.] No? Anyone else care to say something about what you can learn or what they might learn from that experience? [pause] I'll give you my story. My impression of it is that when I was talking to those students one thing they said to me before is how they didn't seem to be working well together. They were all doing it on their own, all doing their own individual problem, and I've got the feeling that at the end of the day, or the end of that problem, they were trying to solve it themselves and were getting frustrated they were all getting different answers. I think we can see that each group has got slightly different answers from other groups, but it's the way you tackled the problem that's been nice and interesting and similar. . . .

Questioning the Lessons

1. *What sort of language (style, structure, organization, register, and so on) does the student reporter use?*

   How can the teacher help resolve the tension between the fact that increased preparation time can improve performance but can also emphasize the public-speaking aspect of the event, with its attendant social pressures on performance? Spontaneous requests, however, without much time for reflection or organization, though offering a potentially less formal context for speaking, can merely result in recollections of the last thing students were doing.

   Steve's class was still using informal, context-specific spoken language. This may have been due to the nature of what they were trying to say but also may involve the OHP as a means of amplifying a local conversational setting (where such informality might be more appropriate) into a global one.

2. *How can the tension between wanting the students to say themselves what they have done and wanting to use what they say to make general remarks about how to undertake investigative work be handled?*

   This tension can be particularly strongly felt in cases where teachers have seen something that they value as a higher-order process (specializing systematically, developing notation, coping with getting stuck or whatever) while circulating around the small groups. The process may not have been a salient one for students (unless the teacher made a big point of it at the time), so they probably have little idea either of what to emphasize or why this particular incident is being focused on. If the teacher has made the point to them, why are they being asked to repeat it?

   Teachers are reluctant to say for the students what one of the students has done, instead of prompting with something like "Why don't you say it, because you did it?" A truthful response from the student might be "Because it is *your* anecdote. I don't know what significance it has for you." Students do not have the teacher's reasons for highlighting particular parts of what they have done. It is far better to say something like "I want the rest of the class to know about something related to what you did. At the moment, I am the only one who knows what that is. What I saw you doing was . . . and it struck me because. . . ."

Peter Gates made a particular point of this in his last interchange where he told the group what *he* thought they might have gotten from the activity. Steve Wilson had a number of examples where he made general points out of the particular report-backs he had heard, and he, on occasion, asked the general audience whether they had experienced something similar to what had just been reported.

3. *How can students develop the skills of selection of and reflection on what to report? How can they acquire a sense of audience?*

Obviously, this is contingent on the perceived audience and purpose behind carrying out the reporting back. Who knows what might be worth telling about? The teacher has no control over what comes out. He or she can only work on it after it has emerged from the student, though it is likely that what they (student and teacher) then do with the response will influence later reporters.

One important skill is the ability of students to disembed their discourse from the knowledge of the group who saw the work developed, so that someone who was not there can follow what is being described. The tendency is for the reporter to assume that everyone will know what he or she is talking about.

One difference between Peter's and Steve's classes is the students' willingness to ask each other questions after a report-back. Steve, by rebroadcasting, has already done it for them.

4. *To whom is the reporter talking?*

Students often address themselves to the teacher, the person who, besides their own group, probably knows most about what they have done—and the students know this. How might the teacher deflect the reporter's attention to the rest of the class?

In Barbara Jaworski's account (1985) of a poster lesson she comments:

> Most pupils, on taking the hot seat, started off addressing their comments to Irene [the teacher], stopping now and then in what they were saying to allow her to comment, or to solicit her comments. In some cases she replied to them or prompted them directly, which then encouraged a two-way exchange with the rest of the class as audience. In order to get the rest of the class to participate she then had to overtly invite comments from them. What in fact started to encourage more general discussion was Irene's deflecting of the invitation to comment to others in the group.

If the teacher reinterprets what the student says for the rest of the class (possibly by playing a role, which in television interviews is known as "audience's friend"), what effects might this have on the reporter? By playing "audience's friend," the teacher can ease the strain on reporters, by taking the focus off them, possibly by reinterpreting or expanding for the audience and then moving into more general questioning of the class. This role was particularly apparent in the pattern of interchange in Steve Wilson's class. But it also may result in the teacher being looked to as broadcaster and interpreter of the person reporting back and thus acting as an intermediary between reporter and audience which may get in the way. After all, if reporting back were an effective technique in and of itself, there would be little need for the teacher to intervene—the reporting back would do its own work.

If the teacher has adopted this role, then he or she may be asking questions on behalf of the audience or more directly explaining the student's words to the others. Whether the teacher says "tell us" or "tell them" may be an important difference in cuing reporters as to whom they should be facing and speaking.

Where is the teacher standing and where is the control? If there is a silence, whose responsibility is it to fill it? Where is the audience's attention and who are they asking questions of? If it is predominantly a conversation between the reporter and the teacher, what is the intended role for the students who are being invited to listen?

What are the students supposed to be doing while the report-back is being given? What would you (as their teacher) like to be happening in their heads and what could you do to help bring it about? What techniques do you employ for deflecting or involving the audience? Are students too concerned about the fact that their turn is coming up (and are perhaps rehearsing what they are going to say) to attend to what the current reporter is saying? How can they be encouraged to be active listeners?

5. *What justifications does the teacher explicitly offer the class for reporting back on their work to the rest of the class?*

   In the two excerpts, the teachers were trying to say why they value reporting back. In particular, look back to what Steve said at the beginning to set up the reporting back. How does what the students did relate to this? How does what he said differ from the guidance Peter gave to his older students?

   What covert justifications does the teacher have? What are various student views about why they have been asked to engage in this activity? Do they mirror what the teacher has said?

   There can be some difficulty conveying to students what it is they are being asked to do: If the activity is too vague, the students do not know why they are being asked to carry it out and can flounder; if too precise, students will tend to do exactly that, thereby constraining what might happen. Overspecification also retains the teacher's control and initiative rather than handing this over (at least in part) to the students.

   This is a familiar tension which lies at the heart of teaching. One general formulation [referred to there as the didactic tension] runs as follows (Mason 1988, 33):

> The *more* explicit I am about the behaviour I wish my pupils to display, the more likely it is that they will display that behaviour without recourse to the understanding which the behaviour is meant to indicate; that is, the more they will take the *form* for the substance.
>
> The less explicit I am about my aims and expectations about the behaviour I wish my pupils to display, the less likely they are to notice what is (or might be) going on, the less likely they are to see the point, to encounter what was intended, or to realise what it was all about.

Conclusion

Madeleine Goutard (1968) claimed that one role of the teacher when working with students on their own mathematical activity was to "help the students follow their own intentions through, strengthen their own intuitions and carry their own creations to a higher level." Reporting back can assist in these aims, but considerable care needs to be taken to ensure that this is achieved. Not least, this requires students to take on the activity of reporting back as one *they* choose to undertake, rather than merely another task that they are doing "for the teacher."

Reporting back can place some quite sophisticated linguistic demands on students in terms of communicative competence—that is, knowing how to use language to communicate in certain circumstances. Here it includes how to choose what to say, taking into account what you know and what you believe your audience knows. Educational linguist Michael Stubbs (1980, 115) claims, "A general principle in teaching any kind of communicative competence, spoken or written, is that the speaking, listening, writing or reading should have some genuine communicative purpose." Yet this is at odds with my earlier comment about the classroom being an avowedly unnatural, artificial setting, being precisely the place where all the necessary learning that does not take place naturally and spontaneously has to be confronted. Nonetheless, provided the pupils are willing to take on whatever constraints the activities entail, then there is the chance of their learning. It is still an open question as to whether or not working directly at increasing the formality of spoken mathematical language, prior to working on improving written control of language, will assist written fluency. I feel it is a worthwhile prospect that deserves further exploration.

Students learning mathematics are acquiring communicative competence in mathematical language, and classroom activities can be examined from this perspective to assess the opportunities they offer for learning. Teachers cannot make students learn—at best, teachers can provide well-thought-out situations that allow students to engage in mathematical ideas and develop skills that use spoken and written language to that end.

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Note: The videotapes transcribed in this article form part of the Open University mathematics education course ME234 "Using Mathematical Thinking." For more information about this course, contact the Centre for Mathematics Education, The Open University, Milton Keynes, U.K.

Part III

What Kind of Teaching Can I Practise?

Chapter 7
The Converse Structure of Communicative Classrooms

Daiyo Sawada

The two stories in Part III may be of particular interest to those who have so far concluded that there are many effective ways to bring communicative activities into the classroom (Part I) and that these activities do enhance mathematics learning in ways that are unique to these activities (Part II) and also to those who may be ambivalent about whether or not their own teaching style is compatible with a communicative approach.

More specifically, if as you consider how that communicative nature of your teaching can be enhanced you get the nagging feeling that teaching mathematics communicatively is not simply a matter of adding communicative activities to your repertoire, then the two stories in this chapter and the contrasts between them are for you.

"Where am I going to get the time for all this language activity during math class?" I shall answer this question (and others as well) by contrasting the structure of traditional classrooms with what I call communicative classrooms by telling two stories.

Telling the Stories

Here are my two stories told in brief form:

The Traditional Classroom Story

What, When, How, Why, Who, Where

The Communicative Classroom Story

Where, Who, Why, How, When, What

I did say these versions were brief! Nevertheless, several interesting points emerge immediately. For example, the communicative story reverses the traditional one; the communicative story is the *converse* of the traditional. However, the stories are equivalent in the following ways:

1. Both are the same length.
2. There is a one-to-one correspondence between the story elements. In this sense, each story has the same "cardinality."
3. As "sets," each story contains not only the same number of elements but also exactly the same elements, and as such they are not only equivalent but also identical.

From the perspective of set theory, these two stories are identical, and contrasting or comparing them would be pointless. However, from a topological perspective, these stories are not equivalent. For example, in the traditional story When comes after What, but in the communicative story When comes after How. The topological connections are different. Although we do not normally think of classrooms in topological terms, topological relations in the communicative story, such as "betweeness," "proximity," "closure," "neighborhood" (and should we add "community"?), are very appropriate in considering communication.

To discuss these topological notions without sounding too technical, let me transform the word *set*. From the point of view of sets there is little to talk about. If we transform *set* into *setting* however, we have a very topological-like word—a setting is like a neighborhood and a neighborhood is a supportive context for communication and community. Now if we transform *setting* into its use in drama, then we have a very supportive setting for telling a story. Consider the brief stories above as settings for telling stories about classrooms. These settings could even be considered topological versions of story structures.

Retelling the Stories

I will retell each story using the brief version story structures. Because the drama metaphor seems appropriate to this setting, I will cast the stories in dialogue form.

Cast

You (a teacher at Atrebla Elementary School)
Me (author of this chapter)

Setting

Staff room

Scene I Setting the Scene

You: Can I get you a cup of coffee?

Me: Could you? Black, please. [settling into a comfortable chesterfield]

You: Thanks for taking the time to come to Atrebla. When you said you could come anytime, I didn't think it would be this afternoon.

Me: Usually it takes us longer, but you caught me just before lunch; besides, Atrebla is just an hour and a half out of Edmonton, and I wanted to catch you while you were still reading this monograph.

I'll begin by retelling the traditional classroom story and follow up with the communicative classroom story.

Scene II Retelling the Traditional Classroom Story

Me: The traditional classroom story begins with the What, the *what to teach.*

You: You mean the curriculum?

Me: Yes, the content objectives determined by Alberta Education and laid out in five basic curricular strands.

You: Yeah, I know: problem solving, numeration, operations, geometry, measurement and data management. If we know what's good for us, we had better cover the curriculum.

Me: You make an important point which underscores why the What comes first. Coming first, it means that the curriculum has priority. The traditional story begins with What, with the curriculum, with Alberta Education. These are taken as givens.

You: This is like problems structured with "the Given" and "the Required." We start with the Given.

Me: It seems obvious, doesn't it?

You: Almost common sense. Let's add the When.

Me: By bringing in When at this point, we sequence the What with respect to time.

You: You mean we make a scope and sequence chart!

Me: Yeah! In curriculum jargon, What and When together make a scope and sequence chart. It's not surprising that this chart has been the dominant storyline of all program development in school mathematics—program development can't be done without one.

You: I hope you're not going say that communicative classrooms won't need scope and sequence charts!

Me: You're already anticipating! But that will come later in the flow and won't determine the storyline as it does here.

You: I'm just making a mental note to come back to this point. Let's bring in the How. This seems to me to be the nuts and bolts of teaching.

Me: Once the scope and sequence chart is taken as a given (the What and When), then the next step is to tell teachers in some detail how to implement this chart on a day-to-day (lesson-to-lesson) basis (the How or How to).

You: If the What and When are *curriculum development*, then the How is *curriculum implementation.*

Me: Exactly. Some people call it *instruction*.

You: "I want something (the What) I can use (the How) on Monday morning (When)." I hear teachers saying this every time they go to an inservice session or a conference. If they don't get that, they feel it's a waste of time. I know I'm getting ahead of myself again, but are you going to say that teachers in communicative classrooms will want something different?

Me: You be the judge of that when we get there.

You: You know if the What, When and How are curriculum and instruction, then the Why has got to be rather redundant.

Me: You're right. In the traditional story the Why is generally taken for granted or presumed to be the responsibility of the program of studies or textbook writers. If a student asks "Why are we learning this?", the standard response is that it is needed for next year's work. In other words, the answer to this Why question is already taken to be given in the scope and sequence chart (the What and When)—the Why is reduced to the What and When. In this sense, the What and When have the greatest priority, powerful enough to account for Why.

You: But this kind of answer to a Why question is inadequate. To me, Why questions ask for something deeper than sequence. I'm making a mental note of how this will be different in communicative classrooms because then the answer will have to be given in terms of the Who and Where. Am I correct?

Me: Structurally you have to be, but there may be more to it than structure. If we tell a story backwards, maybe it is not just a reversal but as well a change in the nature of the story. For example, if in a detective story the conclusion is already revealed as the story begins, that story will be quite different from one that doesn't reveal who did it until the very end. We need a literary theorist!

You: I don't think literary theorists are interested in backwards stories, and maybe curriculum theorists aren't interested in backwards pedagogy either! [chuckle, chuckle]

Me: I don't think curriculum theorists are interested in a backwards pedagogy, but maybe, just maybe, teachers might be and maybe children might be even more interested!

You: You're getting me interested. Let's add the Who. I'm particularly interested because of my early childhood background.

Me: Great. So now we have What, When, How, Why and Who.

You: Do you mind if I do a little anticipating? I think the Who like the Why is going to be reduced to the four Ws that come before it. Am I right?

Me: May I answer with a question? What led you to your prediction?

You: Well, I'm just continuing the pattern of the Why being reduced to the When of the What. Second, I was thinking of individualizing instruction as a way of trying to overcome differences among children—we individualize as a way of dealing with their uniqueness, and in this sense the Who is reduced to a How.

Me: You're saying that in the traditional approach we teachers aren't so much interested in children for *who* they are but rather as some differences, mainly psychological, that need to be accommodated through individualizing instruction (the How)?

You: It's my early childhood bias showing through again, but I think that children are interesting because of who they are and that who they are isn't simply something to deal with through some diagnostic procedure. Our society is dominated enough by technology, the "how to" so to speak. As teachers, we don't need to reduce children to a particular sequence of instructional treatments.

Me: That's even more interesting! Now you're saying that the problem of Who is reduced to a problem of How to teach What When.

You: You could say that. And that brings us to the Where.

Me: Yes, the What, When, How, Why, Who and Where.

You: Now I see the pattern. The Where, like the Who and Why that come just before it, is just the continuation of the storyline determined by the priority accorded to the What, When and How in that order. The Where in this story is literally anywhere. The Where is just a place where the Who are often placed into five rows of desks to be taught the What. The classroom is the ubiquitous Where, and I guess all classrooms are pretty much the same—just places for the other Ws to happen.

Me: Aren't you being a little harsh? Is the Where that trivial in the traditional story? It is the last of the Ws and therefore the least in importance, but will just anyplace do?

You: I think I'm justified in being extreme. I don't recall much from the methods courses I took at university, but I do vividly remember one day when the prof asked if any of us could remember where we were the last time we divided fractions. No one could remember the details, but we all knew it was in school. We don't use school math for much outside the classroom. When did you last factor polynomials? I bet it was in math class. The Where has become so irrelevant that it makes what is learned disconnected from anything else.

Me: Maybe that is why school mathematics is so often criticized as having little to do with the so-called real world. Despite recent emphasis on real-world problems, children still have difficulty making applications to real situations. If we keep ignoring the Where, any approach trying to relate mathematics to the real world can only be superficial.

You: With that statement, I am sure we have arrived at the backwards sequence which begins with the Where. But, first things first—your coffee mug is empty!

Scene III The Communicative Classroom Story

Me: Yes, the communicative story begins with Where one learns rather than with What one is to learn. It begins with the setting, much as novels often do. There has never been a story that wasn't set in a particular place. Indeed, without a place, a story is essentially meaningless and cannot be told. A story always happens somewhere, and that particular somewhere sets the tone, sets the context for what happens. In communicative classrooms, the context is more important than the text.

You: Two months ago, I would have said hogwash! How can context be more important than text? But I attended a language experience workshop recently and kept hearing that meaning resides in the context, that meaning is between the lines rather than on the lines. Are you saying it's the same in math?

Me: Precisely, if I may use such an inappropriate word. Traditionally, with priority on the What, meaning was thought to reside dominantly in the text, in the words themselves, rather than the context. The new language-learning philosophy is a severe critique of this tradtition.

You: But we need more than a critique. Teachers need more than talk about context or place. How exactly does one go about setting up a place?

Me: You used the word How.

You: Yeah, I guess I'm getting a bit ahead of the story, but I'm concerned because I don't know of any resources that I could use to set up my classroom as a particular place. I know about material on classroom environments, but most of that is theory. For example, I don't know of any math or commercially available programs that start with place.

Me: If you're right, it shows that the communicative story really hasn't been told in math classrooms.

You: So how does one establish the communicative story structure in the classroom as a way of teaching mathematics? How does one go about establishing a place? What kind of place is it?

Me: Suppose you were to put on an old trench coat, one that smelled as crumpled as it looked, and were to walk into your classroom peering here and there and muttering as you hovered close to a window, "There don't appear to be any footprints here," and were to examine the doorknob carefully and whisper, "There don't appear to be any fingerprints here," and were to look inquisitively at the class and ask, "Who do you think I am?" and suppose, as the hands shot up, you were to continue examining the door and exclaim, "Yes, this lock has been tampered with!" and then, as you pulled out a magnifying glass to take a closer look, you were to notice that nearly all of the students were now waving their hands wildly and shouting exhuberently, "I know, I know!"; suppose, to prolong the ambience, you looked up and uttered the complete innanity, "Let me give you a clue; I am not a dentist," well, the students would be standing up dying to blurt out what they know; the classroom would be a sea of excitement. Finally, if you were to look at a student who rarely volunteered to answer anything in class and say, "   , who do you think I am?", he or she would be shouting "A detective!" before you were finished. All the hands would come down somewhat disappointed at not having been asked. Perhaps one boy would still have his hand up and you would think to yourself somewhat apprehensively, "Maybe I haven't been able to set the scene authentically enough.   - doesn't think I'm a detective." So you would say, "Yes,- ——?" And he would say, "I think you're a spy!"

You: It sounds too real.

Me: That's rather perceptive of you. Two years ago in a Grade 3 class, it's exactly what I did. I'm just retelling the story.

You: Tell me some more.

Me: Sure. Can I tell it as a dialogue?

You: Why not? It would be in keeping with "communication."

Me: Okay. I'll use "S" for the students. This was not my own classroom; I was collaborating with the teacher on a problem-solving project. The children didn't know me at all.

S: Are you a real detective?

Me: I can't really tell you too much. I can show you my card.

[I had made a special ID card on the computer. All it had on it was "D. Sawada, TPD". The teacher had introduced me as Dr. Sawada so the card at least was consistent with what she had said.]

S: What's the TPD?

Me: I really can't reveal that.

[The students accepted this. Such acceptance was part of setting the scene: Detectives can't be totally open about their work. The answer was Tokyo Police Department but, because I wanted a tone of mystery to hover within the room, I chose not to reveal this at that point. Instead I asked:]

When do detectives do most of their work?

S: At night.

Me: Why at night?

S: Because that's when crimes are committed.

Me: Sounds reasonable. Detectives have to be good at detecting at night when it's too dark to see well. If it is really really dark, how can a detective see?

S: With a flashlight.

Me: That's an idea.

S: But if you use a flashlight, you will be a sitting duck!

Me: You've got a point there.

S: Maybe you could use your hands and feel your way.

Me: Yes, you could use your hands to "see"! A terrific idea. Guess what I have in these bags?

[I had selected two identical copies of each of six different solids (wooden prisms, cone, ellipsoids and so on) and had placed one of each pair in individual cloth bags. I placed the other six solids on the table at the front of the room and asked:]

In each bag is one of these [pointing to the blocks], but you can't see which one with your eyes.

S: But we could see it with our hands!

Me: What do you mean you can "see" with your hands?

S: We can feel it with our hands.

[Each student came up and felt a bag, then selected one of the solids on the table and pulled the solid out of the bag to verify his or her vision. I could continue this story, but perhaps I've said enough to set the scene.]

You: I see that in setting the place you also smuggled in some geometry.

Me: Yes, but the kids weren't thinking particularly about the geometry even though they were learning about it. At that point the geometry wasn't important. Setting the scene was important—establishing the Where. Over the next few lessons, the classroom was transformed into a detective agency with each student a detective. At the end of my lesson, I congratulated the students for being able to "see" with their hands and for having the promise of becoming real detectives.

You: Sorry for being skeptical, but did all the children succeed in seeing with their hands?

Me: I was surprised as well, particularly because three of the children were special education students mainstreamed for mathematics. Children are pretty good at seeing with their hands.

You: I can see how a detective agency would be a particular place where children could learn mathematics. Detectives solve mysteries; in math class, students solve problems. Solving cases and solving problems aren't very different. Being a detective and being a problem solver are quite compatible.

Me: A detective agency is just one such place. Many other places could be used in a classroom as well, for example, a collectors' club where children are collecting hockey cards, sea shells, postcards, whatever they collect or a trading post, space station, firehall and so on.

You: Each setting lays out a place: a toy factory, a detective agency, a firehall, a collectors' club, a press room, a trading post, a space lab, an animal farm. These are places (the Wheres) where mathematics (the What) occurs in real ways, or should I say natural ways?

Me: Places where mathematics occurs in culturally meaningful ways. The What (mathematics) arises naturally in a contextually meaningful way. For example, at a toy factory, toys have to be packaged and shipped. How do you package toys? Do you box them in three rows of four or two rows of five? And what if you stack these boxes into crates? How do you arrange the boxes in crates? How many boxes in a layer? How many layers in a crate? And how should we stack the crates on the truck? How many boxes can a truck hold? Mathematics galore, and it all arises in real form in the toy factory.

You: You're using the word *natural* or *naturally* as if the classroom, when set up as a detective agency or toy factory, is a natural setting in school. Certainly, it isn't "real"; it is by no means a real detective agency. I could even say that it's contrived. It has to be. Dectective agencies don't exist naturally in schools!

Me: Believe it or not, I concur. I concur not just to sound positive but also because there is something very artificial about school and, more importantly, something very artificial about mathematics. Both require an attitiude of "let's pretend." Schooling is just one big "let's pretend"—we take kids out of their daily life and force them to come to school from 8:30 a.m. to 4:00 p.m. five days a week. What they encounter in school is one big "if": If all that we do in school is valid and important, then it will be useful for students sometime in the future. Often, however, this usefulness doesn't happen and schools are then criticized for being irrelevant or out of touch with reality.

You: It strikes me that what you are calling the "if" is simply the curriculum or the What and When of traditional schooling.

Me: Precisely, and when the "if" is found wanting, we don't question the traditional classroom structure, we merely fiddle with the What and When. *Vision for the Nineties* is a good example of such change.

Mathematics is also one big "if." The "if" part is usually called "the premise(s)" and the "then" part, the "conclusion." Even the advice we may give students in problem solving takes on this form: "First determine what is given (the "if"), then try to connect this to what's required (the conclusion)." Mathematics is one big if-then sequence written large (axioms giving rise to theorems). We might say that mathematics is the Land of If. A very contrived Land of If because any "if" is okay as long as it leads to interesting results. So a great mathematician is one who can contrive powerful "ifs."

You: If I understand what you're saying, then in the communicative classroom, places such as the toy factory are also Lands of If.

Me: And each needs to be a powerful Land of If to generate natural, rich mathematical results. Each is highly contrived. As a teacher, I would encourage students to participate fully in setting up this contrivance. This is the important point: Once the contrivance is set up, what happens within the context is the creation of mathematics in ways natural to the contrivance.

You: Actually in any walk of life, business, politics or schooling, whatever context we set up will also be somehow contrived; otherwise, we would not have to set it up. But within this contrivance, within this game, some natural things can happen; things that are natural to our game.

Me: I like your metaphor of a game. It is much like playing a game and, if you are going to be a good player, you must understand the rules well. It would be even better if you could participate in designing the rules, in designing the game. Then not only would you understand the game better but also you would probably be a good player as well.

You: But if mathematics is in the game or the place, how do we get it out?

Me: Why don't we consider that when we get to the How part of the story.

You: Let me raise a different concern. I have seen early drafts of the new 1994 Program of Studies for mathematics, and there is a strong emphasis on integration, continuity and real-world connections as well as the use of manipulatives. These concerns were problematic in the traditional approach, and the new Program of Studies is placing a priority on them again (or perhaps still). What seems so exciting to me about starting the communicative story with Where is that by doing so many of these problems disappear.

Me: Is that right?

You: Sure. Consider integration. We talk about integrating math with science or language with social studies and so on. We do it by integrating the What, the content. We look for common themes or topics. In contrast to this, if we focus first on Where, we find the content already integrated in situ, in the "if." For example, in a trading post setting, social studies and mathematics would be there together as would language arts. Literally no integration of subjects needs to be done. The subjects are already together in the setting, like a topological network of relations.

Me: So the priority on Where/Who brings forth integration as a topological network? How about another example?

You: We spent only a few minutes talking about the toy factory, but I can already see how the kids are going to get right in there as if they were operating a toy factory themselves, deciding what to construct, how many to construct, how to package the goods, how to market them, how to prepare advertising and receive orders, what quanities to ship, how to receive and reply to complaints, how to fill out order slips and on and on. The language arts and social studies are already there.

Me: If you're right, then the problem of integration is just an artifact of having begun with the What and then having developed and packaged each What or subject matter separately, so separately that integration is now seen as a big problem. We shouldn't lose sight of the fact that this problem is a product of the traditional classroom story.

You: The same holds true for continuity with and connection to the real world. It's already in the context of this situation. It's only in the text that the What is separated from reality. In settings like a space lab, trading post or animal farm, ongoing experiences are situated in real-world settings, contrived though these may be at first.

Me: That's enough of extolling the virtues of Where. Let's bring in the Who.

You: With this integration and continuity, I expect it will be difficult to separate the Who from the Where.

Me: In communicative classrooms, the *nature of the Where—the nature of the place*—is a context which is taken to be jointly created by students and teacher. Introductory teaching activities begin with scene setting in activities that draw upon students' experience with detective stories and television shows, such as "Rescue 911", "Star Trek" and so on, as a way of developing such a place in the classroom. Because the context is given by the students, it is automatically related to their experience. By starting with Where, relevance and meaningfulness are properties of the learning setting. Traditionally, by starting with What rather than with Where and Who, the problem of making the What meaningful and relevant was acute. We tried to "make" the activities interesting by making them colorful or fast paced and so on. In communicative classrooms, we incorporate interest by focusing first on a place developed out of children's everyday experiences whether those be watching detective movies on TV, reading mystery novels or reading about police work in newspapers. As this continues, the place becomes a community of children learning mathematics (the What).

You: So you're saying that a communicative classroom is as much a place created with and through children's experiences as it is a program?

Me: Yes, a place which as much as anything is created out of children's experience in the world, a place is where people are. That's why we begin with Where/Who so that the classroom can become a particular kind of place that contextualizes the very learning that creates it. In constructing the place, children learn mathematics; or more generally, in constructing the context, they learn the text.

You: I like that last turn of phrase. It says it so compactly. But the communicative story isn't all context, is it? Children are also doing something in the classroom. How does this kind of doing differ from the usual activity in a normal classroom?

Me: That question moves us right into the Why. You've already put your finger on it in the way you asked the question: The activity in a *place* is quite different from the traditional activity in a math classroom. Consider the usual scenario where a student is given a task card perhaps with some manipulatives and is to do the activity. The activity could be done anywhere—at home, in a group with other children, alone at a desk, after school during detention. The place in which the activity is done in not stressed. What is stressed is that it is important to have activities of a concrete nature, as well as those involving the pictoral and symbolic modes. We are told over and over again to use manipulatives. This is the important principle.

You: How does context change the action?

Me: In communicative classrooms, I would spend a lot of time setting the scene—establishing the nature of the place. The classroom becomes a detective agency. Activity in the classroom is guided by the sheer fact that it happens in a detective agency. Children become detectives. Their actions are actions of detectives trying to solve cases. The actions of detectives are quite different from those of barbers or salespeople or principals. Questions such as "Why are you doing this?" or "Why is it important to do this?" are answerable within the detective frame. If a child asked, "Why do I need to write up this problem? I've already done it with the materials," the chief detective (perhaps the teacher) might say, "When a detective is solving a case, does she keep a record? Why? Is it important to keep a record? Does she need to file a report?" The class might diverge into a discussion of what kind of files detectives keep on a case. What goes into the file? They could even visit the police station and observe not only what reports are routinely kept but also the particular format of the reports and what sort of information is gathered.

You: So there is activity that is indigenous to the place and its indigenousness makes the activity different from ordinary classroom activity?

Me: Precisely. Indigenous activity is quite the opposite of activity imposed from the outside in an arbitrary way. It is activity that arises naturally and appropriately in the place. In a collectors' club, the action of exchanging or trading would be indigenous as would be displaying and safekeeping. Activity indigenous to a trading post would include bartering, stockpiling, packing, visiting and so on.

You: I just thought of another example of a Why question that would be quite different. I like children to be careful and organized when they work, and I find it difficult to get children to appreciate why. One of my weaknesses is to become impatient when a student does sloppy work. Now instead of becoming irritated, I could simply ask, "Why do detectives need to be careful when they gather evidence?"

Me: An excellent example and, as detectives, children can generate answers to this question through discussion. However, trying to teach students to be careful or mindful of the need to be careful is like pulling teeth. Why should they care about being careful as long as they get the answer? In the context of a detective agency though, being careful is *endemic* to the place. Otherwise, the case may be tossed out of court on a technicality. Trying to *impose* carefulness can be next to impossible, yet in the traditional story, this is what we are often forced to do.

You: I suppose so. Returning to the trading post for a moment, setting an authentic scene for that would involve activities normally called social studies.

Me: And language arts as well, particularly stories of early settlements such as Fort Macleod, Fort Saskatchewan or Fort Edmonton and of communication between the Chief Factor and the Hudson's Bay Company and with the trappers too.

You: This would be integration that would happen as an integral part of setting up the *place* and running it. In fact, it is in the running of the place that the How of the story takes place. If the class were to become a space lab, there would be a natural integration with science. It could get exciting, particularly from the kids' viewpoint!

Me: I think all this leads naturally into the How. In communicative classrooms, How is not so much a question of how to teach as it is of how to be a detective or a fire fighter or a space traveller. The How is guided by the Where, Who and Why. In a strong sense, questions of How become questions of *learning* moreso those of *teaching*: The teacher becomes a learner along with the students as both participate in running the place. Moreover, these questions of how to learn find their answers in actions and activity endemic to the place. Criteria of what is appropriate are already in place. Authority alone is never appropriate. Ask "What would a detective do now?" rather than say "Here is the right way to do it. Now do it carefully."

You: It finally seems clear to me that in doing the How (running the place) the students learn the What, the mathematics content embedded in the place.

Me: That's the hope. In communicative classrooms, the content (mathematics) is learned by doing things (the How) that come naturally in the place (the Where). Mathematics is not so much taught as it arises in the place through the actions and activities that occur there. What occurs there is *theirs* (the students').

You: You used the word *hope*, but when it comes to covering the curriculum, hope isn't enough.

Me: Your point is well taken. Just because the What occurs last in the sequence doesn't mean that it has no priority. Alberta Education and parents would hang us out to dry if we were to ignore or belittle the What. Nevertheless, in communicative classrooms covering the curriculum doesn't happen because the teacher teaches each and every objective. It happens because the selection of places provides a set of situations in which mathematics (the What) is encountered in contextually meaningful ways as an activity that solves problems arising in such settings. In planning the selection of places, we would have to ensure that all of the curriculum objectives would be covered in situ.

You: I would love to see someone develop a set of settings or places that would cover the curriculum at each grade level. That would be invaluable if ever I were to live out the communicative story in my classroom.

Me: Why don't you and I develop such a set of resources? Do you think other teachers would be interested?

You: It would be the equivalent to what traditional teachers are looking for when they go to conferences, as I mentioned earlier. When communicative teachers go to conferences, they would be interested in such resources.

I have to get back to my classroom now. I have a sub there now, but I want to see the students off. There goes the bell. Perhaps we can finish these stories another time. Actually, I think I could almost finish this one myself! Still, could you leave me with a parting summary?

Me: I'll give it my best shot in 50 words or less. In communicative classrooms, the Where and Who jointly become the medium (place) in which and through which the What is learned. We don't begin by teaching the What and then try to be sure all students are paying attention and staying on task. Rather, with the help of students, we begin to build a place in which the What will arise spontaneously, and to be on task is simply to do what is appropriate in such a place. Because each student is an original settler and creator of the place, appropriateness is something indigenous to their understanding.

Let me leave the following chart with you—it says it in less than 50 words:

Traditional	
What	(Program of Studies)
When	(scope and sequence chart)
How	(instruction)
Why	(fits scope and sequence)
Who	(children)
Where	(classroom)

Communicative	
Where	(a place)
Who	(people in the place)
Why	(appropriate to the place)
How	(indigenous action)
When	(as it happens)
What	(mathematical literacy as well as the Program of Studies)

You: One sequence is very top-down, and the other is very bottom-up. Communication thrives in a natural way only if it supports itself from the bottom up.

If you are interested in developing resources for constructing places so that the communicative story can be lived in each mathematics classroom in Alberta, contact Daiyo Sawada, 11211 23A Avenue, Edmonton T6J 5C5; phone (403) 436-4797 (res.) or (403) 492-0562 (bus.).

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