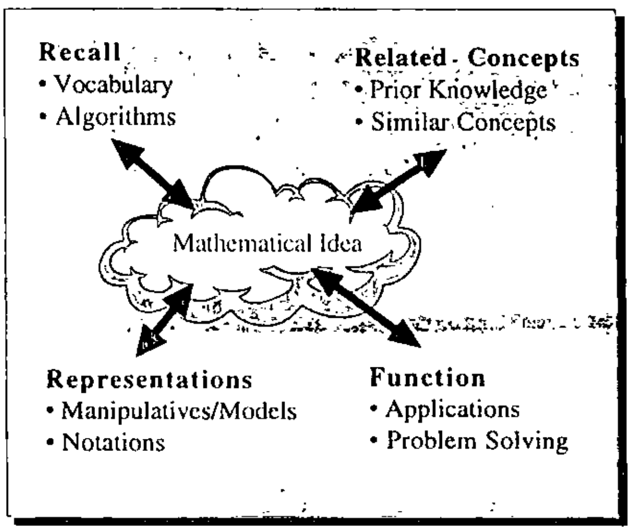
Understanding Mathematics

Some Ways of Knowing

Learning without understanding is a frustrating experience. But, what do we mean when we say we do not understand a mathematical idea? Or, more to the point, what does it mean that we know or are able to do when we say that we *do* understand mathematics? Can you actually learn something and not understand it?

Think back to a time when you were challenged to learn a new idea or complete a given task that you simply did not understand. No doubt that experience carried with it a great deal of aggravation and maybe a large element of problem solving as you learned to compensate or somehow work through your difficulty.



When we are faced with a task for which we have little understanding we often find ourselves trying to memorize the steps to complete the task, merely as a way of compensating and getting past a difficult moment. Unfortunately many of our students have the unpleasant experience of encountering mathematics without understanding, and as a result, they too compensate by memorizing formulas and routines. These students compensate by trying to emulate every step their teacher takes in solving example questions, hoping that those steps are appropriate for every problem to be encountered. Usually, however, the steps are not identical and there is enough difference between questions, tasks and problems that memorization and mimicry are not the keys to success — understanding is *required*.

But, what does it mean to understand? The National Council of Teachers of Mathematics (NCTM) has attempted to describe understanding in terms of "connections." Connections can be made between two or more ideas, between ideas and models, between ideas and notations, between ideas and their uses, between ideas and words, between ideas and algorithms, etc. In short, there are many different kinds of connections which connect our ideas to what we know and are able to do. It stands to reason that the more connections we have for each idea we retain, the greater the understanding we have of that idea. When we have the vocabulary to express the ideas, when we can articulate ways to use the idea, and when we can describe how ideas follow from earlier ideas, or even how they are similar or different from other ideas, then we gain confidence that we do in fact understand the concept. There are many ways to know an idea, but the greater the number of connections we have, the greater the level of understanding.

So, can you learn something but not understand it? In short, no. If understanding a new idea has to do with developing "connections," then so does learning. Insofar as only learning how to complete a task (e.g., carry out an algorithm or process) is at least some accomplishment, it is really difficult to argue that such learning is sufficient when the learner is incapable of applying the task to any situation other than the immediate one and sees no way to connect the learned process to any related or similar tasks. No, all learning requires connections, and the more connections attained, the more effective that learning becomes.

This conceptualization of understanding has important applications to our elementary mathematics classrooms. If we can accept that connections are key to understanding, then we must teach in such a way that students have many opportunities to develop such connections, and we must accept that these connections actually constitute the main goal of our teaching. We can encourage the development of connections through allowing structured exploration and experimentation with mathematical ideas integrating manipulatives, problem-solving experiences and writing activities in our daily instruction (among other activities). In other words, we help students learn mathematics by pointing them toward the key attributes of a wide variety of learning activities all of which are designed to encourage connections between ideas and the world around the student.

This conceptualization of understanding represents the underpinnings of this monograph. The purpose of this monograph is to provide a range of meaning-based activities for the primary mathematics classroom, focusing on manipulatives, problem solving and writing experiences. The activities are designed to help students develop connections between ideas, between ideas and the real world, between ideas and various models and manipulatives, and between ideas and their applications in problem-solving situations.

Manipulatives
Are Powerful Mathematical Models

In the description of meaning given in the section above, it was argued that manipulatives can play an important role in the learning of mathematics. Manipulatives provide a concrete