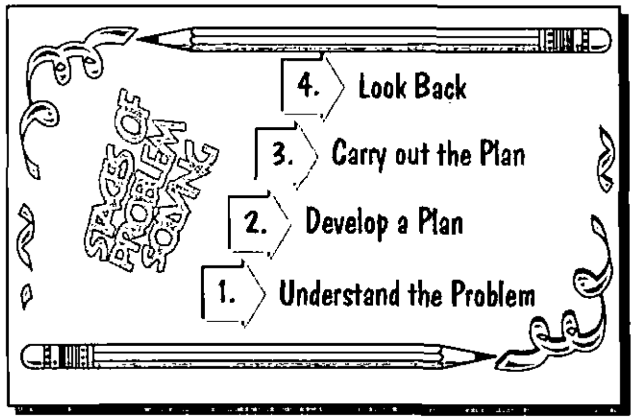
Problem Solving

Is More than Just Solving Problems

It is probably true that the best way to becoming a good problem solver is by solving many varied problems. However, such advice is not particularly helpful to teachers who must plan and organize for mathematics instruction. What is it that we teach when we teach problem solving? Or is simply giving students many different problems to solve over a period of time good enough?



When we teach problem solving we obviously must expose students to many different problems, but we can structure those experiences to ensure that students encounter a certain body of knowledge. In essence, while solving a variety of problems, we would like students to become familiar with three different elements of problem-solving knowledge: the *stages* of problem solving, the *strategies* of problem solving and the *skills* of problem solving.

STAGES and STRATEGIES

There have been many different people who have tried to define the various stages of problem solving, but probably the simplest description of this process was given by Polya who defined the following four (successive but non-linear) stages: understand the problem, develop a plan, carry out the plan and look back.

In the first stage, understand the problem, the solver simply reads and studies the information given until (a) s/he knows and understands what information is given, and (b) can clearly identify the task that is meant to be completed. This may involve listing the information given, looking up unfamiliar terms (e.g., how many are in a *gross*?), or even reading information from a table, graph or chart. All of these activities help the solver better understand both the task at hand and the context of the problem itself.

The second and third stages of problem solving really occur together as the solver develops a plan and then tries it out. At these stages it is helpful for the solver to consider whether or not s/he has seen a similar problem before and what strategies were successful in earlier experiences. There are many different strategies from which solvers may select, including elimination, modeling (drawing a picture, acting it out, constructing a model), looking for a pattern, constructing a table, solving a simpler question and even guessing and testing. All of these strategies are useful, but not all are useful in solving all problems, and some may be more efficient than others. In other words, we must teach students when an elimination strategy may be useful and when it is not. Likewise we need to teach students how to selectively choose strategies and how to recognize problems which obviously lend themselves to particular strategies. Often there is more than one appropriate strategy for solving a given problem and, typically, the strategies are most effectively applied when they are combined (e.g., looking for a pattern within a table of values). The six strategies can be applied to any of the problems in the activity pages that follow.

The final stage of problem solving is the looking back stage. At this stage the solver has found a solution, checks to ensure that his/her solution meets all of the conditions in the problem and checks to ensure that no arithmetic errors were made. More important, the solver studies his/her route to the solution and reflects on the process. Was there a better way? What made this problem easy or difficult? How could this problem be typified so that a person could more easily identify a useful strategy in the future? Such reflection is extremely important because it represents the true learning in the problem-solving experience.

It is obvious that problem-solving strategies need to be conscientiously introduced, modeled and practised with students. No single strategy works every time, so confidence with a range of strategies is imperative. However, the purpose for articulating the stages of problem solving is less transparent, but the stages are important. The stages serve as a metacognitive tool; that is, they serve to help students manage and control their own thinking. Ideally we would like to encourage students to ask themselves questions such as "Do I understand what is happening in this problem?" and "Are there some obvious strategies that would be helpful?" and "Does my answer make sense?" Being able to ask and answer such questions represents an ability to make conscious choices, and surely this must represent the real goal of problem solving instruction.

SKILLS

There are many different problem-solving skills that help the solver become more efficient and effective. Individually these skills cannot guarantee success but collectively they represent an extremely important body of abilities. A partial list of skills includes

- identifying given and wanted information;
- identifying extraneous information;
- identifying missing or hidden information;
- identifying key words and operations;
- reading from a table, chart or list;
- identifying multiple step problems;
- rewriting a problem in your own words;
- drawing a picture or graph; and
- describing the action in a problem.

One can see how each individual skill could aid a solver from time to time, but obviously no one skill constitutes the secret to successful problem solving. It is important to remember that, while these skills can be taught in isolation, their purpose is really to support the solver in a variety of possible problem-solving situations.

The development in students of this body of knowledge (the stages, strategies and skills of problem solving) represents the major purpose or goal of problem-solving instruction. However, these elements need to be routinely and carefully *integrated* with daily instruction, not taught as a separate program. Problem-solving instruction needs to be integrated because it simply does not stand on its own — it has no purpose in and of itself. Its purpose is found in its application to given and real life tasks.

Writing Activities

In the Mathematics Classroom

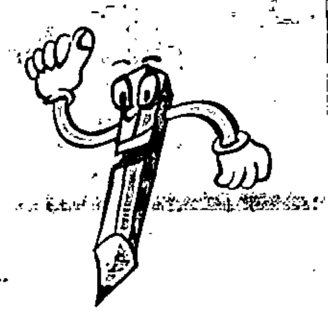
Writing in the mathematics classroom is a topic which has gained increasing support and interest over the past decade. Writing and learning mathematics seem to go together naturally. Writing about ideas while learning mathematics seems to add a dose of purpose both to the learning of mathematics and to the process of learning how to write.

The very act of writing forces the writer to think carefully about his or her topic, and may even serve a metacognitive function by asking the writer to reflect on his or her own understanding of concepts. At the same time it provides a variety of legitimate activities that allow students to practise important writing skills.

In short, students need to learn how to communicate both orally and in writing and, furthermore, they need to learn how to communicate mathematical ideas. There is no place in our school curriculum in which writing activities cannot be included, and there is no grade prior to which we can say writing activities are inappropriate. How much easier the writing and communicating process will appear to our students if it has been regularized and emphasized at all stages of the learning process in all subject areas. The question becomes, what types of writing activities are possible and appropriate in our classrooms at the earliest grades?

Some Writing Activities:

- **Keep a journal ...**
- **Write a problem ...**
- **Make a list ...**
- **Write a letter to explain ...**
- **Write a story or poem about ...**
- **Describe what you like about ...**
- **Describe your feelings about ...**
- **Construct your own dictionary of math terms ...**



Many authors have written about the various kinds of writing activities which can be integrated with the study of mathematics. Some ideas are provided in the inset on the previous page. But most writing activities seem to fall in one of two categories: *reflective* writing and