# Making Math Make Sense 

 in the Primary Classroom
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- Writing Corner


in the Primary Classroom

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Some Ways of Knowing

Learning without understanding is a frustrating experience. But, what do we mean when we say we do not understand a mathematical idea? Or, more to the point, what does it mean that we know or are able to do when we say that we do understand mathematics? Can you actually learn something and not understand it?

Think back to a time when you were challenged to learn a new idea or complete a given task that you simply did not understand. No doubt that experience carried with it a great deal of aggravation and maybe a large element of problem solving as you learned to compensate or somehow work through your difficulty.

When we are faced with a task for which we have litule understanding we of en find ourselves trying to memorize the steps to complete the task, merely as a way of compensating and getting past a difficult moment. Unfortunately many of our students have the unpleasant experience of encountering mathematics without understanding, and as a result, they too compensate by memorizing formulas and
routines. These students compensate by trying to emulate every step their teacher takes in solving example questions, hoping that those steps are appropriate for every problem to be encountered. Usually, however. the steps are not identical and there is enough difference between questions, tasks and problems that memorization and mimicry are not the keys to success - understanding is required.

But, what does it mean to understand? The National Council of Teachers of Mathematics (NCTM) has attempted to describe

understanding in terms of "connections." Connections can be made between two or more ideas, between ideas and models, between ideas and notations. between ideas and their uses, between ideas and words, between ideas and algorithms, etc. In short, there are many different kinds of connections which connect our ideas to what we know and are able to do. It stands to reason that the more connections we have for cach idea we
retain, the greater the understanding we have of that idea. When we have the vocabulary to express the ideas, when we can articulate ways to use the idea, and when we can describe how ideas follow from carlier ideas, or even how they are similar or different from other ideas, then we gain confidence that we do in fact understand the concept. There are many ways to know an idea, but the greater the number of connections we have, the greater the level of understanding.

So, can you learn something but not understand il ? In short, no. If understanding a new idea has to do with developing "connections," then so does learning. Insofar as only learning how to complete a task (e.g., carry out an algorithm or process) is at least some accomplishment, it is really difficult to argue that such learning is sufficient when the learner is incapable of applying the task to any situation other than the immediate one and sees no way to connect the learned process to any related or similar tasks. No, all learning requires connections, and the more connections attained, the more effective that learning becomes.

This conceptualization of understanding has important applications to our elementary mathematics classrooms. If we can accept that connections are key to understanding, then we
must teach in such a way that students have many opportunities to develop such connections, and we must accept that these connections actually constitute the main goal of our teaching. We can encourage the development of connections through allowing structured exploration and experimentation with mathematical ideas integrating manipulatives, problem-solving experiences and writing activities in our daily instruction (among other activities). In other words, we help students learn mathematics by pointing them toward the key attributes of a wide variety of learning activities all of which are designed to encourage connections between ideas and the world around the student.

This conceptualization of understanding represents the underpinnings of this monograph. The purpose of this monograph is to provide a range of meaning-based activities for the primary mathematics classroom, focusing on manipulatives, problem solving and writing experiences. The activities are designed to help students develop connections between ideas, between ideas and the real world. between ideas and various models and manipulatives, and between ideas and their applications in problem-solving situations.


[^0]can play an important role in the learning of mathematics. Manipulatives provide a concrete
experience which is designed to help students actually sec (and usually touch) a mathematical idea through the physical objects used to represent it.

But, why are seeing and touching mathematical ideas so important? The seeing and touching of these mathematical models often instil a sense of believability and confidence in the ideas which are encountered. We find it casier to accept and include as part of our cognitive structures things that we see and touch.

While manipulatives are important from the perspective of providing the mcans to visualize mathematical ideas, they are also useful from the perspective of problem solving. When students have models, they also have a mechanism for exploring ideas, visualizing problem situations. and guessing and checking to search for solutions or other means of solving given problems.

While manipulatives are extremely important from the perspective of encountering, exploring and developing new ideas, they do suffer from a variety of drawbacks. For example, teaching with manipulatives drastically increases management concerns within the classroom. Using manipulatives requires the teacher to do extra planning and organization prior to instruction and, with some grades and classes. increases the risk of off-task behavior. None of these represent significant reasons for not incorporating manipulatives. However, these limitations do make clear the need for monitoring student work during instruction and implementing management routines and structures for distributing and collecting the manipulatives.

One of the more pragmatic difficultics associated with manipulatives is their of en
prohibitive cost. Purchasing class sets of manipulatives can become extremely expensive, often impossible as a result of today's budgetary restrictions. Schools have implemented a variety of measures to counteract this difficulty, including purchasing class sets to be shared among several teachers, purchasing small group sets - useful when organizing the class around centre-based activities - and creating alternative manipulatives often made from common and available materials. Some suggestions for finding or making alternative manipulatives for those used in this book are given below.


Base Ten Blocks can be created using popsicle sticks and beans. A single bean is the same as a single block, ten beans glued to a popsicle stick represents a long block and a raft built from a collection of ten sticks covered in 10 beans each would represent a flat block. For some activities, photocopied pictures of base ten blocks may be sufficient (sce duplication masters at the end of the book).


Cuisenaire or Counting Rods can be cut from colored construction paper. Cut strips 1 cm wide ranging in length from 1 cm to 10 cm long. All strips of the same length should also be cut from the same color of paper.


Pattern Blocks can also be cut from colored construction paper according to their various shapes: hexagons can be cut from yellow paper, trapezoids from red paper, etc. (see duplication masters at the end of the book).


Attribute Blocks can be cut from paper just like the other manipulatives above. To make a complete set of blocks, cut a large and small triangle, a large and small circle, a large and small rectangle, and other shapes from each of red, blue, yellow and green paper. The thickness of the shapes can also be varied by cutting them from paper or heavier cardboard.


Color Tiles are also easily made by cuting one inch squares from colored paper. If thicker tiles are preferred, glue the construction paper to corrugated cardboard before cutting. Ceramic tiles are also readily available at minimal cost.


3-D Shapes are nice to have for tracing activities, but they can easily be made out of plasticine or clay, or cut from wood by a carpenter with moderate skill.


Dice are not particularly expensive unless purchased in quantity (check dollar or games stores). Most dice could be made from plasticine (even the four-sided dice) if necessary. These dice could also be easily made using paper nets as shown below. The paper versions are less durable, but are, however, casily adapted to create biased dice (replacing the values on any given side with new values).

(a regular die)

(abiased die)


Is More than Just Solving Problems

It is probably true that the best way to becoming a good problem solver is by solving many varied problems. However, such advice is not particularly helpful to teachers who must plan and organize for mathematics instruction. What is it that we teach when we teach problem
who defined the following four (successive but non-linear) stages: understand the problem, develop a plan, carry out the plan and look back.

In the first stage, understand the problem, the solver simply reads and studies the
solving? Or is simply giving students many different problems to solve over a period of time good enough?

When we teach problem solving we obviously must expose. students to many different problems, but
 information given until
(a) s/he knows and understands what information is given, and (b) can clearly identify the task that is meant to be completed. This may involve listing the information given, looking up unfamiliar terms (e.g., how many are in a gross?), or even reading information from a table, graph or chart. All of these activities help the solver better understand both the task at hand and the context of the problem itself.

The second and third stages of problem solving really occur together as the solver develops a plan and then tries it out. At these stages it is helpful for the solver to consider whether or not s/he has seen a similar problem before and what strategies were successful in earlier experiences. There are many different strategies from which solvers may select, including elimination, modeling (drawing a picture, acting it out, constructing a model),
looking for a pattern, constructing a table, solving a simpler question and even guessing and testing. All of these strategies are useful, but not all are useful in solving all problems, and some may be more efficient than others. In other words, we musi teach students when an elimination strategy may be useful and when it is not. Likcwise we need to teach students how to selectively choose strategies and how to recognize problems which obviously lend themselves to particular strategies. Often there is more than one appropriate strategy for solving a given problem and, typically, the strategies are most effectively applied when they are combined (e.g., looking for a pattern within a table of values). The six strategies can be applied to any of the problems in the activity pages that follow.

The final stage of problem solving is the looking back stage. At this stage the solver has found a solution, checks to ensure that his/her solution meets all of the conditions in the problem and checks to ensure that no arithmetic errors were made. More important, the solver studies his/her route to the solution and reflects on the process. Was there a better way? What made this problem easy or difficult? How could this problem be typified so that a person could more easily identify a useful strategy in the future? Such reflection is extremely important becaus: it represents the true learning in the problem-solving experience.

It is obvious that problem-solving strategies need to be conscientiously introduced, modeled and practised with students. No single strategy works every time, so confidence with a range of strategies is imperative. However, the purpose for articulating the stages of problem solving is less transparent, but the stages are important. The stages serve as a metacognitive tool; that is,
they serve to help students manage and control their own thinking. Ideally we would like to encourage students to ask themselves questions such as "Do I understand what is happening in this problem?" and "Are there some obvious strategies that would be helpful?" and "Does my answer make sense?" Being able to ask and answer such questions represents an ability to make conscious choices, and surely this must represent the real goal of problem solving instruction.

## SKILLS

There are many different problem-solving skills that help the solver become more efficient and effective. Individually these skills cannot guarantee success but collectively they represent an extremely important body of abilities. A partial list of skills includes

- identifying given and wanted information:
- identifying extraneous information;
- identifying missing or hidden information;
- identifying key words and operations;
- reading from a table. chart or list;
- identifying multiple step problems:
- rewriting a problem in your own words;
- drawing a picture or graph; and
- describing the action in a problem.

One can see how each individual skill could aid a solver from time to time, but obviously no one skill constitutes the secret to successful problem solving. It is important to remember that, while these skills can be taught in isolation, their purpose is really to support the solver in a
variety of possible problem-solving situations.

The development in students of this body of knowledge (the stages; strategies and skills of problem solving) represents the major purpose or goal of problem-solving instruction. However, these elements need to be routinely
and carefully integrated with daily instruction, not taught as a separate program. Problemsolving instruction needs to be integrated because it simply does not stand on its own it has no purpose in and of itself. Its purpose is found in its application to given and real life tasks.


## In the Mathematics Classroom

Writing in the mathematics classroom is a topic which has gained increasing support and interest over the past decade. Writing and learning mathematics seem to go together naturally. Writing about ideas while learning mathematics seems to add a dose of purpose both to the learning of mathematics and to the process of learning how to write.

The very act of writing forces the writer to think carefully about his or her topic, and may even serve a metacognitive function by asking the writer to reflect on his or her own understanding of concepts. At the same time it provides a varicty of legitimate activities that allow students to practise important writing skills.

In short, students need to learn how to communicate both orally and in writing and,
furthermore, they need to learn how to communicate mathematical ideas. There is no place in our school curriculum in which writing activities cannot be included, and there is no grade prior to which we can say writing activities are inappropriate. How much easier the writing and communicating process will appear to our students if it has been regularized and emphasized at all stages of the learning
 process in all subject areas. The question becomes, what types of writing activities are possible and appropriate in our classrooms at the earliest grades?

Many authors have written about the various kinds of writing activities which can be integrated with the study of mathematics. Some ideas are provided in the inset on the previous page. But most writing activities seem to fall in one of two categories: reflective writing and
creative writing.
In reflective writing we ask students to contemplate their own understanding of mathematical ideas and write about the things that they understand and the things that confuse them. We may ask students to describe what they think something means or what a symbol stands for. We might even ask students to talk generally about their experiences of learning mathematics, such as their enjoyment of, or frustration with, math class.
"Why" questions work well to encourage students to display their understanding or the meanings they have built, as in

Why do $5+4$ and $6+3$ have the same sum?

Other questions that focus on having the students define terms and key ideas also help in the assessment of student learning, for example,

In your own words, what is a pattern?

Students could also be asked to apply their understandings to a specific task, such as

Describe how you could tell if two shapes have the same area.

Creative writing can also be used to probe students' understandings of mathematical ideas, but such questions tend to add a fun twist. For example, we could ask students to generate a story or poem.

> Write a story telling about something a crazy cat did each day for a week.

We could also ask students to construct problems for others to solve.

> Write a riddle for your classmates to solve! Make a list of 5 clues describing an object in your classroom. Ask a friend to solve your riddle.

In summary, we have attempted to define two different, though non-distinct, types of writing activities: reflective (which includes meaning-based questions, definition questions, and application tasks) and creative (which includes problem-posing activities and a variety of other activities, such as writing stories and poems).


Each of the following activity pages is divided into several sections. It must be noted that the descriptions in each of the boxes are written for teachers the reading level required on each of these pages surpasses that reasonably expected at the lower elementary level.

At the top of the each page you will find the Objective behind each of the activities on that page. The code in the top right comer of the page shows the strand and substrand in which the objective is found, as well as the number of
greater challenge, and downward, for students who might find the activity too challenging.

Each page also contains a Problem Solving box with a single problem. It is intended that each problem require the student to engage the objective listed at the top of the page in order to successfully solve the problem. The problems range widely in level of difficulty, with some being fairly easy for Grade 2 students while others represent a more significant challenge. In the balloon superimposed over the problem box, the teacher will find a suggested strategy the objective. For example, the code $\mathrm{N}(\mathrm{NC})-04$ stands for the fourth objective in the Number Concepts substrand of the Number strand. Each objective is quoted directly from the Grade 2 list in the Western Canadian Protocol document.

The largest box on the page (checkerboard pattern) contains a hands-on Representation or Activity for each objective. The graphic in the top right comer of the page identifies the materials used in this activity/representation.

Some activities may be completed as a whole class; some may be more effective when completed individually or in small groups. The box at the bottom left of the page provides two Adaptations: upward, for students needing a

for solving the problem, and the answer. Complete solutions (where possible) are listed at the back of the monograph.

Finally, in the bottom right corner of each activity page the teacher will find a Writing Corner.Each writing activity represents a sample activity that could be

Count to 1000 by $1 \mathrm{~s}, 2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s and to 100 by 25 s using starting points that are multiples of $1,2,5,10$ and 25 respectively.

Materials: base ten blocks, six-sided die.
Roll the die. Make a set of that number using flat blocks (hundreds). Roll again and add that many long blocks (tens) to your set.

Write the number for the set you have created.
Add 2 single blocks (ones) to your set. Regroup if you can. Write the new number. Repeat this several times, building to your set and adding to your list. Can you make it all the way to 1000 ?

Adaptations: build your set a single block at a time, or 5 single blocks at a time, or by a long block (tens) each time.

Jerry started counting by tens at 630. Erica started counting by fives at 875 . Johnel started counting by ones at 936 . How many numbers did each have to say before reaching 1000?


While building sets, do not progress above the value 100 , and use a 100 chart along with the manipulative. Build the set and color the number showing the value of the set on the chart. Describe the pattern which emerges.


Model the numbers on the calculator as well as build the set with manipulatives. To model the numbers on the calculator, repeatedly, add the number you are counting by to the value on the calculator screen.

STIBATHEAM: Make a List ANSWEIS:
Jerry: 37, Erica: 25, Joḥnel: 64, including the starting number.
 she said while counting by fives. Make the same list as Sandy's. Describe any patterns you see in the list.

Estimate. then count the number of objects in a set (0-100) and compare the estimate with the actual number.

Materials: linking metric cubes, metre stick.
Take a large handful of the linking blocks. Estimate the number of blocks in your handful. Write your estimate.

Link the blocks together and place the train along the edge of the metre stick.
Count how many cm the end of your train is from the estimate you recorded.
Repeat. Try to get closer the next time.
Which student in your class can take the largest handful of blocks?
Play a game with a friend: Repeat the above process five times, summing the differences between the actual lengths and the estimated lengths for all 5 rounds. The player with the lowest sum wins.

A bag is filled with small blocks. Each student in your class is asked to draw 1,2 or 3 blocks from the bag. Estimate how many blocks will be drawn in all. What is the largest number possible? The smallest?

sTÍRATBiGiv: Act lt Out ANEWERE:

Answers vary.

In the problem above, how did you make your estimate?

Work with a partner. Each of you take a handful of blocks. Estimate the size of your set, your partner's set and the set that would set and the set that wou
be created if both your sets were combined.


Recognize, build, compare and order sets that contain 0-100 elements.

Materials: deck of cards (ace, 10, and face cards removed), base ten blocks.

- Begin by shuffling the cards. Turn the first 2 cards face up to determine a value less than 100 . For example, the cards shown make 46.

| 4\% \% | $\cdots$ |
| :---: | :---: |
|  | - ${ }^{\text {a }}$ |
| ¢f+ | 4 |

- Take as many long (tens) blocks as the value shown on the first card and as many single (ones) blocks as shown on the other card. What is the value of your set?
- Repeat to create a second set. Which set has more long (tens) blocks? How do you know? Which set has more single (ones) blocks? How do you know?
- Which set is larger? How much larger? pulled out a handful of bingo chips. Janna took out out 74 chips.

STIE ATEEGY: Act it Out A.NEWEER:

Janna has 42 bingo chips. Susan has $3^{n}$ bingo chips.

How many bingo chips did Janna have? How many bingo chips did Susan have?


Start with sets less than 10 and use one-to-one correspondence to sequence sets and to determine which set is the largest or smallest.

Using base ten blocks create sets, each with a value greater than 100 . Order the sets by comparing place values. Which set has more hundred blocks? Ten blocks? Which set has the greater value?


Materials: calculator, base ten blocks, popsicle sticks, 100 board, bingo chip, paper, pencil.

- Work in partners. Have one partner put a single digit in the calculator. The other partner will enter another digit in the calculator. For example, assume the display now reads:
- One partner now builds the set (47 in this example) out of popsicle sticks while the other builds the set out of base ten blocks.
- Use the 100 board and place the bingo chip on the number created.
- Write sentences to describe the sets that have been built. For example,
- 47 is 10 less than 57 .
- 47 is 3 more than 44 .

When I build my number, I use more long blocks than single blocks. The number of long blocks used is shown. What is my number?


Use only values less than 10. Play a game to see how many different ways you can describe the value chosen.


Work in groups of three and use the same process to create values less than 1000. Omit the 100 chart and popsicle sticks from the activity to increase difficulty level and make conceptualization of the set more abstract.

## STIBATEAV: Make a Model: INSWEIR:

Possible answers include 30,31 and 32.

Write a problem like the one above that would have someone try to guess your secret number.

Demonstrate place value concepts to give meaning to numbers up to 100 .

Materials:" paper, pencil, popsicle sticks, string, bingo chips, 100 numberboard.

- Work with a partner for this activity.
- Create a loop out of the string. You will work with your partner to build a set of popsicle sticks that has a value of 100 inside this loop.
- On a turn you can add one or two stick bundles (tens) or 1 or 2 single sticks (ones) to the set in the loop.
- After adding the sticks, write the new value of the set. Make a list of all the values you create on your way to 100 . Move the bingo chip marker on the 100 numberboard to illustrate the new value (e.g., if you added two bundles, you will need to move the marker down 2 rows): How many turns does it take to get to 100 exactly? Can you get to 100 in exactly 12 turns?

Nicole can use only single base ten blocks and long blocks. She must use exactly 8 blocks. What values can Nicole make?


Work with a partner. Create a 3-digit number on a calculator (e.g., 335) and build the set using base ten blocks. Take turns adding or removing a block of any size from the set and perform the comparable operation on the calculator (e.g., add a flat then +100 ).


Materials: 6-sided die, counting rods, metre stick, beans, paper, pencil.

- Roll the die. Take the same number of rods (of any color - all different, all the same or a mix) as the number rolled.
- Place the rods in a train along the edge of the metre-stick. Start your train at the number 'zero'. How long is your train?
- Place one bean on the ' 10 ' on the m-stick. Place another on the $20,30,40$ and so on. How can you describe these numbers?
- Which of these beans is closest to the end of your train?
- Write a sentence like the one below describing your train:
' 26 rounded to the nearest 10 is 30 .'

How many numbers between 23 and 58 are there which would be rounded down if rounded to the nearest 10 ? How many are there which would be rounded up?


## STIBATEGil: Make a List

 NNGWEIR:There are 16 numbers which would be rounded down and 18 which would be rounded. up.

Play a game with a friend:
Write some numbers between I and 100 on cards. Flip the cards over one at a time. The first player to say the number to which this card would be rounded (when rounded to the nearest 10) keeps the card.

The player with the most cards wins.
 Write a letter to a friend explaining how to round to the nearest 10. Finish your letter with a rule to know whether to round down or up.
Include examples! two players.

- Both players place 2 cards in order down on the table at the same time to create 2 digit numbers less than 100. Example: 75 .

- Each players says his or her number out loud. The player whose cards make the largest value takes all 4 cards. Making a list, keep track of the largest number each player makes during the game.
- Continue to play until all the cards have been used, then count the number of cards each player has collected during the game, scoring one point for each card. The player who has made the single largest number during the game scores 10 bonus points. High score wins.


## What is the largest number less than 100 you can

 make using only the following numbers?$$
5
$$

What is the smallest?

Using a calculator, construct a list of all of the numbers in order from 1 to 100. Generate the list by repeatedly adding one on the calculator. Write 3 sentences about the numbers in the list.


Roll a die 3 times, keeping track of the values rolled. How many different 1, 2 and 3 digit numbers can you make using those numbers?

STiBATLifil: Guess \& Check ANSWERS:

Largest is 66 , smallest is 22 . Without using a digit twice, the largest is 65 and the smallest is 23 .

Writing Ca
Write any number between 40 and 60 . Write a sentence using that number and the words ' 10 less than.'
Write another sentence using the words ' 3 more than.'

Materials: 100 numberboard, bingo chip, die (6-sided).

- Place the bingo chip on the square marked with a zero.
- Roll the die, and move the bingo chip forward that many spaces on the numberboard.
- Write the number word for the space on which the bingo chip lands.
- Repeat until you reach or pass the number 20 , then set your marker on the 20 space and work your way back down.
- Which number word requires the most letters to write?
- Play a game with a partner. Roll the die and move the chip as above, but see who can write the most number words before passing the 20 space on the numberboard.

If you wrote out all the number words from one to twenty, which letter would you use the most often?

## Which letter is used only once?

Which vowel isn't used at all?



Draw 3 cards from a deck (A, 10, J, Q, K removed) and turn them face up in order. Write the number word to go with the number you drew.

Challenge: what is the first number word to use the letter $a$ ? letter $b$ ?
 ANSWER:
most common - letter e only once - letter y unused vowel - Letter a

$$
\begin{aligned}
& \text { Writing Comer: } 00000 \\
& \text { Write sentences to } \\
& \text { : } \\
& \text { describe the students in } \\
& \text { your class. Say how many } \\
& \text { students there are in total; } \\
& \text { how many girls; how many } \\
& \text { boys. Write } 2 \text { more } \\
& \text { : sentences to describe your }
\end{aligned}
$$ classmates.

Materials: spinner mat. overhead spinner, 31 various different small objects.

- Prepare the spinner mat by dividing the mat into 4 sections. Draw (or write the name) of one of the 31 objects in each section of the mat.
- Place the 31 objects randomly in a line.
- Twirl the spinner.
- Find the object identified by the spinner. Identify out loud the position that object holds from the front of the line (e.g., "twentieth from the front"). Now, switch that object with any other object in the line. Repeat several times.
- Adaptations: (a) instead of placing the objects in a line, place them one each on the spaces on a calendar page, (b) identify positions both from the front and back of the line, or (c) write as many sentences as you can describing the position of the object.

Joey noticed that there was a pattern to the weather in October. On the first day it rained. On the second day it was windy. On the third day the sun shone, and then the rain, wind and sun came again in that order. What will the weather be like on the 17 th , 22nd and 31st if the pattern continues? How many sunny days this month?

## STinITRAGY: Make a List ANSWEIR:

17th - windy
22nd - rainy
31st - rainy, but 10 sunny days: in all.

Play a tic-tac-toe game on the month calendar page. Player one will say "put my X on the fifth." Player two selects and verbalizes another date on which to place his/her $O$. The first player with 4 in a row wins.

> Writing Comer: $\bullet 0.0 .0 \cdot$
> Construct a list of 3 places you hear or see ordinal numbers used. Give an example of each.

Explore the representation of numerals $(0-100)$ using a calculator or a computer to display numerals.

Materials: 100 board, bingo chips (two colors), calculator.

- Play a game of tic-tac-toe with a partner.
- First player picks any number on the 100 board and enters that number on the calculator.
- After entering that number on the calculator she can place his/her colored marker on that space on the 100 board.
- The second player now picks a number, enters it and claims that space on the 100 board.
- Players continue taking turns and claiming spaces until one player has 4 pieces in a row.

Jackie entered all the numbers from 1 to 50 in her calculator one at a time. How many numbered keys did she press?

How many times did she press the 3 key?


Find numbers which, when entered into the calculator and the calculator is turned upside down, spell the following words:

## H 505 She 69 9E H5

STIEATEAV: Act it Out ANSWIEIE:

Jackie pressed 91 keys in all. She pressed the 3 key 15 times.

- Writing Comer:

Write a short story using words that can be spelled on the calculator. Replace those words with numbers or equations, and then let a friend read your story.

## Demonstrate if a number from 1 to 100 is even or odd.

Materials: metric cubes, metre stick, deck of cards (10, J, Q, K, A removed). - Begin by turning over 2 cards to create a number less than 100 . Find this number along the edge of the metre stick. This is the target number. Assume we turned over the following two cards and our target number is 59 :


- Create a train along the edge of the metre stick by adding two cubes to the train at a time (starting at zero).
- Continue adding 2 blocks at a time until your train exactly reaches the target number (in which case your number is even) or until you pass the target number (in which case your number is odd, as in our example above).
- Adaptation: as you build your train, color all the various train lengths you made on a 100 board. Do you see a pattern?


Illustrate and explain halves, thirds and fourths as part of a region or a set.

Materials: pattern blocks, pencil crayons, pencil.

- How many red blocks does it take to cover a yellow block? What part of a yellow block does a red block cover?
- How many blue blocks does it take to cover a yellow block? What part of a yellow block does a blue block cover?
- How many green blocks does it take to cover a red block? What part of a red block does a green block cover?
- How many green blocks does it take to cover a blue block? What part of a blue block does a green block cover?
- Use any pattern blocks to create shapes which would contain:
- four blue blocks
- four green blocks
- four red blocks

With her pattern blocks, Jocelyn built a shape using eight blocks. A yellow block covers one third of her shape. Build a shape like Jocelyn's.

Can you make such a shape without using any green blocks?


Take one pattern block. and trace around it twice (not overlapping, but touching on one side). Color one of the shapes to show one-hall. Repeat with several different shapes. Trace around a block 3 times, shading one to show thirds, etc.


Try representing parts of a set using pattern blocks. Take 4 of the same pattern block laying 3 of them face down on the table and standing one on its edge. What part of the set is standing on its edge?

## SHIBITEAGV: Make a Modët ANSWERE:

There are 8 different combinations of any green: 6 blue and 2 red.blocks.....

Use manipulatives, diagrams and symbols to demonstrate and describe the processes of addition and subtraction to 100 .


Materials: deck of cards (blacks: 2, 3 and 4 s only, reds: 2 to 9 only), bingo chips, 100 board, paper, pencil, base ten blocks.

- Separate the red and black cards. Draw one black card and one red card to make a two-digit number (black is the digit in the 10 s place). Repeat to construct a second number. The task is to find the sum of these two numbers. Assume we have drawn the values 35 and 42 .
- Place a bingo chip on the zero of the 100 board. Move the chip forward the same number of spaces as the value of the red card in the first number ( 5 in our example). Now move the chip forward the same number of spaces as the value of the second red card ( 2 in our example). Add the value of the two red cards. What do you notice?
- Move your chip downward the same number of rows as the value of the first black card ( 3 in our example). Repeat for the second black card (4 in our example). The space where the chip lands represents the sum of the two numbers ( 77 in our example).
NOTE: To model addition, we move the marker right $(+1)$ and down $(+10)$ starting at zero. To model subtraction, we move the marker left $(-1)$ and upwards $(-10)$ starting at the minuend. It is recommended that students model their equations with base ten blocks at the same time they write the traditional notations and work the 100 board.

Eighteen people came to listen to the concert on Monday and 39 came on Tuesday. How many in all came to the concert on these two days? How many came Wednesday if the total for the three days was


Talk about what a move on the 100 board represents:

- to move right one space means to add 1.
- to move left one space means to subtract 1.
- to move down one row means to add 10 .
- to move up one row means to subtract 10 .
A. Use base ten blocks to model 3 digit addends with sums to 1000 .
B. Use the 100 board to model the addition of 3 or more smaller addends.

STİRTMEGY: Draw a Picturé ANSWEIR:
The Monday and Tuesday total was 57. On Wednesdav 39 people came to the concert.

Explain what it means to "regroup" when adding or subtracting.

Apply and explain multiple strategies to determine sums and differences on 2-digit numbers, with and without regrouping.

Materials: paper, pencil, counting rods, calculator, base ten blocks, metre stick, 6 -sided die.

- Work with a partner. Each person rolls the die and takes the same number of rods (any color or combination of colors) as the value rolled.
- In turn, each person creates a train of his or her rods along the edge of the metre stick to determine the length.
- The task is to determine what the overall length of the two trains would be if placed end to end along the edge of the metre stick.
- One partner calculates the length by modeling the addition using base ten blocks. The other partner adds the two lengths with paper and pencil and then on the calculator.
- Working together, the partners now place their trains end to end to check the total length against the values determined using the paper/pencil, calculator and base ten blocks. Repeat, but switch roles.

Clarissa's shoe is 19 cm long. Joel's shoe is 24 cm long. How far would their two shoes stretch if placed end to end?
If you wanted to make the longest train possible using two different shoes of students in your class, whose shoes would you choose?


## sTisATEXiV: Write an Eqüation:

 - NSW DEB:Clarissa's and Joel's shoes together are 43 cm long.

Write a letter to a creature from another planet explaining how we would use base ten blocks to show $15+17$. Draw a picture of you and your space friend.

> Apply a variety of estimation and mental mathematics strategies to addition and subtraction problems.


Materials: popsicle sticks.

## Strategy One: Count on Tens, then Ones

- Pick two numbers less than 50 (assume 38 and 25 ). Create two sets using popsicle sticks. To the largest number ( 38 in our example), add the tens of the second number and model by joining the tens bundles of the second number to that of the first number (in our example we would join to the 38 the two tens from the set of 25 thus creating $38+10+10=58$ ). Now count on the remaining ones (in our example we have the 5 remaining singles from the 25 , thus starting at 58 we would count $59,60,61,62$ $63 \ldots$ for a sum of 63 ).


## Strategy Two: Borrow

- Pick two numbers less than 50 (assume 38 and 25 again). Create two sets using popsicle sticks. Take the right number of singles from the second set and add them to the first set to complete a group of 10 (in our example we would take 2 singles from the 25 and add them to the 38 to create two new sets the first set now has 40 and the second set has 23). Count all of the groups of ten and all of the singles to find the sum. Work with a partner to make up a problem and then find the sum mentally and with the popsicle sticks.

- Play this game with a partner.
- Shuffle the cards and spread them out face down.
- Player one turns over a single card. Player two now turns over any 2 other cards hoping to find two cards with a sum or difference equal to the card turned over by player one.
- If unsuccessful, all cards are turned face down again and players switch roles. If successful, player two scores a single point and the cards are removed from the game. Players now switch roles.
- First player to score 3 points is the winner.


## 

## How many addition equations are there which have a sum of 10 ?

## STIEATUEGV: Constructa Z İst ANEWEIR:

There are six in all.

If players are successful turning over the cards, ask them to provide one other addition or subtraction sentence with the same sum or difference. This must be done before they can collect their cards.

Demonstrate the processes of multiplication and division using manipulatives and diagrams.

Materials: two six-sided dice (one red, one white), linking cubes, paper, pencil.

- Let the red die represent the number of sets of blocks, and the white die represent the number of blocks in each set.
- Roll the dice. Create an array of blocks: (Example: red rolled 3, white rolled 4)

- Draw a picture of your set. Write a multiplication statement to go with your picture.
- Create a bulletin board display of all the different arrays you can make using these two dice.
How many different ways can you find to arrange exactly 24 blocks into columns and equal sized


Replace the 6 -sided dice with 8 or 10 -sided dice to maker larger numbers of larger sets.

Write addition sentences to go with each model.
Write two division sentences to go with each model.

There are 4 ways: $1 \times 24,2 \times 12$,


$$
3 \times 8 \text {, and } 4 \times 6 \text {. }
$$

How can you tell if one number is divisible evenly by another? For example, can 14 be divided by 3 evenly?
How do you know?

Sort objects and shapes, using one or two attributes.

Materials: attribute blocks, overhead spinner, blank spinner mat.

- Create a spinner mat with the following regions: more than 3 corners, large, thin, (and a very small region which reads 'select any shape').
- Play this game with a partner. Create a set of any 11 attribute blocks placed in a pile between two players.
- The first player twirls the spinner and takes one shape of the type identified by the spinner (assuming there is one).
- Play passes to the second player who likewise tries to claim a block from the pile.
- Players continue taking turns and claiming blocks until all blocks are taken. The player with the most blocks wins.


## How many objects in the room can you find which are both blue and rectangular?



Make a second spinner mat with each region showing one color.

Play the game described above, but players must spin one attribute from each spinner mat before claiming a block.

Materials: attribute blocks, string.

- Work with a partner.
- First partner creates a loop with the string. This player silently decides upon a rule for sorting the blocks (e.g., "only squares").
- This partner now adds one block to the set in the string loop. The second partner guesses what the rule is for sorting the blocks. If incorrect, the first partner adds another block, and the second partner makes another guess.
- This process continues until the second partner guesses, correctly at which time the partners begin again, switching roles.

Identify and describe patterns, including numerical and nonnumerical patterns.

Materials: 100 chart, pencil crayon, calculator.

- Start with zero in your calculator. Generate your pattern by repeatedly adding 3 to the value on your calculator.
- Color on your 100 chart each number which the calculator displays.
- Repeat until you reach or pass 100 (or until you can guess the pattern and finish coloring the remaining boxes on your chart).
- Write a description of the pattern on your 100 chart.
- Repeat with a new 100 chart, but add 2 each time.
- Repeat with a new 100 chart, but add 5 each time.
- Repeat with a new 100 chart, but add 9 each time.
- Repeat with a new 100 chart, but add 1 , then 2 , then 1 , then $2 \ldots$

If you create a train (along the edge of a metre stick) of a white and a blue counting rod repeated over and over, you can make a train exactly 50 cm long.

What other sets of two different rods can be used to make trains exactly 50 cm long?

## STIBATEGY: Make a List

## ANSWEIR:

The following sets of rods will work: white + blue, red + brown, light green + blac:゙范,
dark green + purple, white + purple, Predict: using the number board as described above, what pattern would you see if you add 5 , then 6 , then 5 , then 6 , and so on.

Check and see!


Make a list of 5 patterns you see on your 100 chart. Give an example of each.

Create, extend and describe patterns including numerical and nonnumerical patterns.

Materials: deck of cards.

- Play this game with a partner.
- Take any red card and any black card from the deck laying them face up on the table. These two cards are the start of a pattern (red, black, red, black, red ...). Split the remaining deck into two equal parts, and give one to each player.
- Players now take turns drawing the top card from their pile and playing it on the pattern already started (if it fits the pattern).
- If it does not fit the pattern, the card is returned to the bottom of that player's deck.
- The first player to play all of his/her cards on the pattern wins.
- Adaptation: Play the same game, but alter the pattern. For example:
- red-red-black-black
- spade-heart-diamond-club

Jonas started with 7 blocks in his bucket. On the first day he added one block, on the second day two blocks, on the third three blocks and so on. On which day had Jonas collected 100 blocks?



Have students work on their own. First sorting the cards (all reds in one pile, all blacks in another).

Now have students build a pattern by drawing one card from each pilc laying them on the table in turn.


Play the same game as above, but give one player all the red cards and one player all the black ones.

Each player plays one card to start the pattern (e.g., a six followed by a 9 ). This pattern of digits ( 6 and 9 in our example) must now be repeated until one player has successfully played 3 more cards.

STIB ITEEIV: Act it Out ANSWEIR:

On the 14th day Jonas had a total of 112 blocks in his hucket.

Translate patterns from one mode to another: manipulatives. diagrams, charts, calculators, words, symbols.

Materials: paper, pencil, pattern blocks, 100 chart, crayon.

- Make a chart of the different pattern blocks, listing one of the following activities beside each type of block: blink, wink, snap fingers, tap toe, clap, touch nose.
- Now make a pattern with any 2 pattern blocks and then act it out. For example,

- Draw your pattern on a piece of paper. Write the actions under each block.
- Now write the color under each block. Now pick a different letter for each type of block and write that letter under each one.

Jarrod created a train of pattern blocks that looked like this:


What color was the 40th block?

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STIRITIEANV: Make a Modet ANSWEIB:
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Assign a value to each block as follows: white (1), green (1). blue (2), red (3), orange (4). yellow (6).

Create a pattern using 2 kinds of blocks. Use a calculator to add up the value of each block coloring the sums as you go on your 100 chart.

Describe any patterms you see.

The 40th block was blue.

Construct items of specific lengths ( $\mathrm{cm}, \mathrm{dm}, \mathrm{m}$ ).

Materials: overhead spinner, spinner mat, dice, metre-stick.

- Play a (team) race game in the gym!
- Create a spinner mat like the one shown:
- The first player (or team) twirls the spinner and rolls the die, and then measures that distance from one edge of the gym floor toward the opposite edge.
- Play now passes to the other player (or team).

- Players (or teams) keep taking turns until one reaches the opposite wall, and that player (or team) wins!

Natalie used only black and purple rods to make a
 train 31 cm in length. How many of each rod did she use? Can you draw a line 31 cm long?


HOWN
Play a similar game, but do not mix the various units (i.e., have students roll the die and measure across a page in cm only, or up the filing cabinet in dm only).

Construct a list of objects which would be measured using $\mathrm{m}, \mathrm{cm}$ or dm .


Ask students to keep track of how many $\mathrm{m}, \mathrm{cm}$ and dm they used during the game and use it to estimate the length of the gym floor using each of the three measures. Use a tape measure to check the reasonableness of the estimate.

## STIBATEGV: Act It Out NNSWEIS:

1 black and 6 purple rods were used.

Select the most appropriate standard unit (cm, dm, m) to measure a length.

Materials: paper strip ( 1 m in length), white counting rod, orange counting rod, eraser. - A white rod is 1 cm in length. An orange rod is 1 dm in length. The paper strip is 1 m in length.

- How many white rods fit in an orange rod? How many orange rods fit along the paper strip?
- Try to use each rod and the paper strip to measure the length of the eraser. Which unit works best? Why?
- Try to use each rod and the paper strip to measure each of the following. Which unit works best? Why?
- the top of your desk
- the door to your classroom
- the floor in your classroom
- the length of your thumb


# Who must walk farther to their washroom door from your classroom door-the boys or the girls? 

Estimate, measure, record, compare and order objects by length. height and distance around, using standard units ( $\mathrm{cm}, \mathrm{dm}, \mathrm{m}$ ).

Materials: tape measure, objects in the room, paper, pencil.

- Create a spinner like the one shown. Work with a partner for this activity. Select 3 different objects of different lengths, and place them on the table top in order from largest to smallest (or longest to shortest).

- Ask your partner to identify one of the three objects. Now twirl the spinner. If the spinner lands on the "+" symbol you must find an object in the room larger (or longer) than the selected object and the next larger object. f the spinner lands on the "." symbol you need to find an object in the room smaller (or shorter) than the identified object and the next smaller object. If your partner selected the largest object and you spun a " + ", then any larger object will do. Likewise if your partner selected the smallest object and you spun a " - ", then any smaller object will do.
- Reverse roles. Continue building your set of objects until you have seven in all. Create a list of the seven objects. Estimate the length of each and then measure to check the accuracy of your estimate. Repeat the activity but measure for height, and then for distance around.

Sondra measured a candle, a pencil, an eraser and a comb. Using the clues below, find out how long each was:

- the candle was 10 cm longer than the pencil.
- the eraser was 9 cm shorter than the comb.
- the pencil was twice as long as the eraser.
- the eraser was 5 cm long.


Select only 2 objects at a time, measuring each and comparing the lengths.


Game: have students record the estimated height, length and distance around for each of 3 objects. Now measure each object. Score 1 point for each cm your estimate is over or under. Low score wins.

Estimate, measure, record and compare the area of shapes using non-standard units.

Materials: pattern blocks.

- Take a large handful of pattern blocks. Use these blocks and put them together to cover the table without leaving any gaps or spaces.
- How many yellow blocks would it take to completely cover your shape? Record your estimate, then try it and see.
- How many red blocks would it take to completely cover the shape? Record your estimate, and then test to check.
- Create a shape that would be covered by 10 yellow blocks placed together not leaving any gaps or spaces. After building the shape, try to cover it with 10 yellow blocks.


## Which collection of pattern blocks would cover a larger area?

Cover a scrap of paper with several different kinds of objects such as cubes, paperclips, thumbtacks, tiles, etc.

Does it require the same number of each of these objects to cover the paper? Why?


Take a yellow block and cover it with green blocks. How many green blocks are needed? Repeat with the red and blue blocks.

How can you use this information to predict how many yellow blocks are necessary to cover a shape.

STIBATEGI: Act lt Out ANSWEIR:

The first set.

Writing Corner: $\cdots \cdots \cdots$
Describe how you could tell if two shapes have the same area.

Materials: 6 -sided die, pattern blocks, color tiles, construction paper, scissors.
(A) • Roll the die. Take the same number of green pattern blocks as the value rolled and place them on top of your construction paper such that one side of each block touches the side of another block.

- Continue rolling the die and adding green pattern blocks until you have used exactly 15 blocks.
- Trace around your blocks and cut out the shape.
- Repeat.
(B) - Using the shape created above, and rolling the same die, see how many rolls it takes to cover each of the paper shapes (you made above) using color tiles.

Knowing that all tiles must touch each other on at least one side, how many different shapes can you make that have an area less


Have the class create a bulletin board display showing shapes with an area of 15 green pattern blocks.


Estimate then check: How many green pattern blocks will it take to cover a piece of paper? Note: you probably will not have enough green pattern blocks - time to problem solve!

Hint: How many green pattern blocks fit in a yellow pattern block?

STIEATEGV: Guess and Check: ANSWEIR:
There are 21 in all: 1 using 1 tile, 1 using 2 tiles, 2 using 3 tiles, 5 using 4 tiles, and 12 using 5 tiles.

Will a shape built from 10 green pattern blocks have an area greater or less than a shape built from 10 color tiles? How do you know?

Materials: linking metric cubes, several small boxes, cans and bottles, paper, pencil.

- Take one container and place as many small metric cubes inside it as you can. Remove the cubes and count them.
- Take a different container. How many cubes do you think will fit in this container? Record your estimate. Check and see how close you were!
- Repeat until you have tried all of the containers.
- Sort the objects from largest to smallest (the one which can contain the most cubes down to one which can contain the least).


> June's box can hold 15 cubes. Marissa's box can hold 23 cubes. Kim's box can hold all of June's cubes and all but 7 of Marissa's. How many can Kim's box hold?


Repeat the activity above. but use more uniform shapes (e.g., only boxes or only cans). Have students fill several containers before making estimates on the remaining containers.

Place several hexalink cubes (c.g., 48) together to make a large rectangular solid. Wrap the solid in paper. Have students estimate the number of blocks inside. Can you make different arrangements of the same number of blocks which look to be different sizes?


Estimate, measure, record, compare and order the mass (weight) of objects, using non-standard units.

Materials: simple balance, color tiles, paper clips, linking metric cubes, several small objects.
(A) Select one small object. Weigh the object three times: once with color tiles, once with paper clips and once with metric cubes.

- how many color tiles does it take to balance the object?
- how many paper clips does it take to balance the object?
- how many linking metric cubes does it take to balance the object?
(B) Select 3 of the small objects (not the one used above). Make a list of the three objects and record an estimate for each: how many cubes would be required to balance each object? Check and see! Now sort the three objects from heaviest to lightest.
(C) Select 2 new small objects. Estimate how much they would weigh together. Record your estimate. Check and see!



It takes 4 paper clips to balance one color tile. It takes 3 color tiles to balance one hexalink cube. Joel's ball can be balanced using 5 cubes. How many paper clips are needed to balance his ball?
A.Noweit:

It will take 60 paper clips. (measured in blocks).
Knowing which object weighs more in blocks, which would weigh more measured in paper clips? Explain
your answer.

Recognize that the size and shape of an object do not necessarily determine its mass (weight).

Materials: plasticine, wooden block ( 2.5 cm on a side), hexalink cube, clay, piece of styrofoam, simple balance.

- Using the materials, create five blocks, approximately 2.5 cm on a side:
- shape a block using the plasticine, shape another using the clay.
- cut a block of the same size from the styrofoam.
- you now have five blocks: wood cube, plastic cube, plasticine, styrofoam, and clay.
- Using the balance, sequence the blocks from lightest to heaviest.
- Answer the questions:
- Which block was the heaviest? Which block was the lightest?

- If you were to make blocks 5 cm on a side, would the sequence still be the same? Why?


# Which weighs more, a kg of wood chips, or a kg of styrofoam? 

Which pile would be bigger?

Use a large stryofoam chunk and a smaller clay ball to explain how size does not necessarily determine mass.

Use two balls of the same size (one of styrofoam and one of clay) to explain how shape does not necessarily determine mass.

STEBATEAV: Ľogical Reasoring: ANGWEIB:
They weigh the same - 1 kg each! The styrofoam pile would be bigger.

Place a large styrofoam ball on one side of a balance. Use plasticine to create a ball to balance it on the other side.

How many styrofoam balls of a given size are needed to balance a plasticine ball of the same size?

## Writing Cormer: <br> Do all objects with the same shape have the same mass? How do you know? Draw a picture to explain your answer.

> Materials: color tiles, stopwatch.

- Take a collection of 100 color tiles and place them in one mixed pile.
- Have one volunteer sort the 100 tiles into like-colored piles, 5 to a pile.
- Estimate how long it will take to stack the tiles. Record your estimates. Let the person try, and time him or her. Who had the best estimate?
- Let everyone in your class try the same activity. How much time did you spend on this activity altogether?
- Make up your own task like the one above. Estimate how long it will take to complete and then try it to test your estimate.


## How long would it take you to count



Play a game:
Teacher times out one minute exactly. Students stand up when they think 1 minute has passed. After all students are standing, teacher announces who was closest to 1 minute. This student reveals his/her strategy.


Have students keep track of how many hours and minutes they spend on each subject over one week.

Create a bulletin board or bar chart showing how time is distributed among the school subjects.

Which activities take up the most time of any given day?

## STHEATEGY: Act It Out

 ANSWEIt:Answers vary.


Now describe a job that would take you about 1
hour to complete.

Select the most approporiate standard unit to measure a given period of time.

Materials: blank spinner mat, overhead spinner.

- Place the following activities on the spinner mat:
 walk the dog, cook supper, vacuum the house, build a house, rake the leaves, make a bed, set the table, do your homework, clean the garage, write your name, count to 10 , eat a french fry.
- Have students take turns twirling the spinner, and describe how long it would take to complete the given activity. Example: it might take only seconds to write your name.
- Suggestion: as students complete one turn, have them add a new event to another blank spinner. When it is complete, move to the next spinner and play again.


Materials: paper, pencil, one 4-sided die, two 6 -sided dice, one 12 -sided die.

- Play this game with your friend! Race against your friend to construct a list of the months of the year, in order.
- On a turn you may choose to roll 1 die or any 2 dice. If you choose 2 dice you must add the values together to determine the value rolled.
- Once you roll a 1 , you can write January on your list. o add February to your list you must roll a 2 , etc. The first player to make a list of all 12 months wins.
- If your roll is successful (that is, you roll the value needed to list the next month), you may take another turn. If you do not roll the value needed your turn ends.

Adaptation: Players roll at the same time as many times as necessary adding months when they can. First player to list December wins.

Braden, Mark, Melody and Kara were all born the same year. One was born in June, one in July, one in April and one in December. Using the clues below, find out who is the oldest:

- Melody is younger and taller than Kara.
- Mark was almost a New Year's baby.
- Braden was born in a short month.
- Braden was born before Kara.

STHIATEGY: Make a List ANSWEIt:

From oldest to youngest they are Braden, before the value rolled!

Kara, Meiody and Mark. Braden is oldest.

Use the 12 -sided die to play
a game with a friend. Take
Use the 12 -sided die to play
a game with a friend. Take turns rolling the dice, then first person to name the month indicated by the value rolled scores a point. High score wins.

Play again, this time naming the month which comes

in your class and their birthdays. Make a list of whose birthday is next, the next, and so on.

Relate the number of days to a week, months to a year, minutes to an hour, hours to a day.

Materials: large clock face (drawn on poster paper, minutes and hours marked off), calendar page (marked off in days and months), dice, small markers.

- Play this game with a friend:
- The game has 2 game boards: a clock face and a calendar page. The objective is to race your way around the clock face (first around the outside to count off the number of minutes in an hour, then along the numbers on the face - twice, to show hours in the day), then to race through one week of the calendar ( 7 days in a week), then through the 12 months of the year. The first player to reach December wins.
- Players take turns rolling the die and moving their markers. As they complete each stage (e.g., minutes, hours, days, months) they should write a sentence to describe the relationship.

Adaptation: if you land on a space held by any opponent, you send them back to the beginning of that stage of the game. For example, if your opponent is sitting on Wednesday, and you land on Wednesday, your opponent must start the days of the week over.

How many times does the minute hand on a clock point to a 3 in one day? during your school day?

## How many times does the minute hand on a clock point to a 1 in one day?



## STIRATEGV: Make a List ANSW:

The hand will point to a 3 a total of 24 times in one day. It will point to a 1 a total of 120 times!.

Using a calculator, compute each of the following:

- days in 2 years
- hours in a week
- minutes in a day
- hours in March

Materials: calendar page with all 12 months on a single page, pencil, ruler.

- Create a calendar dot-to-dot!
- Make a list of dates which are to be connected by a line. Once all of the correct lines are drawn they will result in a picture.
- Example: connect all of the following dates on the 2000 calendar provided to make a letter of the alphabet:

$$
\begin{aligned}
& \text { January } 10 \rightarrow \text { October } 14 \rightarrow \text { May } 24 \rightarrow \\
& \text { December } 3 \rightarrow \text { March } 17
\end{aligned}
$$

Note: activities such as the one above will be specific to a particular calendar. Use the calendar templates appended.
OU what day of the week is the
following
September 24th? week is the
following
September 24th? week is the
following
September 24th?

STHETTEGY: Construct a Modè ANSWERt:

September $24^{\text {th }}$ will fall on a Thursday.


Using the activity above, write out instructions such that the lines would spell your name.

 JII

Use a thermometer to determine rising and falling temperatures.

Materials: beakers, water, ice, black construction paper, thermometer, paper, pencil.

- Begin by placing the water in the beaker. Use the thermometer to record the temperature of the water.
- Place the ice in the water. Record the temperature every few minutes. How cold does the water get before the ice melts?
- Remove any remaining ice.
- Split the water equally into two beakers. Place both beakers in the sunlight: one on a piece of white paper and one on a piece of black construction paper. Record the temperature every few minutes in each beaker. Does one beaker of water warm up faster than the other?


The temperature rose 5 degrees before noon, and then another 6 degrees by 2 o'clock. It stayed at that temperature until 4 o'clock, when it started to fall. By 6 o'clock it had fallen 8 degrees to $16^{\circ} \mathrm{C}$. What was the temperature that morning?


Use a metre stick as a model of the thermometer. Place linking metric cubes along the edge, adding and removing blocks to show the temperature rising and falling. Tell a story that involves temperature change and model it using the metre stick.


In the activity as it is described above, have the students create graphs to show the temperature as it rises or falls in the beakers. Alternatively, have the students graph the temperature each day and at different times during the day. Ask students to predict temperatures at given times tomorrow.

## STHEMTEGY: Work Backwăraś ANSWEIS:

It was $13^{\circ} \mathrm{C}$ that morning. the temperature is high. Now describe something you enjoy doing outside when the temperature is near freezing.

Materials: money manipulative, real or play money (coins).

- Work with a partner for this activity. Have one partner place a collection of coin cards on the money board, leaving no gaps or spaces. Determine the value of that collection of coins.
- The other partner now removes some coin cards and replaces them with equivalent coins (e.g., remove a dime card and replace it with two nickel cards, remove a nickel card and replace it with 5 penny cards, etc.). Determine that the value has not changed.

- Repeat this many times. How many different ways can you find to make the same value as the one with which you started?

Note: if preferred, the same activity can be done without the money manipulative, substituting a set of real or play coins.


How many different ways can you make $36 \not \subset$ using only pennies, nickels, dimes and quarters?

How can this be done using exactly 10 coins?


In the activity above have students place real or play coins on top of the coin cards. The coins can be removed at any time to create a set with the specified values.


Keep track of how many different ways there are to make up each of the values up to 25 ¢.

Describe any patterns you might find.

## STIBATEGV: Make a List

 ANGWEIR:There are 24 ways in all. It can be done with 10 coins by using 6 pennies, 2 dimes and 2 nickels:


Estimate, count and record using the cents symbol only, the value of collections of coins up to $\$ 1.00$.

Materials: real or play coins (mostly pennies, some nickels, a few dimes, no more than 3 quarters), overhead spinner, blank spinner mat.
(A) - take a handful of coins from the given collection of coins.

- estimate the value of the coins.
- count the coins to determine the actual value. How close were you?
- try this again to see if you can make an even better estimate.
(B) - create a spinner mat with 4 equal sections: penny, nickel, dime, quarter.
- twirl the spinner 6 times, taking the coin spun to create a set of 6 coins.
- estimate the value of the set, then count to check how close you were.
(C) - using the spinner above, play a game with a friend.
- take turns twirling the spinner and taking the coin spun.
- after 5 turns, who has the most money?



# Angela has 3 of one kind of coin, 2 of another, and 5 of another. The value of her set of coins is 45c. Which coins does she have? 



## STIB:ITEGY: Guess \& Chêck CNEWEAS:

Angela has 3 dimes, 2 nickels and 5 pennies.

Play a game with a friend like the game described in part (C) above. Twirl the spinner taking the coin spun. If taking that coin would put you over $\$ 1.00$ you must give up one of that coin instead. First person to reach $\$ 1.00$ exactly wins.

Explain how you estimate the value of a set of coins.

Materials: money manipulative, real or play money.

- How many of the penny cards does it take to cover the quarter card? Write a sentence to explain how many pennies there are in a quarter.
- How many quarter cards does it take to cover the money board (or, in other words, to make up $\$ 1.00$ )? Write a sentence to explain how many quarters there are in one dollar.
- How many penny cards would it take to cover the entire money board (to make up $\$ 1.00)$ ? Write a sentence to explain how many pennies in one dollar.

- Use what you know to figure out each of the following: How many pennies in a $\$ 2.00$ coin? a $\$ 5.00$ bill? a $\$ 10.00$ bill?


Tim decided to trade all of his money in for pennies at the bank. After trading he had 775 pennies. Assuming he had as few bills and coins as possible, what did he have before going to the bank?


## NTIATT日Xiv: Make a List ANSWER: <br> Tim had a $\$ 5$ bill, a $\$ 2$ coin, and three quarters.

Use a deck of cards (2 through 9). Turn over 3 cards to make a number less than 1000 . How many pennies, nickels, dimes, quarters, $\$ 1$ coins, $\$ 2$ coins. and $\$ 5$ bills are necessary to equal this number of pennies? Use as few of each as possible.

Materials：solids，paper，pencil．
－Take your 3－D solids one at a time and trace around all of their flat surfaces（faces）．
－Sort the solids into groups：
－those which can be used to draw circles，and those which cannot．
－those which can be used to draw squares and rectangles，and those which cannot．
－those which can be used to draw triangles，and those which cannot．
－those which have faces which cannot be traced and those whose faces can all be traced．
－Place the solids in order from the one with the most faces to the one with the least．
－Place the solids in order from the one with the most vertices to the one with the least．
－Place the solids in order from the one with the most edges to the one with the least．

If you could walk all around（and even under）these figures how many faces could you see on each？


Can you build a figure with exactly 20 faces？

Take a separate page for each solid．Trace all the faces for that solid on that page．Set the solid on top of its page．

Now try sorting the solids in the activity above．


By tracing around the edges of your solids，draw a picture of：
（a）a child playing with a ball at a beach，
（b）a child sledding down a very long hill，
（c）a cat made out of only triangles．

STILITEGI：Make a Modéty： ANSWEER：
The first figure has 18 ，the second 36 ．


Identify, name and describe specific 3-D objects as cubes, spheres, cones, cylinders, pyramids.

Materials: solids, small boxes, cans, balls, Toblerone chocolate boxes, etc., paper, pencil.

- Begin by folding your paper in half and unfold it again. Select one object you think has the same shape. Write the name of the solid at the top of the page.
- On the left side of the page trace all sides of the solid.
- On the right side of the page trace all sides of the object selected.
- With a line, connect one face on the left side of the page with a similar face on the right side. Do the objects have the same numbers of the same kinds of faces? Do the objects have the same number of edges? of vertices?
- On the back of the page write two sentences about what your solid can do (ie., can it stack? can it roll?). Can your chosen object do the same things?


## I have 3 solids, only two of them are alike. Altogether these solids have 12 faces, 16 edges and 8 vertices. What solids do I have?

## STIBITEAV: Guess \& Check ANSWER:

I have two cylinders and one rectangular prism.

## Writing Corner: 00 Oh my oh me, What can I be? <br> live only one face <br> That you can't even trace! <br> Write a poem like one

above that would get a friend to guess what kind
of solid you are describing.

Build a skeleton of a 3-D object, and describe how the skeleton relates to the object.

Materials: solids, toothpicks, marshmallows.

- Use toothpicks and marshmallows to construct a rectangular prism:
- How many toothpicks did you use? How many edges does your rectangular prism have?
- How many marshmallows did you use? How many vertices does your rectangular prism have?
- Repeat the above process, but construct a rectangular pyramid.
- Repeat the above process, but construct a triangular prism.
- Repeat the above process, but construct a triangular pyramid.

Francine wants to create a figure using toothpicks and marshmallows. She wants the figure to look like a clock tower with a cube on the bottom and a square pyramid on the top. What is the fewest number of
toothpicks and marshmallows she needs?

## STIEATEGY: Act it Out ANSWERE:

Francine will need 9 marshmallows and 16 toothpicks.


Use the toothpicks to create two dimensional shapes (squares and rectangles) first and then join them together to make three dimensional shapes.
A. Build compound shapes such as a triangular prism with a triangular pyramid at each end.
B. Play a game: race to construct a cube. If you roll a 1 or 2 you can add a
marshmallow. Roll a $3,4,5$ or 6 to add a toothpick. You must start with a marshmallow.


Materials: color tiles.

- Work with a partner for this activity. Place 3 tiles on the table side-by-side:

- Pretend this is the start of a pattern. Build the pattern taking turns adding blocks. See who is first to make a mistake in the pattern.
- After you have added several tiles, rearrange your train by dividing it into rows where each red tile starts a new row. How many of each color did you use?
- Repeat as above, but use each of the following to start new patterns:

Red Green Yellow Blue Green Red Green

## Corey lined up several buttons in order:

blue-red-black-blue-red-black, and so on.
How many buttons of each color are there in the first 25 buttons in the pattern?

命定

Begin with simpler patterns (e.g., use only an AB pattern) and have the students write several sentences about their patterns:

- how many of each color are needed?
- are there more of one color than another? etc.



## ANSWEAR:

 FTIBITEGV: MakeaListCorey will use 9 blue buttons, 8 red buittons, and 8 black buttons-

Have the students:

- write sentences about their patterns.
- chart their patterns.
- act their pattern out (e.g.. every red tile you snap your fingers. clap for every blue (ile, ctc.).
- record the pattern using the $A B C$ notation.
- look for objects in the classroom which show patterns.


## 

Match and make identical (congruent) 2-D shapes.

Materials: pattern blocks.

- Use as many pattern blocks of any kind as you wish to create a shape that looks like a dinosaur. Have a friend create a second dinosaur exactly like the first. How do you know the dinosaurs are identical?
- Now use the pattern blocks to make a shoe. Make the mate to the shoe.

- Use the pattern blocks to build a tree. Make three more and pretend it is a forest. What kind of animals live in your forest?


# The silhouette of Teresa's shape built with pattern blocks is shown. She used exactly 7 blocks and exactly 4 of them were green. Can you build a shape like Teresa's? 




Play a game of 20 questions:
The first player builds a shape using three pattern blocks. hidden from the second player.

The second player now tries to build the same shape of the same blocks by using clues earned by asking up to twenty yes/no questions.


Communicate and apply positional language in oral, written or numerical form.

Materials: OH spinner, 2 blank spinner mats, color tiles.

- Create 2 spinners:
- Color Spinner - 3 sections: red, green, blue
- Position Spinner - 5 sections: above, behind, below, in front, beside
- Take 3 tiles (a red, green and blue) and arrange them on your table top. The tiles may be randomly placed in a row, side by side, or separated, but they may not be stacked.
- Twirl both spinners and place a yellow tile on the table according to the instructions spun. Example: assume you spun "below green," then lift the green tile and place a yellow tile beneath the green.


## Sandra has stacked four color tiles. Create a stack like hers using the clues below:

- no 2 like colors are next to each other.
- the only red tile is above the only blue tile.
- there is no yellow tile.
- the tiles on the top and the bottom of the stack are different colors.

Using only the positional spinner as described above, have students spin and then pick two objects which can be described by the position identified. Example, the clock is above the blackboard, or Jason is beside Ryan.


1. Have the students make 5 spins and then using construction paper squares build a model of their tile arrangements. Write a sentence for each yellow tile.
2. Play Simon Says using the positional spimer. Example: Simon Says put your hands behind your knees.

STiBATEGY: Act ltout

## ANSWHER:

red green blue $\%$ green

Create symmetrical 2-D objects by folding and reflecting.

Materials: paper, scissors, color tiles, mirror.
[1] To cut a heart from a piece of paper, first fold the paper in half, and then along the folded edge cut half of a heart. This way you know the two sides of the heart will match.


Using the same method, make a spade, diamond and club!
[2] Take 4 color tiles (no more than 2 of the same color) and arrange them in a square as shown. Along one edge stand a mirror. Use the mirror to build the reflected image. Remove the mirror and write two sentences about the rectangle you have built.

## How many of the capital letters have a horizontal line of symmetry? How many have a vertical line of symmetry? Can you make any words with a line of symmetry using these letters?

 etc.

STIS ITEGV: Drawa Pictüre: NNSWNER:
Eleven letters have a vertical line of symmetry. Nine letters have a horizontal line of symmetry. BED, BOX and DICE are examptes of wonds withe:a: Tine of symmetrate.

Have students fold the paper twice (into quarters) before cutting - save both folded edges when you cut!

After unfolding you will have figures with 2 lines of symmetry.

What shapes and figures can you make?

## - Writing Comer: 0 ••• Make a list of 5 objects in the room which are

 symmetrical. Pick one object and explain how you know it is symmetrical.Formulate the questions and categories for data collection，and actively collect first－hand information．
jty，
Materials：blank spinner mat，overhead spinner．
－Create a spinner with the following six categories：
jYy ${ }^{2}$ Materials：blank spinner mat，overhead spin
－Create a spinner with the following six categories： pets，movies，shoes，sports，hair，eye．
－Twirl the spinner．
－Create a question in the category spun．Examples：
How many people in your class have a cat as a pet？
Do more people like to watch a movie at home or in a theatre？
What is the most common shoe color of the students in the Grade 2 class？
What sport or game will the students in your class play at recess？
What is the most common eye or hair color of the students in your class？
－Conduct a survey to answer your question．
－Create a poster to display your question and the answer．


# In your class，what is the favorite flavor of ice cream？ 

Start by giving the students a question and collect the information together．

Now，ask the students to generate a follow－up question and proceed to collect the data．


Have students brainstorm a list of possible questions which can be researched．

For each question brainstorm a list of possible ways to collect the data．

Answers vary．


Choose an appropriate recording method, such as tally marks, to collect data.

Materials: popsicle sticks (or strips of colored paper), crayons, paper bag.

## Part A

- Start by coloring several popsicle sticks, some red, some green, some blue and some yellow.
- Conduct a poll: go ask several friends which of these four colors is their favourite color. As they select their favourite color, have them place that popsicle stick in the bag.
- After you have finished your poll, empty out the bag and arrange the popsicle sticks in sets of up to 5 of the same color as shown:
- What color was most popular? least popular?


## Part B



- Conduct another survey, this time recording individual responses on a blank popsicle stick before putting it in the bag. Group the sticks to build a tally graph.

Darren conducted a survey of the eye color of everyone in his class. Darren has 24 classmates. How many more people must he ask to complete his chart?

## STIEATBEGV: Use an Equättờn ANSWEIE:

He must ask 2 more people.

In Part A above, start by grouping the like-colored sticks together. Count the number of each color.

Now make a list of each and its frequency.

Now arrange the sticks into a tally graph. Show how the tally graph makes it easy to count the number in each set.


Work with a friend. One of you should conduct a poll as in Part A above. The other should ask the same people the same question, but record the answers with a paper and pencil.

Build a popsicle stick tally chart and a paper and pencil tally chart.

How are they different? the same?



Organize data using graphic organizers such as diagrams, charts and lists.

Materials: 4-, 6-and 8 -sided dice, paper, pencil.

- Conduct an experiment:

Is it easier to roll a with a 4-, 6 - or 8 -sided die?

- Roll all three dice at the same time. Keep track of which dice show a $\%$ on each roll. After 20 rolls, which die produced the most ${ }^{\bullet \cdot} \cdot \mathrm{s}$ ?
- Repeat the experiment two more times.
- Create a chart to show how many ${ }^{\bullet} \cdot$ s were rolled with each die for each time you tried the experiment.


Construct and label concrete/object graphs, pictographs and bar graphs.

Materials: color tiles, blank spinner mat, overhead spinner, pencil.

- Work with a partner. Start by making the two spinner mats as shown.
- Have each partner twirl a spinner at the same time. If both spinners point to the same color, take one tile of that color and set it aside to build a concrete graph later. Repeat until you have set aside ten tiles.
- Use the ten tiles to create a concrete graph.
- Construct a bar graph and pictograph to show your results.



## Kara drew the incomplete pictograph shown. Using the clues below, finish the graph:

- the number of red and green buttons was the same as the number of blue and yellow.

| red | (๑) (๑) |
| :---: | :---: |
| green | @ © ® $\odot$ |
|  |  |
|  |  |

- there are more than 10 buttons in all.
- there is at least one button of each of the 4 colors.
- there are more red than yellow buttons.


##  ANSWUEIS:

There are 5 blue buttons and 1 yeilow


Discuss data and draw and communicate appropriate conclusions.

Materials: paper bag, color tiles, paper, pencil.

- Work with a partner for this activity.
- While your partner is not looking, place zero, one or two tiles of each color in the bag. Try to make sure you have at least 3 tiles in the bag.
- Now draw one tile from the bag (no peeking!) and show it to your partner.
- Return the tile to the bag. Do this three times and then let your partner guess how many of each color tile you put in the bag.
- Your partner will want to keep track of what is drawn by making a list.
- If his/her guess is correct, switch roles. If incorrect, draw and replace three more tiles and let him/her guess again. Keep going until the partner guesses correctly.
- Who can figure out what is in the bag first?

Jackie twirled her spinner 15 times and made a list


Generate new questions from displayed data.

Materials: 6-sided die, linking metric cubes, paper, pencil.

- Roll a die 5 times.
- On each roll take a collection of cubes (the same number as the value rolled).
- Sort the cubes into groups by colour. Link the cubes together to make trains of cubes made from the same colour
- Create a tally chart to display the number of each color.
- Create a bar graph to display the number of each color selected. How is the bar graph like the trains of cubes you created above?
- Write three questions you could ask about your tally chart.
- Write three questions you could ask about your bar graph.


Justin kept his marble collection in a bag. He pulled the marbles out of the bag one at a time and made a tally chart to show how many he had of each color. Which of the following questions can be answered using the information in the tally chart?

| red | HH H | 1 |
| :---: | :---: | :---: |
| green | HH |  |
| blue | 11 |  |
| yellow | HH II |  |

- Were there more red or green marbles in the bag?
- How many different colors of marbles were in the bag?
- How many marbles were in the bag?
- Which color of marble did Justin draw first?


## STIBATEGV: Logical Reašorinğ. NSW曋:

The last question cannot be answered with the clues given.

Have students collect charts and graphs from newspapers and magazines. Construct one or more questions that can be answered using the information in those charts and graphs. Create a bulletin board display.


Describe the likelihood of an outcome, using such terms as likely, unlikely, expect, probably.

## Materials: deck of cards.

- Work with a partner to play this game.
- Decide who will collect hearts and who will collect kings.
- Shuffle the cards and leave them face down in a pile.

- Take turns flipping over a single card. If you are collecting hearts and you flip over a heart then you get to keep the card. Otherwise you discard it.
- Take turns flipping over cards looking for your type of card until all cards have been claimed or discarded. The player who collects the most cards in his/her set is the winner.
- Answer the questions:
- are you more likely to draw a heart or a king.
- is it likely or unlikely that you will draw a king?
- should you expect to draw a heart every turn?


With which spinner will you probably spin a 1 ? With which spinner are you likely to spin a 2?


STIRITEGV: Guess \& Chẽk NNSWEIR:
You are likely to spin a 1 with the third spinner. You are likely to spin a 2 with the second spinner.

How does this help
explain what you are explain what you are likely to draw?

Play the game again, this time drawing for 'red' cards and "black' cards.

How do the results of this game compare to game described above? Which game could be described as fair? How do you know?


Make a prediction based on a simple probability experiment.

Materials: blank spinner mat, overhead spinner.

- Create a spinner like the one shown.
- Play this game with a friend.
- On a turn: - predict where the spinner will land when twirled.
- twirl the spinner.
- if your prediction is correct, score that number of points.
- First player to collect 8 or more points is the winner.
- Adaptation: change the game so that if you predict incorrectly, your opponent scores the number of points indicated by the spinner.



## Are you more likely to roll a 2 with a 6 -sided die or with a 4 -sided die?



$\mathbf{N}(\mathbf{N C})$-01. Page 10
Jerry: $630,640,650,660,670,680,690,700,710$. $720.730,740.750 .760,770,780,790.800$, $810,820,830,840,850,860,870,880,890$. $900.910,920,930,940,950,960,970,980$.
990. Jerry says 37 numbers.

Erica: $875,880,885,890,895.900,905.910,915$. $920.925 .930,935,940,945,950,955,960$. 965, 970. 975, 980, 985, 990. 995. Erica says 25 numbers before reaching 1000 .
Johnel: 936, 937, 938, 939, 940. 941, 942, 943, 944. $945,946,947,948,949,950,951,952,953$. 954. 955, 956. 957. 958. 959. 960. 961. 062. 963. 964, 965, 966, 967, 968, 969, 970, 971. 972, 973, 974, 075, 976, 977, 978, 979, 980, 981. 982. 983, 984. 985, 986, 987. 988, 989. 990. 991, 992, 993, 994, 995, 996, 997, 998. 999 . Johnel says 64 numbers before 1000 .
$\mathrm{N}(\mathrm{NC})-\mathbf{0 2}$, Page II. Answers vary.
Answer varies depending upon class size. Largest number is triple the number of students in the class. Smaliest number is the same as the number of students.

## $\mathrm{N}(\mathrm{NC})-03$. Page 12

Split the set of bingo chips between Janna and Susan, giving each girl 37 chips $(37+37=74)$. To ensure Janna has 10 more chips than Susan. give 5 of Susan's chips to Janna:

Janna: $37+5=42$.
Susan: $37-5=32$.
Total: $42+32=74$.
$\mathrm{N}(\mathrm{NC})-\mathbf{0 4}$. Page 13
We may use 0.1 or 2 single blocks, therefore we can make the numbers 30.31 or 32 . However, the problem does not limit blocks larger than the long blocks. so any combination of flat blocks or large cubes together with the 3 longs and singles would also be correct! Other possible solutions: 130, 530 . 3432. 731, etc. In all, there are 300 solutions using only the single, long. flat and large cube blocks!
$\mathrm{N}(\mathrm{NC})$-05. Page 14
Any value less than 100 where the digits have a sum of 8 would work:

$$
8,17,26,35,44,53,62,71.80 .
$$

$\mathrm{N}(\mathrm{NC})$-06, Page 15
Rounded Down: 24. 30, 31, 32, 33. 34, 40, 41, 42 , $43.44,50,51,52,53,54$. There would be 16 numbers which would be rounded down if you count 30,40 and 50.
Rounded Up: 25, 26, 27, 28 29, 35, 36, 37, 38. 39, $45,46,47,48,49,55,56,57$. There would be 18 numbers which would be rounded up.

## N(NC)-07, Page 16

The largest number you can make is by puting the largest digit in the tens place, and the same digit in the ones place, therefore 66. The smallest you can make is 22 using the same process. The largest you can make without repeating a digit can be made by putting the largest digit in the tens place, and the second largest in the ones place, hence 65 . Using the same process the smallest that can be made is 23 .
$\mathbf{N}(\mathbf{N C})$-08. Page 17
Letter E: used 33 times Letter F: used 5 times
Letuer G: used 2 times Letter H : used 4 times
Letter I: used 9 times Letter L: used 2 times
Letuci N : used 17 times Letter O : used 4 times
Letter R: used 4 times Letter S: used 4 times
Letter T: used 15 times Letter U : used 2 times
Letter V: used 5 times Letter W: used 3 times
Letier X: used 2 times Letter $Y$ : used 1 time.
The letter E is used the most often. The letter Y is used only once. and the vowel $A$ is not used at all.

## $\mathrm{N}(\mathrm{NC})$-09. Page 18

1 -rain, 2 -wind. 3 -sun, 4 -rain, 5 -wind, 6 -sun. 7 -rain. 8 -wind, 9 -sun. 10 -rain, 11 -wind. 12-sun, 13 -rain. 14 -wind, 15 -sun, 16 -rain, 17 -wind, 18 -sun, 19 -rain. 20 -wind. 21 -sun, 22 -rain, 23 -wind, 24 -sun, 25 -rain, 26 -wind, 27 -sun. 28 -rain. 29 -wind. 30 -sun. 31 -rain. There are 10 sumny days in all.

## $\mathbf{N ( N C )}$-10. Page 19

To enter 1-9 requires pressing 9 numbered keys. To enter 10-19 requires 20 key strokes. 20-29 takes 20 key strokes, 30-39 takes 20 key strokes, 40-49 takes 20 key strokes. To enter 50 requires 2 key strokes. $9+20+20+20+20+2=91$ strokes.
The following numbers require pressing the 3 key: 3 , 13, 23, 30, 31, 32, 33 (iwice!), 34, 35, 36, 37, 38. 39. and 43. Jackie will press the 3 key 15 times.

## $\mathbf{N}(\mathbf{N C})-11$, Page 20

Recognize that any number 1.3,4 or 8 can go in the tens place, but only the 4 and 8 can go in the ones place if it is to be an even number. Therefore, we can construct this list:
$4,8,14,18,34,38,44,48,84,88$.
We can make 10 different numbers.

## $\mathbf{N}(\mathbf{N C})-12$, Page 21

The solutions shown below represent the various combinations of blocks which can be made having an area 3 times that of a single yellow hexagon. The arrangement of the blocks may vary.









The last solution above is the only combination of blocks not using any green blocks.
$\mathrm{N}(\mathrm{NO})-13$. Page 22
On Monday and Tuesday: $18+39=57$ people. $96-57=39$ people came on Wednesday.

## $\mathbf{N}(\mathbf{N O})-14$. Page 23

Clarissa's shoe and Joel's shoe together would make a train $19 \mathrm{~cm}+24 \mathrm{~cm}=43 \mathrm{~cm}$ long.
Answers vary.

N(NO)-15, Page 24, Answers vary.

## N(NO)-16. Page 25

Assuming that you use only whole numbers, and that $1+9$ counts as the same equation as $9+1$, then there are six in all.

## $\mathbf{N}(\mathbf{N O})$-17. Page 26

Assuming you create only simple arrays, and assuming that a $2 \times 12$ array is considered the same as a $12 \times 2$ array, there are 4 different ways:


## AAAAAAADAAAB AAAMAAAAAAAA

## AAAAAABA AAAAAAAA АААААААА

## AAABAB AAAAAA AAAABA AABAAB

PR(P)-01, Page 27. Ansuers vary
$\mathbf{P R}(\mathbf{P})-02$, Page 28
The names of each of the first set of objects contain the letter A: bat. guitar, glass, paper, and eraser. None of the objects in the second set have names which contain the letter A: clock, spoon, desk.

## $\mathbf{P R ( P ) - 0 3 , ~ P a g e ~} 29$

Any number which is a factor of 50 and which can be built using a set of two counting rods would work. The factors of 50 are 1,2,5,10,25 and 50 . Of these only the values 2,5 and 10 can be built from a train of two counting rods. A train of length 2 can only be built from two white rods, and they are not different from each other. so we are left with trains of length 5 and 10.

Trains of length 5 can be made from:

(length $1+$ length 4)
(length $2+$ length 3 )
Trains of length 10 can be made from:


PR(P)-04, Page 30
Day One: 7 blocks +1 block $=8$ blocks.
Day Two: 8 blocks +2 blocks $=10$ blocks.
Day Three: 10 blocks +3 blocks $=13$ blocks.
Day Four: 13 blocks +4 blocks $=17$ blocks.
Day Five: 17 blocks +5 blocks $=22$ blocks.
Day Six: 22 blocks +6 blocks $=28$ blocks.
Day Seven: 28 blocks +7 blocks $=35$ blocks.
Day Eight: 35 blocks +8 blocks $=43$ blocks .
Day Nine: 43 blocks +9 blocks $=52$ blocks.
Day Ten: 52 blocks +10 blocks $=62$ blocks .
Day Eleven: 62 blocks +11 blocks $=73$ blocks.
Day Twelve: 73 blocks +12 blocks $=85$ blocks .
Day Thirtcen: 85 blocks +13 blocks $=98$ blocks .
Day Fourteen: 98 blocks +14 blocks $=112$ blocks.
Jonas had collected 100 blocks on the fourteenth day.
PR(P)-05, Page 31
Construct a list of blocks by color until you reach the fortieth element in the list. The blocks are grouped below in sets of ten:

BRRBRRBRRB
RRBRRBRRBR
R BRRBRRBRR
BRRBRRBRRB
The fortieth block is blue.

## SS(M)-01, Page 32

A black rod has a length of 7 , and a purple rod has a length of 4 . Create a table of the numbers of each rod used and their total length:

| Black |  | Purple |
| :---: | :---: | :---: |
|  |  |  | | Total Length |
| :---: |

Therefore. 1 black rod and 6 purple rods together have a lengh of 31 cm .

SS(M)-02, Page 33, Answers vary.

## SS(M)-03. Page 34

Eraser is 5 cm long.
Pencil is twice as long as eraser, $2 \times 5=10 \mathrm{~cm}$.
Comb is 9 cm longer than eraser, $9+5=14 \mathrm{~cm}$.
Candle is 10 cm longer than pencil $10+10=20 \mathrm{~cm}$.

SS(M)-04, Page 35
In a set of pattern blocks, a hexagon has an area equal 106 green triangles, a trapezoid has an area equal to 3 green triangles, and a blue rhombus has an area of 2 green triangles. Therefore, the total area covered by the first set would be: $6+3+2+1=12$ green triangles. The total area covered by the second set would be: $3+3+2+2+1=11$ green triangles. The first set covers a greater area.

SS(M)-05. Page 36
There are 21 possibilities in all, as follows:
Using one tile: $\square$
Using two tiles: $\square$

Using three tiles: $\square$ $\square$

Using four tiles: $\qquad$ $\square \square . \square$.


Using five tiles:

$\mathbf{S S}(\mathbf{M})-\mathbf{0 6}$, Page 37
$15+23-7=31$ cubes. Kim's box can hold 31 cubes.
$\mathbf{S S}(\mathbf{M})-\mathbf{0 7}$. Page 38
$4 \times 3 \times 5=60$. It will take 60 paper clips to balance the ball.


SS(M)-08. Page 39
Each pile (styrofoam and wood chips) must weigh the same if they each weigh 1 kg . Because styrofoam is a less dense material, the pile of styrofoam would be much bigger.

SS(M)-09. Page 40, Answers vary.
SS(M)-10. Page 41, Answers vary.

## SS(M)-11. Page 42

Because Mark was almost a New Year's baby, he must be the one horn in December. Braden was born hefore Kara who was born before Melody. The order is:
April - Braden
June - Kara
July - Metody
December - Mar

SS(M)-12, Page 43
The minute hand will point to a 3 at quarter past every hour of the day. Given there are 24 hours of the day, the minute hand will point to the 3 a total of 24 times. There are live 1's on an analog clock. so the minute hand will point to a 1 five times every hour. There are 24 hours in the day, so the mimute hand will point to a 1 a total of $5 \times 24=120$ times each day.

## SS(M)-13, Page 44

In 1998, July 17th was a Friday and September 24th was a Thursday. Because the number of days between July 17th and September 24th can never vary from one year to the next. whenever July 17th falls on a Friday, September 24th will fall on a Thursday.

## SS(M)-14, Page 45

Construct an equation by working backwards: $16^{\circ}+8^{\circ}-6^{\circ}-5^{\circ}=13^{\circ} \mathrm{C}$. It was $13^{\circ} \mathrm{C}$ that morning.

SS(M)-15. Page 46
Construct a table to show all the possibilities:


There are 24 ways in all to make up $36 e$. The arrow marks the combination using 10 coins.
$\mathbf{S S}(\mathbf{M})-16$, Page 47
In the problem it says that Angeia has three different kinds of coins, at least two of each kind. Therefore, Angela cannot have any quarters as $2 \times 254=504$.
There are then six possibilities, and only the last combination works:

3 pennies. 2 nickels, 5 dimes $" \rightarrow 634, \mathrm{X}$
3 pennies. 2 dimes, 5 nickels $\rightarrow 484$, X
3 nickels, 2 pennies, 5 dimes $\rightarrow 674$. X
3 nickels, 2 dimes. 5 pennies $\rightarrow 404$, X
3 dimes, 2 pennies. 5 nickels $\Rightarrow 474, X$
3 dimes. 2 nickels. 5 pennies $\Rightarrow 45 \%, V$
SS(M)-17, Page 48
775 pennies have a value of $\$ 7.75$. To have the fewest possible coins and bills. Tim would have a $\$ 5$ bill, a $\$ 2$ coin. and three guarters.

SS(3D\&2D)-18. Page 49


On this figure you can see 4 faces on the top, and similarly 4 faces on the bottom. If you count the perimeter of the top (as one way to count the number of faces showing on all the sides) you count a total of 10 . This figure-therefore has a total of 18 faces showning.


On this Cigure you can see 8 faces when looking down from the top or up from the bottom. You can also count 16 faces along the sides, and 4 faces showing on the inside. $8+8+16+4=$ 36 faces in all.

Then are many differen arrangements which can be built to show 20 faces, including


SS(3D\&2D)-19, Page 50
Construct a table of faces, vertices and edges, then guess and check to look for a possible combination.

| Shape | Faces | Edges | Vertices |
| :---: | :---: | :---: | :---: |
| Sphere | 1 | 0 | 0 |
| Cylinder | 3 | 2 | 0 |
| Cone | 2 | 1 | 1 |
| Tri. Pyramid | 4 | 6 | 4 |
| Tri. Prism | 5 | 9 | 6 |
| Rect. Pyramid | 5 | 8 | 5 |
| Rect. Prism | 6 | 12 | 8 |

Two cylinders and one rectangular prism have a total of 12 faces, 16 edges. and 8 vertices.

## SS(3D\&2D)-20, Page 51

Construct the cube first, which requires 8 vertices (marshmallows) and 12 edges (toothpicks). Now add the pyramidal shape to the four marshmallows on the top, which requires four more toothpicks and one more marshmallow. A total of 16 toothpicks and 9 marshmallows will be needed.


SS(3D\&2D)-21. Page 52
Construct a list of all of the buttons by color (grouped here in sets of 5):

Blue Red Black Blue Red
Black Blue Red Black Blue
Red Black Blue Red Black
Blue Red Black Blue Red
Black Blue Red Black Blue
In the first 25 buttons there are 9 blue, 8 red, and 8 black buttons.

SS(3D\&2D)-22, Page 53
To construct the exact same shape requires the use of one yellow hexagon pattern block, 4 green triangles, and 2 blue diamond blocks. These blocks could be arranged in a variety of ways to create the shape.


## SS(T)-23, Page 54

We know from clues two and three that we have one red. one blue and two green tiles in our stack. With the four tiles, there are 12 different arrangements:

| GGBR | GGRB | $G R G R$ | GRBG |
| :--- | :--- | :--- | :--- |
| GBRG | GBGR | BRGG | BGRG |
| BGGR | RBGG | $R G B G$ | RGGB |

Of the 12 arrangements. only 2 meet all of the conditions, shown in italics above.

## SS(T)-24. Page 55

The chart below shows the letters which have vertical and/or horizontal lines of symmtery:


Any combinations of letters all of which either have horizontal lines of symmetry or vertical lines of symmetry would work to form words:

BOX, DICE. BED. TOYOTA, WOW, KICK, MOW, DID, HOW, WHAT, WHO, OW, TOW, MUMMY, HAM. MITT, HIT, etc.

SP(DA)-01, Page 56, Answers wary.

## SP(DA)-02, Pagc 57

So far Darren has asked $6+10+4+2=22$ people. If Darren has 24 people in his class he must ask 24-22 = 2 more people.

SP(DA)-03. Page 58, Answers vary.

## SP(DA)-64. Page 59

We know from the first clue that there must be a total of six blue and yellow buttons. We know from the last two clues that there must be only one yellow. therefore the other five buttons are blue.


SP(DA)-05, Page 60. Answers vary.
This list of results could be created by any spinner which had some red, some green, and some blue spaces!

The spinner could also have any other combination of colors along with red. green and blue spaces (for example, if there were a very tiny space of orange, it very likely would not have shown up in the first 15 spins). If the 15 results are a fair sampling of the spinner, then the spinner probably has 5 parts where one part is red, two are green, and two are blue, as below.


SP(DA)-06, Page 61
Assuming Justin has finisled pulling the marbles from the bag, the first question can be answered: there are more red marbles. The second question can be answered: there are four different colors of marbles. The third question can be answered by finding the total for each color: $11+5+2+7=25$ marbles. The last question cannot be answered.because a tally chart does not specify the order in which information is collected.

## SP(C\&U)-07, Page 62

The likelihood of spinning a 1 is determined by the area covered by the 1 spaces. More than half the area must be covered by a 1 for it to be a probable outcome as seen in the third spinner. Likewise. you are likely to spin a 2 with the second spinner.

## SP(C\&U)-08, Page 63

Students should solve the problem experimentally. by rolling each die several times and counting the number of 2 s rolled.

Theoreticalty. there is one 2 on a 6 -sided die. so the chance of rolling a 2 is $1 / 6$. There is one 2 on a 4 . sided die. so the chance of rolling a 2 with this die is $1 / 4$. Because $1 / 4>1 / 6$, you have a better chance of rolling a 2 with the 4 -sided die.

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| 23 | 24 | 25 | 26 | 27 | 28 | 29 |
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| 23 | 2.4 | 25 | 2.6 | 27 | 28 | 29 |
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 -80-

Pattern Blocks: Green Triangle


Pattern Blocks: White Rhombus


- 82 -


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Pattern Blocks: Red Trapezoid


Pattern Blocks: Yellow Hexagon


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\hline 36 & 37 & 38 & 39 & 40 \\
\hline 41 & 42 & 43 & 44 & 45 \\
\hline 46 & 47 & 48 & 49 & 50
\end{array}
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| 51 | 52 | 53 | 54 | 55 |
| :--- | :--- | :--- | :--- | :--- |
| 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 |
| 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 |


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[^0]:    In the description of meaning given in the section above, it was argued that manipulatives

