The Nonsense in My Little Girl's Geometry Program

The current elementary school geometry program will be reviewed, some nonsense pointed out, and suggestions made for more 3-D geometry in the early grades.

W. GEORGE CATHCART

University of Alberta
Edmonton, Alberta

Mathematics educators have devoted considerable energy to justifying the inclusion of geometry as a legitimate content area in the elementary school mathematics program. (Vigilante, 1965; Robinson, 1966; Inskeep, 1968; and Vance, 1973 are some examples.) "Children greatly enjoy working with this aspect of mathematics", "geometry is encountered in everyday life", "geometry can extend and enrich the study of arithmetic", "geometry should be taught ... because of its inherent beauty and vast utility in everyday life" are examples of the kind of arguments used by these writers.

While we must be ready to justify everything we include in the curriculum, geometry has become so generally accepted that our efforts should change from justifying to evaluating. It is time we took a careful look at the geometry program in the elementary school.

TABLE 1

GEOMETRY CONTENT OF FIVE ELEMENTARY MATHEMATICS TEXTBOOK SERIES

Grade \ Textbook	T1	T2	T3	T4	T5
1	Open and closed curves. Interior and exterior of curves. Betweenness for points	Incidental	Simple closed curves, segments. Plane figures	Identification of plane figures. Symmetry.	3-D shapes, lines, segments, curves, 2-D shapes. Similarity, congruence.
2	Points, lines, segments, intersecting and parallel lines. Polygons.	Incidental	Circles. Congruence by tracing. Diagonals. Symmetry by folding and tracing. Square corners.	Open and closed figures. Segments, betweenness. Congruence of segments and figures.	More 3-D and 2-D shapes. Open and closed figures. Transformations. Patterns. Intersecting shapes. Sorting shapes.
3	Sets of points. Rays, angles. Congruence for segments and angles. Triangles, parallelograms, rectangles, squares, circles.	Angles, triangles, quadrilaterals, polygons. Parallel lines right angles. Circles. Coordinate system.	Rigidity. Correspondence in congruence. Solid shapes (cube, pyramid, sphere).	Interior, exterior. Angles, right angle. Parallel lines. Similarity. Corresponding parts.	Characteristics of solids. Symmetry. 2-D derived from 3-D. Intersecting and parallel lines. Angles.
4	Planes. Perpendicular. Right angles. Classification of polygons. Congruence of polygons.	Simple closed curves. Cubes, triangular pyramids, cylinders and cones.	2-D Figures (extended). Rotations. Scale. Ray, angle, plane, right angles. Perpendicular and parallel lines.	Triangles and other 2-D figures. Perpendicular lines. Space figures.	Compass constructions. Tiling. Congruence - Extension.
5	Open and closed surfaces. Plane and curved surfaces. Space figures. Similarity. Congruence by sliding.	Sets of points. Planes. Congruent segments. Perpendicular lines. Congruent triangles. Space figures.	Classification of triangles. Transversals. Regular polyhedron. Compass constructions. Curve stitching.	Classification of angles and triangles. Circle. Compass constructions. Scale. Coordinate system.	Cross section of solids. Planes of symmetry. Scale. Construction of solid shapes. Planes.
6	Classification of angles and triangles. Congruence by reflecting. Intersection of planes and surfaces. Space figures.	Congruent angles and triangles. Compass constructions. Pythagorean theorem. Cross sections. Solid shapes.	Regular polygons, interior and exterior angles. Planes of symmetry.	Extension of previous ideas.	Rotations of regular polygons. Figures as sets of points. Bisection Classification of triangles. Angle sums.

WHAT ARE WE DOING?

Before I level some criticism against the geometry program, let me review the program outlined in a number of textbooks. I do this on the assumption that the textbook is the basic curriculum guide in most classrooms.

Table 1 is an abbreviated sequence chart for five modern textbook series for Grades I through VI. T_1 and T_2 were published between 1965 and 1969, while T_3 , T_4, T_5 are 1970 or newer publications.

With the exception of T_5, the geometry program outlined in Table 1 can be generalized into a sequence something like this: Points, lines, shapes in the plane (including classification, similarity, and congruence), angles, shapes in space. While programs differ in degree of integration of these ideas and with respect to other topics such as curve stitching, compass constructions, transformations, and other concepts, the sequence stated above seems to be a common one.

There are programs which deviate from the above sequence - T_5 for example, which integrates solid and plane geometry from Grade I through to Grade VI - and many teachers add to and subtract from the program in a textbook. However, the sequence represented by textbooks T_1 through T_4 in Table 1 represents the most common type of geometry program presented to elementary school pupils today.

WHAT IS WRONG WITH WHAT WE ARE DOING?

At least three major criticisms can be levelled against our current elementary school geometry program. These will be discussed in order of increasing severity and seriousness.

First, some of the concepts being taught are not really very important. Perhaps the only justification for including them is that they are prerequisite to more advanced work in geometry. For example, I question the value of the point-set approach in the primary grades. While the concept of a point is basic to much of our advanced geometry and the idea of a set is certainly a unifying factor in mathematics, the meaningfulness of these ideas to a young child is questionable.

We place far too much emphasis on definitions based on the point-set approach. For example, defining polygons as the union of certain kinds of line segments is not satisfactory to many pupils in the primary grades. Another example of a concept of little importance is the distinction between open and closed curves.

Of more serious concern is that we start our geometry program with the abstract rather than with the concrete. All but one of the textbook series outlined in Table 1 introduce geometry "logically" with lines and various polygons defined as sets of points. Points, lines, polygons are all abstract concepts.

Beginning with the abstract contradicts the best theories of learning which we have. Bruner (1966), for example, has found that children first code and represent the world around them in an enactive way, later in an iconic or visual way, and finally, only after sufficient experience, they can make use of a symbolic coding system. According to Piaget, most children are not able to operate at this symbolic or abstract level until they reach the stage of formal operations near the *end* of the elementary school years, not at the beginning as we assume in our geometry program.

Dienes (1967) maintains that we need to embody mathematical concepts in many different physical materials before we can expect children to abstract the mathematical concept.

Beginning with the abstract as we do in geometry ignores the child's conception of space. Piaget and Inhelder (1963) and others who have replicated their work have found, for example, that young children do not believe that a line segment can be a set of points. If asked what the smallest line segment is, they insist that it is still a line segment until they are approximately 11 years of age.

Thus, as Copeland (1972) says:

> to begin in first grade with the notion of the basic element in geometry as the "point" and that lines, squares and so forth are "sets of points" ignores the child as a prelogical rather than a logical person, assuming instead that he has the logical apparatus of an adult mind [p. 23].

The third and most serious criticism is that most of the geometry taught in the primary grades does not build on the child's previous knowledge and experience. We start off teaching points, lines and polygons without asking what kinds of geometric experiences children have had when they come to school. What kinds of experiences have children had with geometric concepts before we see them in school? While they have *not* played with points, lines, triangles, squares and so on, almost every child has built castles out of blocks or cubes, thrown balls or spheres aound the house, licked ice-cream cones, and helped his mother by taking a tin or cylinder of soup out of the cupboard. The child lives in a three-dimensional world. He is frequently manipulating solid shapes. In fact, a very high proportion of a child's out-of-school geometry experiences revolves around solid shapes.

In failing to build on the three-dimensional experiences which a child has had prior to coming to school, we are violating a basic principle of good learning theory which says that new knowledge should be built upon previously learned knowledge.

Delaying the teaching of 3-D geometry to the fourth or fifth years of school, as most of the textbooks in Table 1 do, is pure nonsense. Thus, the nonsense in my little girl's geometry program lies in the poor sequencing of the topics of geometry.

WHAT SHOULD WE BE DOING?

What can be done to improve the geometry program in our elementary schools? If the major criticism that the sequence is backwards, or, at best, all screwed up can be overcome, the other criticisms may also disappear.

What we should be doing is starting in the early grades with three-dimensional geometry and graduate to two- and one-dimensional geometry in the upper elementary grades. This is the reverse of our present order. The remedy, however, is not simply reversing the order. That is too easy and naive. We can't simply take the 3-D ideas and materials presented in the fourth and fifty years of school and put them into the first. The whole approach has to be changed.

Since children's experience with three-dimensional geometry has been at the active and experiential level, we need to continue this approach and build on that experience. "Action on objects precedes perception and, of course, conception." (Skypek, 1965, p.443)

Activities in the primary grades involving three-dimensional geometry might fall into the following broad categories.

Getting a feel for solids

Young children should have opportunity to simply play with various solids. A useful activity is for one student to hide a shape in a cloth and another student to feel the shape and try to guess what it is. His description can be general rather than involving technical terms.

Relating solids to familiar objects in the environment

Let the child find objects in the classroom, his home, the grocery store, or other places, which look like some of the regular solids we want him to become familiar with.

Figure 1, on the following page, is an example of a possible assignment card for such an activity.

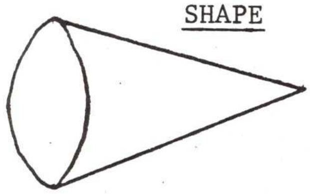
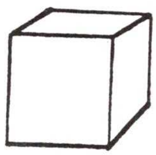
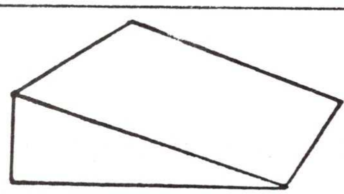
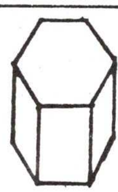
Examine properties

Another useful activity for primary pupils is to have them examine some of the properties of the various solids. For example, they could count the number of edges, faces, and corners (vertices) of the solids. This is not a trivial exercise because children must devise some kind of a scheme for keeping a record of which edges, for example, have been counted. They frequently lose track of which ones they have counted. Pupils can also examine the kinds of faces (curved or flat) and the kinds of edges (curved or straight) which various solids possess.

Figure 1

FAMILIAR OBJECTS

Find some familiar objects at home, school or in the grocery store which remind you of each of the solid shapes on the left.

	SHAPE		REMINDS ME OF
1)		1)	____________ ____________
2)		2)	____________ ____________
3)		3)	____________ ____________
4)		4)	____________ ____________
5)		5)	____________ ____________

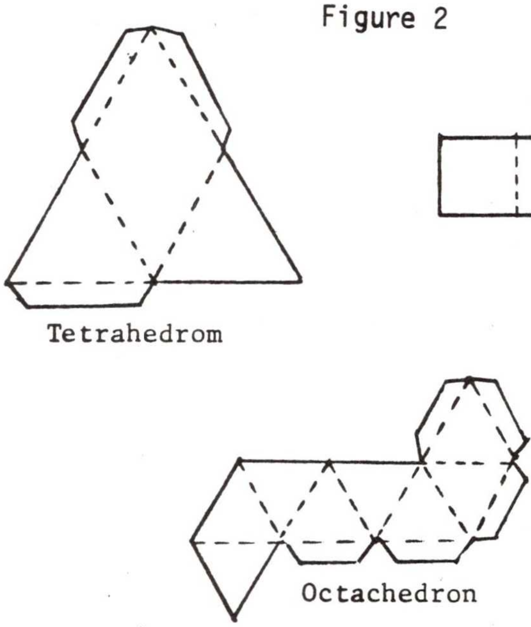
Classification

There are many criteria which young children could use to sort and classify a set of solid shapes. For example, they could sort them on the basis of the number of faces, number of edges, number of corners, kinds of faces (flat or curved), whether the objects roll easily or not, and so on. They should be encouraged to sort the same set of objects several times using a different criterion each time.

Build

Children should be given an opportunity to construct various solid shapes. They can do this in two ways. First, they can use a pattern and fold it into a solid. Some examples of patterns are given in Figure 2.

Figure 2



A second activity involving construction makes use of small sticks and modelling clay or marshmallows. Pupils can use these to make skeleton models, a few of which are illustrated in Figure 3. Sticks and rubber bands also work well, especially if larger models are desired. (See Scott, 1970.)

Figure 3

Having had these kinds of experiences, it should be easy for children to make a transition to two- and one-dimensional geometry in the upper elementary grades. For example, the faces of the solids are two-dimensional shapes. Lines can be demonstrated by extending the edges of solids, including parallel and perpendicular lines. The corners of the solids serve as illustrations of points.

SUMMARY

Geometry has been a very valuable addition to the elementary school mathematics program. However, sequencing of topics within the geometry section should be improved through some reorganization. We begin with the abstract and irrelevant when we should begin with the concrete and relevant. We should begin our program with activities revolving around three-dimensional shapes because these are the things with which children are familiar and, in fact, have already had experience. Such a reorganization is one way of contributing toward "excellence in mathematics education".

REFERENCES

Bruner, J. *Toward a Theory of Instruction.* Cambridge, Mass.: Harvard University Press, 1966.

Copeland, R.W. *Mathematics and the Elementary Teacher* (second edition). Toronto: W.B. Saunders, 1972.

Dienes, P. "Some basic processes involved in mathematics learning", in M. Scandura (ed.) *Research in Mathematics Education.* National Council of Teachers of Mathematics, 1967.

Inskeep, J.E. Jr. "Primary-Grade instruction in geometry", *The Arithmetic Teacher,* 1968, 15, 422-426.

Piaget, J., and B. Inhelder. *The Child's Conception of Space.* Boston: Routledge & Kegan Paul, 1963.

Robinson, G. "The role of geometry in elementary school mathematics", *The Arithmetic Teacher,* 1966, 13, 3-10.

Scott, J. "With sticks and rubber bands", *The Arithmetic Teacher,* 1970, 17, 147-150.

Skypek, D.H. "Geometric concepts in grades 4 - 6", *The Arithmetic Teacher,* 1965, 12, 443-449.

Vance, I.E. "The content of the elementary school geometry program", *The Arithmetic Teacher,* 1973, 20, 468-477.

Vigilante, N.J. "Why circumvent geometry in the primary grades?" *The Arithmetic Teacher,* 1965, 12, 450-454.