Motivating Number Fumblers

The acquisition of arithmetical skills often depends on the students' interest and motivation. To get the students interested, tasks must be appealing. To motivate students, the problems must be relevant to the students' environment. A collection of tasks, projects, and problems to interest and motivate the student will be discussed.

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The low achievers, underachievers and reluctant learners need motivation more than anything else in mathematics. Frequently, these students are mentally capable of performing the required tasks but are not interested and, consequently, neither concentrate nor apply themselves.

For the past three years, we have been gathering materials which, we hope, will help the number fumblers to concentrate, to acquire some of the manipulative skills, and be sufficiently motivated to take part in the class activity. These materials are to be given to the students after the regular classroom presentation of the corresponding topic and the accompanying minimal drill. In place of the extended and dull pages of unmotivated problems, may we suggest the following problems and others similar to these.

PROBLEMS FOR WHICH CONCENTRATION IS REQUIRED

MAXI-COLUMNS

Place the numbers 1, 2, 3, 4, ..., and so on in column 1 or in column 2. There is only one rule: no number may be put in a column if it is the sum of any 2 other numbers already in that column.

Example

Column 1	Column 2
1	2
3	4
5	

We placed 1 in column 1, 2 in column 2, 3 in column 1. Notice that we had to put 4 in column 2, since we could not put it in column 1 (1 and 3 are already in column 1 and $1 + 3 = 4$). We placed 5 in column 1. Now we cannot place 6 in either column 1 (since $1 + 5 = 6$) or column 2 (since $2 + 4 = 6$). With the above arrangement of numbers, we have gone from 1 to 5. Can you go higher by arranging the numbers differently? (Answer: yes, you can go to 8.)

Problem 1: How high can you go with 3 columns? (Answer: 23)

Problem 2: How high can you go with 4 columns? (Answer: 65)

In solving these problems, the student will see a pattern developing as he goes from two columns to three and four columns.

PROBLEMS FOR WHICH CONCENTRATION IS REQUIRED AND OPPORTUNITY FOR DRILL IS PROVIDED

HAPPY NUMBERS

Take a number, say 13.

Square each digit and add. $1^2 + 3^2 = 10$

Repeat the above with 10. $1^2 + 0^2 = 1$

A number for which this pattern yields finally a 1 is a happy number.

The number 13 is a happy number.

Take the number 2.

Square it and add. $2^2 = 4$

Repeat the above with 4. $4^2 = 16$

Square each digit in 16 and add. $1^2 + 6^2 = 1 + 36 = 37$

Repeat.

$$3^2 + 7^2 = 9 + 49 = 58$$
$$5^2 + 8^2 = 25 + 64 = 89$$
$$8^2 + 9^2 = 64 + 81 = 145$$
$$1^2 + 4^2 + 5^2 = 1 + 16 + 25 = 42$$
$$4^2 + 2^2 = 16 + 4 = 20$$
$$2^2 + 0^2 = 4$$

Notice that we are back to the number we started with. The pattern will cycle forever.

The number 4 is not a happy number.

Problem 1: Find all the happy numbers less than 100.

Problem 2: Let A = 1, B = 2, C = 3, D = 4, ... and so on through the alphabet. Take a name: ALLEN. Add the numbers corresponding to each letter. Thus,
ALLEN 1 + 12 + 12 + 5 + 14 = 44
Test whether or not 44 is a happy number. If 44 is a happy number, then ALLEN is a happy number name.

Problem 3: Problem 2 can be adapted to the days of the week, to the months of the year and to the years themselves.

Thus, take 1973. $1^2 + 9^2 + 7^2 + 3^2 = 1 + 81 + 49 + 9 = 140.$

Continue with 140. Is 1973 a happy number year?

PROBLEMS FOR WHICH CONCENTRATION IS REQUIRED, DRILL IS PROVIDED, AND WHICH MOTIVATE SOLUTION STRATEGIES

PATTERNS FROM THE ADDITION AND MULTIPLICATION TABLES

Using the addition table, solve the following problems:

+	0	1	2	3	4 ....
0	0	1	2	3	4 ....
1	1	2	3	4	5 ....
2	2	3	4	5	6 ....
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

1. Take 3 consecutive numbers in a row (column) - for example $\boxed{2}\boxed{3}\boxed{4}$

   What is the sum? Is it 3 times the middle number? (yes. 3 x 3 = 9)

2. Take 5 consecutive numbers in a row (or column) $\boxed{2}\boxed{3}\boxed{4}\boxed{5}\boxed{6}$

   What is the sum? Is it 5 times the middle number? (yes. 5 x 4 = 20)

3. Take 7, then 9 consecutive numbers in a row (or column). What is the sum? Did you use the same pattern as in (1) and (2)?

4. Take 4 consecutive numbers in a row (or column) - for example $\boxed{2}\boxed{3}\boxed{4}\boxed{5}$

   What is the sum? Is it twice the sum of the first and last number? (yes. 2(2 + 5) = 14)

5. Take 6 consecutive numbers in a row (or column) $\boxed{2}\boxed{3}\boxed{4}\boxed{5}\boxed{6}\boxed{7}$

   What is the sum? Is it 3 times the sum of the first and last number? (yes. 3(2 + 7) = 27)

6. Try the above for 8 and 10 consecutive numbers in a row (or column). What is the sum? Did you use the same pattern as in (4) and (5)?

7. Discover the rule for finding the sum of the numbers in the addition tables that appear in the following patterns.

(a)

2	3
3	4

Square 2 x 2

Rule: Number of numbers in the square (4) times the number on the dotted diagonal.

(b)

1	2	3
2	3	4
3	4	5

Square 3 x 3

Does the rule in 7(a) hold here?

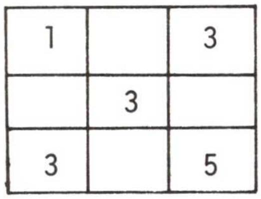
(c)

+	0	1	2	3 . . . 10
0				
1				
2				
.				
.				
.				
10				

Challenge: What is the sum of all the numbers in this addition table (do not consider the numbers here or here)?

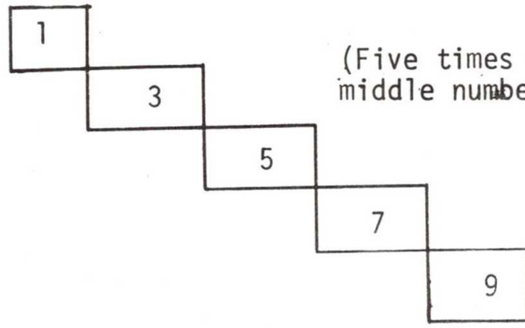
8. Find a rule for the sum of the numbers in the addition table that appear in the following patterns.

(a)



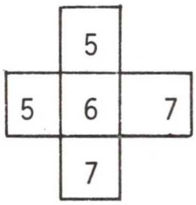
(Five times the number in the center. 5 x 3 = 15)

(b)



(Five times the middle number

(c)



(Five times the middle number)

9. Challenge: Find other patterns in the
   (a) addition table, and
   (b) in the multiplication table.

PROBLEMS WHICH INTER-RELATE TOPICS

HOW DO YOU FIND THE GREATEST COMMON DIVISOR OF TWO NUMBERS?

A. Intersection of sets of divisors

We illustrate this method by an example.

Find the GCD of 6 and 4.

The set of divisors of 6: $D(6) = \{ 1, 2, 3, 6 \}$

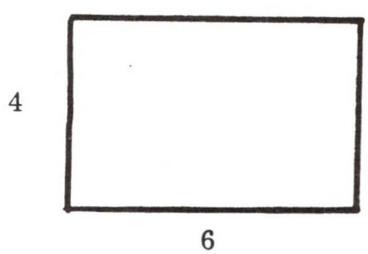
and the set of divisors of 4: $D(4) = \{ 1, 2, 4 \}$

Now $GCD(6,4) = D(6) \cap D(4) = \{ 2 \}$

B. Geometric method

Take the numbers 6 and 4.

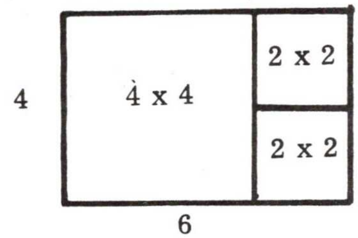
Construct a rectangle with sides 6 and 4 units.



Partition the rectangle according to the following procedure:

1. Find the largest square in the rectangle (there may be more than one).
2. Repeat step 1 for what is left in the rectangle after each partition.

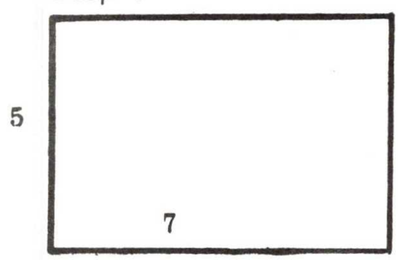
Thus,



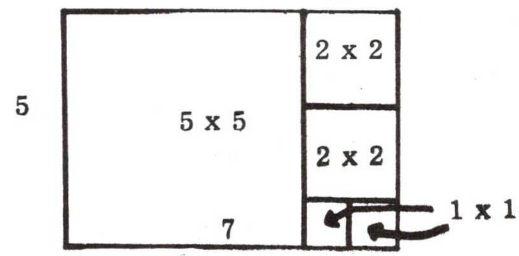
The dimension of the smallest square(s) is the greatest common divisor of 6 and 4. In the above, GCD(6,4) is 2 since the dimension of the smallest square is 2.

Example: Find the GCD(7,5)

Step 1



Partition into squares



GCD(7,5) is 1 since the dimension of the smallest square is 1.

C. Division method

Example: Find the GCD(7,5)

1. Divide the smaller number into the larger number according to the following procedure

5 | 7 — 5 goes into 7 with a remainder of 2

1 R 2

2. Now divide the remainder (2) into the previous divisor (5):

5 | 7

1 R 2 | 5

2 R 1

3. Repeat the step (2)

5 | 7
1 R 2 | 5
2 R 1 | 2
2 R 0

The divisor (1) which leaves a remainder zero (0) is the GCD of the two numbers 5 and 7.

Problem 1: Use the above procedure to find the GCD of

(a) 8 and 13

(b) 21 and 34

(c) 35 and 55

Note: The above numbers are consecutive numbers in the Fibonacci sequence. These numbers give the largest chain of steps in the division algorithm above.

D. Compare the division algorithm in (c) with the rectangle partition procedure in (b).

5 | 7
one 5 x 5 square — 1 R 2 | 5
two 2 x 2 squares — 2 R 1 | 2
two 1 x 1 squares — 2 R 0

The division algorithm gives the partition into squares of the rectangle consisting of sides 5 and 7.

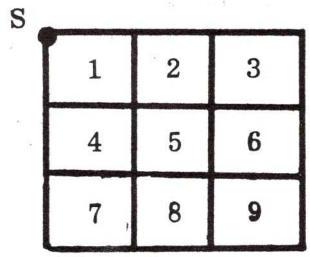
PROBLEMS FOR MODULAR UNITS AND PROJECTS

A student can enjoy working on a project if it is challenging and interesting. The following project is best done by using inch squares rather than trying to show the squares on graph paper.

PROJECT

1. Make a 3 x 3 square out of 1 x 1 unit squares.

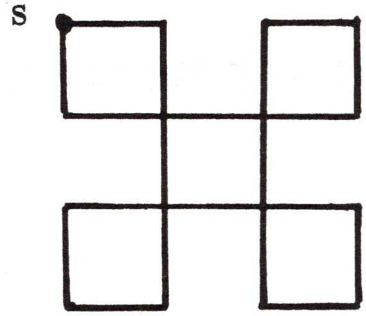
The perimeter is 12 units.



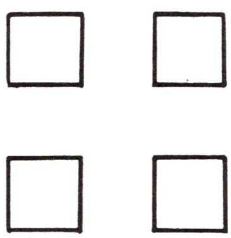
Now remove squares to make the perimeter larger. The rule is:

> you must remove squares in such a way that starting at any point S you can trace a path back to S.

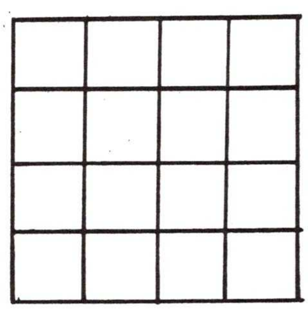
The maximum perimeter for a 3 x 3 square is made by removing squares 2, 4, 6, and 8 to get



The perimeter here is 20 units. Notice that starting at any point S of the above arrangement, we can trace the perimeter and return to S. Figures such as the following are excluded.

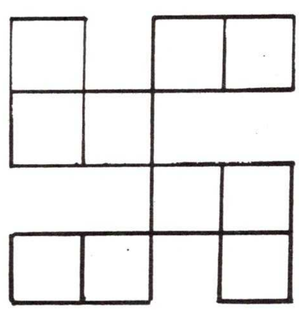


2.



4 x 4 square

Perimeter 16 units



Maximum perimeter

28 units

Problem 1: Take 5 x 5, 6 x 6, 7 x 7, 8 x 8 squares.
Find maximum perimeter.

Problem 2: Can you find formula(s) that will give you the answer for the maximum perimeter for an n x n square where n is an element of 1, 2, 3, 4, ...

Challenge: Apply the above to unit cubes and find arrangement of n x n x n unit cubes with the maximum surface area.