The Last Twenty-Five Years -- What Have We Learned?

During the past 25 years, much significant work has been done to improve the mathematics programs of junior high schools. This discussion will describe how some of the outcomes of this work can give direction to the present and future efforts to continue improving programs.

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It was about 25 years ago that the movement to weigh the values in mathematics seriously began in this country. It was then that the works of promise for the teaching of the subject began to appear. It was only a little later that the textbooks first attacked successfully the unreal topics and problems of the past and set a standard for the real ones for the present and the future. Since the movement began, there has been a veritable revolution in the subject matter, in its arrangement, in the spirit with which it is presented and in the textbooks in which the work is set forth.

To give you some feeling for the way things go in mathematics, the above statement was made in 1923 by Eugene Smith who was reporting on the progress of teaching mathematics in the 25 years prior to 1923.

Many good things have been happening in mathematics, and our modern programs are better because of them, but we ought to look at how they were done so that we can learn something for the future.

WHAT HAS HAPPENED?

In the late 1940s, there was considerable conflict about what really ought to be done in mathematics. People like Brownell and others were talking about teaching mathematics for meaning so that children would understand it and have a rationale for what they did. Then came the war, which proved conclusively that America was a mathematically illiterate nation. For example, over 1,600 out of 2,000 officer candidates from colleges across America failed a mathematics examination which covered material through Grade VIII or the early part of Grade IX. These people came through a mathematics program in which the skills were the dominant thrust. They performed well in order to get to college, but after they had been away from the skills for a while, they forgot much of what they had learned. When candidates for electronics training in the navy were writing a test which included problems dealing with division of fractions, you could depend on someone to say, "If you tell me which one I turn upside down, I can do this problem." They learned the skills, but they never thought very much about their meaning. So the conflict in the late 1940s was between understanding and being able to perform certain competencies such as the 29 competencies set forth by the National Council of Teachers of Mathematics (1945) which every intellectual citizen ought to know or be able to perform.

In the 1950s and early 1960s, new programs came along to replace the old ones. They were not designed to enhance the old or replace part of the old, but to completely take out the old ones and put in the new ones in one swoop.

These programs got their impetus in America when Sputnik flew because America became frightened by the notion that it was not as technically competent as the Russians. This situation was viewed as a national emergency, and the federal congress allocated all kinds of funds for the design of these new programs.

One consequence of the new programs was massive teacher training programs. The National Science Foundation paid people to come in and learn about the new mathematics programs. Unfortunately, teachers think that children ought to know exactly the same about mathematics as they do. If teachers know it, everybody ought to know it; if teachers don't know it, it probably shouldn't be learned by anybody. The teachers who came to these training courses learned the new mathematics but went back and put it into the schools the way they had learned it. This created problems. The teacher training was only moderately successful at best.

In the 1960s and early 1970s we began to move away from an emphasis on technology and science. There is now discontent with the programs which were designed to meet the national emergency of the late 1950s. Evidence of discontent is widespread. Journals such as *Wall Street Journal*, *Time* and other similar magazines have had articles in the last year about the failure of mathematics programs.

Many promises were made with the new mathematics programs. For example, we were promised that if you really stressed the concepts, the skills would take care of themselves. So the emphasis was on content. The discontent started when the test results began to come in. The old programs were not sufficient, because results showed that children could not do certain kinds of things. So new programs came into being, and then the old tests which measured only skills were administered again. Children went down on computation, and the argument was that the tests do not measure the things we are teaching.

One of the problems was that the people who are paying for the programs were not told what the goals of the new mathematics programs were. The new mathematics programs were never designed to make children better computers but to help them become better at understanding mathematics and hopefully better at problem-solving. Many people think that if children cannot compute, the program must be a failure. Consequently, new mathematics is in disrepute in the United States today. This illustrates the point that if you are doing something,you better know why you are doing it and what you can expect from it.

Some discontent is caused by the fact that new programs make it possible for students who learn mathematics well to learn more while the kind of students who were marginal students in the early 1950s are learning less. New mathematics programs are more appropriate for the brighter students than for others. So the enrollment in high school mathematics classes in the United States is decreasing substantially. Why? If we have a good mathematics program, one that accommodates all pupils, the number of pupils who enroll will increase. A corollary of that is an apparent lack of concern for the mathematical literacy of the nation. I have very little regard for the mathematics programs which makes it possible for fewer and fewer children to learn more and more because the function of the school with respect to mathematics should be to make as many students mathematically literate as it possibly can. This should be the major criterion in evaluating a mathematics program.

Discontent has also arisen out of lack of attention to application and problem-solving. The idea that if the students get the concepts, they will be able to solve the problems, has proven false. The National Council of Teachers of Mathematics and the Mathematical Association of America have a joint proposal before the National Science Foundation to prepare a source book for junior and senior high schools on applications in mathematics. Its content is to be compatible with all programs so that every teacher can supplement his program with the source book.

Another reason for discontent is that mathematics educators have not provided the people, who are paying the bill, the kind of information they need in order to make judgments about whether the program is successful in achieving its objectives. Beckmann (1970) was one of the fortunate people who took the 29 competencies when they first came out and designed a test to see how well students in Nebraska performed. In 1965-66 he gave the test again. He found that students of the modern mathematics programs were learning the 29 competencies better than those of mathematics programs 15 years earlier. In addition, the modern math students learned more mathematical ideas. There are many studies such as this, but we don't hear much about them because we haven't made a very good effort of disseminating them.

WHAT HAVE WE LEARNED?

One of the things we have learned from the events of the last 25 years (I think we knew most of them in the first place) is that it is almost always impossible to completely take out one program and put in a completely new one with a new set of objectives and with new emphasis. A program has to be sold. Selling a new mathematics program is just like selling a refrigerator. The first thing you do is to establish a need for it. The people who buy it - *not* the people who make - have to believe that need exists. Once they recognize the need, they have to be shown that the product will satisfy their need for it. They also have to be shown that it is economically feasible for them to place it into their house, that they can afford to buy it and cannot afford *not* to buy it. The same applies to selling a mathematics program.

Very few teachers had any input into the new programs, very few teachers were ever consulted, very few teachers ever really saw the need for new programs. Regardless of what textbooks people were given, everything went on very much as usual when they closed the door to the classroom. We talked about how important it was for children to understand and get a feel for the spirit of mathematics, but when you collected the tests that were given, you found that those ideas were not tested. In fact, tested was how well they could get the answer or compute. Students soon learned that they didn't have to understand it in order to get good grades, they just had to deliver the solution to the equation or perform the computation.

Another thing we learned is that any national emergency will pass, one way or another, but a product designed to meet some national emergency will outlive the emergency. This results in discontent which puts pressure on teachers to report to the recommendations of incompetents. What has happened is that all kinds of panaceas are now coming across our educational threshold which are supposed to eliminate all the difficulties and make children mathematically literate. Every one of these proposals has enough truth in it to make it salable to some people.

Individualized instruction is an example of one of these panaceas. There are many definitions of individualized instruction, but it often degenerates into something where students are reacting to materials only, to materials on a machine, or some other way on a one-to-one basis. It is impractical to select a myth such as individualized instruction and make it a blanket which covers the whole mathematics program. The best strategy is to decide first what it is that we ought to be teaching, then decide which of those topics can best be taught by individualized instruction, which of those are best handled by group instruction and which ones can best be taught with just a text and workbook approach. There is no single method I would like to recommend as a blanket to impose over the whole mathematics program. To me, individualized instruction means making arrangements for every child to get his best shot at learning whether that is prescribed by the teacher or chosen by the student himself.

The next time someone wants you to do the whole program by individualized instruction, pose the following hypothetical situation to him. The Province of Alberta made a ruling that every junior high school teacher, in order to maintain his credentials beyond the end of the current school year, must pass an examination on the undergraduate mathematics program. The government also said that there are two ways you can prepare to pass that examination. One is that you can come to a class with a competent instructor and discuss the content on which you were to be examined. You can work with the instructor, do the problems in the usual way, discuss, and work by yourself when it is appropriate. The other way is to pick up a package of materials, go home, study it on your own, and take the examination. Which method would you choose? There are probably some who would learn individually, but that is an art. Many people need the interchange with an instructor. The place where you hone ideas is in a good discussion with somebody who understands them well. If individualized instruction means that discussion is at a minimum and workbook work is at a maximum, then I doubt if many students will be successful learners. The function of the school is to make students successful learners.

Very little research is being done on the characteristics of topics which can best be learned individually. Some skills can be learned individually, but there are many things about proof, for example, that students can learn better with some help.

Another panacea is accountability. Usually accountability means that the teacher is going to be held accountable for the achievement of children. I believe in accountability, but it has to apply to the whole system, not just one segment of it. Accountability ought to be with parents to send children to school who are willing and ready to do the work required to achieve the objectives. The superintendent and principals ought to be held accountable for providing all the necessary materials to achieve the objectives in the most efficient way.

Out of accountability came behavioral objectives which were intended to lay out the whole thing so that people would know what they had achieved. I have no objections to behavioral objectives. I was writing them in 1942, but I wasn't writing them about ideas - I was writing them for skills. In America, we have people next to each other writing behavioral objectives for the same topic. Everybody is inventing the wheel.

Teachers should not be required to write behavioral objectives because this does not make the most productive use of their time. However, every teacher ought to know specifically what objectives he is trying to achieve in any lesson. In Michigan, we asked the teachers not to write the objectives (because so much time is wasted in arguing over the correct verb) but to write down what students should learn in terms of exercises, problems, attitudes, and so on. These exercises were used as the criterion for objectives, and we hired a technician to write objectives to fit the exercises. You have to have a criterion in mind, and teachers can write the criterion well. This procedure takes less time.

There should be at least two lists of objectives. One list related to skills and certain elgorithms could be stated behaviorally because they can be measured that way. Another list would contain goals related to such things as proof and problem-solving. We might be able to write objectives in these areas, but when I have them all checked off I still cannot certify that a student is a problem-solver or that he can do proof. Teachers ought to make it clear what, in their opinion, it is that can be stated and measured behaviorally and what it is that cannot. The whole program need not be defined by a set of behavioral objectives.

We need more data on children's problem-solving abilities. It seems that children who can solve problems in Grade III can still solve them in Grade VII and students who cannot solve them in Grade III still cannot solve them in Grade VII. We need to collect more observations over time which will enable us to make statements such as, for example, "John is better in problem-solving now than he was last year for these reasons". But we do not assign a number to it. Teachers who go to school four or five years to learn how to do business with children should be able to make subjective judgments about their achievement. To do this, they have to know the goals, but they ought to know what they can state behaviorally and demonstrate achievement in that area and what it is that they can only *say* students are making progress in.

Another subject falling into the category of panaceas is laboratories. I have regard for laboratories in mathematics, and yet, I have been in places where students were very good in laboratories (setting up experiments and following directions) but didn't learn much mathematics. Certain things in mathematics can be learned in a laboratory setting, but I know schools in which the entire program is built around the laboratory. Many things cannot be taught that way.

What we also see happening today is that publishers, as a result of pressure partially from curriculum committees, are producing textbooks with much less reading in them than found in earlier books. The new texts are much more skill-oriented. When you take reading out of the textbook, you do two things: (1) you destroy the students' chance to hear the ideas discussed on the page by somebody who ought to understand the ideas; (2) and more important, you deny the student the opportunity to learn to read things technically. Adults have a hard time reading technical things because it is not taught in the schools. Most mathematics teachers use the textbook as a problem source. The developmental work is usually done at the board. The students read only the exercises and not the material itself. It is imperative that students learn to read something technical, because one of the major goals of mathematics should be to make students independent learners. They will not always have someone laying it out for them on a chalkboard.

We still don't know at the grass roots what the impact of the new mathematics programs has been. We ought to pressure organizations like the Mathematics Council, The Alberta Teachers' Association, and the National Council of Teachers of Mathematics to begin to generate ways of determining the impact of new programs. They need to make the objectives and goals of the new programs known so that we can determine what we have achieved in terms of the predetermined goals.

We should establish a large item bank from which people could choose, at random, items to determine the achievement of certain objectives. If a single test is to be given at the end of the year, I teach that test because my reputation depends on it. But if some items are chosen at random from a bank of, say, 1000 items, I am a little more general in my teaching.

After a student has completed a topic or a program, we should be able to say that he can demonstrate performance of the skills required in mathematics, that he is making progress in problem-solving according to some criteria set up by us. To say that a student is at the 98th percentile doesn't give me any useful information. In other words, we must answer the question "how many can" rather than establishing percentile ranks.

Performance-based teacher education is being tried in America today. There are some things which teachers ought to know, but we may not agree on what these things are. One skill which teachers ought to be able to perform is to conduct a class in the discovery mode for some competencies and to do a good exposition of other topics because this is how they can be handled best. Many teachers could certify that they have accomplished all of the performance tasks I have seen and still be poor teachers. Others may find it impossible to do some of those things, and yet they may be good teachers.

Many universities in the United States have been slow in developing performance-based teacher education. In some of these situations, the state is doing it for them. The lesson is that we should be aggressive and exert our influence before the time comes when we have no influence to exert.

IMPLICATIONS FOR MATHEMATICS TEACHERS

We need to be firm on some matters about any program or product to be implemented in our district. First, we should request that goals be reasonably well specified. We should also request that evaluation be available and that the evaluation and the goals have a good match with what we think is important in our school system. If we were firm on these things, we would avoid the situations I have seen where the same textbook is sold in one district for the good students and in another district for the slow learners.

We have a responsibility to experiment with new programs, but we don't involve the whole school system. No experimental program should be carried out unless it has first been tested with the smallest population possible to apply the results to a larger population. It is a catastrophe to introduce the whole school to a new program without testing it first to see if it is going to work.

Another matter I would insist on is that in any new program, equal attention to method and content is given. We were caught in a trap in 1950s and 1960s. with too much emphasis on content. A knowledge of content is necessary but not sufficient for a competent teacher. A new program should make it possible for the teacher to use the material in a variety of ways.

Teachers ought to be responsible for long- and short-range planning. Whenever they don't carry out their responsibilities in this area, somebody else will do it for them. It will be done at the state or provincial level, or the university people will do it for the teachers.

An organization such as the Mathematics Council ought to project what the needs and major goals in mathematics are. This should be done in realistic terms, not to meet some national emergency or to react to some criticism that is in the forefront.

Teachers should insist on mathematical literacy for the greatest number of people. Every year a student ought to have a mathematics program available to him in which he has at least a 0.8 probability of success if he is willing to do a reasonable amount of work. Teachers who like to teach the mathematics instead of working with children to make them achievers bother me. The school is measured by how many literate people it turns out and by how good they are. The teachers' organization should require that all colleges, universities, and other organizations help plan and carry out that mission.

The amount of a mathematics teacher's knowledge has little impact on the achievement of students. The attitude of the teacher toward mathematics has little impact on the attitude of students. In spite of all this, it turns out that the teacher is still the most important element.

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