Mathematics and the Low Achiever

Many studies have been designed to give us information on low-achieving mathematics students. The results of these studies will be summarized. Also some conjectures will be presented on operational procedures that seem to be effective.

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I have been working on the problem of slow learners for 25 years, and I have a few conjectures which I will share with you. I also feel that I have had some success in working with slow learners.

I will define the low achiever as a person who, for some reason or other, is two or more grade levels below those on which we would expect him to be. These individuals may be categorized further as under-achievers, slow learners or persons with emotional problems. For the purposes of this paper, I will not make these distinctions. We can plan a suitable mathematics program for the low achiever without these more restrictive categories.

A SUMMARY OF THE CHARACTERISTICS OF LOW ACHIEVERS

Generally low achievers do not enjoy competition in the mathematics classroom. They do not seek challenges. They lack inventiveness and a questioning nature. They have a poor self-concept and poor study habits. They are easily confused. They are unable to express their ideas in writing. It takes a longer time for them to learn a concept, and they forget very easily. They have a short attention span and a low frustration level. They have reading difficulties. Often they have social and emotional problems. They have short-range goals. There is a tendency for them to leap to conclusions, and it is difficult for them to generalize. They are unable to follow directions. They are frequently absent, and they don't bring their materials to class.

We can't blame the students for having these characteristics. We must love the students in spite of their deficiencies. If we don't care for the students, they have a very small chance of succeeding.

SUGGESTIONS FOR TEACHING SLOW LEARNERS

Immediately after Sputnik, nearly all mathematics educators were primarily interested in programs for the capable mathematics student. Max Sobel was an exception. Here are some suggestions he had for the classroom teacher.

> Whereas all students crave security and have a need to succeed, the slow learner is especially vulnerable in this respect. Years of consistent failure in the early grades make him prey for any sort of meaningless trial-and-error scheme just to get an answer and satisfy the demands of the teacher.
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> It is important that we make an effort to motivate the student. At ages thirteen to fifteen there is resistance to learning unless the subject matter is of interest to the student and meaningful to him; again, the slow learner can become especially resistant unless his interest, dulled by years of failure, is aroused.
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> Junior high school youth in general, and slow learners is particular, are eager to grasp and adopt patterns of work providing them with security and independence.
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> To summarize briefly: the slow learner in the junior high school has the same characteristics as other pupils of the same age, the same basic needs and interests. However, more than the average child, he needs to be given the chance to experience success and approval; more than the others he needs to feel that he is a member of the group with a contribution to make; he needs status; his confidence must be reestablished, his interest stimulated, his attitude towards mathematics made favorable, his ego flattered [Sobel, 1959, p.348].

Sobel (1959) goes on to outiline 12 specific suggestions for guiding the learning activities of the slow student.

1. Because of a very short attention span, the activities of the slow learner must be varied.
2. Concrete presentations must be emphasized.
3. An emphasis on practical applications is important.
4. These students must be allowed to compete with themselves, and their achievement should be measured in terms of individual growth.
5. Topics must be taken up in spirals - not taught once and then forgotten.
6. Where possible, subject matter should be correlated with work in other classes.
7. Drill is essential, but it must be meaningful and not rote.
8. Verbal materials in the text must be developed orally.
9. Frequent reviews are necessary.
10. In his need of security, the slow learner appreciates and does best in a situation where classroom management is routine.
11. Successful student materials should be exhibited to provide a feeling of success.
12. The final item concerns the procedure used to start the school year, whether it be in grade seven, eight, or nine. There is little doubt but that most slow learners in the junior high school are in dire need of a meaningful re-teaching of arithmetic [p.349].

In reference to the last suggestion, don't make the mistake of spending all of the first week in school reviewing. If a student returns to school with even a little enthusiasm and is asked to do the same "old stuff" in which he failed before, his enthusiasm will quickly die. Give him something a little different so that his interest, as little as it is, will last as long as possible.

Greenholz and Keiffer (1970) made some recommendations and comments about inner-city children. Since inner-city students are often low achievers, I will quote quite liberally from their article. Occasionally I will react to some of the things they say.

> The teacher should give clear explanations and avoid vague generalizations. For example, it is better for the teacher to say, "Only one child may go to the pencil sharpener at a time", than to say, "Let's not have so many at the pencil sharpener at a time." ... A teacher must realize that some words that are shocking to him may be standard terms to inner-city pupils [p.589].

How we handle the "shocking" language can make a big difference. Members of our society are becoming more accustomed to these language variations. If we are going to do anything with the inner-city children, we have to accept them for what they are and for what they say. Perhaps we can do a little to help them develop a more polite language, but we must exercise care.

> Children in the inner-city handle money, do the family shopping, and buy their own clothes earlier than middle-class children do. Practical problems involving the prices of purchases and sizes of various

> containers may be the foundation on which the teacher will build a further development of mathematical skills and understandings [Greenholz & Keiffer, 1970, p.589].

For example, I had a low-achieving boy in one class who had a paper route. One day I experienced some problems relating to a paper route. This boy solved them quickly. Other students were working with paper and pencil and wondered how that boy was able to do so well. Whenever possible, we should select problems consistent with the learner's experience, something he can do, something in his language, and something he is interested in. This means we have to get close to the child in some way and then build the lesson on what we learn about the child.

> Precise diagnosing of each pupil's strengths and weaknesses must precede plans for his instruction [p.589].

Jockey for position. We should try to find out where the students are before we make a frontal attack on their ignorance of mathematics.

> He does not respond well on standardized tests. These tests are formulated for, and standardized on, a middle-class population [Greenholz & Keiffer, 1970 p.589].

> It matters greatly that each student experience sufficient success to strengthen his confidence and pride in himself, to improve his self-esteem, and to encourage him to exert effort [Greenholz & Keiffer, 1970, p.590].

> Teacher expectation is a strong motivational factor [p.590].

If we expect a student to learn, he will probably learn something. If we treat him as if he were a "knucklehead", he will probably fulfill that expectation as well.

> Success or failure in mathematics is closely related to a student's ability to develop the reading skills required by the subject.
>
> It is not enough to drill the student on a collection of discrete rules, manipulations, and procedures prescribing how to accomplish certain specific tasks. He must learn why the procedures are appropriate and which ones to select in the problems he encounters [Greenholz & Keiffer, 1970, p.590].

As a general rule, if we drill the low achiever on material he doesn't understand, he will forget it in a few weeks and then our efforts will have been wasted.

> Let the pupil measure, experiment, try out his ideas and reach generalizations as much as possible on his own [Greenholz & Keiffer, 1970, p.590].
>
> Healthy competition is excellent when both success and failure are possible. It cannot exist where the work of the highly capable and

> that of the very limited are compared. School competition thus is a daily punishment for the less favored who can never win [p.591].

For example, I asked some low-achieving Grade II pupils if they cared to play a mathematical game. They expressed a strong interest. One excited little girl lost twice in a row, and her interest immediately waned. You must use games carefully if they are to help in your teaching. Games are effective learning tools if the reason for losing a game can be blamed on poor luck rather than low ability.

> The kind of school organization that permits teachers and pupils to work together with the fewest interruptions is the most effective. Time to give attention to mathematics each day achieves better results than does an irregular schedule with the passing of several days between successive class meetings [Greenholz & Keiffer, 1970, p.591].

In reference to classroom techniques, Greenholz and Keiffer (1970) add this:

> The classroom teacher must operate on the assumption that a fairly positive and forceful attitude toward classroom descipline will allow, in the long run, greater opportunity for meaningful teaching [p.592].

We cannot let every student "do his own thing". We need to be somewhat rigid, but this can be done with kindness.

> Teachers continue to study every proposed instructional aid as they search for better ways to teach urban children. No machine, however, can teach gentleness, respect, and understanding. These come only from human interaction [p.595].

I am in complete agreement with this final quote. We become educated by working with people, not by working with machines. We should remember this as we use computer-assisted instruction and other forms of programmed materials.

RESEARCH ON LOW ACHIEVERS

During the past 10 years, low achievers in mathematics have been the subject of many research studies, but many of the conclusions have been inconclusive. This state of affairs is especially noticeable in studies involving the use of teaching machines, self-study techniques, calculators, flow charts, vocational mathematics, contract and team-teaching, small group instruction and the use of older elementary students as tutors for younger students. However, Suydam and Weaver (1971) report some research which help to answer the following questions. The remarks which follow are for the most part quotes from their report.

Do special mathematics programs for environmentally disadvantaged students make a difference?

It is not at all surprising to find studies which report that special

programs designed to provide special treatments and emphasis for disadvantaged pupils result in higher achievement when compared with "regular" programs which include no special provisions for such pupils.

A mathematics program "specially designed for culturally disadvantaged pupils" emhasizing success experiences, careful development of concrete to abstract levels, use of simple language, reduced reading level and load, and such techniques as discovery, inquiry and experiments in the fourth grade in inner-city schools was compared with a "regular" program. Significant differences favored the experimental group on measures of concepts and overall achievement, and gains for the experimental group were greater than for the regular group on computation and application measures (Hankins, 1969).

Are programs for low achievers effective?

The findings of research on grouping on the basis of achievement have been much more variable than those for grouping on the basis of ability. Differentiated instruction appears, however, to be more effective than total class instruction.

Sherer (1968) found that low-achieving pupils in Grades III through VII taught by author-developed materials, using instructional aids such as drawings, counters, and number lines and charts, showed significantly greater gain in arithmetic achievement than those taught by a traditional procedure.

Hillman (1970) found that Grade V pupils given immediate knowledge of results, either with or without candy reinforcement, scored significantly higher than pupils not given knowledge of results until 24 hours later. Low achievers may profit more than high achievers.

Hillman's study brings to mind a rather effective procedure I occasionally used when teaching low-achieving classes in high school. Before class, I would solve every problem worked out in detail and organized rather neatly on a sheet of paper. When I noticed a student struggling unduly with a problem, I would ask him to show me how he was trying to solve it. If he was making no progress, I gave him the paper with the solution and he could immediately compare my work with his. I also used this technique if students were visiting too much with their neighbors. The talkative student usually took the solution key, stopped talking, and started working. It is important for a teacher of low achievers to find such methods for keeping students "at task" rather than the usual more verbal approaches.

What teaching procedures are most effective for slow learners?

Herriot (1968) found that when pupils in Grades VII and IX who were classified as slow learners studied SMSG materials for two years, they achieved a greater gain than a higher ability control group achieved in one year. Time does seem to make a difference, but the optimum time needed by slow learners to reach a satisfactory level of achievement has not been answered. Nevertheless, this research does seem to suggest that to be effective we should slow down the pace.

Are different materials appropriate for the disadvantaged?

It has been suggested that the use of varied aids, media and materials, along with real life experiences and laboratory techniques, is especially effective with disadvantaged groups. Schippert (1965) found that, in an inner-city school, use of a laboratory approach in which Grades VII students manipulated actual models or representations of mathematical principles resulted in significantly higher achievement on measures of skills than students taught by a discovery-oriented approach using verbal or written descriptions of these principles. Howard (1970) used mathematical laboratory experiences, planned "to facilitate a hierarchy of needed concepts", with environmentally and academically disadvantaged rural children. Such experiences resulted in both achievement and attitudinal gains.

What remedial procedures have been effective?

Most research reports do not give specific information about the nature of remedial programs. We do know, however, that diagnosis and individualization are effective remedial procedures.

Olsen (1969) reported that use of volunteer tutors with boys in Grades II, III, and IV who were under-achievers and who were achieving two or more months below grade level resulted in no significant differences on most measures of self-concept, achievement and intelligence. At the Grade III level, however, those tutored in arithmetic achieved significantly more than those not tutored. Possibly in the junior or senior high school a judicious use of tutors, especially university students, may be helpful.

Summary

The disadvantaged, as well as all other pupils, profit from special attention. This may be in the form of attention from the teacher, the content of the program, the instructional materials, the organization for instruction or other ways.

Rate of learning is but one variable to be considered in providing effective instruction for slow learners. Methods and materials of instruction also must be adapted to these pupils.

Social relevance appears to be more crucial to consider in the case of disadvantaged students; however, little research has attended to this topic. Active physical involvement with manipulative materials, which is believed to be important for all children, may be even more so for the disadvantaged.

MASTERY LEARNING

Before terminating this account of research on the low achiever, I should also say something about mastery learning. Bloom (1971) claims that mastery learning, if practised as he and others propose, could eventually eliminate individual differences. At present, as students progress through our mathematics programs, the span of individual differences gets greater. Bloom is suggesting

that we can make these differences vanish if we employ certain precedures. First we must determine the objectives we wish to accomplish and arrange them into a sequence or hierarchy. The next step is to divide the sequence into manageable and fairly discrete levels. Each level would have entry behaviors which students must have mastered before being allowed to go to the next level. If this is done and we pay careful attention to certain factors in the affective domain of the student relating to success and failure, give consideration to the development of the self-concept of students, use good instructional strategies such as a variety of explanations and cues, let the students become actively involved, provide reinforcement at the right time, then the individual difference as pictured in Figure 1(a) for the first level will become in the last level the individual differences as pictured in Figure 1(b). Bloom also suggests that the time spent by students at a particular level would vary depending upon when they attain mastery. Of course, if this plan were followed throughout the elementary school, the common practice of homogeneous grouping according to *age* would no longer be possible.

Figure 1(a)

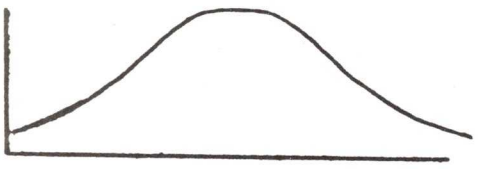
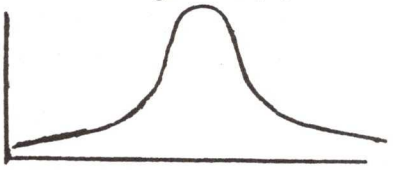


Figure 1(b)



TEACHERS OF LOW ACHIEVERS

Who teaches the low achievers in your school? Is he the best teacher with the most experience or is he the inexperienced first-year teacher who has not yet learned how to defend himself? Usually it is the latter teacher who is given this difficult assignment and often without assistance or moral support from the experienced teachers in the building. I am afraid that we, the experienced teachers, rather than the administrators are to blame for this unfair practice. If the mathematics staff in a building were to suggest to the administrator that low-achieving classes should be shared by the experienced teachers including the department chairman, I am certain the administrator would be pleased to comply. Apparently we feel that getting an easier assignment is, in some way, a promotion. Somehow we must eliminate this "upside down" value system. Really a person who teaches low achievers should earn the right to teach them and should then receive recognition from the profession for doing so.

Here are some characteristics of teachers of low achievers. He has an innate respect and concern for the pupils, he firmly believes that the pupils are capable of learning and that learning results from interaction of pupils with the teacher. He is patient and is determined to provide pupils with some success experiences. A teacher of low achievers must also have a sense of humor and a high frustration level. He must be satisfied with small gains. Even though he may be the best teacher in the school, he recognizes that he needs assistance with multi-media teaching techniques, pacing and sequencing, and in other areas.

Assistance is needed in multi-media techniques because if we are not careful, a visual device may lead students to a generalization somewhat different from the "textbook" generalization which we want them to learn. For example, we can

teach subtraction of integers to low achievers using a number line and relate it to football games and other things in which students are interested. If a student comes up with a rule, it will probably not be the "change the sign and add" rule that we may want to teach.

We need assistance in pacing and sequencing not because we do not pay attention to the developmental aspect of the concepts from the point of view of the students. For example, we use the repeated addition concept of multiplication in the lower grades. We indicate 2 x 5 means two fives, but we cannot treat multiplication of fractions that way. It does not make sense to say that 2/3 x 3/4 means two-thirds three-fourths. Somewhere along the line we have to gradually change the concept of multiplication to one which generalizes to include fractions.

The use of diagnostic techniques and designing realistic course goals is another area in which we need help.

Teachers of low achievers should not have to teach them all day. They should have at least one class of high achievers. They need the change of pace!

SUGGESTION TO TEACHERS OF LOW ACHIEVERS

Experience in Mathematics Ideas, published by the National Council of Teachers of Mathematics, contains many ideas for teaching low achievers. Any material based upon activity, individuality, success, meaning and novelty will probably be successful.

A teacher of mathematics for low achievers should know something about teaching reading. He has to pay careful attention to the readability of materials. Students need help with notation, but we must not burden them with notation.

A frequent change in activity is needed. It is wise to plan about three different activities for a 50-minute period. You might have everyone together for the first activity and then give students a choice for the next two.

Plan sequential instruction based upon feedback from students. For example, one student told me that he didn't know anything about fractions but he needed to learn to work with them so he could learn more about engines. I designed some lessons based on his interest and level of understanding, and he learned enough about fractions to do the things he wanted to do.

Local color makes a big difference. Textbooks don't provide it, *you* have to provide it. For example, I prepared a unit on my bank chequing account. I used one month's transactions. My cheques were written to local business concerns. I "overdrew" my account to provide some experience with a negative balance. The students liked the unit, I think, because they were familiar with the business establishments who were getting paid. They probably were also interested in the way their teacher spent his money. I was willing to sacrifice my privacy on financial matters if students would work in class.

Other suggestions worthy of consideration are:

- Have a planned maintenance program.
- Use non-verbal approaches as much as possible.
- Develop a classroom environment which will help students respect themselves and their classmates.
- Use a laboratory approach to instruction.

COMMENTS CONCERNING A SCHOOL'S TOTAL MATHEMATICS PROGRAM

Every school should have mathematics classes in which a student can achieve success and yet be challenged. Of course, this should apply to low achievers as well as other students. There should be a minimum number of courses in the school solely for low achievers, and in most of these the students should be prepared to take courses in the regular sequence. However, every effort should be made to be certain that students have the necessary entry behaviors before enrolling in a regular course. The purpose of at least some of the special courses for low achievers should be to help them attain the entry behaviors required for the regular courses. It is cruel to place students in classes where the chances of failure are almost certain.

Classes for low achievers should not have an enrollment of more than 20 students. Many of these students do need, at least part of the time, individualized instruction administered individually or in small groups. I would keep these groups small even though the enrollment in some of the classes of more capable students might then become as high as 35 or 40 students.

Classes for low achievers should be provided at several different levels. In general, students should be directed into these classes as soon as deficiencies are detected.

In the Eugene Public Schools we have provided students with a variety of regular and mini-courses to suit individual needs of all students. We feel that this move has resulted in no appreciable loss during the past five years in total enrollment in a secondary school's mathematics enrollment. Students are not required to take more than nine years of mathematics. Nevertheless, over 90 percent of the students who graduate do take at least one course beyond the minimum requirement. We feel that the emphasis on courses for low achievers has helped to keep enrollment from falling.

If your school has not made a significant effort to provide courses for low achievers, I would like to challenge you to try. There are more and better materials available today than five or ten years ago. I also think you will find that the effort will be rewarding.

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