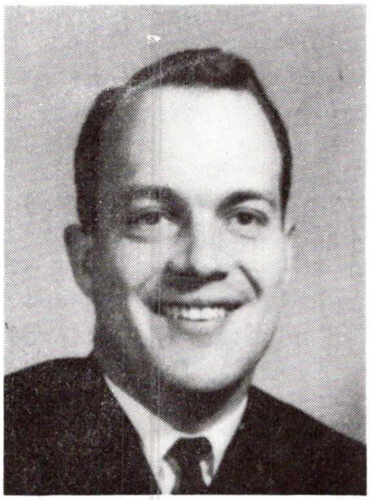
Part II

Linear Measurement in the Early Grades



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THE CHILD

Perhaps the greatest quantity of work in the field of intellectual development and concept formation has been conducted by the Swiss psychologist, Jean Piaget. He has concerned himself not only with the learner, but also with the substance to be learned and its structure or logical organization. Piaget set himself the task of investigating empirically the basis for man's conception of the real external world. With his ingenuous methods and experiments he has probed into the growth of children's concepts in a variety of areas.

Since Piaget's description of intellectual development is a significant contribution to the understanding of how a child thinks and learns, his name is frequently cited by educators developing curricula for the pre- and elementary school child. Although Piaget has not written extensively on the educational value of his developmental theory, other authors have attempted to state the important educational implications of his work. Sullivan, for example, shows how Piaget's description of development could be used as an assessment of intellectual capacity and as an aid in assessment of learning outcomes in a curriculum. Further, he describes how Piaget's stage observations could be used as an aid in the structuring and sequencing of subject matter in a curriculum, and how these observations are suggestive of certain types of learning atmospheres. Harrison in his article showed how Piaget's theory is applicable to secondary school situations and particularly to mathematics teaching and learning. Copeland presents most of Piaget's experiments that are related to mathematics instruction in the early grades. He describes how notions of number, operations with numbers, geometry and measurement arise in children and points out the developmental limitations that might be expected from children at certain age levels. Implications for the classroom teacher are discussed and illustrations show the teacher how to use laboratory or manipulative materials which help the child to learn mathematical concepts at the concrete operational level. Lovell claims that Piaget's work provides the best conceptual framework we yet have, inside which we can discuss the growth of children's understanding of mathematics. His book deals essentially with children's understanding of mathematical concepts and he includes a variety of activities which foster this comprehension of mathematics in the early grades.

In this part of the article, some of Piaget's experiments pertaining to length and linear measurement will be described, and some of his findings will be stated.

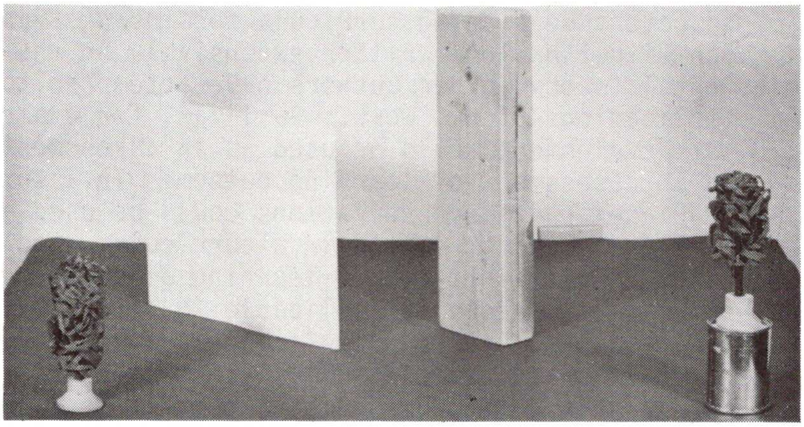
Children come to school with certain conceptions about length. They have had many experiences with it and some of its related concepts. Taller than, shorter than, smaller than, or bigger than are just some of the concepts that many children have at their command. However, it can be shown that a child's understanding of length and measurement is very different from an adult's understanding of these concepts.

To demonstrate how children come to understand the concepts of length and measurement in conventional terms, Piaget (1960) has outlined some interesting little experiments. The experiments consist of simple tests that are described under four separate subheadings.

RECONSTRUCTING RELATIONS OF DISTANCE

Usually no sharp distinction is made between the concepts of distance and length. Psychologically they point to quite different situations. "Length" denotes a measure (linear) of an object. "Distance" refers to linear separations of objects or empty space. The notion of distance is essential to both the development of measuring and viewing spatial relations between objects. Interpreting relations in terms of distance heralds the construction of a coordinate system.

To find out how children judge distances, Piaget and his experimenters use two trees or lead figures [picture #1].



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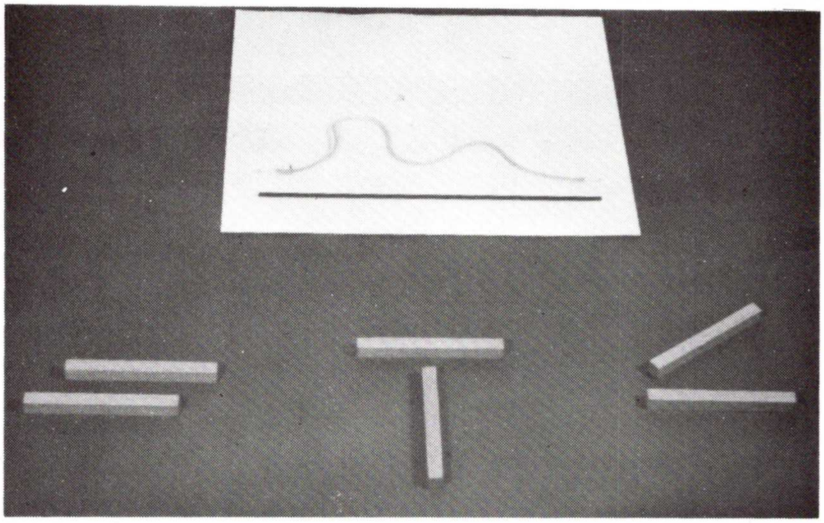
These are placed about fifty centimetres apart. The child is asked whether they are "near one another" or "far apart." The setting is then changed. For example, one tree is raised, screens (with and without doors) or a cube are placed between the trees. In each case the child is asked if the trees are still as "near" or as "far apart," depending on his previous reply. Responses from children of different ages show three distinct developmental stages: the concept of distance as defined above is not present and the distance in one direction is greater or less than the distance in the opposite direction; overall distance is recognized but not when an object is placed between the trees; children realize that distance between two stationary objects remains invariant and the same in either direction, no matter what is placed between them.

CHANGE OF POSITION AND CONSERVATION OF LENGTH

In describing the child's development of operational structures, Piaget distinguishes four distinct stages: the sensory-motor stage (up to 18 months), the pre-operational stage (18 months to 7 years), the concrete-operational stage (7 to 11 or 12 years), and finally, the stage of formal operations. One of the most important components of the transition from pre-operational to concrete-operational thought is the acquisition of various conservations, that is, the cognition that certain properties (in this case, length) remain invariant under certain transformations. According to Piaget (1952), conservation is an important step in the development of logical thinking. It represents intellectual potential and is necessary for all rational activity.

The subjects are faced with different arrangements [picture #2] and the questions are designed to determine whether they are conservers or not.



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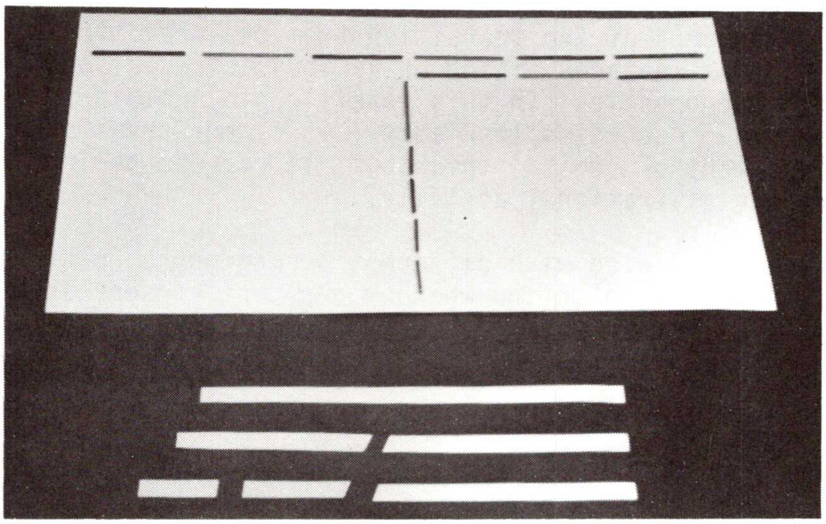
For one part of the experiment the children are faced with a stick and an undulating thread of plasticine (string), their endpoints coinciding. They are asked, "Which of the two is longer, or are they both the same length?" If the children identify the two objects to be the same length, the string is extended and the question repeated. Then the string is twisted once more to make it coterminous with the stick and the same question is asked again.

For another part the subjects are shown two sticks with their extremities facing each other. One of the sticks is then moved forward one or two centimetres or into different positions [picture 2]. For each position the quesion is asked, "Are the sticks the same length, or is one longer than the other?" Only when the children reach the age of seven are they able to relate the objects, any of their parts and the empty sites, and therefore show an understanding of conservation of length.

CONSERVATION AND MEASUREMENT OF LENGTH

Two of the experiments involve distortion of shape. Twelve to sixteen matches (sticks) are placed end to end in two parallel rows so that their equality is obvious. One row is modified by the introduction of angles, zigzags, or some of the matches are broken. Also, two 30 centimetre strips of paper are used. One of the strips is cut, cut again, and arranged in a variety of ways [picture 3]. The question is always whether the two rows or strips are still the same length.



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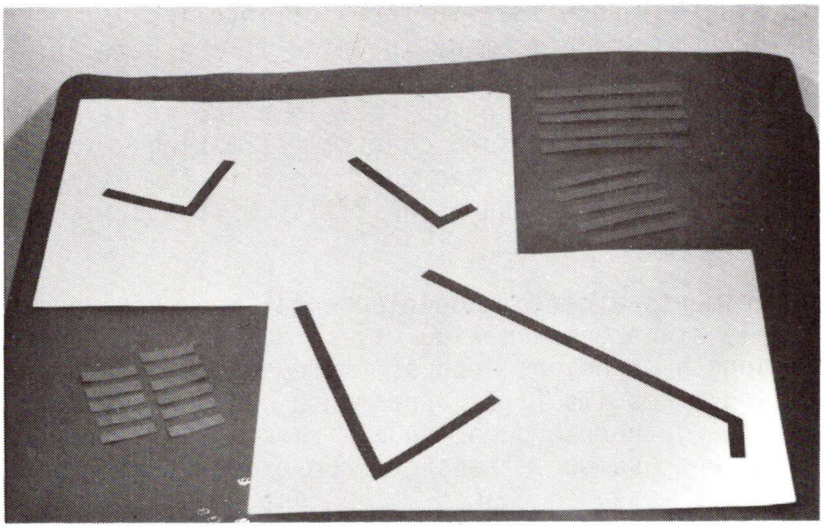
In order to answer the question correctly, both subdivision and order or change of position must be considered. Coordination between these two comes after the age of seven, for most subjects.

Another experiment involves measurement of length. Strips of paper are pasted on cardboard in a variety of linear arrangements: right, acute and obtuse angles [picture 4]. The child is asked to compare two of these at a time, state whether he thinks one is longer than the other or the same length, and then verify his answer. Strips of cardboard are available to serve as "unstandardized" units.

Most young children fail to see the necessity for using unit measures. While many seven-year-olds conserve length, the insightful operational fusion between subdivision and change of position is achieved only by eight-year-olds.



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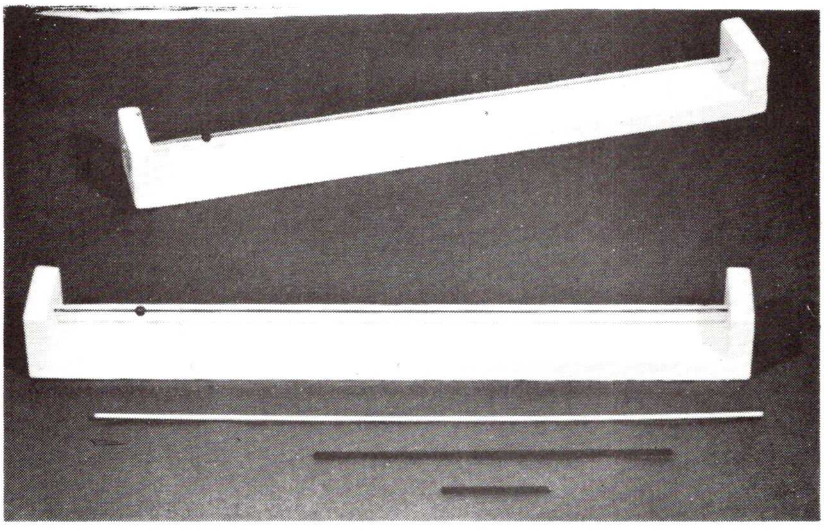
SUBDIVIDING A STRAIGHT LINE

In this experiment the children are asked to locate a segment on a straight line equal to a segment given on another straight line. Beads on strings are used. The setting is changed by staggering the strings (wires), [picture 5] beginning at opposite ends, using strings of different length, or having the strings neither parallel nor in alignment. A blank ruler, two sticks, strips of



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paper, a pencil, or threads are available for measurement purposes. Young children fail the task because they neglect to take into account the points of arrival and departure. They see no need for accurate subdivision. Their measurements are merely visual or manual estimates. Measurement beyond trial and error is not achieved until the age of eight.

The results of the experiments by Piaget (and other researchers who have replicated his studies) show that in terms of intellectual or cognitive development, most children in Grade I, many in Grade II and some in Grade III are not ready for the concepts of length and linear measurement. They lack a reference system and they can only interpret objects in space in terms of their own perspective. Their ability to measure consists of making visual comparisons between endpoints. They fail to see the necessity for using a unit and exact subdivision; unit interaction and transitivity of the common term (unit) are meaningless.

From experiments a certain developmental sequence appears evident. Measuring at an early stage means making visual comparison, and objects are judged to be equal in length or height because "my eyes are good" or "I have good eyes." Change of position makes its first appearance when children attempt to bring perceptual fields together through manual transfer. At a later stage they discover the "middle term" and use body transfer; for example, part of an arm, or finger spans to imitate the measured object. The last stage is reached when the children see the need for using a unit and accurate subdivision, and when they are able to consider the point of departure as well as that of arrival.

The responses young children will make to the questions stated above indicate that they are not "miniature adults" and that they structure the world around them in quite a different way. Their replies imply that many pupils in the primary grades may be able to verbalize measurement terms, but they do not comprehend the underlying concepts, in fact, they are not ready to deal with these. Thus Piaget's methods of observation and inquiry present an excellent way to assess whether a child is ready to be taught certain concepts and understands what has been presented, or is just parroting the teacher. Knowledge of the developmental stage of the pupils is also an invaluable aid in appraising texts or other material that is presented to children in their day work in the classroom. While it might be impossible to test each pupil individually, some insight might be gained by probing the children who have difficulties with some particular concept. This new insight could then be used to provide activities related to the pupil's level of development.

Copeland suggests that a teacher cannot tell children how to measure. Pupils should be provided with materials similar to those described and be allowed to experiment and try to solve measurement problems for themselves. He states -

> . . . if systematic measurement is to be "taught" it should not be presented before the latter part of what is usually the third grade. Even then, for most children it will have to be an experimental or trial-and-error readiness-type experience. . . . The necessary concepts will develop (1) when the child is old enough (eight to eight-and-a-half, according to Piaget) and (2) when he is allowed to operate on (experiment with, manipulate) objects used in measurement. Both conditions are necessary to perform measurement.

Only one particular example has been discussed here. Similar experiments for other geometrical concepts are presented in the same book by Piaget (1960). Also of special interest to the elementary teacher would be Piaget's observation of the child's conception of number (1952), space (1963) and time (1955).

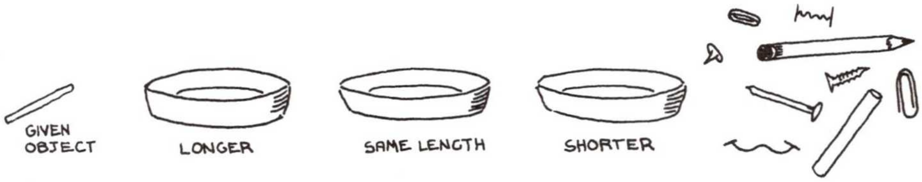
THE TEACHER

The experiments and results from the previous section indicate that, for the young child, linear measurement is a highly contrived process. A certain level of development or understanding is required before young children can attempt systematic and meaningful measurement. Since some children may not reach this level of understanding before they are eight years old, the early part of an instructional sequence should be based on problems and activities which allow the young child to experiment and manipulate in a setting which could be labelled as "trial and error."

In the paragraphs that follow, suggestions for an instructional sequence will be made, some of the important learning outcomes will be stated and sample activities and problems will be described. In terms of most pupil texts or curriculum guides (objectives), the suggested sequence includes activities for the first three years in school. During the first year, emphasis would be on the first part of the sequence. Children in the second year would be taken through the initial parts of the sequence, but then the emphasis would be on the activities suggested in the middle part. Third year children would, after having discussed the initial problems, spend most of their time on the learning outcomes suggested in the last part.

The first task involves the definition of the characteristic to be measured, which in this case is length. To accomplish this a group of objects is presented. One of the objects, such as a paper clip, is put aside or placed upon a plate. The child is asked to sort the given objects into two groups - those that are 'just as long' and those that are 'not as long' as the paper clip. To justify his solution the child is asked to respond to the questions: "How do you know these are as long as the paper clip?" and, "How do you know these are longer?"

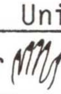
Again objects are presented. One of them is selected and this time the child is asked to separate these objects into three groups: those 'just as long as,' those 'longer than,' and those 'shorter than' the given object. The follow-up discussion would be as for the previous problem.



Five or six children are selected. They are arranged from shortest to tallest.

The remaining members of the class are asked to describe similarities, differences and reasons for the assigned positions. Problems are presented that involve adding to either end of the ordered sequence or inserting into the sequence. The children are then asked to solve similar problems with dowels, pieces of string, pieces of paper, or other objects. These classifying or sorting and ordering problems give the young child the opportunity to make all kinds of relative comparisons and, as a result, will lead him to understand the meaning of length.

Now the problem of describing the characteristic called length without making a comparison to a given object is presented. It is suggested to the children that they use a part of their body or a "body unit" (hand, thumb, arm, etc.) to guess and measure the lengths of various objects. In order to measure correctly, the children will have to be shown the need for, and taught the skill of, accurate subdivision. To accomplish this, the initial measurement problems should be solved by using as many children or "body units" as it takes to completely cover the object to be measured. After having solved a few problems in this fashion, the children are challenged to simulate the action involving many units by using just one of them. Correct and incorrect procedures are illustrated for them and by them. By providing activity sheets that require the child to guess and measure and to record both of these results, the teacher can gain some insight into how familiar the child has become with the "unit" he is using. At this stage guessing should be encouraged. However, in the next part of the sequence, an attempt should be made to teach the child something about the logic of estimation, or the difference between guessing and estimating. During this part of the instructional sequence the activities involve guessing, measuring, recording, comparing, and discussing. The children could also be given a chance to make a decision as to which "body unit" is most appropriate for the measurement tasks under consideration before proceeding with the tasks listed in the previous sentence. The results are recorded in a table.

Object	Unit	Guess	Result
	A or  or 		
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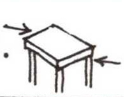
2. 			

The possible learning outcomes for this part of the instructional sequence include knowing how to accurately subdivide the object to be measured, selecting an appropriate unit, and attempting to have the child become familiar with certain units. The final problems or tasks should consist of assigning a measurement task, soliciting results which differ (including the teacher's answer) and displaying these on the blackboard. Reasons for the differences are discussed and the children are asked to make suggestions as to how these differences could be overcome or corrected. The aim is to have the children see the need for a "standard" unit.

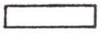
For the next series of estimation (guessing) and measurement problems, the following are examples of "standard" units which could be adopted: new erasers, pencils, crayons, paper clips, pennies, sticks, or pieces of string. To reinforce the necessity for accurate subdivision, the initial problems are solved by supplying the children with a sufficient number of units for covering the object to be measured. Most children may be ready to simulate this action with just one of the units. However, some children may need to use both procedures for awhile or use one method to check the result for the other.

To teach the child something about the logic of estimation, a game setting can be used (Mathex, 1970). For the game of MEST (Measure - Estimate), the children are assigned to groups. A "standard unit," such as a piece of chalk, is shown. A path consisting of two or more parts ( or  or ) is drawn on the blackboard (behind a map) or on an overhead projector. After the children have had a chance to look at the path for five or ten seconds, they are asked to record on a piece of paper how many "units" (pieces of chalk) they think will fit onto the path. One child is chosen from each group and asked to record his/her estimate in a table on the blackboard.

	Estimates					
Problem	Group 1	Group 2	Group 3	Group 4	Group 5	Results
1.						

The path is measured and it is decided which child (group) gave the best estimate. Then the child is given a chance to tell how the estimate was arrived at. To determine a winner, points could be given for each estimate; for example, five points for the closest estimate, four for the next closest, etc. Another table on the blackboard could show the standings. One simple but interesting variation to the setting consists of having each member in a group record the estimate, but the results are compared and discussed within each group before an agreed upon value is reported or recorded on the blackboard. For another possible setting, each child (group) is given a piece of paper, the "standard unit" (eraser) is shown, and the problem of drawing a path three (six, etc.) erasers long is presented. Rules and scoring procedure may have to be revised or decided upon to make this into a game. The responses provide two types of information: how familiar a child (group) has become with a particular unit, and how well a child (group) is able to use the information from previous problems to solve the estimation task on hand.

The follow-up session for activities with "standard units" should include a discussion on how the length of an object can be communicated to someone in a different classroom or town (or country). As a result of this discussion, the children see the need for a "standardized" unit.

A piece of cardboard () is presented to the children and they could be told that when people hear or see the word "centimetre," that is the length they think of or refer to. Assuming that most children have learned how to accurately subdivide and know the difference between guessing and estimating, the main task now is to have them become familiar with this unit. "Thinking metric" or in this case "thinking centimetre" will mean solving various estimation and measurement problems and playing the game MEST (or variations thereof) with this new unit. As activity sheets are prepared for the children, an occasional problem is presented which would make the use of the centimetre rather awkward. As a result, the children may inquire about, or see the need for other "standardized units" and the decimetre and/or metre are introduced to them. Hopefully, they will be given a chance to make their own cardboard ruler or metre stick by drawing or placing onto these marks or numerals the children think appropriate. (For some of them, the first attempt will likely result in the discovery that there are 99 cm in one metre; a valuable lesson in terms of accurate subdivision or using a sharp pencil.)

The main part of the suggested instructional sequence is introducing or defining the characteristic to be measured, using "body units," "standard units" and "standarized units." This sequence and the learning outcomes stated can easily be adapted to other areas of measurement, such as capacity, area, volume, or even time. [See other articles in this monograph, or Liedtke (1974).]

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