## Area Measurement



James H. Vance
University of Victoria

The metric system is a simple, logical way of expressing measures, and less instructional time will be required for teaching the common units of area and for computation and conversion involving these units than was needed under the British system. Thus, more time will be available for developing the major ideas of measurement as they relate to area, and for activities designed to allow students to gain a feeling for the everyday units in real situations.

## CONCEPT OF AREA

Area is the amount of surface bounded by a closed curve or covered by a plane region. The adoption of the metric system will have little effect on the procedures and activities used in the early development of the concept of area and how it is measured. The measurement process first encountered in the study of linear measurement is applied to area. A suitable unit is selected and the object to be measured is compared to it. When the question of a standard unit arises, metric measures are introduced rather than the square inch and square foot.

Following are some suggested activities for developing the area concept.
Arbitrary units - Students find the number of hands, textbooks, etc. needed to completely cover the top of a desk or table.

Tiling - To determine which shapes are suitable for use as a unit of area, students try to completely cover (without overlapping) a notebook using various regular and irregular shapes.


Counting unit squares - Using an arbitrary square unit, students determine the approximate area of non-rectangular shapes by counting the number of squares needed to cover each shape. This can be accomplished by placing squares on the shape, placing a transparent grid over the shape, or drawing the shape on grid paper. Some shapes can also be made on the geoboard. Other tasks would involve using a given number of unit squares to form different shapes with the same area.


The square centimetre $\left(\mathrm{cm}^{2}\right)^{1}$ - is introduced as a standard metric unit for expressing the area of small objects. Students should have many opportunities to estimate and then determine the approximate area of a variety of familiar items. A good supply of centimetre grid paper and/or a transparent centimetre grid should be available for these activities.

Trace each object on grid paper and find its approximate area in square centimetres. Estimate first.

Estimate Measured Area
Your hand
A dollar bill
$\qquad$

A stamp
An envelope
$\square$
${ }^{1}$ The symbol for square centimetre, $\mathrm{cm}^{2}$, should be introduced as just that a symbol. No reference to exponents is needed at first. In later grades it can be pointed out that the symbol refers to $(\mathrm{cm})^{2}$, not $\mathrm{c}(\mathrm{m})^{2}$.


## DEVELOPING AREA FORMULAS

Formulas and procedures for finding the area of a rectangle, parallelogram, triangle, circle, and the surface area of various solid figures are developed in the usual way. Although the activities leading to the formulas normally would be carried out using an arbitrary unit, the square centimetre might be used to provide added exposure to this metric unit.

THE SQUARE METRE ( $\mathrm{m}^{2}$ )
Once standard procedures for determining the area of a rectangle have been developed, problems requiring students to find the area (in square metres) of surfaces in their immediate environment can be posed. Note that, if linear measurements are made to the nearest tenth or hundredth of a metre, multiplication of decimals will be needed. Students should be reminded to estimate before measuring and computing.

1. How many square metres of carpet are needed to cover the classroom floor (or a room in your home)?
2. Find the amount of glass needed to replace all the windows in the classroom (your home).
3. How much paint would be required to paint the four walls of a room (given the number of square metres covered by a litre)?

## LAND MEASURE

The square metre is small for practical purposes of land measure. A square 10 metres by 10 metres (a square decametre) is easy to visualize and was adopted as a metric unit suitable for describing the area of gardens, etc. This unit was called the are (a).


The "are" is not an official SI unit, however, and although it may continue to be used in certain countries, it is not recommended for use in Canada.

The size of the are makes it an appropriate unit for outdoor activities. Rather than learn the term are, students can be asked to estimate and determine area in hundreds of square metres.

1. On the playground, mark off a square 10 m by 10 m . Think of the area contained by this square when you think of $100 \mathrm{~m}^{2}$.
2. Find the area of the athletic field (or a nearby lot, your backyard, etc.). Make linear measurements to the nearest metre.

The hectare (ha) is equal to 100 ares, as the name suggests, and is the area of a square 100 m by 100 m (a square hectometre). Thus $1 \mathrm{ha}=10000 \mathrm{~m}^{2}$.


Like the are, the hectare is a non-SI unit which may continue to be used for a limited period of time. Because of the need for a unit similar to the acre, the hectare will be recognized in Canada for use in surveying and agriculture.

Keeping in mind that the playing field in Canadian football is approximately 100 m by 60 m , the area of two fields side by side would measure a little more than a hectare (about 1.2 ha).

A typical word problem might be as follows:
A rectangular plot of land measures 400 m by 350 m . Find the area in hectares.

Solution: Using metres, Area $=(400 \times 350) \mathrm{m}^{2}=140000 \mathrm{~m}^{2}=14$ ha Changing to hectometres first, Area $=(4 \times 3.5) \mathrm{hm}^{2}=14 \mathrm{hm}^{2}=14$ ha.

The square kilometre $\left(\mathrm{km}^{2}\right)$ is used to describe the area of large tracts of land. The province of Alberta, for example, has a total area of $661188 \mathrm{~km}^{2}$.

Student activities involving this unit would consist of paper and pencil exercises with linear measurements given in kilometres.

## AREA EQUIVALENTS WITHIN THE METRIC SYSTEM

Using the set of metric prefixes, a series of metric units for area are named. (Note that some of these are not commonly used.) A basic idea is that, where linear units differ by a factor of 10 [e.g., $1 \mathrm{dm}-10 \mathrm{~cm}$ ] the corresponding area units will differ by a factor of 100 [e.g., $1 \mathrm{dm}^{2}=100 \mathrm{~cm}^{2}$ ].


Many exercises requiring conversion between metric units of area can be formulated. However, the teacher should be aware of the objectives of such exercises. ! !hile they do provide a review of the system of prefixes and practice in multiplying and dividing by powers of ten, such activities are not essential to developing the understanding and skills relating to area measure at a practical level. Several exercises of this type are now illustrated.

1. How many square centimetres are there in a square metre?

Solution: $1 \mathrm{~m}=100 \mathrm{~cm}$
therefore $1 \mathrm{~m}^{2}=(100 \times 100) \mathrm{cm}^{2}=10000 \mathrm{~cm}^{2}$.
2. What is the relationship between a hectare and a square kilometre?

Solution A:
$1 \mathrm{~km}^{2}=1000000 \mathrm{~m}^{2}$
$1 \mathrm{ha}=1 \mathrm{hm}^{2}=10000 \mathrm{~m}^{2}$
therefore: $1 \mathrm{~km}^{2}=100 \mathrm{ha}$
Solution B:
$1 \mathrm{~km}=10 \mathrm{hm}$
therefore: $1 \mathrm{~km}^{2}=100 \mathrm{hm}^{2}=100 \mathrm{ha}$.
3. $470 \mathrm{dm}^{2}=$ $\qquad$ $\mathrm{mm}^{2}=$ $\qquad$ dam².

One does not learn to "think metric" by converting between the British and metric systems. However, for students who are already familiar with conventional units of area, comparisons provide useful reference measures. For example:

- There are more than 6 square centimetres to a square inch.
- A square metre is a little larger than a square yard.
- A hectare is about 2.5 acres.
- A square kilometre is less than four-tenths of a square mile.

Students should understand how area conversion factors can be obtained from the corresponding linear equivalents.

For example, what is the relationship between a square centimetre and a square inch?


From the diagram, it appears that there are between 6 and 7 square centimetres in a square inch. Computing,
$1 \mathrm{in} .=2.54 \mathrm{~cm}$
therefore 1 sq. in. $=2.54^{2} \mathrm{~cm}^{2}=6.45 \mathrm{~cm}^{2}$
(to two decimal places)

