B. MEASUREMENT AND RELATIONSHIP

B.1 Probability

Probability *(Florence)*

PURPOSE To reinforce and refine students' basic concepts of probability. These activities will provide practice in sampling and graphing. They will enable students to give meaning to number facts upon which we base enquiries, and make interpretations (since they know where the data were collected and how) to investigate the effects of the various types of bias that could have been introduced.

MATERIALS One red and one white dice; a coin; a bag containing six yellow and four red marbles.

WORKSHOP 1. Flip a coin and move one pace to the right if it comes up heads or one pace to the left if it comes up tails. Flip the coin ten times following the directions it gives you each time. How far away are you from the starting point? Do this whole procedure several times. Do you always come out at the same place? Pool your results with several others and make a graph showing (vertically) the total number of times that any of you come to a given final position, against (horizontally) the final position measured from the starting point. What is your average distance in paces in ten flips? What other comments do you care to make?

WORKSHOP 2. Place ten marbles (6 yellow, 4 red) in a bag. Draw a marble; replace it and record the result. Do this 30 times. Now draw three marbles without replacement and record the results, perhaps in a table similar to the one below. Repeat 30 times.

single marbles		*three marbles*	
draw	*color*	*draw*	*colors*
1		*1*	
2		*2*	
3		*3*	
etc		*etc*	

WORKSHOP 3. Make a table showing the various ways numbers can come up when you throw a pair of dice (one red, one white). How many different ways do they come up? In how many different ways do the numbers add up to 7? Throw the dice 25 times. Record and graph the results. From your record compute the fraction

$$\frac{\text{number of throws with 7}}{\text{total number of throws}}$$

Do this for all possible sums. Compute the values you would expect to get for these fractions and compare them with the answer you got in (3).

[Obviously this work could be preceded or followed by more work in probability and statistics involving ideas such as frequency, modes, median, mean, average, etc. The workshops used are really based on game situations themselves so a game to be used as reinforcement was not included. However, there are many games using dice which could be incorporated: spiners, snakes and ladders, roulette, probability in baseball, football probability.]

Probability *(Robinson)*

I. We are given a set of red discs numbered 1, 2, 3, and 4, and a set of blue discs 1, 2, 3, and 4. Each set is placed in a bag and one disc is drawn from each bag to produce an ordered pair.

1) Generate the set of all possible ordered pairs that might be obtained in this way.

2) Do the same thing for sets of three, numbered 1, 2, and 3, and sets of five, numbered 1, 2, 3, 4, and 5.

3) Suppose that we have the same number of members in each of the red and blue sets. Can you find the total possible number of different ordered pairs that you could draw from any size of set?

4) If the red and blue sets have different numbers of members, how could we determine the total possible number of different ordered pairs that we could produce? (Hint - try generating ordered pairs from sets with different numbers of members.)

II. We call the total possible number of different ordered pairs that we can generate from two sets the sample space.

1) If we have a set of three red discs and a set of three blue discs, each set numbered 1, 2, and 3, what fraction of the sample space will be ordered pairs that have the same number for both components?

2) Do part (1) again for pairs of sets numbered 1, 2, 3, and 4, and for pairs of sets numbered 1, 2, 3, 4, and 5.

3) If we have red and blue sets of the same size, how can we find the fraction of the sample space which has ordered pairs with the same components, for any size of set?

4) If the red and blue sets have different numbers of members, how can we find the fraction of the sample space which has ordered pairs made of the same components? (Hint - try generating some sets with different numbers of members.)

III. The fraction that any set of ordered pairs is of the sample space is called the probability of drawing a member of that set from the sample space.

1) Take a set of red discs and a set of blue discs, each numbered 1, 2, and 3, and generate a sample space from the two sets. If we add the first component of any ordered pair to its second component, what is the largest sum we could obtain? What is the smallest sum we could obtain? What is the probability of drawing the smallest sum?

2) Try part (1) again for a pair of sets each numbered 1, 2, 3, and 4, and a pair of sets numbered 1, 2, 3, 4, and 5.

3) For the sets in Part I and Part II, what would be the probability on any one draw of drawing either the largest or the smallest sum of components?

4) For the sets in Part I and Part II, what would be the probability of not drawing either the largest or the smallest sum of components on any one draw?

5) Can you find any patterns in the probabilities you have determined?

IV. 1) Generate a sample space from a set of red discs numbered 1, 2, and 3, and a set of blue discs numbered 1, 2, and 3. For each ordered pair, add its first component to its second component. What is the probability of choosing each of the sums of the components on any one draw?

2) Do part (1) again for sets of discs numbered 1, 2, 3, and 4, and sets of discs numbered 1, 2, 3, 4, and 5.

3) Using the sets from parts one and two, can you find a pattern in the possible sums?

4) Using the sets from parts one and two can you find a pattern in the probabilities of each sum being chosen?

V. A Coin-Flipping Experiment - Please *guess* the answer to each question before you carry out the experiment. If heads comes up one time in a row we say that we have a run of one. If heads comes up two times in a row we say that we have a run of two, and so on.

If you flipped a coin 400 times -

1) How many heads would you expect to get?

2) How many tails would you expect to get?

3) How many runs of one, either heads or tails, would you expect to get?

4) Answer question #3 for runs of 2, 3, 4, 5, 6, 7, 8, 9, 10, and more than 10.

Carry out the experiment by flipping a coin 400 times and carefully recording your results in order.

5) How many heads did you get?

6) How many tails did you get?

7) How many runs, either heads or tails, did you get of: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and more than 10?

8) How do the results of the experiment compare with your guesses?

9) Can you find any pattern in the results of your experiment?

VI. Flipping Coins - If we flip one coin once, its sample space has two members, heads and tails. The probability of getting heads is ½. If we flip two coins at once, the sample space has four members, (H,H) (H,T), and (T,T).

1) What is the probability of getting two heads when we flip two coins at the same time?

2) Can you make a sample space for flipping three coins at the same time? What is the probability of getting three heads if we flip three coins at the same time?

3) Do #2 again for four coins.

4) Can you use the number of coins flipped at one time to determine the probability of getting all heads, or all tails?

5) What can you say about the probability of getting any number of heads from 0 to the number of coins that are flipped?

VII. Another Coin Flipping Experience - Guess about the answers before you do the experience.

1) If you flip two coins at the same time, how many times would you expect to get both heads if you flip them 100 times, 200 times, 300 times, 400 times, 500 times?

2) Carry out the experiment. Flip two coins 500 times and carefully record, in order, your results.

3) How many times did you get two heads in the first 100 flips, first 200 flips, first 300 flips, first 400 flips, first 500 flips?

4) How does what you actually got compare with what you guessed?

5) Does the number of time that you flipped have any effect on the accuracy of your guesses?

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