

C. GEOMETRY

C.1 Concept of Size

Summary of Shapes in the World Around You (*West*)

CLASSIFICATION OF TWO-DIMENSIONAL SHAPES

For each card:

- a) Find a solid with a face like that shown on the card.
- b) Examine the face carefully.
- c) Answer the questions.
- d) Record the answers.
- e) Make a chart to display the information you have gathered.

Questions:

- a) How many sides (faces) does the shape have?
- b) How many corners (vertices) does the shape have?
- c) How many curves does the shape have?
- d) What are some other properties of the shape?
- e) Make a list of the things around you that make use of the shape.
- f) Is there anything else you might say about all the shapes?

CLASSIFICATION OF THREE-DIMENSIONAL SHAPES

Use the same directions as in two-dimensional shapes.

Questions:

- a) How many faces does the solid have?
- b) How many corners (vertices) does the solid have?
- c) How many edges does the solid have?
- d) Make a list of the things (every-day objects) that make use of the solid.
- e) Can you find relationships among the variables (edges, faces, vertices) you have worked with in the above questions?
- f) Is there anything else you might say about all the solids you have worked with?

FIVE REGULAR POLYHEDRA

The hexahedron, tetrahedron, octahedron, dodecahedron, and icosahedron belong to a special class of solids. Find out why these five solids belong to a special class. You might find the necessary information in the library.

MATERIALS Set of cards with two-dimensional shapes, set of cards with three-dimensional shapes, various models of solids.

INVESTIGATION OF TWO- AND THREE-DIMENSIONAL SHAPES

DIRECTIONS Make successively larger models of the unit cube or pyramid.

- a) What things are changing?
- b) What remains the same?
- c) Find the patterns of the sequences you obtain.
- d) Either draw a graph or make a table and continue the patterns as far as you can.
- e) Compare the lengths of successive edges, successive volumes, successive surface areas.
- f) What do you notice about these patterns?
- g) What is the sequence of the ratios of the surface area to the volume?

MATERIALS Sets models.

LAKE AND ISLANDS

Questions

- a) Can you arrange the islands in order according to their perimeters?
- b) Can you arrange the islands in order according to their areas?
- c) Can you find a relationship between areas and perimeters of the islands?
- d) Have you considered the lake in your investigation?
- e) Can you display in some way the information you have gathered?

MATERIALS Graph paper, lake and islands board. [Make the island irregular in shape but keep the perimeter constant. Perhaps the island perimeter could be a fractional part of the lake perimeter.]

A PROBLEM FOR YOU

You are planning to keep chickens in your garden. You have fifty feet of wire fence and you wish to make the pen as large in area as your fence will allow. Work out the length and width that will enclose the largest space. Use graph paper to prove that you are right.

MATERIALS Graph paper.

Density, Specific Gravity: Grades VIII, IX (Fisher)

[Based on research with this type of activity, it should not be taught below the Grade VIII level. The following is designed to lead the student to the ability to predict whether an object will float or sink in water. This depends on two concepts: density and specific gravity. As a criteria to help determine whether your students are ready for this kind of activity, you might use Piagetian-type tests based on conservation of weight and conservation of volume. As a follow-up to these activities, introduce ideas and activities dealing with the principle of flotation, and Archimedes' Law.]

PROBLEM How do you know when an object will float or sink in water?

Make an hypothesis or guess. (An hypothesis is sometimes called an educated guess.) Proceed with an investigation to test your hypothesis. If it is correct, well done. If not, you can make another guess, or rule.

STATION 1 Examine the four blocks and the hollow plastic cube. What do you discover? You should notice that the four blocks are all of equal size or volume and that this volume is equal to the inside volume of the plastic cube.

Now make a guess as to which of the four blocks will float or sink. Record your guesses. Put each block in water and find out how many of your guesses are correct. How did you do?

You might like to discover a better way of determining whether an object is going to float or sink in water.

- a) Weigh each of the four blocks and record (grams).
- b) Weigh the cube filled with water and determine the weight of the water.
- c) Compare the weight of each block with the weight of the water, remembering which blocks floated or sank.

Do you notice any relationships or patterns? Discover anything? Can you make a rule which tells you when an object will float or sink? Try.

STATION 2 Complete the data table (next page) by -

- a) checking the weights of the blocks and the water (nearest 1/10g),
- b) finding the volume of the blocks and water (use graduated cylinder and record your answer to the nearest ml),
(do you need to measure five different volumes?)
- c) compare weight to volume for each of the blocks and the water by dividing the weight by the volume. What units will your answers be in?

| <i>BLOCK</i> | <i>WEIGHT (g)</i> | <i>VOLUME</i> | <i>WEIGHT/VOLUME</i> |
|--------------|-------------------|---------------|----------------------|
| <i>A</i> | | | |
| <i>B</i> | | | |
| <i>C</i> | | | |
| <i>D</i> | | | |
| <i>WATER</i> | | | |

The figures you have just calculated have a special name. Do you know it?

Weight/Volume = Density

Density is quite useful in predicting whether an object will float or sink. Can you see a relationship between the densities of the objects that floated or sank and the density of water? Can you state a rule using the term density and whether or not an object will float or sink?

STATION 3 Let us check to see if you really do know the rule. Examine the six bottles in front of you. They all have different weights, but equal volume. Find their weights, volume and density and make a guess as to which will float or sink. Record your answers in the data table below. Did you use your rule to help make your guess or prediction?

| <i>BOTTLE</i> | <i>WEIGHT</i> | <i>VOLUME</i> | <i>DENSITY</i> | <i>GUESS</i> | <i>OBSERVATION</i> |
|---------------|---------------|---------------|----------------|--------------|--------------------|
| <i>R</i> | | | | | |
| <i>S</i> | | | | | |
| <i>T</i> | | | | | |
| <i>W</i> | | | | | |
| <i>X</i> | | | | | |
| <i>Y</i> | | | | | |

Now put each bottle in the water to see if it floats or sinks. Record your observations. How many did you get right? Was your rule correct? Can you make a better rule to predict without fail when a bottle will float or sink? State your rule once more as best you can.

STATION 4 Can you find the density of each of the objects in front of you? Record your calculations in the following data table.

| OBJECT | WEIGHT (g) | VOLUME (c.c.) | WEIGHT OF AN EQUAL VOLUME OF WATER | DENSITY | SPECIFIC GRAVITY | FLOAT OR SINK |
|--------|---------------|------------------|--|---------|---------------------|------------------|
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |

If one cubic centimeter (C.C.) of water weighs 1 gram, what would an equal volume of water of each of the above objects weigh?

Using a reference book, look up the definition of specific gravity. Does the definition agree with or resemble any of your rules?

STATION 5 Try and figure the specific gravity of each of the objects. Does it agree with the numbers you have for density? Can you make a rule about when an object will float or sink using the definition or idea of specific gravity? Try.

FOLLOW-UP ACTIVITIES

- A. *What is your volume in cubic feet?* - You need to find the weight of a cubic foot of water. (Hint - remember that you nearly float in water.)
- B. *Sand and You* - Remembering that you almost float in water, try to work out the comparison in weight between a bucket of sand and a bucket of you.
- C. *Eureka!* - What is the famous principle that Archimedes, the Greek mathematician, discovered, which excited him so that he ran around with no clothes on hollering Eureka! Eureka!
- D. *Hydrometers* - Find out what a hydrometer is and how it works. How can you use a hydrometer to test the strength of the antifreeze in your car radiator?
- E. *Would you float in a tank of gasoline or mercury?*
- F. *A Float* - Weigh a large float or piece of cork. What do you notice about its weight in relation to its size? Think of a way by which you can find out how much water would occupy the same amount of space as the float occupies. When you have discovered the answer, weigh the water. From your answers, find out how many times heavier water is than the substance from which the float is made. (What is the scientific name for this term?)

REFERENCES

- Brown, Gerald W. and Lucien B. Kinney. "Let's Teach Them About Ratio," *The Mathematics Teacher*, April 1973, pp.352-355.
- Duncan, David R. and Bonnie H. Litwiller. "Mathematics in Sports: Examples for General Mathematics," *The Mathematics Teacher*, March 1973, pp.201-206.
- Ercolano, Joseph L. "Remarks on the Neglected Mean," *The Mathematics Teacher*, March 1973, pp.253-255.
- Kulm, Gerald. "Area Ratios in Convex Polygons," *The Mathematics Teacher*, May 1974, pp.466-467.
- Pascale, Marie. "The Percent Game," *The Mathematics Teacher*, December 1974, pp.739-740..
- Pederson, Juan J. "Some Whimsical Geometry," *The Mathematics Teacher*, October 1972, pp.513-521.
- Schild, Albert. "Geometry of the Means," *The Mathematics Teacher*, May 1974, pp.466-467.
- Stengel, Carol Elizabeth. "A Look at Regular and Semiregular Polyhedra," *The Mathematics Teacher*, December 1972, pp.713-719.
- Whitman, Nancy C. "Using the Laboratory Approach to Relate Physical and Abstract Geometry," *The Mathematics Teacher*, February 1972, pp.187-189.