C.2 Regular Polygon Relationships

Perimeter, Area, Volume: Grades VII, VIII *(Gajdos)*

[*The following workshops have been structured to prevent oonfusion for students who may not have been exposed to the workshop approach previously.*]

WORKSHOP 1   INTRODUCTION TO MEASUREMENT

How many students are there in your class?
How many brothers and sisters do you have?
How many players on a baseball team?

To answer these questions you count the number of elements in each set - you match the natural number with each element of the set in the counting squares. But, suppose we ask the questions -

How tall are you?
How far do you live from school?

These questions couldn't be answered by counting elements as we did in the first series of questions. In order to answer questions like "how long," you need to be able to make measurements.

TASK Students experiment with different forms of measurement.

Open-ended questions

1. How many different ways can you measure objects such as the length of our classroom, your table, the blackboard? Make a list.
2. First estimate your answer.
3. Using your new units that you inverted, measure the length of your classroom, table, blackboard.
4. Make a chart.

	Your estimate	First unit of measure	Second unit of measure	Third unit of measure	Actual measure
Length of classroom					
Length of table					
Length of black board					

WORKSHOP 2 PRIMITIVE FORM OF MEASUREMENT

TASK Collecting data in experiments involving measuring: fathom, cubit, digit, hand, foot, span. The results will be put in a chart.

PURPOSE To help students discover the need for standard measurements.

UNIFYING IDEAS Functions and relations, measurement, addition.

MATERIALS Roll of adding machine paper, scissors, yardstick or tape measure, glue.

PROCEDURE For each person in your group cut a strip of paper equal in length to one fathom, one cubit, one digit, one hand, one foot, one span. Is there any relationship between one fathom belonging to one person and one fathom belonging to another? Measure the length of each measure and record your answers.

	Person 1	*Person 2*	*Person 3*	*Person 4*	*Average*
Fathom					
Cubit					
Span					
Hand					
Foot					
Digit					

1. Do you see any difficulties that could occur using these units of measure?

2. Can you relate experiences involving non-standard units of measure?

3. How can we overcome the problems of non-standard units of measure?

WORKSHOP 3 PERIMETER OF TRIANGLES

TASK Collecting data in experiments involving rolling cardboard triangles along a yardstick, winding a thread around the triangle and measuring the thread. For each object or triangle, students record in a table and graph the distance traveled around.

PURPOSE To help students discover the meaning of perimeter; how to use it.

MATERIALS Cardboard triangle, thread, scissors, tape measure, yardstick.

Open-ended questions

1. How many things can go around our triangle?

2. How many different ways can we measure around the triangle?

3. Estimate the distance around.

Triangles	*Measure side 1*	*Measure side 2*	*Measure side 3*	*Distance around*	*Your estimate*
1					
2					
3					
4					
5					
6					

4. What conclusion do you come to about the perimeter of a triangle?

5. Can you suggest a formula that will describe the perimeter of the triangles?

WORKSHOP 4 PERIMETER OF A RECTANGLE

TASK Collecting data in experiments involving measurement of the sides of different rectangles. For each rectangle, students record in a table the sides and the distance around.

PURPOSE To help students discover the perimeter of a rectangle

MATERIALS Rectangles of varying size, ruler, tape measure, thread, paper, scissors.

Open-ended questions

1. How many things can you go around that are rectangular in shape?

2. How many different ways can we measure around? Is there a better way?

3. Estimate the perimeters of the rectangles. How close were you?

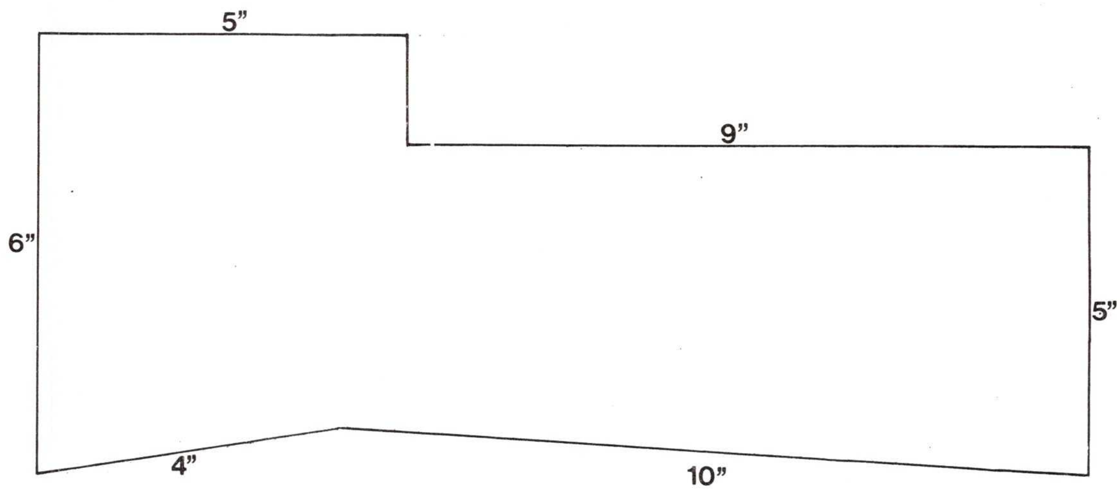
4. Can you find the perimeter of a rectangle without measuring all the way around?

5. Can you determine a formula for the perimeter of a rectangle that will work in all cases?

	Length 1	Width 1	Length 2	Width 2	Perimeter	Your estimate
1						
2						
3						
4						
5						
6						
7						

Open-ended questions

1. Can you find the perimeter of a 5 sided polygon?
2. Can you find the perimeter of a 12 sided polygon?
3. Find the perimeter of the following:



WORKSHOP 5 CIRCUMFERENCE OF A CIRCLE

TASK Collecting data in experiments involving rolling tin cans or other circular objects along a ruler or yardstick; on winding the thread around the round object and measuring the thread. For each object, students record in a table and graph the distance traveled in one turn.

PURPOSE To help students discover that the rate of the circumference of a circle to its diameter is a constant (7); to determine a formula for the circumference.

UNIFYING IDEAS Multiplication and division, functions and relations, measurement. [It is intuitively clear that the longer the diameter of a circle, the longer the circumference will be.]

MATERIALS Yardstick, thread, scissors, round objects of varying diameters, graphing paper.

Open-ended questions

1. How many ways can we measure the distance around the object?

2. Use your method to find the circumference; make more than one measurement and average your results.

Distance in one turn

Diameter	1st	2nd	3rd	4th	Average

Graph Your Results

Average distance in one turn

Diameter of circular object (inches)

Open-ended questions

1. Describe what you notice about the graph.
2. Do you see any relationship between the circumference and the diameter?
3. Is it a constant ratio? If so, what is it?
4. If you were given this ratio and the diameter, could you then find the circumference?
5. Can you suggest a way of finding out what the circumference of a circle is without measuring all the way around?
6. If the ratio of circumference is π can you suggest a formula for the circumference of a circle?

WORKSHOP 6 AREA OF RECTANGLES

TASK Collecting data in experiments involving placing square units on a cardboard rectangle. For each rectangle, students record the number of square units needed to cover each.

PURPOSE To help students discover the meaning of area of rectangles.

UNIFYING IDEAS Multiplication, functions and relations, measurement. [The student will realize intuitively that area means the amount of surface exposed.]

MATERIALS Ruler or tape measure, 50 squares, (exactly 1 inch by 1 inch), rectangles of varying sizes, scissors.

Open-ended questions

1. Estimate the number of squares needed to cover each rectangle.
2. Record your answers.

Rectangle	*Length (inch)*	*Width (inch)*	*No of squares to cover*	*Your estimate*
1				
2				
3				
4				
5				
6				
7				

3. Do you see any relationship among the length, width and the area of a rectangle?

4. Graph your results.

5. Can you think of an easier way of finding the area of a rectangle?

6. Explain your finding in a formula that will work for all rectangles.

WORKSHOP 7 RELATIONSHIP BETWEEN AREA AND PERIMETER

TASK Collecting data in experiments involving the comparison of constant area of a rectangle with perimeter, and comparison of constant perimeter with area.

PURPOSE To help students discover the relationship between area and perimeter.

UNIFYING IDEAS Relations and functions, measurement.

MATERIALS Piece of string 19" (ends tied together to form a band), one-inch square units.

Open-ended questions

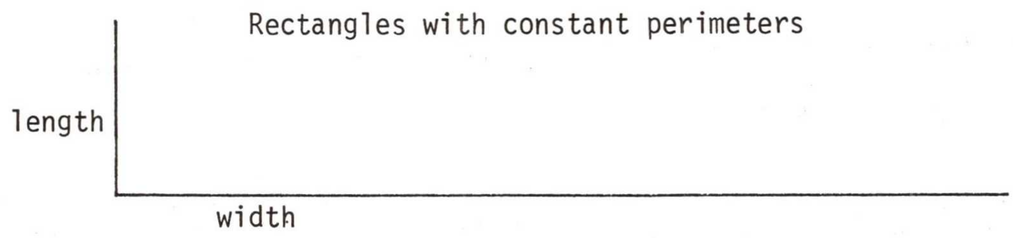
A. Constant Perimeter - find the largest area.

1) What are the measurements of the figure that produces the largest surface area?

2) What is the shape called?

3) Make a table showing how the length changes with the width.

4) How does the area change with the width?



B. Constant Area - find the largest perimeter.

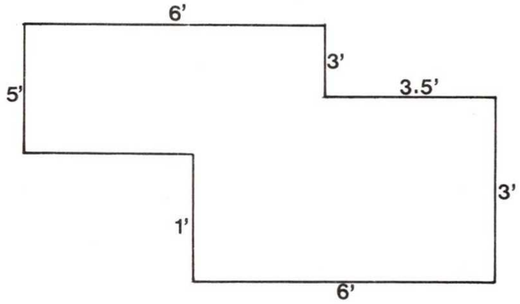
1) Given an rectangle with a constant area of 16 sq. inches, find the largest perimeters available.

2) Make a chart of the possible formations.

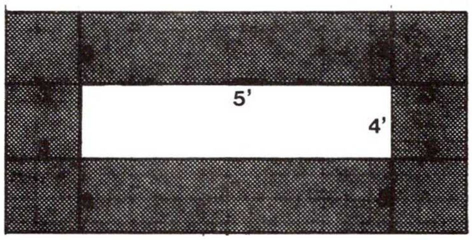
3) Draw a graph to show the relationship between the constant areas ordered by the widths.

Puzzles involving area and perimeter

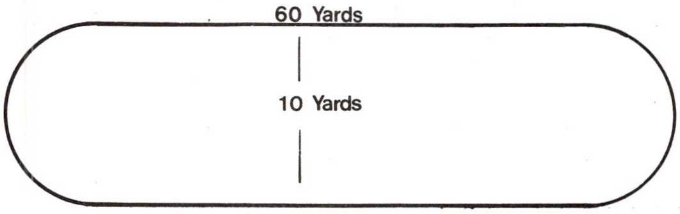
1. Find the area of the figure below.



2. Find the area of the shaded part.



3. Find the perimeter of the figure below.



WORKSHOP 8 VOLUME OF RECTANGULAR BOX

TASK Collecting data in experiments involving filling a cardboard box with salt measured with a one-cubic-inch container. For each box students record the measurements of the box, the surface area of the box, and the volume of the box.

PURPOSE To help students discover the meaning of volume.

UNIFYING IDEAS Multiplication, relations and functions, measurement. [It is intuitively clear that volume means capacity. The student will know that, to determine the volume of a container, he has to measure how much material will go into it.]

MATERIALS Various rectangular containers, salt, one-cubic-inch container.

Open-ended question

1. What is the meaning of volume?

2. Is there a relationship between the measurement of the container and the volume?

3. Is there any relationship between the surface area and volume?

Estimate the number of inches in each box. Fill the boxes with salt. Were your estimates close?

	Width	*Length*	*Height*	*Volume*	*Volume estimate*
1					
2					
3					
4					
5					
6					

4. Can you see any relationship among length, width, height and volume?

5. Can you suggest a formula that will give the volume for all rectangular containers?

Length	*Width*	*Height*	*Surface area*	*Volume*

6. Can you see any relationship between the surface area and the volume?

Regular Polygons: Grades VIII, IX *(Klopp)*

OBJECTIVES To increase familiarity with polygons, to introduce the concept of a limit, to make the construction of regular polygons more meaningful, to provide a problem in which the students can analyze and evaluate data and generalize from them, to introduce a problem with more than two variables, to discover their own formulas for the area of a regular polygon, and to provide an opportunity for creative thinking.

MATERIALS Glassheaded colored pins, roll of thread, protractors, colored felt pens, supply of duplicated sheets with the circle r=12.

1. Find in as many ways as you can the triangle, then the quadrilateral, the hexagon, the octagon, with the maximum area that can be inscribed in the circle given.

2. How are the area and the perimeter of a polygon related to each other?

3. How does the area of a polygon change as you increase the number of sides of the polygon?

4. Find the approximate area of the polygon with 99 sides.

First, students will concentrate on the maximum area of the triangle because of its simplicity. I would expect at least three different approaches: Keeping the base of the triangle constant and

1. changing the measure of the angle:
   a. at one of two endpoints of the base or
   b. (as displayed) the angle at the midpoint of the base,

2. changing the measure of the altitude,

3. changing the measure of the perimeter.

From all three cases, students will conclude that an isosceles triangle will give the maximum area.

Next, they will change the length of the base of an isosceles triangle. From their data the students will see that the area is maximized when the base is congruent to the other two sides. An equilateral triangle is a regular polygon, and it is hoped that the students will generalize that the required quadrilateral is a square, etc.

When finding the area of a hexagon (later an octagon), students are encouraged to develop their own area formulas by dividing the hexagon into convenient areas (triangle, rectangle, etc.) which can be easily computed.

By making a graph of the relationship between the number of sides of a polygon and its area, students will soon discover that the area reaches a limit, the area of the circle. Therefore, the area of the circle is the best and easiest estimate of a 99-sided polygon.

Mapping the School Yard *(Herman)*

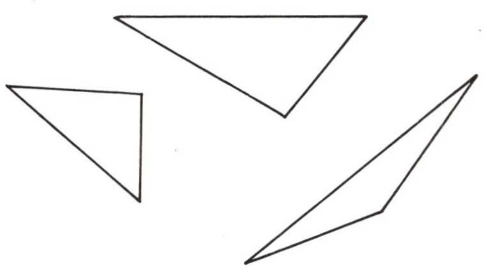
Pupils are always asking of what use is this to us? By mapping their school yard, and building a model to scale, they will find use for scales, fractions, proportions, accuracy of measurement error, probable error, percent in the possible error, decimals, some simple geometry and trigonometry and, above all, the need for careful and accurate work in all computations of basic math.

First the pupils would be taught to use the protractor. Then they would be given time to discover that the interior angle sum of all triangles = 180°; isosceles triangles have two angles equal and two equal sides, whereas an equilateral triangle has all three sides equal and the interior angles all equal 60°. They would also be encouraged to find ratio relationships among the measures of sides of similar triangles.

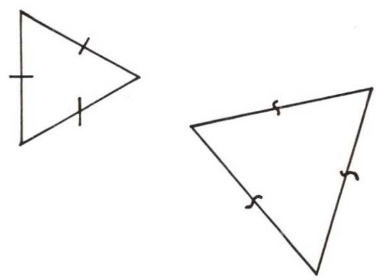
The pupils could then make instruments for measuring angles with these, along with tape measures and by using triangulation, they could map their school yard. By using shadows and ratios they could find heights of any vertical objects. With their measurements they could build a scale model of the school yard.

This project could be correlated with the social studies course, giving stronger reinforcement for the value of math as a useful tool in every-day life.

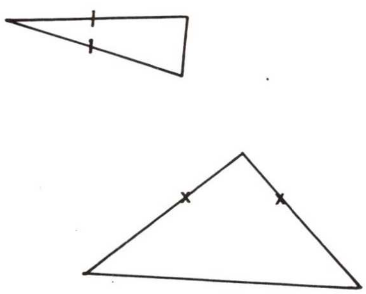
I. Σ interior $\angle$ measures = 180°



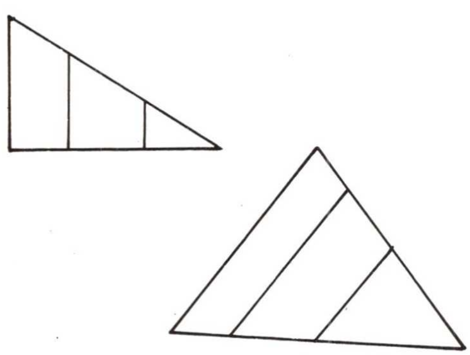
II. Equilateral Δ's



III. Isosceles Δ's



IV. Ratio in similar Δ's



Similarity of Triangles *(Gunn)*

PURPOSE Students acquire the idea of a similarity correspondence between triangles. Hence the idea of "similar." They must be led to the concept that two triangles are similar if and only if there is a one-to-one correspondence between their vertices such that corresponding pairs of angles are congruent and corresponding pairs of sides are proportional. This is similarity.

BACKGROUND Students must already have covered the concepts of ratio and proportion and congruence of triangles (SAS, ASA, AAA Correspondences).

MATERIALS Protractor, ruler, pencil, paper.

ACTIVITY Similar triangles.

A. Using a protractor, draw $\triangle ABC$ and $\triangle KMN$ so that $\angle A \cong \angle K$ and $\angle B \cong \angle M$. Measure the sides of the triangles and $\angle C$ and $\angle N$.

1. Is $\angle C \cong \angle N$?
2. Is $\frac{AB}{KM} = \frac{BC}{MN}$?
3. Is $\frac{KN}{AC} = \frac{KM}{AB}$?
4. If two angles of one triangle are congruent to corresponding angles of a second triangle, are the third angles congruent? Why?

B. Draw a $\triangle ABC$ which has sides with the following lengths. AB=4, BC=5, AC=6. Construct a $\triangle A'B'C'$ with A'B'=8, B'C'=10, A'B'=12. Measure very carefully the angles of both triangles. Within the limits of accuracy you will notice that $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$.

1. Since these congruences exist and since $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$ are the two triangles included similar?

C. From A and B above, does it seem that the congruence of two pairs of corresponding angles is sufficient for the similarity of two triangles? Is an SSS correspondence pattern sufficient to guarantee similarity of two triangles?

D. Investigate other possibilities.

correspondence pattern sufficient to guarantee similarity of two triangles?

D. Investigate other possibilities.

E. Investigation of pairs of similar triangles reveals several possible relations. The angles of one triangle are congruent to the corresponding angles of the other. There also seems to be a relation between the length of the pairs of sides. The angle congruences and relations between lengths of sides do not occur at random. There is a clear cut correspondence between similar triangles which maintains an order of correspondence of vertices, and, therefore, of angles and sides.

F. Can you arrive at a reasonable definition of similar triangles?

G. If two triangles are congruent, does it necessarily follow that they are similar? Are two similar triangles necessarily congruent? Why?

BIBLIOGRAPHY

Nichlas, E.D., et al. *Modern Geometry.* Toronto: 1968.

Rasenberg, H., et al. *Geometry: A Dimensional Approach.* New York: 1968.

Van Engen, H., et al. *Seeing Through Mathematics Book #2.* Toronto: 1968.

Wilson, M. S., *Geometry: A Modern Approach.* Toronto: 1968.

OTHER REFERENCES FROM *The Mathematics Teacher*

Allen, Charles E. "Mission-Tangrams." February 1973, pp.143-146.

Bolater, L. Cary. "Midpoints and Measures." November 1973, pp.627-630.

Cohen, Israel. "Pythagorean Numbers." November 1974, pp.667-669.

Hawkins, Vincent J. "Some Changes in Shop Mathematics Due to Metrication." November 1974, pp.601-603.

Kingston, J. Maurice. "The Unexpected Attracts Attention." November 1973, pp.655-656.

Krause, Eugene F. "Taxicab Geometry." December 1973, pp.695-706.

Moulton, J. Paul. "Experiments Leading to Figures of Maximum Area." May 1975, pp.356-363.

Olson, Alton T. "Some Suggestions for an Informal Discovery Unit on Plane Convex Sets." March 1973, pp.267-269.

Oullette, Hugh. "Number Triangles-A Discovery Lesson." December 1975, pp.671-674.

Pederson, Juan J. "Some Whimsical Geometry." October 1972, pp.513-521.

Rothbart, Andrea and Bruce Paulsell. "Pythagorian Triples: A New Easy-to-Derive Formula With Some Geometric Applications." March 1974, pp.215-218.

Spaulding, Raymond E. "Pythagorean Puzzles." February 1974, pp.143-146; 159.

Trigg, Charles W. "The Volume of the Regular Octahedron." November 1974, pp.644-646.

Whitman, Nancy C. "Using the Laboratory Approach to Relate Physical and Abstract Geometry." February 1972, pp.187-189.