B. GEOMETRY

B.1 Lines

Parallel Lines and Perpendicular Lines *(Dufresne)*

In order to create an atmosphere of active learning with today's students, they must first be "conditioned." The majority of our senior high students will suffer the same restrictions that I feel. That is, they feel inferior when compared with their teacher and fellow students. That fear, that their way is not the correct way or at least not the expected way to attack a given problem, restricts creative work on their part. Since active learning requires creative work on the part of the student, time must first be spent on changing students' attitudes. I feel this is very important and as this is the developement-of-a-topic that I, personally, will find useful, I shall state how I overcome these difficulties.

From the start of the year, I think of the students as "people." I treat them with respect, listen to everything they have to say and value what they have said. If a student knows that you feel this way, he will respond by giving more and more of himself and not just give expected answers or someone else's answers. Since students are more intelligent than we give them credit for, their suggestions and view points are full of merit, and with the correct handling they gradually gain confidence which, in turn, generates more ideas from their own thinking.

In class, wrong and right answers must be handled in such a way as not to work the student's participation. The students must be taught that a wrong answer is valuable. All answers should be proved. When a wrong answer has been disproved, generally the student who gave that answer, and the others, have learned valuable information and perhaps this understanding will lead to a correct answer. A student must be made to feel that his answer is a good one, although not necessarily correct, and that he had grounds to give such an answer. As such, it must be treated with respect and given the same treatment as other answers in proving or disproving them. Above all, students should not be graded on the quality of their responses, but, rather, on the quality of their participation.

The above sounds simple, but I have found that it works. For the lessons, the students have already learned the following mathematical skills. They know what is meant by the slope of a straight line, and how to find the slope and equation of a line. They know what is meant by parallel and perpendicular lines.

Different methods suit different teachers. I have tried the formal group method but have not found it to work satisfactorily and, therefore, I prefer to have students work individually. Through past experiences the students know that they are allowed to confer on their work so, in reality, the situation is one of group work as they do compare answers, argue and agree. The advantage of their method is that each student must record his own results and can't rely on the work of others. Also, if a student doesn't agree on the method used by others around him he can gracefully withdraw and do it the way he wants.

Each student has several sheets of graph paper, a ruler and a hand-out sheet. I would prefer to ask only one question and from there let the students take it on their own. The question would be similar to the following one. "Knowing what you know about the slopes and equations of straight lines, find out everything you can about the slopes and equations of lines that are (i) parallel, and (ii) perpendicular." The student would be given the following instructions: he is not allowed to use his text book, only his graph book, and try to discover the relationships on his own. This simple question is enough for some students, but for others it is frustrating as they don't know how to begin. Therefore, the following sheet of instructions has been prepared.

1. On your graph paper draw several lines that are parallel.

2. Find the equations and slopes of these parallel lines. (Hint - if you draw your parallel lines to intersect a few lattice parts, you can use these points to find the slopes and equations of these lines. This will give a greater degree of accuracy.)

3. Can you find any similarities or relationships in the slopes and equations of these parallel lines?

4. Draw a different sets of parallel lines to check your results.

5. What conclusions can you draw?

6. Repeat 1-5, using perpendicular lines.

   (If you cannot see any relationships try drawing another set of lines.)

This active learning lesson is relatively easy for Math 20 students. A few students found that their curiosity led them further into the slope concept. For example, one student came to me with the following question. "What about the slope of $y = x^2$? I don't think it is constant." I agreed, gave him a quick explanation of how to find the slope by drawing a tangent to the curve and explained that the slope of $y = x^2$ could be found by using a certain formula, and could he find out what the formula was? The student drew the graph to a large scale taking many points to ensure greater accuracy. He took the slope at many points recording both the point and slope, and came to the conclusion that the slope was equal to 2x for any pt. with coordinates (x,y). He also ventured that the slope of any curve $y = x^n$, would be nx. I drew the graph of y x and pointed out that the slope is always positive, even when x is negative. He was able to come up immediately with the answer that the slope of $y = x^3$ must be 3x and then the slope of $y = x^n$ must be nx^{x-1}. He must have done a lot of work on his first answer that he could switch his answer so readily because he did seem to have a clear explanation of it. I was greatly pleased and gave him a pat on the back, explaining that he had developed a "formula" in calculus for the derivative of y, when $y = x^n$. I believe that the best results come from students' own questions and if a teacher can encourage them to investigate for their own answers, the results will be greater.

A student is truly interested to question in the first place and even if the question isn't entirely related to the topic under discussion he should be encouraged to pursue it if there is any mathematical merit in it.

After all of the students had a chance to find the relationships, we discussed them in class and then tried some of the questions in the textbook that used this new information. I find it is necessary to have a discussion period at the end of the lesson to help those students who are unable to formulate all their conclusions until they are told that they are correct.

In concluding I will say that the lesson was satisfactory. The students found it enjoyable and it provided for all levels of students in that the quicker students had questions to ask that led them into further discovery of their own. There was no student that couldn't complete at least a part of the assignment.

Parallelism *(Moynihan)*

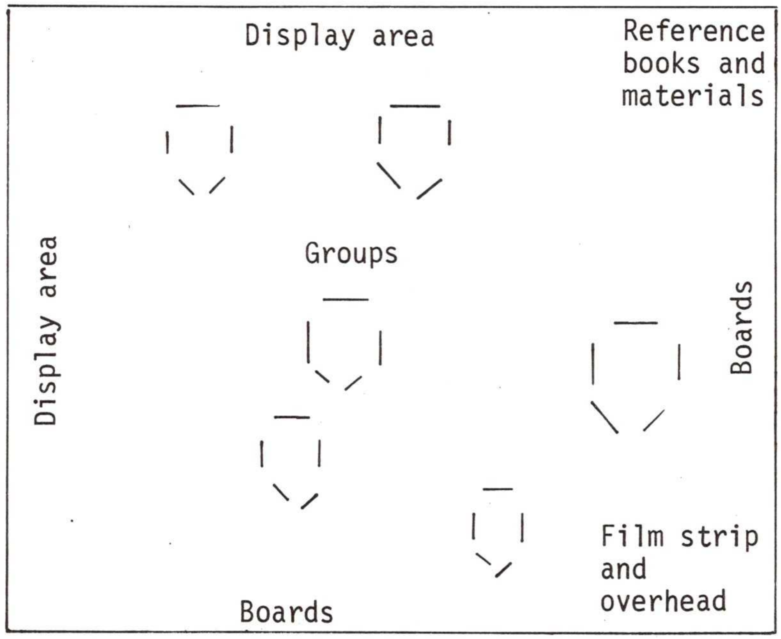
A Workshop Approach to Teaching the Concept of Parallelism

PURPOSE *Activity I* - To discover the conditions which make two or more lines parallel; to apply the knowledge gained to the solution of some problems.

Activity II - To discover the results when two or more parallel lines are crossed by a transversal.

Activity III - To discover the kinds of parallelograms, their relation to each other and to quadrilaterals in general.

CLASSROOM SET-UP



MATERIALS Text: *Geometry - A Modern Approach.* Wilcox. Transparencies to accompany text. Reference Texts: *Geometry, Plane and Solid.* Brown and Montgomery; *Modern School Mathematics.* Jurgensen, Donnelly and Dolcianni; *Glossary of Mathematical Terms.* Gundlach; *Universal Exercises in Geometry.* Charles E. Merrill Books, Inc. Film strip projector. Film strips:

S.V.E. - Modern Elementary Geometry

1. Points, Lines and Planes
2. Angles
3. Parallelism (used as a review after activities completed).

Overhead projector, colored mounting paper, paste, brushes, scissors, colored pencils, magazines (for pictures), graph paper, mimeographing services (for copies of instructions so that each pupil has own copy to work with), geoboards, punched cardboard strips, pins, and elastic bands.

[*For this project, 30 Grade X students enrolled in the Math 10 course were randomly chosen and divided into six groups of five students each. When an activity was finished, each student was required to write up the findings. Before beginning this activity, you may wish to review. There are a number of film strips in the film strip corner. Run them if you wish. Please rewind them carefully when you have finished. Note - the strip on Parallelism was not made available at this time; it was used for an overview when the project was finished.*]

ACTIVITY I

Draw AB. At A in AB draw a co-planar line perpendicular to AB. Call it AC. At B in AB draw BF perpendicular to AB.

Take any point D in AC. What is the measure of AD? Record it on your data sheet.

Measure a distance BE on BF such that AD = BE. Record it on your data sheet.

Join DE and extend it to form DE. Select a number of points on AB. Call them X_1, X_2, X_3. Measure the perpendicular distance between these points and points on DE. Call the point on DE Y_1, opposite X_1, Y_2 opposite X_2, etc. Record your data. What do you notice about your results? What conclusions have you reached concerning AB and DE?

Such lines are said to be parallel. Can you write a definition of parallel lines?

EXERCISES

1. Given the co-planar AB — MN and CD — MN, prove that AB is parallel to CD. Draw conclusions from this exercise and state it as a theorem.

2. Construct a line parallel to a given line at a given distance from it.

3. Given a triangle ABC with the angle B bisected by BD, can you construct a line through A parallel to BD?

ACTIVITY II, Transversals

Draw MN and PQ parallel to each other. Now draw XY intersecting MN at the point R and PQ at S. XY is called a transversal on MN and PQ.

How many angles have been formed? Identify them by numbering them 1, 2, 3, 4, etc. Measure the angles and record your data. What do you notice about your results?

Do you get the same kind of results with other parallel lines crossed by transversals having different slopes? Try several pairs and record your data.

What happens if you use a set of more than two parallel lines? Try it and record your data. What conclusions regarding the angles have you drawn? Discuss them and write them down.

There is an interesting problem involving parallels on page 142 of *Geometry, Plane and Solid*, by Brown and Montgomery.

Alternate Angles - Angles 1, 7 and 2 and 8 are called alternate angles; angles 4 and 6, and 3 and 5 are interior alternate angles.

Exterior Angles - Angles 1, 2, 8, 7, are called exterior angles.

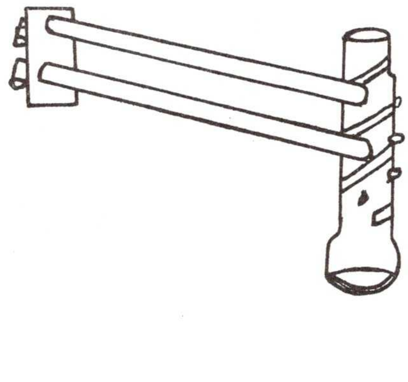
Interior Angles - Angles 3, 4, 5, 6, are called interior angles. Note that alternate angles lie on opposite sides of the transversal.

Corresponding Angles - are those angles which lie in the same relative position with regard to the transversal. Angle 2 and angle 6 are corresponding angles. How many other pairs of corresponding angles are there? List them.

Are there any supplementary angles in Figure 1? If so, list them. Can you prove that the angles you have named are supplementary? If so, write out your proof.

AN INTERESTING PROBLEM

Dave's hobby is photography. At times he needs a dim light. In his dark room, which is unlighted, he can see no convenient way to extend wiring for an electric circuit. From some flat curtain rods, a discarded but usable hinge, tape, and rubber band, Dave devised a flashlight holder as shown below. Now he has a light whose height is adjustable yet the flashlight will remain perpendicular to the floor at any height. What parallel relations did Dave control in his arrangement of materials?



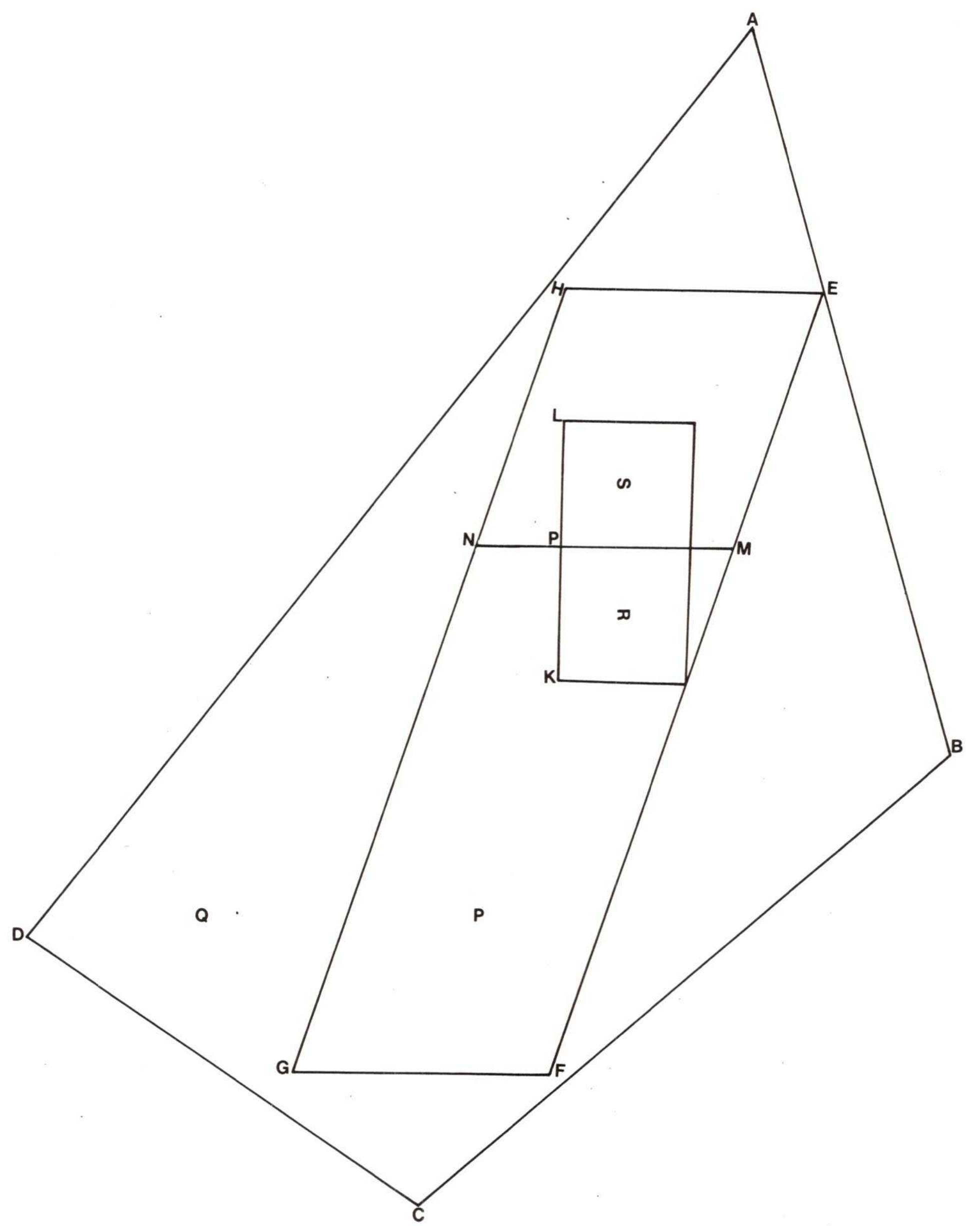
ACTIVITY III, Parallelograms

DEFINITION A parallelogram is a quadrilateral having its opposite sides parallel.

Using the cardboard strips, how many kinds of parallelograms can you make? Try the same thing with the geoboard. Compare your results. Record the results on your data sheet.

What do all the parallelograms have in common?

Draw a large quadrilateral on a sheet of graph paper. Can you fit the different kinds of parallelograms you found into it?

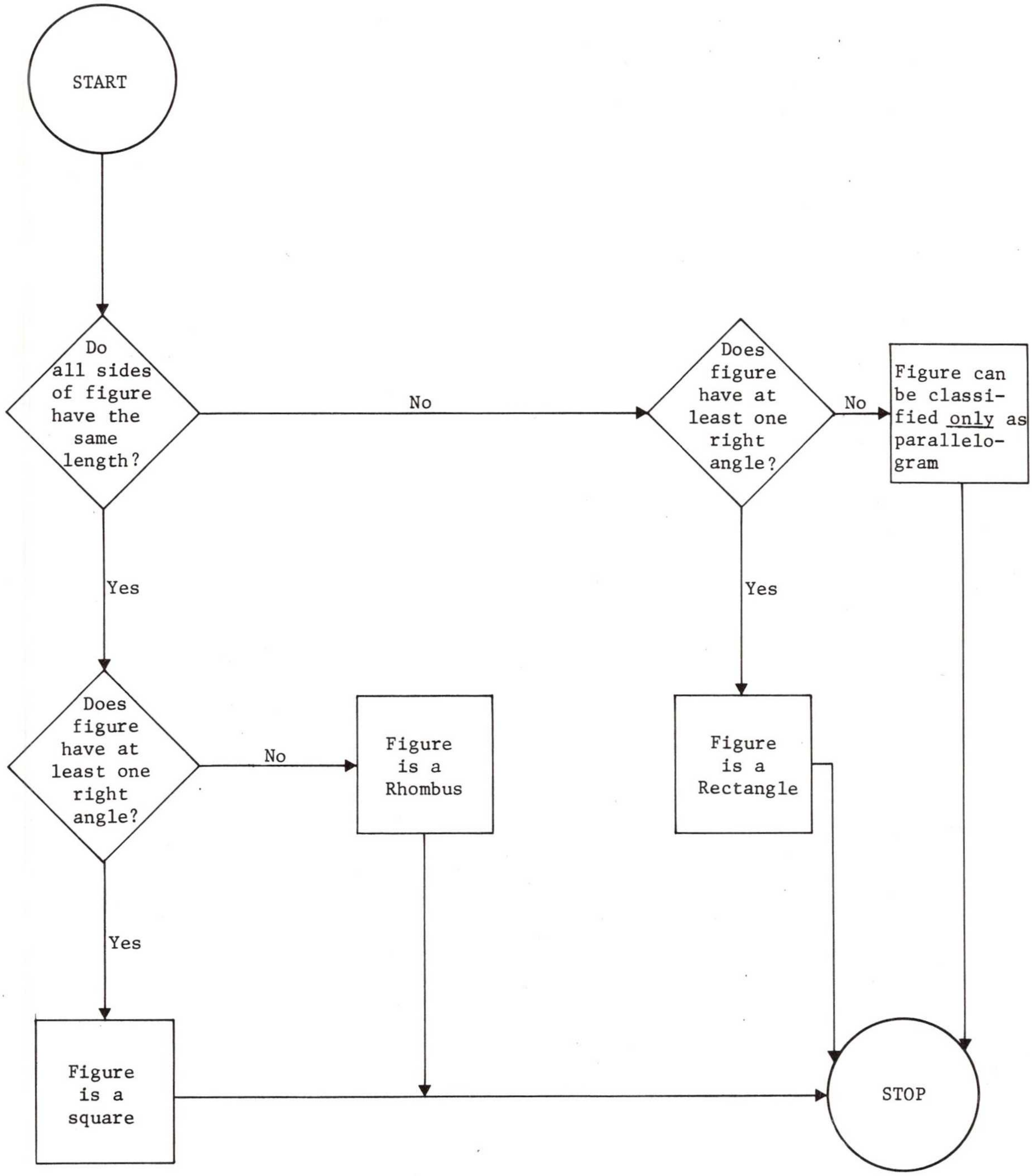


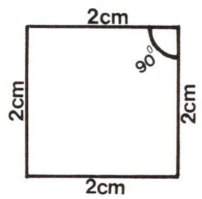
Can you fit any of the parallelograms into any of the others? Can you express your findings in terms of sets? Are any of the parallelograms subsets of any other kinds?

Find some pictures illustrating the use of parallelograms in such fields as architecture, fabric design, furniture design, or others. Mount them. See slide for one of them.

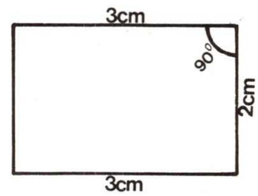
NAMING PARALLELOGRAMS

Using the flow chart below, name each of the figures on the next page.





1 ________



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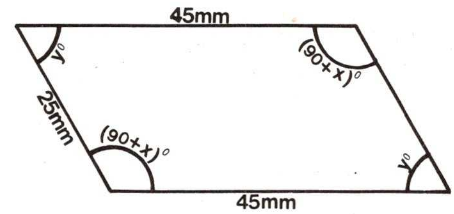

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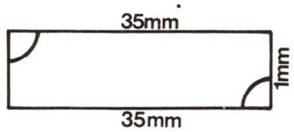

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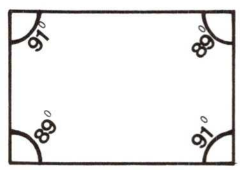

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REFERENCES

Alpart, Bruce Jay. "Auxiliary Lines - A Testing Problem," *The Mathematics Teacher*, February 1973, pp.159-160.

Brieske, Thomas J. "Functions, Mappings, The Mapping Diagrams," *The Mathematics Teacher*, May 1973, pp.463-468.

Farrell, Margaret A. "An Intuitive Leap or an Unscholarly Approach," *The Mathematics Teacher*, February 1975, pp.149-152.

Hawkins, Vincent J. "Some Changes in Shop Mathematics Due to Metrication," *The Mathematics Teacher*, November 1974, pp.601-603.

Hirsch, Christian R. "Graphs and Games," *The Mathematics Teacher*, February 1975, pp.125-126; 131-132.

Hoffman, Nathan. "Partitioning of the Plane by Lines," *The Mathematics Teacher*, March 1975, pp.196-197.

Jordan, James H. "Constructing the Square Root," *The Mathematics Teacher*, November 1972, pp.608-609.

Ranucci, Ernest R. "Of Shoes-and Ships-and Sealing Wax-of Barber Poles and Things," *The Mathematics Teacher*, April 1975, pp.261-264.

Roth, Norman K. "Map Coloring," *The Mathematics Teacher*, December 1975, pp.647-53.

Whitman, Nancy C. "Chess in the Geometry Classroom," *The Mathematics Teacher*, January 1975, pp.71-72.

Winkin, Zalman. "Transformations in High School Geometry Before 1970," *The Mathematics Teacher*, April 1974, pp.353-360.

B.2 Regular Polygons

1. Square - Use the unit squares to find the area of several squares whose sides are: 1 unit, 2 units, 3 units, etc. long. See if you can find an equation that gives the area of any size square.

2. Rectangle - Use the unit squares to find the area of several rectangles whose dimensions are:

Width	Length
1 unit	2 units
1 unit	3 units
2 units	3 units
2 units	4 units
3 units	3 units
3 units	6 units

You can make up a few more rectangles of your own if you like. Can you find an equation that gives the area of any size rectangle? Is there a relationship between the equations for the area of a square and a rectangle?

3. Parallelogram - Use the unit squares to find the area of several parallelograms whose dimensions are:

Base	Altitude
2 units	1 unit
3 units	2 units
3 units	3 units
4 units	3 units
5 units	3 units
6 units	3 units

Draw the parallelogram first if necessary. Can you find an equation that gives the area of any parallelogram? Is there a relationship between the equations for areas of a rectangle and a parallelogram?

4. Triangle - Use the unit squares to find the area of several triangles whose dimensions are:

Type	Base	Altitude
Right angle	1 unit	1 unit
Right angle	2 units	1 unit
Isosceles	2 units	4 units
Isosceles	3 units	2 units
Right angle	4 units	3 units
Scalene	5 units	3 units

Draw the triangles first if necessary. Can you find an equation that gives the area of any triangle? Does the type of triangle make any difference? Is there a relationship between the equations for areas of a triangle and a rectangle?

REFERENCES from *The Mathematics Teacher*

Bolater, L. Cary. "Midpoints and Measures," November 1973, pp.627-30.
Bradley, A. Day. "The Three-Point Problem," December 1972, pp.703-6.
Brumfiel, Charles A. "Generalization of Vux Triangles," February 1972, pp.171-74.

Cohen, Israel. "Pythagorean Numbers," November 1974, pp.667-69.
Ewbark, William A. "If Pythagoras had a Geoboard," March 1973, pp.215-21.
Fehlen, Joan E. "Paper Folds and Proofs," November 1975, pp.608-11.
Gare, Norman, and Sidney Penner. "An Absent-Minded Professor Builds a Kite," February 1973, pp.184-85.
Lepowsky, William L. "The Area of a Parallelogram is the Product of its Sides," May 1974, pp.419-21.
McIntosh, Jerry A. "Determining the Area of a Parabola," January 1973, pp.88-91.
Olson, Alton T. "Some Suggestions for an Informal Discovery Unit on Plane Convex Sets," March 1973, pp.267-69.
Oullette, Hugh. "Number Triangles-A Discovery Lesson," December 1975, pp.671-74.
Smith, Stanley A. "Rolling Curves," March 1974, pp.239-42.
Spaulding, Raymond E. "Hexiamonds," December 1973, pp.709-11.
Stengel, Carol. "A Look at Regular and Semiregular Polyhedra," December 1972.
Trocollo, Joseph A. "Instant Insanity - A Significant Puzzle for the Classroom," April 1975, pp.315-19.
Trigg, Charles W. "Collapsible Models of the Regular Octahedron," October 1972.

B.3 Circles

Properties of a Circle *(Gordon)*

PURPOSES

1) To develop an awareness of some of the properties of the circle to the extent that pupils can solve numerical exercises which require no rigorous proofs,

2) to develop an interest or desire to look beyond the obvious in a diagram,

3) to develop a desire to have some interest in going further than the given question requires and finding why certain conditions exist,

4) to review previous knowledge of geometry.

PROCEDURE Divide the class into small groups (four has seemed to be the most workable number). Each group is given a series of questions which require conditions. Not only are they asked to discover a conclusion to each but also to explain their conclusions. At the end of each activity--which in most cases requires an hour--the groups report their conclusions - some of them obvious to all in the class, others requiring explanations by the group with ensuing discussion.

[*At the beginning of each class, necessary definitions of new terms and a review of basic constructions need to be discussed. This can be done in a class previous to the workshop. However, if it is done at the beginning of each day, it allows the teacher to make comments about the previous day's work and tie up loose ends when there is no heated discussion with which to compete. Also, if done at the beginning of the class, the terms and constructions are new for the day and ready to be worked with. At the end of some activities the groups are assigned numerical exercises, if appropriate, and at the end of the five activity sessions the class is given several numerical exercises which involve the* *use of all properties. (If a more rigorous approach were the aim, at the end of each activity the class could be assigned deductions rather than numerical exercises.) At this point it is the decision of the students to work independently or remain in groups. At the end of the workshops, one class period can be spent discussing the properties of the circle. This serves to answer questions that arise from numerical exercises and gives time for new discoveries to be discussed. This is a fun class, allowing moments of glory for some who find answers to questions that they ask themselves and sorting out troubles for others who need help.]*

MATERIALS Compasses, ruler, protractor, question sheets.

[*In the existing Math 20 course time does not allow for development of theorems and application of them to rigorous deductive proofs. Therefore, the workshops are set up using only compasses, ruler and protractor. Nevertheless, in a situation where the prupose is to develop deductive reasoning, the same workshops can be used only requiring rigorous proofs rather than accepting conclusions based on measurement alone.*]

Each student is given the following series of instructions. The same instructions are given each day, that is: for each condition, construct two or three diagrams of different size and draw a conclusion about the additional information you find. Why is your conclusion correct?

Activity One - Chord Property Theorems

1. Construct the right bisector of any chord of a circle.
   Construct the right bisector of two or more chords of the same circle.

2. Construct a line through the center of a circle meeting the mid-point of a chord.

3. Construct a line through the center of a circle and perpendicular to a chord.

4. Construct two chords of a circle equidistant (perpendicular distances) from the center of the circle.

5. Construct two equal chords of a circle.

6. Construct two chords of unequal lengths.

Activity Two - Sector Angle and Sector Arc Theorems

1. In the same circle, construct equal sector angles.
   In equal circles, construct equal sector angles.

2. In the same circle, construct unequal sector arcs.
   In equal circles, construct unequal sector angles.

Activity Three - Inscribed Angles

1. Construct an angle at the center of the circle (sector angle).
   Construct an angle in the circle (touching the circle) on the same arc as the sector angle.

2. Construct two different angles in the circle subtended by the same arc.

3. Construct an angle subtended by the diameter of the circle.

4. Inscribe a quadrilateral in a circle. What can you say about the angles?

5. Inscribe a quadrilateral. Produce one of the sides. What can you say about the angle so formed and the angles of the quadrilateral?

Activity Four - The Tangent

1. Construct a line from the center of a circle perpendicular to a tangent to a circle at the point of contact.

2. Construct two tangents to a circle from a common point outside the circle. Join the center of the circle and the external point.

3. Construct a chord and a tangent from the same point on the circle.

4. Construct a tangent to a circle. From the same exterior point construct a secant. What can be said about the lengths of the secant segments and the tangent length?

Activity Five

1. Construct two equal angles subtended by the same line on the same side of the line. What can you say about the end points of the line and the vertices of the angles?

2. Construct a circle on the hypotenuse of a right triangle as diameter.

3. Construct a quadrilateral having interior and opposite angles supplementary. What can be said about the vertices of the quadrilateral?

4. Construct a quadrilateral having the exterior angle equal to the interior and opposite angle. What can be said about the vertices of the quadrilateral?

[*The workshops on the circle were designed specifically for the Math 20 course and developed the suggested topics in the course. In comparison to the usual lecture approach, the class seemed to have a better "feeling" for the properties of the circle. When it came to applying the properties to exercises, there was much more enthusiasm about attacking the problems. It was not just another assignment to be completed, but, rather, another time to find new ideas. There was more desire to find* ***why*** *conditions existed rather than merely completing an exercise.*]

REFERENCES from *The Mathematics Teacher*

Einharn, Erwin. "A Method for Approximating the Value of π With a Computer Application," May 1973, pp.427-30.

Hatcher, Robert S. "Some Little-Known Recipes For π," May 1973, pp.470-74.

Hutcheson, James W. "The Circular Geoboard, A Promising Teaching Device," May 1975.

Jenks, Stanley M., and Donald Peck. "Thought Starters for Circular Geoboard," March 1974, pp.228-33.

B.4 Circles and Polygons

Inscribed Angles, Sector Angles and Inscribed Quadrilaterals *(Connolly)*

[*Prior to this activity students have been introduced to circles and some of the basic definitions. In particular, they know the meaning of chord, arc, sector, segment, inscribed angles, sectors angles and inscribed figures. It can be used at whatever level students are introduced to a study of circles. Presently this is at the Math 20 level. The activity is best performed in pairs.*]

MATERIALS (per pair of students) A set of geometry instruments, scissors, paper from which circles may be cut or already prepared circles of various radii.

Activity I

Cut from the paper several congruent circles. Draw in each equals chords AB (Figure 1a) and cut each into congruent segments as illustrated in Figure 1b,, keeping segment AXB for first activity.

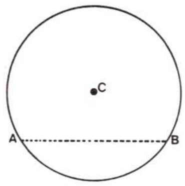

Figure 1a

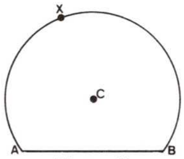

Figure 1b

From segment AXB cut various inscribed angles ACB, Figure 2a and 2b

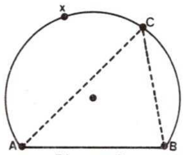

Figure 2a

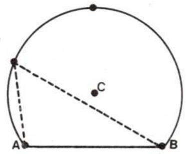

Figure 2b

Measure each angle at C and record your results.
What do you discover?
How would you increase the measure of angle C?
How would you decrease the measure of angle C?
What happens if chord AB is a diameter?

Cut a sector angle AOB from one of the segments used above as illustrated in Figure 3.

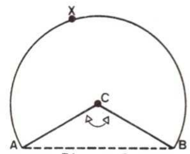

Figure 3

Measure angle AOB.

How does it compare with angle ACB?

Can you make a general statement about the inscribed angle ACB and the sector angle AOB?

Could you develop a formal proof for your conclusion?

Activity II

Using several congruent circles, cut them as illustrated in Figure 4, varying the positions of X and Y.

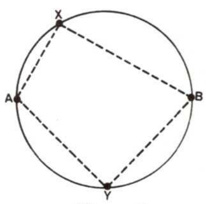

Figure 4

Measure the angles AXB and AYB and record your data.

What do you discover?

What do you think is true of angles at A and B?

What conclusion can you draw about an inscribed quadrilateral?

Is this true for all quadrilaterals?

Make a general statement for your conclusions.

REFERENCES from *The Mathematics Teacher*

Doasey, John A. "What! A Roller With Corners?", December 1972, pp.720-24.
Ehrmann, Sr. Rita (Cordia). "Projective Space Walk for Kirkman's Schoolgirls," January 1975, pp.64-9.
Kingston, J. Maurice. "The Unexpected Attracts Attention," November 1973, pp.655-6.
Krause, Eugene F. "Taxicab Geometry," December 1973, pp.695-706.
Kumpel, Paul G. Jr. "Do Similar Figures Always Have the Same Shape?", December 1975, pp.626-28.
Moulton, J. Paul. "Experiments Leading to Figures of Maximum Area," May 1975.
Pederson, Jean J. "Some Whimsical Geometry," October 1972, pp.513-21.
Schraeder, Lee L. "Buffons Needle Problem: An Exciting Application of Many Mathematical Concepts," February 1974, pp.183-86.
Shwarger, Michael. "Parametric Construction of the Conics," February 1972, pp.105-9.