ACTIVITIES INVOLVING THE GEOBOARD

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"Activities Involving the Geoboard" was prepared by Dr. Coltharp and presented by him at a workshop for elementary school teachers at the forty-ninth annual meeting of the National Council of Teachers of Mathematics, in Anaheim, California, April, 1971. The following is an edited version of that presentation.

The geoboard is a simple tool that allows children to visualize abstract mathematical concepts, develop intuitive thinking, and discover numerical relations and operations for themselves. "Modern" mathematics can be taught through the use of the geoboard in a modern manner involving students in the process of their own education.

Modern educational philosophy advocates the use of many approaches and materials to help children see mathematical relationships and to integrate rather than isolate various branches of mathematics. The geoboard has proven to be an ideal tool for this purpose. While it is widely accepted as a specific aid in explaining basic geometric concepts, its application is by no means limited to this area. We will investigate some of the possibilities in areas other than geometry.

PRESCHOOL AND KINDERGARTEN CLASSROOM ACTIVITIES

Provide each youngster with a geoboard and several rubber bands and let creativity begin. The following "Suggestions" and "Possible Questions" are quoted directly from "Geoboard Geometry for Preschool Children."[1] The article also contains sample responses from children age two to six years.

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[1]W. Liedtke, and T.E. Kieren, "Geoboard Geometry for Preschool Children," *The Arithmetic Teacher*, February, 1970, pp.123-126.

FAMILIAR SHAPES

SUGGESTIONS On your geoboard, show how to make some shapes that look like something in this room. Try to make something that can be found in the kitchen, the basement, yard, grocery store, playground, garage. Show something your dad uses. Show something that is alive.

POSSIBLE QUESTIONS / INSTRUCTIONS Can you tell your friend what you have made? Look at something someone else has made and try to guess what it is. (Ask for a hint where it can be found.) Does your figure look the same if you turn the geoboard around? How many corners does your figure have? (Are there more corners or more sides?)

PLANE FIGURES

SUGGESTIONS Try to make figures with three sides that are small, large, "skinny," "fat." Try to make figures with four sides that are long, short, long and wide, long and narrow, short and wide, short and narrow, "like a square," "not like a square." Try to make figures with "many sides."

POSSIBLE QUESTIONS / INSTRUCTIONS What does the figure you have made remind you of? Does it look like anything that is familiar to you? (Where did you see something like it before?) Does the figure change if you turn your geoboard? Make two figures that (1) do not touch, (2) touch, (3) cut into each other. Look at the figures you have made.

SEGMENTS

SUGGESTIONS Try to make segments that are short, long, straight, "crooked." Try to make segments that do not touch, touch, cross each other (intersect), will never touch (parallel), are exactly on top of each other. Try to make various segments leading to two (or more) points; various numbers of segments (for example, two that are equal, two that are not equal, many different segments).

POSSIBLE QUESTIONS / INSTRUCTIONS How would you make a road? Can you make a very narrow road? Can you make one that is long and narrow? Make a railroad track. (If possible provide rubber bands of different colors.) Can you make a road and a train track that cross, do not cross, will never cross? Look at two pegs in different corners. How many different roads--crooked or straight, with few or many corners--can you build between these two pegs? Which road would you like to travel on? Why?

NINE-PIN GEOBOARD[2]

"A nine-pin geoboard is easily constructed from 3/8-inch plywood and 5/8-inch panel pins, the pins being about two inches apart (see Figure 1). The nine-pin board has the advantage of being so simple that the child believes any mathe-

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[2]Suggestions and questions from "Creating Mathematics With A Geoboard," Peter Wells, *The Arithmetic Teacher*, April 1970, pp.347-349.

matics that may arise can soon be sorted out."[3] One board and a pile of elastic bands between two children is ideal - a partner is there to encourage, correct, and argue with. During a free activity stage the teacher can watch for situations that particularly interest the children and that are suitable for development and extension.[4]

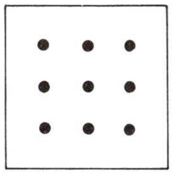

Figure 1

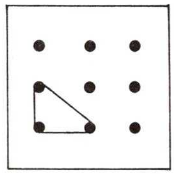

Figure 2

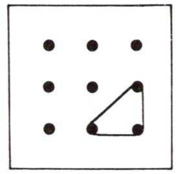

Figure 3

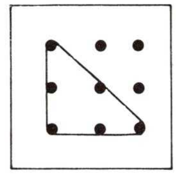

Figure 4

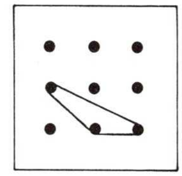

Figure 5

One activity might involve the smallest triangle possible (Figure 2). How many triangles the same as this one can you make on the board? A problem arises immediately. Is the triangle in Figure 3 the same as the one in Figure 2. Some will say yes, others no. Some will want to include Figure 4. The discussion which evolves will challenge mathematical interpretation. Definitions and classifications, equivalence relations, and ideas of sets are all involved, and the teacher should be aware of this although the children may not be.[5]

In answering the question "How many of the smallest triangles are there?" the children will normally experiment with elastic bands. Some will generalize and come up with an answer quickly. For these sharper ones, perhaps the question "How many triangles like figure 5?" will challenge them.

Some other questions which involve classification of shapes on a nine-pin board are listed. These will prove to be spring boards to further mathematical discoveries.

1. How many different squares can be made on the boards? How many of each type?

2. Consider the four-sided shapes. Which ones have an axis of symmetry? How many can you give a familiar name to?

3. Look at the triangles you can make. How many are isosceles? How many are right-angled? How many of each type are there on the board?

4. Make shapes with more than four sides. What is the largest number of sides you can get?

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[3]Wells, *op. cit.,* p.347.

[4]*Ibid.*

[5]Wells, *op. cit.,* pp.347-348.

5. How many straight line segments of different length can be made?

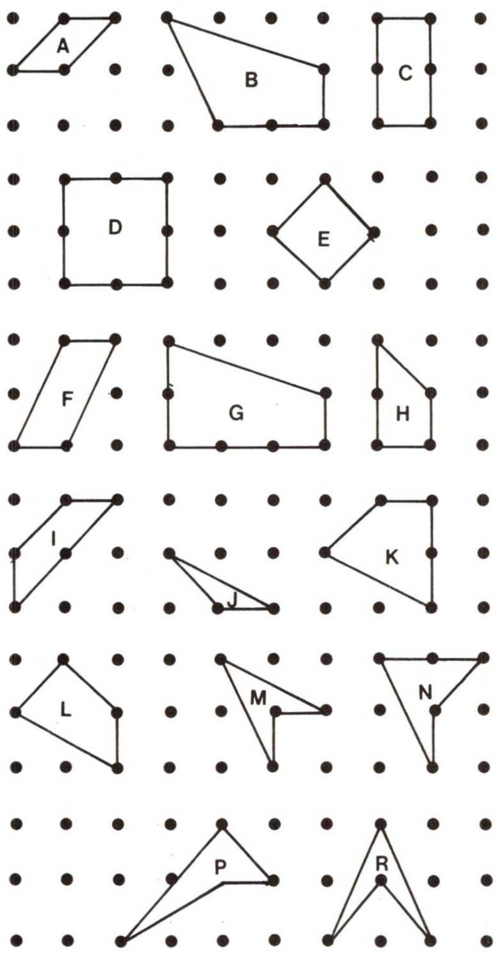
6. What shapes cannot be made on the board?

WORKSHEETS INVOLVING THE GEOBOARD

The following worksheet is an example of an activity-oriented program you may want to investigate further.

Specific behavioral objective - Identify the diagonals of various plane figures and define the idea of a diagonal.

Learning opportunity - Make the following shapes on your geoboard. Transfer each figure to dot paper and draw in all the diagonal lines using a red pencil. In which shapes are the diagonals of equal length?



Test Item - Define what you mean by the term "diagonal." Draw a shape that has no diagonal and tell why it doesn't by applying your definition.

Test Item - Does a diagonal necessarily bisect the angle at that vertex?

[*A follow-up discussion might develop the idea of whether or not the definition given would work for space figures or for a line joining two vertices which is not a side*[6]]

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[6]Carol H. Kipps, "Topics in Geometry for Teachers - A New Experience in Mathematics Education," *The Arithmetic Teacher*, February, 1970, p.166.

AREA AND PERIMETER

The concepts of area and perimeter can be introduced through the use of the geoboard, pegboard, graphing paper or the T-board. The following activities and questions are adapted from the article "Geometry Via T-Board,"[7] which you may want to read to determine what constitutes a T-board.

First, enclose a rectangle two units long and one unit wide. How many square units are enclosed by the rectangle? (See Figure 6.) Find the distance around the rectangle by counting the units forming the sides. (See Figure 7.)

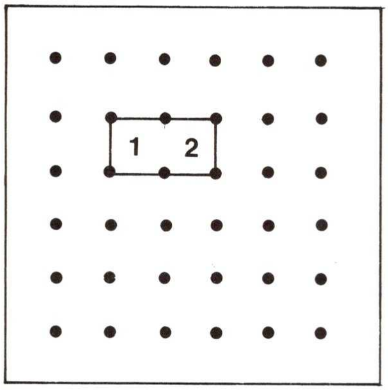

Figure 6

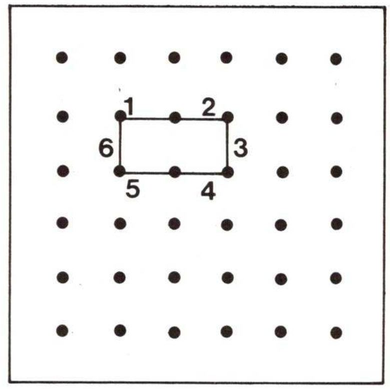

Figure 7

Next, double the length of the rectangle and maintain the same width. How many square units are enclosed by this rectangle? (See Figure 8.) Find the distance around the rectangle again by counting the units forming the sides. (See Figure 9.)

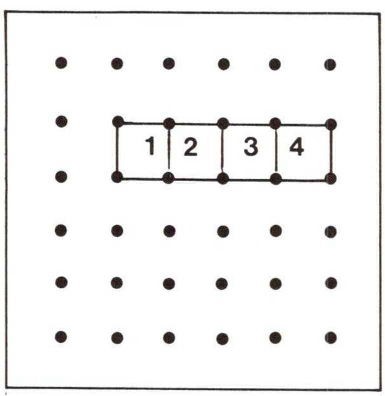

Figure 8

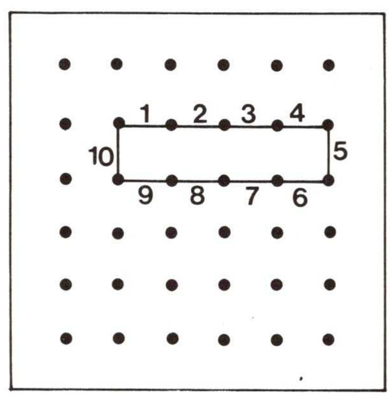

Figure 9

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[7]Donald O. Teegarden, "Geometry Via T-Board," *The Arithmetic Teacher*, October, 1969, pp.485-487.

Use your geoboard to determine the number of square units enclosed by the rectangle formed when the original length is maintained and the width is doubled. What will be the distance around such a rectangle? Consider the same questions when both the length and width are doubled. Display your results in the following table.

Length	*Width*	*No. of square units enclosed*	*Distance around*
2	1	2	6
4	1	4	10
2	2	□	△
4	2	□	△

Starting with a different rectangle, perform the same cycle of doubling the length, the width, and then both length and width. Then ask the following questions:

How could you find the number of square units enclosed by any size rectangle quickly from just knowing the length and width?

Does the distance around double when the number of units enclosed is doubled by making one dimension twice as great?

How do you make the distance around twice as large as the original?

Is there a way of figuring the distance around without counting each item?

Use the geoboard to determine the missing measures indicated in the following table.

Size of Rectangle	*No. of square units enclosed*	*Distance Around*
3 X □	△	10
3 X 3	△	○
2 X □	8	○
4 X □	△	14
4 X □	16	○
5 X 2	△	○

If the term "area" is substituted for "number of square units enclosed" and "perimeter" for "distance around" we find that many important concepts have been developed in this activity.

PRIME AND COMPOSITE NUMBERS[8]

A technique based upon the manipulation of square regions may be used to help children discover that the counting numbers may be classified as prime or composite numbers. The geoboard (or pegboard) may be used to introduce this technique.

"Considering only horizontal and vertical rectangular arrangements, in how many different ways can a given number of square regions be arranged?"[9] For example, how many different arrays can be made with only one unit region? One region may be arranged in only one way, as a 1 x 1 array (see Figure 10).

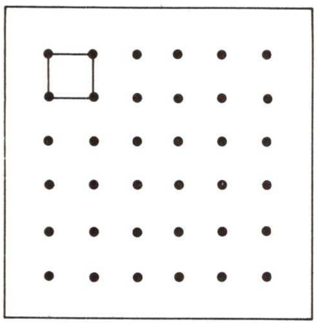

Figure 10

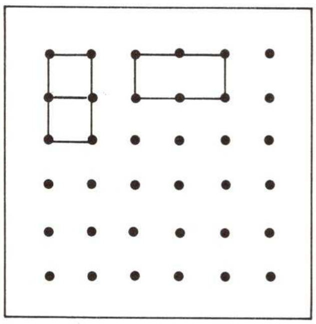

Figure 11

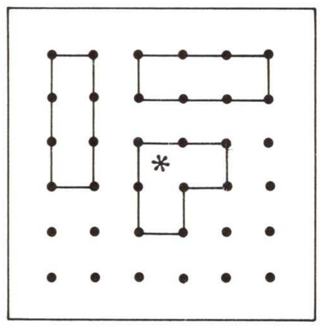

Figure 12

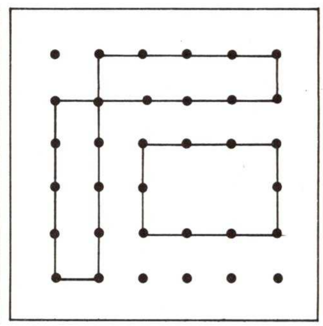

Figure 13

How many different arrays can be made with only two unit regions? Two regions may be arranged in only two different ways, as a 2 x 1 and as a 1 x 2 array (see Figure 11).

How many different arrays can be made with only three unit regions? Why isn't the array marked with * (in Figure 12) considered?

How many different rectangular arrays can be made with only four unit regions? Four regions may be arranged in three different ways, as a 4 x 1 array, as a 1 x 4 array, and as a 2 x 2 array (see Figure 13).

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[8]This activity was taken from: "Pattern for Discovery: Prime and Composite Numbers," Frances Hewitt, *The Arithmetic Teacher,* February, 1966, pp.136-138.

[9]Hewitt, *op. cit.,* p.136.

With the limitations of the typical geoboard, you may wish to resort to a pegboard with more range, coloring on graphing paper, or cut out square regions. Continue the activity for five, six, seven, eight, and possibly even more unit regions. Each time ask the question, "How many different rectangular arrays can be made with the given number of unit regions?"

The first step may be merely to determine whether the number can be represented by only one rectangular array, by only two, or by more than two different rectangular arrays.

In any discovery exercise such as this, it is important that the data be arranged in some pattern so that a generalization is possible. The information may be recorded in a table. The table below has been completed for 1, 2, 3, and 4; continue the process for several more counting numbers.

Only one array	*Only two arrays*	*More than two arrays*
1		
	2	
	3	
		4

After children are familiar with the process and have had experience with numbers, perhaps through 20, you may wish to extend the table as indicated.

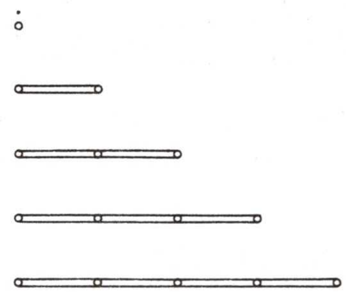
Only one array	*Only two arrays*	*More than two arrays*	*Pairs factors*	*Different factors*
1			1 x 1	1
	2		2 x 1	1, 2
			1 x 2	
	3		3 x 1	1, 3
			1 x 3	
		4	4 x 1	1, 2, 4
			2 x 2	
			1 x 4	

The final step in this discovery activity would be to find a rule that allows us to classify the counting numbers. Up to this point the children can discover everything but the names of the classes, namely *prime* and *composite*. See if you can devise the rule used to classify the counting numbers. What about the number 1?

FRACTIONS[10]

"The following exercises can be adapted to the needs of the class, and more exercises of the same sort can be easily developed by the teacher."[11]

EXERCISE 1



Prepare the geoboard in the manner above with rubber bands around two pegs, three pegs, four pegs, and five pegs. Although any of the three may be considered the unit, use a different colored rubber band around the four pegs to indicate the unit measure for this discussion. Ask questions such as the following:

How long is the unit strip? (One unit)

How long is the top strip? (One-third of a unit)

Why? (Because three of these one-third units will be as long as the unit)

How long is the next strip? (Two-thirds of a unit)

Why? (It is twice as long as the one-third strip)

How long is the bottom strip? (Four-thirds of a unit)

Why? (It is four times as long as the one-third strip, or one-third more than one unit)

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[10]George S. Cunningham and David Raskin, "The Pegboard As A Fraction Maker," *The Arithmetic Teacher,* March, 1968, pp.224-227.

[11]Cunningham, *op. cit.,* p.224.

INSTANT GRAPHS[12]

With the aid of rubber bands, "instant graphs" can be quickly constructed on your geoboard. It may be necessary to use a larger pegboard and movable pegs but much of the data gathered can be displayed on the geoboard.

The bar graph (Figure 14) "... could be a record of the number of days of school four boys missed during the last month. The line graph (Figure 15) could show the number of pupils who were absent from the classroom for each of the five days last week.

"Pieces of cardboard could be used to label the parts of the graphs, and challenges can be added by requiring the students to calculate and construct various scales."[13]

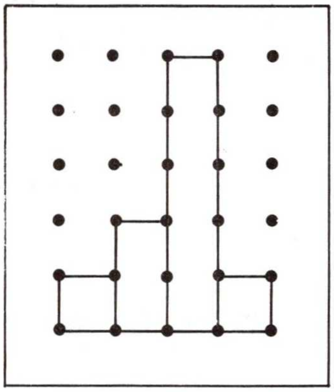

Figure 14

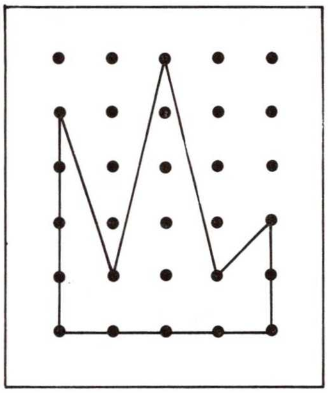

Figure 15

LIST OF MATERIALS AND SOURCES

Cuisenaire Company of America, 12 Church Street, New Rochelle, New York 10805

Plastic Geoboards. Two pamphlets also available: *Notes on Geoboards; Geoboard Geometry* by Caleb Gattegno.

Houghton-Mifflin Company, 1900 S. Batavia Avenue, Geneva, Illinois 60134.

Geoboard Kit. Each kit contains a 7" by 7" geoboard, assorted rubber bands, and 40 student activity cards.

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[12]What Can You Do With A Geoboard?" Werner Liedtke. *The Arithmetic Teacher.* October, 1969, pp.491-493.

[13]Liedtke, *op. cit.*, p.491.

Selective Educational Equipment Company, Inc., 3 Bridge Street, Newton, Massachusetts 02195.

Plastic Shapes Board.

Sigma Enterprises, Inc., Box 15485, Denver, Colorado 80215.

The Sigma Geosquare. Geosquare Classroom Kit includes 30 Geosquares, large assortment of colored rubber bands, teacher's manual, pad of Geosquare dot paper. Geosquare Teacher's Manual available separately for $1. Also available are three kinds of Geosquare dot paper: 2-inch scale, 1-inch scale, and 1/2-inch scale.

Walker Teaching Programs and Teaching Aids, 720 Fifth Ave., New York, N.Y. 10019.

Geoboards. Plastic Geoboard for overhead demonstrations. Book by Donald Cohen, *Inquiry in Mathematics Via the Geoboard.* Geo-Card Math Lab, introducing approximately 150 problems and experiments. *The Geoboard: A Manual for Teachers* by Irving Kreitzberg. Designed for instruction in Grades K-8. $2.45.

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