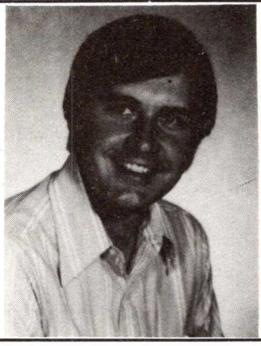
GRAPHING



by *Ed Irmis* North School Glencoe, Illinois

The following paper was prepared and presented by the author as supplementary material for his elementary school "graphing" workshops at the forty-ninth annual meeting of the National Council of Teachers of Mathematics, in Anaheim, California, April 15-17, 1971.

The ideas presented in this paper are primarily at an introductory level and detailed development has not been provided. Further development of these initial activities can evolve from the discoveries, questions and interests of the children this material is intended to serve. Many important graphing topics (inequalities, three dimensional graphing, circular graphs, etc.) have not been included. Therefore, what has been included merely scratches the surface of the subject of graphing.

BAR GRAPHS

The following activities provide an opportunity for children to tell about themselves which they so often seem to enjoy.

CLASS ACTIVITY 1 - LIVING GRAPHS

The children in the class form a type of bar graph (in this case the proper term probably is "histogram") as an introductory experience in graphing.

PREPARATION Place the names of the twelve months across the wall or bulletin board.

DESCRIPTION Each child stands in line according to the month of his birthday. To keep a uniform starting place for each of the lines (bars), the first child in each line stands directly in front of the name indicating the month of his birthday.

SAMPLE QUESTIONS

(A) Is there anyone in your month with the same birth date?
(B) Which month has the most birthdays in our class?
(C) If you switch the order of people in your line, would some other month have the most birthdays?

(D) Which month has the fewest birthdays?
(E) What is the difference between the month with the most and the month with the fewest birthdays?
(F) Did any months end in a tie?
(G) Did anyone notice something interesting about our "living graph? that he would like to tell the class?

CLASS ACTIVITY 2 - GISMO GRAPH

Sometimes being an actual part of a graph makes it difficult for people to see clearly the results of the information collected. So, let's try a different type of bar graph (histogram).

PREPARATION Number several cards and place them in numerical order - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, across a table or across the floor.

DESCRIPTION This time we are collecting some information about the number of pets each child has at home. Each child places his math book in the pile that tells the number of pets in his home. In other words, if you have two pets at home, place your book in the stack of books labeled by card number 2.

SAMPLE QUESTIONS

(A) Did anyone else put their math book in the same pile as yours?
(B) What is the meaning of the tallest stack?
(C) What ideas can you tell the class from our "gismo" book graph?

CLASS ACTIVITY 3 - USING A GRID

It can be difficult to carry around a "living" graph or even a stack of books, especially if you want to show your graph at home. Here is another way to make a bar graph.

PREPARATION Draw a heavy dark line across the bottom of a grid marked off in one-inch squares. Below this line, number the vertical columns - 0, 1, 2, 3, 4, 5, 6, 7, etc., from left to right. Have crayons available.

DESCRIPTION The children are asked to count the number of blocks it is from their house to school. Each child colors one of the squares in the column telling the number of blocks he lives from school. Example - if you live 5 blocks from school, you color a square in the column marked with the "5." (You might have each student write his name in the square he colored to demonstrate that his square represents the number of blocks he lives from school.)

SAMPLE QUESTIONS

(A) How many other people in this class live the same distance from school as you?
(B) What can you tell from our graph?

Other topics for activities 1, 2 and 3

- height of each student in the class,
- weight of each student in the class,

- number of brothers and sisters each student has,
- temperatures during any one month,
- number of vowels in each student's full name,
- number of glasses of milk each student drinks per day,
- number of TV programs watched by each student in one week,
- number of hours each child sleeps,
- foot lengths of the children in the class.

(Also see Ida Mae Heard, "Making and Using Graphs in the Kindergarten Mathematics Program," *The Arithmetic Teacher*, October, 1968, pp.504-506.)

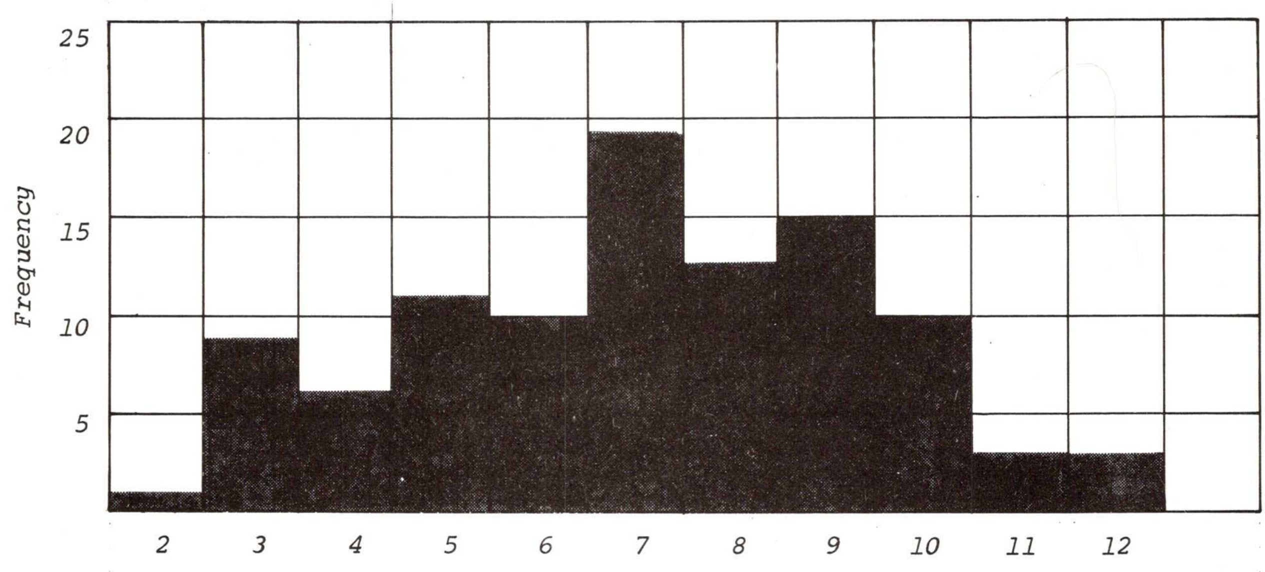
Here are some opportunities for children to work together in small groups, collect data, and exchange results with other groups by comparing graphs.

SMALL GROUP ACTIVITY A - Rolling Two Dice

MATERIALS Paper marked off in 1 or ½-inch squares, and a pair of dice for each group.

OBJECTIVE To collect data and help predict the frequency of occurrence of numbers 2 through 12 using a pair of dice.

Sample graph (100 throws)

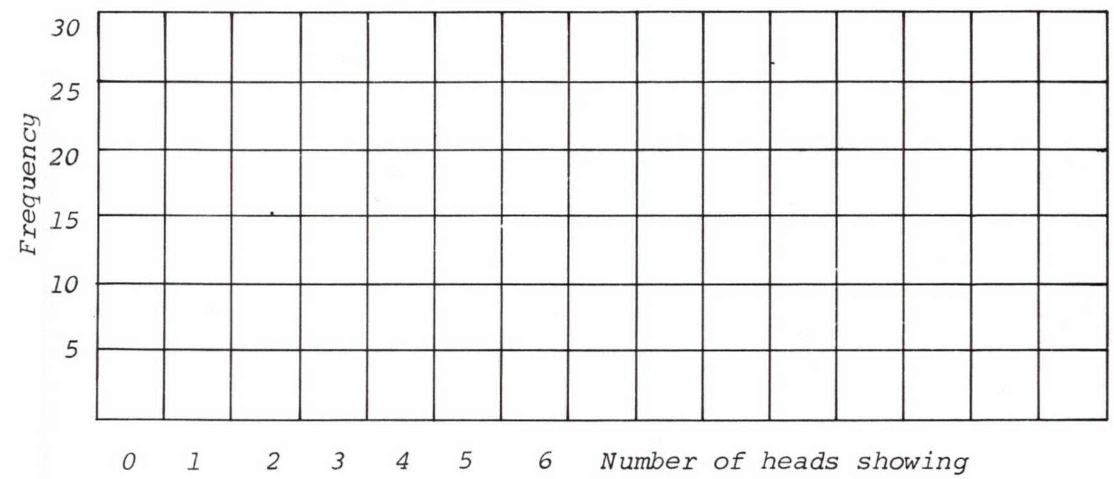


SMALL GROUP ACTIVITY B - Coin Toss

MATERIALS Paper marked off in ½ or 1-inch squares, and six coins for each group.

OBJECTIVE To collect data (by throwing six coins each toss) to determine the frequency of occurrence of the number of heads showing per toss.

Sample graph (try this one yourself on 100 tosses)



(For additional materials or activities see also *School Mathematics Project* (SMP) Book B, Chapter 9, "Statistics"; *Freedom to Learn*, Chapter 6, Sections 6-9, pp.87-93.)

GRIDS AND COORDINATES

As I was walking down the hall at school last week, John came up to me and said that he had found my five dollars. "What five dollars?" I inquired. He said, "Remember the five dollars you lost in town last October when you were teaching us about graphing?" "Oh, yes," I replied. "Where did you find it?" "Well," he said, "it must have blown around for a long time because I didn't find it where you said you lost it." We both agreed at that point that it probably was not my five dollars, so he kept his newfound treasure. But, what about the story of the lost five dollar bill that John remembered so long?

Story of the Lost Five Dollar Bill

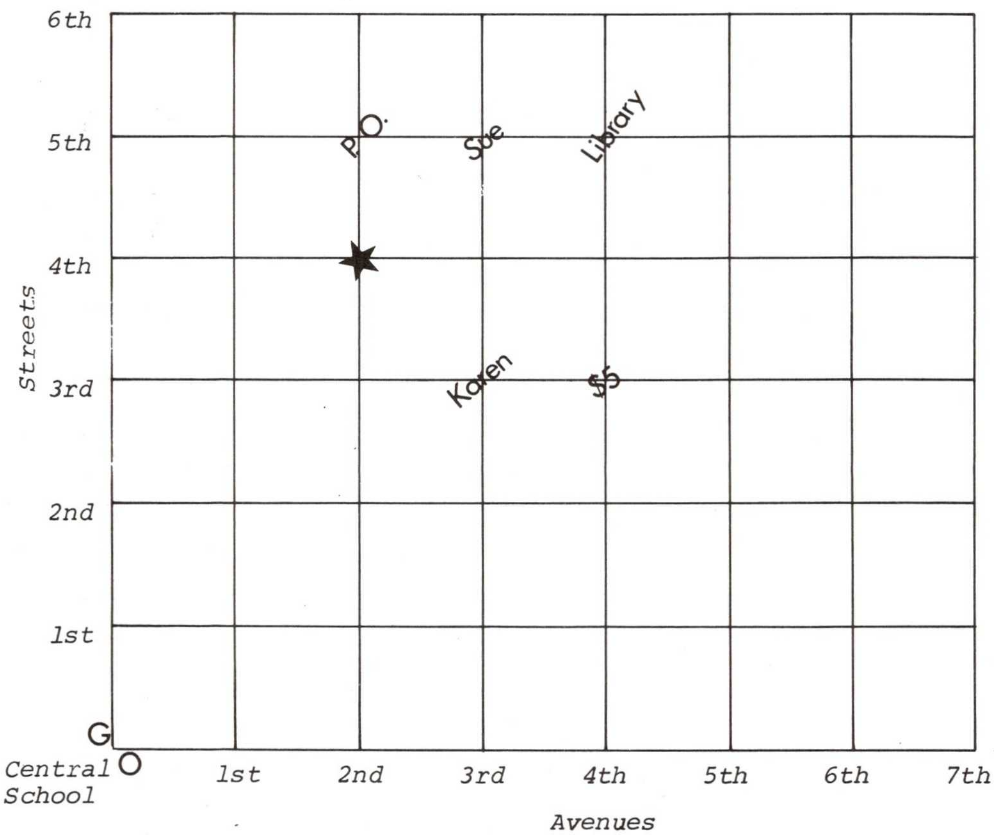
During noon hour one day last October, I left school to borrow a book at the library in town. While waiting for the light to change on the corner of Fourth Avenue and Third Street, I took out my wallet to look for my library card. It must have been on that corner when the five dollar bill fell out of my wallet.

I had planned to look for the lost bill after school but several students came in to ask some questions about their homework assignment just as I was leaving.

A good friend of mine (a mathematician, of course) was going into town and he said he would be glad to look for the money. "Where did you lose it?" asked Mr. Matt A. Maddox (he likes to be called by his full name). Not wanting to let everyone listening know where the money might be found, I used a secret code that my friend also knew.

I said, "If Central School is 'GO,' then look at four, three." My ex-friend must have found the money because I haven't heard from him since.

Map of Your Town



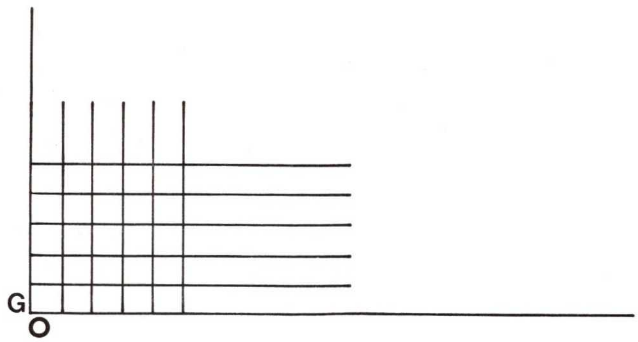
Do you know how the secret code works? The library is at (4,5). Use our secret code to tell where Karen is standing, (___,___). Put your name on the map at (2,3). Where is the star of our baseball team standing? (___,___). Dirk is standing in a straight line with Karen and our baseball star *and* he is also in a straight line with the post office, Sue and the library. Where is Dirk? (___,___). (Put Dirk's name in the correct location on the map.) Put an X on the map in a straight line with you and the library *and* also in a straight line with my $5 and the post office. What are the code numbers for the X? (__,__).

(See also *School Mathematics Project* (SMP), Book A, Part 2, "Maps," pp.15-17; *Graphs Leading to Algebra*, Chapter 1, "Introduction to Co-ordinates," pp.2-11.)

THE GAME OF FOUR-IN-A-ROW

The game of Four-In-A-Row is similar to Tic-Tac-Toe. Its primary purpose is to provide experiences in graphing ordered pairs (our "secret code") onto a grid. It also helps to develop game strategy skills. [Make a copy of the game board with large squares on the blackboard.]

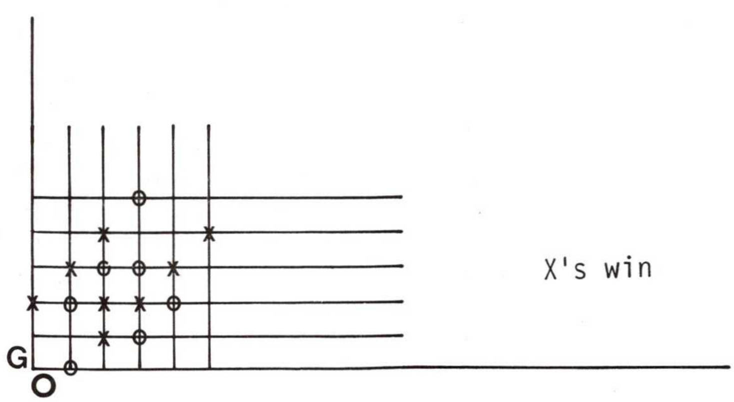
Game Board for Four-In-A-Row



HOW TO WIN Place four of your team's marks in a straight line without any of the opposing team's marks in between them.

HOW TO PLAY Divide the class into two teams: X's and O's. A member of the X's gie s the teacher an ordered pair (according to our "secret code"). The teacher places an X in that location on the game board. Then a member of the O's gives an ordered pair. The teacher places an O in that location on the game board. Teams alternate giving ordered pairs until one team wins with "four-in-a-row."

PENALTIES If a player gives a location already occupied, or a location off the game board, his team loses that turn.



X's win

Can you name the four winning locations? (___,___) (___,___)

(___,___) (___,___)

(Other games to promote skill and enthusiasm for graphing on grids are "Grid Football" (Activity 5), *Activities in Mathematics*, Graphs, Scott Foresman and Company, 1971, pp 13-14; "Hide-a-Region N< 2 Can Play," *The Arithmetic Teacher*, October 1969, pp.496-497; "Shifts," *School Mathematics Project* (SMP), Book B, p.116.)

PICTURES DRAWN ON GRIDS

Among the many activities that graphing provides, drawing pictures on grids is popular with most children. Students are generally enthusiastic about drawing pictures or designs made from a list of ordered pairs (our "secret code") to be graphed on grids. The children enjoy the "drill" and at the same time gain a proficiency for coordinate graphing.

(For coordinate picture and design drawing activities see *Activities In Mathematics*, Second Course, "Graphs," pp.15-17, Activity 6, "Making designs on a grid"; *School Mathematics Project* (SMP), Book A, Exercise B, pp.18-19.)

[*Encourage the children to make up some of their own designs or pictures for other members of the class to try. A class card file containing student-created problems, ideas, puzzles, and discoveries is a possible method of organization.*]

A spin-off from this activity provides children with some experiences with the concepts of congruency and similarity. For example, after several children have attempted the same drawing, the teacher might ask, "How does your picture compare with your neighbor's?"

[*Note - If all students use the same size grid, they should all have "matching" (congruent) figures. If the members in the class are using different size grids, some figures might be congruent while others will have exactly the same shape but will be different in size, that is, not congruent but similar.*]

(See *Freedom to Learn*, pp.153-157, for additional activities with congruency and similarity.)

Another possible activity for improving the individual's skill in graphing, is Activity 9, "Changing Size and Shape with Graph Paper," in *Activities in Mathematics*, pp.31-34, "Graphs." [If the *Activities in Mathematics* texts are not available, the teacher should draw a figure that can be fitted to a standard size grid. The children trace this prepared drawing with tracing paper, then fit their tracing to the graph paper. Using the appropriate ordered pairs, the design or picture is reproduced on larger or smaller grids. This type of activity can be quite valuable especially when the same picture is drawn on several different size grids.]

LET'S GET DOWN TO THE "FACTS"

"...So the kids have a great time drawing pictures and playing games, which is fine, but that won't help them learn their 'facts'!" This comment, made by a fifth grade teacher, is a common reaction at this point and understandably so. However, the potential dividends from introducing children to graphing in the elementary grades is almost unlimited. The real benefits of graphs often result from making graphing a vehicle for reaching the goals of your present mathematics program. Before discussing some of these "dividends," let us take time out here for a few important agreements.

5x9 = 45 is a *true* sentence.

$\frac{1}{5} \times \frac{2}{5} = \frac{3}{10}$ is a *false* sentence.

(5x□) - 3 = △ is an *open* sentence (because you can't be sure what will be substituted for the □ and the △).

Which of these sentences are *true / false / open ?*

(a) (5x6) + 3 = 5 x (6+3)

(b) □ x △ = 24

(c) 3 x (4+5) = (3x4) + (3x5)

This is a Truth Table

□	△
0	1
1	3
2	5
3	7
½	2
etc	

. . . it is a Truth Table for this *open* sentence (2x□) + 1 = △

Can you guess the *open* sentence for this Truth Table?

□	△
1	19
2	18
3	17
4	16
5	15
10	10
1/4	19-3/4
etc	

Each pair of numbers in the Truth Table is called an "ordered pair." Can you name several more ordered pairs that belong in the last Truth Table?

(For proper development of these ideas, see *Discovery in Mathematics*, A Text for Teachers, pp.23-24 and 75-94; *Graphs Leading to Algebra*, Chapter 2, pp.11-22; *Explorations in Mathematics*, A Text for Teachers, Chapters 25 and 26; *Activities in Mathematics*, Second Course, "Graphs," Activity 17, pp.67-70.

We will begin with a very simple activity to illustrate how information is collected, recorded in a Truth Table, graphed on a grid, and finally generalized by an *open* sentence.

GUESS MY RULE

SAMPLE ACTIVITY ONE - Rectangles and Corners

MATERIALS Unlined, rectangular (sharp cornered) paper
Graph paper
Sharp pencil

Fold the plain piece of paper down the middle and tear it along the fold so that you get two rectangles. How many corners are there altogether in the two rectangles? How many corners were in the paper before you tore it?

Now, take *one* of the rectangles. Fold it down the middle and tear along the fold to get two new rectangular pieces. How many pieces of paper do you have now? How many corners do you now have altogether?

Nancy started a table to keep a record of this information. Can you complete Nancy's table by continuing to fold and tear the paper in the same way?

Number of Rectangles □	*Total number of Corners* △
1	4
2	8
3	12
4	___
5	___
10	___
100	___
___	80

On the graph paper, locate "GO" and graph the ordered pairs (1,4) (2,8) (3,12), etc., from Nancy's table. Do all the ordered pairs fit on your graph paper? What can you say about the points you graphed? Can you use the graph to predict three more ordered pairs for Nancy's table?

Can you GUESS MY RULE? [ ]

[This can be done as a class discussion activity or a small group activity if the children have the necessary reading skills. If in written form, place on two sheets of paper.]

Another beginning activity for children to try is Activity 8, "Squaring Off," *Activities in Mathematics*, "Graphs," pp.25-26.

[It would be best to continue with the □ and △ for a long time before introducing letters such as "l" and "p." For Activity 8, use □ for "l" and △ for "p." Use a full sheet of graph paper to graph the ordered pairs from this activity so that it is not necessary to distort the graph. It is too early at this point to use a grid where the units for □ are not equal to the units for △ .]

For something a little bit different, try Activity 21, "The Dog Pen," *Activities in Mathematics*, "Graphs," pp.83-86.

ADDITIONAL INSTRUCTIONS

(1) Write □ above the word "width" and △ above the word "length" in the table on page 83.

(2) Instead of using the grid provided on page 83, use the graph on page .

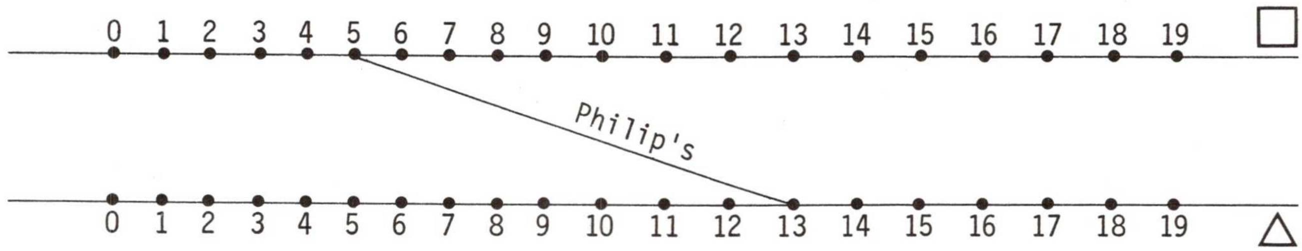
(3) See if you can (a) graph 20 different points on this grid, (b) write 20 different ordered pairs in the Truth Table, and (c) Guess My Rule.

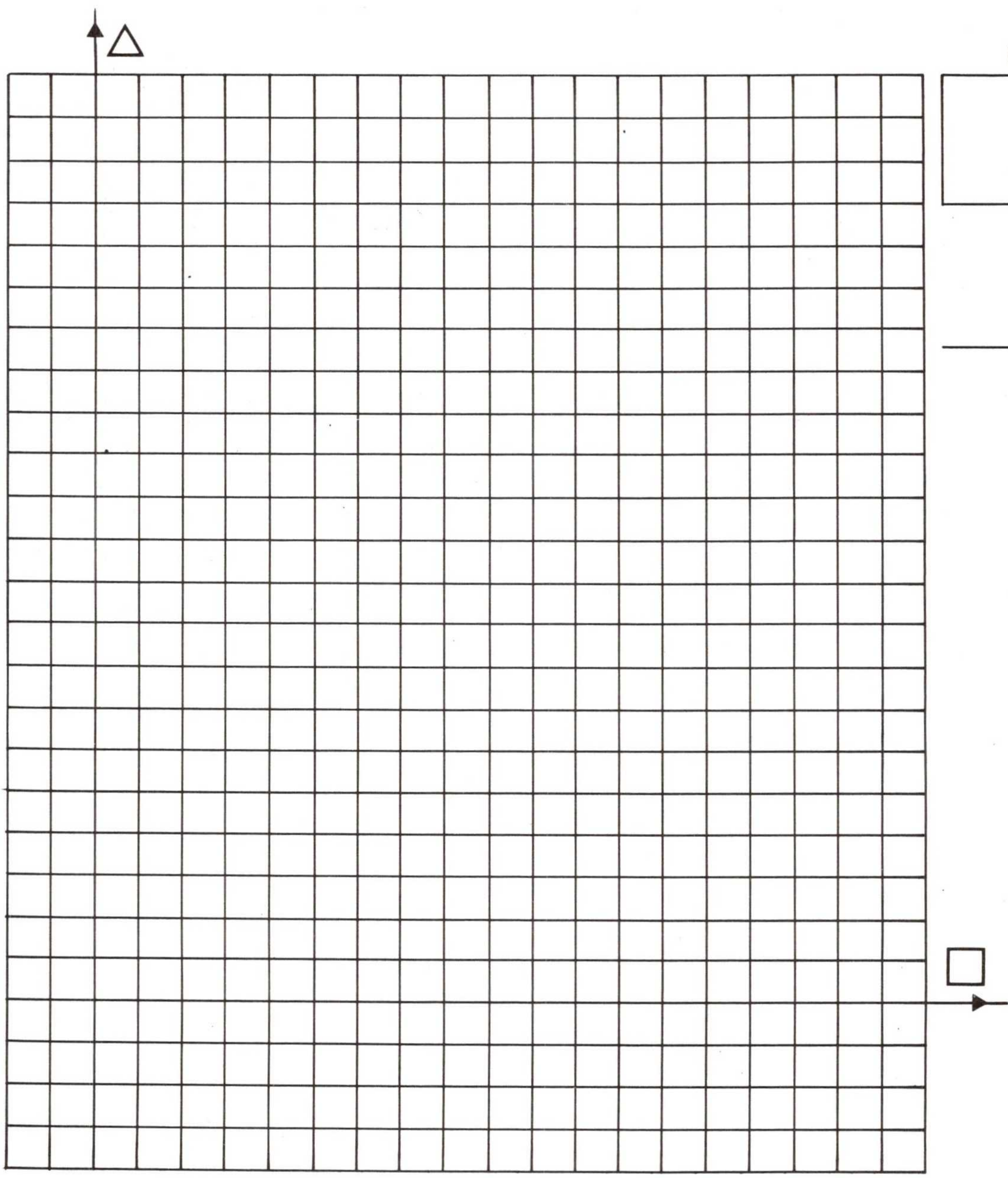
Did you include the ordered pair (0,18) or (18,0)? What would the figure described by these ordered pairs look like? Did you use a number that is *not* a whole number?

You probably noticed two parallel number lines above the grid on page 15. The top line is labeled □ while the bottom line is labeled △ . Using these number lines, it is possible to make a different type of graph.

GRAPHING ON PARALLEL NUMBER LINES

Pick any ordered pair from your Truth Table in Activity 21. Use a ruler to connect the locations suggested by your ordered pair. Example: Philip chose the ordered pair (5,13). Therefore, Philip connected the point labeled 5 on the □ number line with the point labeled 13 on the △ number line. Now, make all of the connections shown in the Truth Table. What does this graph look like?





Guess my rule

□	△

The following two situations result in graphs of a different nature, when compared to the three earlier activities.

SAMPLE ACTIVITY TWO - Inside the Rectangle

On a piece of graph paper, draw several different rectangles so that each rectangle you draw has exactly 24 squares inside. Record the length (□) and the width (△) of each of your rectangles in a Truth Table. Graph the ordered pairs from your Truth Table on both graphs (parallel and perpendicular axes).

Can you "Guess My Rule?" If not, draw more rectangles with 24 squares inside to get more information for the Truth Table. After you find the *open* sentence, see how many more ordered pairs you can get for the Truth Table, and graph the new ones you find.

[*Although the 4x6 rectangle is congruent to the 6x4 rectangle, encourage the use of both figures as "different" rectangles since each gives a unique location on the graph. For those children not aware of the Commutative Principle, this activity strongly hints at the fact that 4x6 = 6x4, 3x8 = 8x3, 2x12 = 12x2, 1x24 = 24x1, 1½x16 = 16x1½, etc. If the child uses only whole numbers, the graph will have large spaces between points. The teacher could ask how these spaces might be filled in. This suggestion is intended to promote the use of numbers other than whole numbers.*]

SAMPLE ACTIVITY THREE - Adding Odd Numbers

Suppose a nice lady down the street gave you and your best friend a bag of gumdrops to share. After you and your friend divide them up equally, it turns out that there is one gumdrop left over. Could the bag have held 16 gumdrops? How about 25 gumdrops?

Janice started a list to show how many gumdrops there could have been in the bag. Can you continue Janice's list? 1,3,5, ___, ___, ___, ___, ___, ___.

Let's try something with the numbers in Janice's list. How much do you get when you add the first *two* numbers in her list? What is the sum of the first *three* numbers in Janice's list? Now try adding the first *four* numbers in her list. What did you get? Use this table to keep a record of the totals (sums) you get each time.

Number of addends □	*Sum* △
1	___
2	4
3	9
4	___
5	___
6	___
7	___
10	___

Graph these ordered pairs on both kinds of graphs and Guess My Rule!

Can you predict how much you would get by adding the first 25 numbers in Janice's list?

[*This activity might provide a basis for discussion of odd versus even numbers. If this activity is presented in written form, place on two half sheets. See "How Squares Grow"," Freedom to Learn, p.129.*]

(For additional materials and activities, see *School Mathematics Project* (SMP), Book C. Chapter 3, "From Relation to Graph," pp.39-53; Madison Project Shoe Box Kits; *Activities in Mathematics*, "Graphs," Activity 7, pp.19-23, Activity 19, pp.75-76, Activity 22, pp.87-92; *Graphs Leading to Algebra*, p.18; *Mathematics Through Science* (SMSG), Part I, II, and III; *Freedom to Learn*, Chapter 7 (Section 7-4), pp.127-130.)

APPLICATIONS

The graphing situations mentioned here (and the many more like them) provide opportunities for children to discover the need for using non-whole numbers, to practice and increase their skills with all types of numbers and operations, to allow for some individual creativity, to acquaint themselves with a variety of math subjects (area, perimeter, patterns), including their interrelationships, and to experiment with making "diagrams" of their results through graphs.

There are several other important areas in mathematics where graphing is a valuable learning tool. Graphing techniques are helpful in solving "practical" and "word" problems. Here is one example.

A sixth grade class was working on a phosphate pollution project. One leading brand of detergent recommended 1/4 cup of detergent per wash load, which released 20 grams of phosphates. One of the questions in the project asked, "If a box of this detergent releases 1000 grams of phosphates, how many cups of detergent are in the box?" A frequent inquiry by the children is, "What are you supposed to do?" or "How do you do it?"

One way to answer this is to have the children make a table showing the amount of detergent (in cups- □) and the units of phosphates (in grams- △). The first item in their table, therefore, becomes 1/4 for □ , and 20 for △.

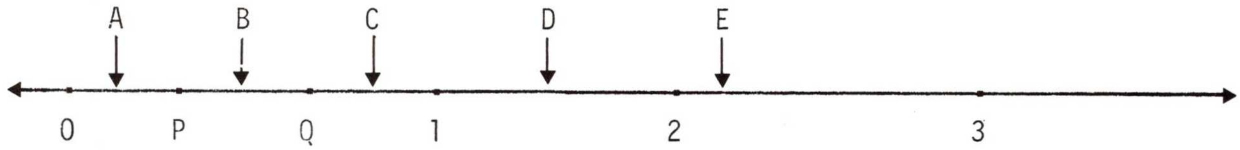
After several more pairs of numbers are placed in the table, the question becomes, "How far do we have to go before we can stop?"

As the children work on the table many will "discover" the *open* sentence $80 \times \square = \triangle$, which leads them to the *open* sentence $80 \times \square = 1000$ and, for some, the division problem, $1000 \div 80$.

[*All children cannot be expected to discover these relationships easily. Those having difficulty might be encouraged to make* accurate *graphs (parallel and perpendicular axes) of the ordered pairs in the table, and use the graphs to help them predict the answer.*]

(Graphing activities, providing unique experiences in comparing fractions (relationships between equivalent and non-equivalent fractions through graphing), can be found in *School Mathematics Project* (SMP), Book B, Chapter 5, "Comparison of Fractions," pp.51-62.)

P and Q are numbers represented on the number line located as shown below. Which arrow (A, B, C, D, or E) points to a location that could represent the product of P and Q (PxQ)?



Frequently C or D is chosen as the answer. Children often conclude from their experiences with whole numbers that the product of two numbers (say P and Q) is *always* going to be "bigger" than the two numbers multiplied. So, when asked to give the product of 1/2 times 1/2, a common reply is "one."

Graphing can help provide "models" to illustrate what actually happens with multiplication. To begin this project, the ordered pair (2,3) was graphed on the grid in Figure A. Using this point and "GO" as opposite corners of a rectangle, we have a "model" in Figure A for the multiplication problem 2x3.

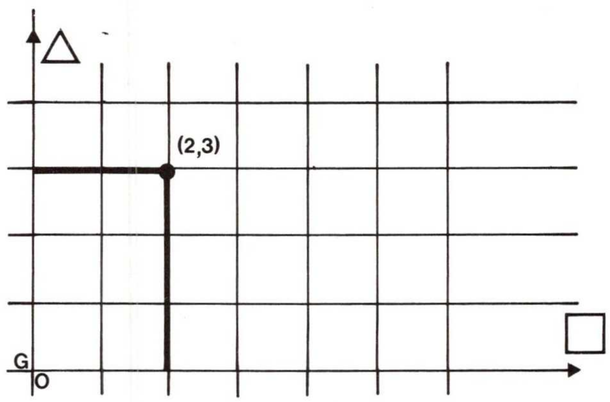


Figure A

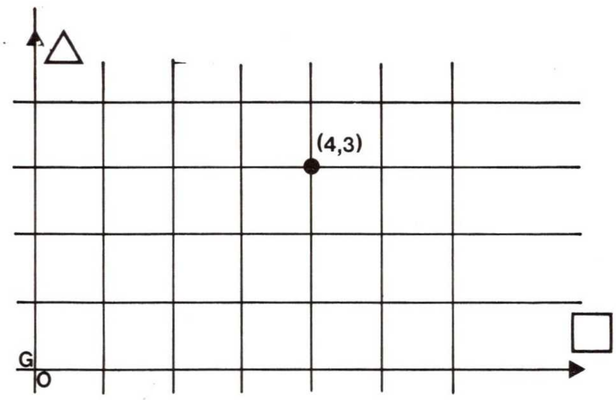


Figure B

(a) Make the rectangle that serves as a "model" to illustrate the multiplication problem 4x3 on the grid in Figure B.

(b) How can you find the answer to 4x3 from your rectangle?

[Children require many experiences with finding the area of rectangles before being introduced to the ideas in the following activity. Geoboard activities, measuring areas of rectangular objects in the classroom, and graphing situations like those mentioned on the previous pages, for example, Sample Activity Two, are appropriate experiences to encourage children to discover the "multiplication - rectangular area" relationship.]

Use the grid in Figure C to make a "Model" to illustrate 1/2x1/2, that is, use the ordered pairs (1/2,1/2) and (0,0) as opposite corners of a rectangle.

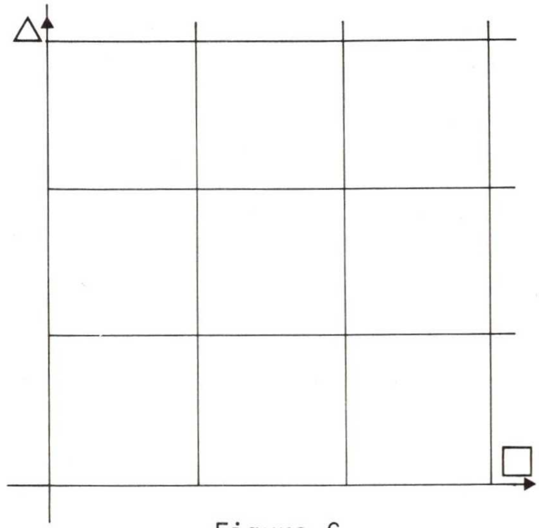


Figure C

[Some children may argue this figure is not a rectangle. This provides the teacher with an opportunity to talk about squares as special kinds of rectangles.]

On graph paper ruled off in 1-inch squares, try making a "model" to illustrate 1-1/2x2-1/2.

TRY THIS Cut out 1-inch squares from graph paper ruled off in 1/4-inch squares then glue them onto 1-inch ruled graph paper. Now make a "model" to illustrate 1/4x1/4.

What do you notice?

Make a "model" to illustrate 1-1/4x2-3/4. What is the product?

Try to illustrate on this same grid the multiplication problem 2-1/4x1-1/2. Do you notice anything different this time?

[Children will discover shortcuts (algorithms) to eliminate the need to illustrate each multiplication problem when given enough opportunity to invent them.]

Further development of these situations lead to equivalent fractions, fractions with their equivalent decimals, multiplication with decimals, division with fractions or decimals.

LIST OF MATERIALS AND SOURCES

(1) Nuffield Foundation Mathematics Project Publications

Graphs Leading to Algebra (1969)

John Wiley and Sons, Inc.
605 Third Avenue
New York, New York 10016

In Canada: Longman Canada Ltd.
55 Barber Greene Road
Don Mills, Ontario

(2) Members of the Association of Teachers of Mathematics

Notes on Mathematics in Primary Schools (1968)

Cambridge University Press
American Branch
32 East 57th Street
New York, New York 10022

/ Cuisenaire Company of America, Inc.
12 Church Street
New Rochelle, New York 10805

(3) *Freedom To Learn* (1968), Edith Biggs and James MacLean

Addison-Wesley Publishing Co.
106 West Station Road
Barrington, Illinois 60010

In Canada: Addison-Wesley (Canada) Ltd.
67 Gervais Drive
Don Mills, Ontario

(4) The Madison Project

Discovery In Mathematics, A Text for Teachers (1964), Robert B. Davis;
Explorations In Mathematics, A Text for Teachers (1967), Robert B. Davis,

(5) *School Mathematics Project* (SMP), Books A, B, C, and D (1969-70)

Cambridge University Press
and Cuisenaire Company of
America, Inc. (as above)

(6) *Activities in Mathematics,* Second Course, "Graphs" (1971),
Johnson, Hansen, Peterson, Rudnick, Cleveland, Bolster

Scott, Foresman and Company
1900 East Lake Avenue
Glenview, Illinois 60025

In Canada: Gage Educational
Publishing Ltd.
P.O. Box 5000
164 Commander Blvd.
Agincourt, Ontario

(7) Introducing Mathematics: 3

The Search for Pattern (1970), W.W. Sawyer

Penguin Books Incorporated
7110 Ambassador Road
Baltimore, Maryland 21207

(8) One-Inch (ruled graph paper) - R305

Oravisual Company Inc.
321 15th South
St. Petersburg, Florida

(9) School Mathematics Study Group (SMSG)

Mathematics Through Science, Part I "Measurement and Graphing," Part II "Graphing, Equations and Linear Functions," Part III "An Experimental Approach to Functions." (Although too difficult for Grades IV-VI, these are intended to suggest possible graphing situations.)

A.C. Vroman, Inc.
367 South Pasadena Avenue
Pasadena, California 91105

(10) *Shoe Box Kits* (Independent Exploration Material - Madison Project): (a) Geoboard, (b) Tower Puzzle, (c) Centimeter Blocks, (d) Discs, (e) Peg Game, (f) Weights and Springs.

Math Media Division
H and M Associates
Post Office Box 1107
Danbury, Connecticut 06810

REFERENCES

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