

## B.2 Ratio and Rate Concepts

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### Experience With Ratio: Grades V, VI (*Woodeye, Wong*)

**OBJECTIVE** To give students some experiences related to the concept of ratio. An overhead projector is used to project an image of an object. The length of the object and the length of its image on the wall form a pair of numbers which illustrate the concept of ratio. Students are asked to tabulate pairs of measurements (for object and image) in a two-column table.

#### *Experience 1 - Find the Image*

This experience serves to introduce the idea of a ratio by the use of an overhead projector. Students are asked to match different colored sticks, or straws, with their images for a given ratio. Ratios such as 1:2, 1:3, and 1:4, can be used.

*Materials* Envelopes, four colors of sticks (or straws), overhead projector, worksheets.

#### *Experience 2 - Sketch the Image*

Students are asked to construct images of specific objects rather than to identify the image. A ratio of 1:2-1/2 can be used. The stations should each have a box of several small objects.

*Materials* Unlined paper, small boxes, small objects.

#### *Experience 3 - Measuring the Image*

Students use a centimeter ruler to measure line segments and their images. In some cases, either the length of the object or the length of the image cannot be determined. Students record information in ratio tables and determine the ratio.

*Materials* Overhead projector, transparency, metersticks, centimeter rulers, worksheets.

#### *Experience 4 - Recognize Like Shapes*

Students work with regions rather than with one-dimensional segments. Six stations are established. Students are required to match objects with their images.

*Materials* Graph paper, centimeter rulers, worksheets.

### Experience 5 - Enlarge a Printed Circuit

Students construct drawings, to scale, of printed circuits.

*Materials* Graph paper, worksheets.

### Experience 6 - Shrink a Head

Students construct an image that is smaller than the object. A scale of one centimeter to 1/4 inch is used.

*Materials* Graph paper, centimeter rulers, worksheets.

(Source: "Experience in Mathematical Ideas," *NCTM* 1970, Vol.2, pp.69-95.)

## Concept of Ratio Proportion: Grades V, VI (*Woodeye, Wong*)

**OBJECTIVE** To introduce ratio and proportion to students through a discovery situation. (This approach can also be used for junior and senior high general mathematics.) The discovery method will, hopefully, help students find patterns that will enable them to verbalize algorithms for the solution of proportion.

**PROCEDURE** As a starting point, have two students each take an arbitrary number of objects from a container. A comparison is then made of the number of objects drawn by each of the students. A table is used to keep a record of the number of objects selected.

Example	<i>Girl</i>	3	6	9	...	15
	<i>Boy</i>	4	8	12	...	20

Each time the children come to the container, they are instructed to withdraw the same number of objects that they had drawn on their first turn. The objects drawn are displayed where they can be seen by the entire group. The girl in the example cited above drew three objects while the boy drew four. Thereafter, and with each successive drawing, the total number of objects is recorded in the table. (If the same number of objects are drawn by both, these numbers should be recorded, but another withdrawal is made to get different numbers. After three or four selections, the students should be able to supply new values without actually drawing objects from the container. If the children have trouble supplying the values, they should return to the container for additional drawings.)

After completion of the first table, a second table is constructed without having the children draw objects from the container. At this point the word "rate" is introduced. The students are asked to think of the table as a record of the rate at which these objects were selected from the container.

Example	<i>Girl</i>	3	6	12	15	...	( <i>G,B</i> )
	<i>Boy</i>	5	10	20	25	...	(3,5); (6,10); (12,20); (15,25)



In this example, the girl drew three to the boy's five (expressed as a ratio of 3 to 5). The children are then asked to fill in the table. Once again it should be stressed that the table can be verified by having the children actually draw the number of objects specified.

At this stage, the notation of ordered pairs is introduced. The tables are constructed vertically to make this transition easy. The values listed in the second table are written using ordered pairs which were referred to as "rate pairs." Once the concept of rate pairs is established, the next step is to show equivalence between rate pairs. At this point, a student is asked to explain to his classmates how the objects were drawn from the container and how the tables were constructed. The children are asked to volunteer other rate pairs that show the same rate of drawing.

Example  $(3,5) \rightarrow (9,15)$   
 $(3,5) \rightarrow (21,35)$   
 $(3,5) \rightarrow (6,10)$

The students should see that equivalence exists between other rate pairs from the same table. They are asked to find a rate pair that is equivalent to  $(21,35)$ , but they are instructed not to use  $(3,5)$ . A second rate pair is introduced and the students supply rate pairs. The term "equivalent rate pairs" is used to express the same rate. The pair  $(3,5)$  express the same rate as  $(6,10)$  even though different numbers are used.

As a further exercise on finding equivalent rate pairs, examples can be devised which call for replacements supplied by the students.

Example  $(2, \square) = (6,9)$   
 $(12,16) = (\square,4)$

The symbol of replacement should be used in all positions.

A game based on equivalent pairs can help the children see the relationship between the products of the means and the extremes.

The rules of the game are as follows. The girls pick a pair of numbers that would yield equivalent rate pairs. They give one value and let the boys find the other. In a similar fashion the boys could pick a pair of replacement values.

Hopefully, the children will soon realize that the product of the replacements listed in the table is the same as the product of the numbers in the examples. This might set the stage for a verbalization of the rule. It is possible to see that replacement could also include fractions.

*[This brief development of proportion, as outlined above, would, with extension, lend itself to problems involving percent, scale drawing, conversion, similar figures, area, and volume.]*