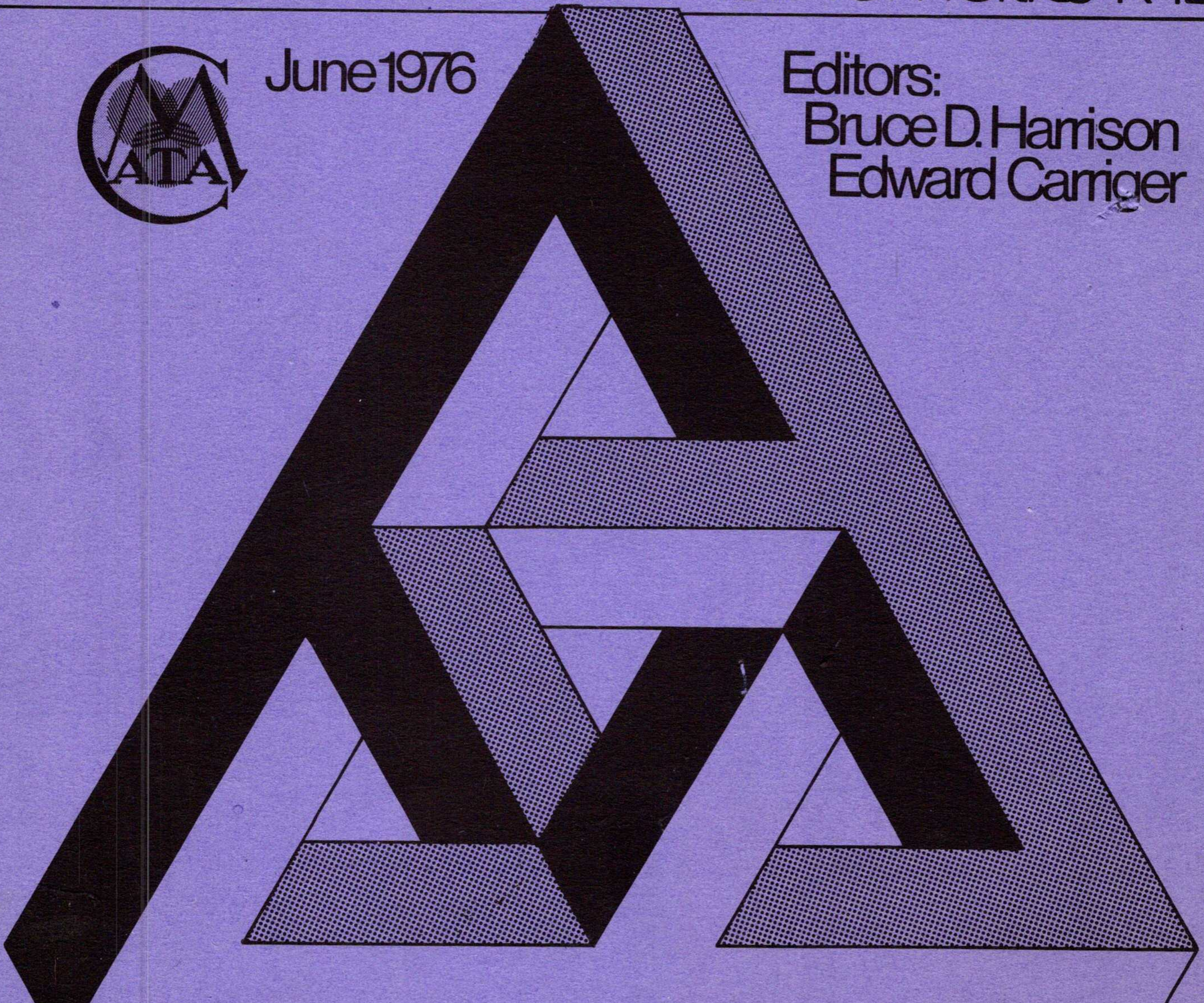
MATH
MONOGRAPH NO.4

timeless activities for mathematics k-12

June 1976

Editors:
Bruce D. Harrison
Edward Carriger



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Executive, 1975-76

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PART ONE

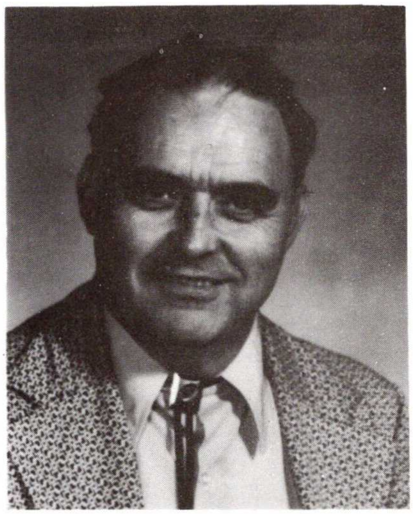
FEATURE ARTICLES

Introductory Note

The present monograph was undertaken by Bruce Harrison in the fall of 1971 and proved to be too ambitious an undertaking to be completed that academic year under the time constraints imposed on him at that time. Consequently, when the manuscript was still not complete in the spring of 1975, on the threshold of a year-long sabbatical leave, Edward Carriger graciously consented to complete the project in the absence of the initiator.

Editor, *Dr. Bruce D. Harrison,*

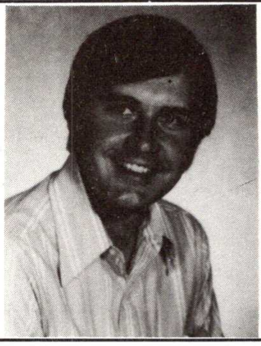
The intent of the monograph is to make available, in one place, a collection of mathematics activities put together by classroom teachers and found in Alberta mathematics curriculum guides. The monograph consists of the excellent lead articles, followed by sections on elementary and secondary school mathematics activities drawn from teacher-developed activities collected over the past four years.



Co-Editor, *Edward Carriger,*

Grateful thanks are extended to Mr. Carriger for bringing this project to its logical conclusion. We hope Alberta school teachers will find the material contained herein useful in the demanding day-to-day challenges they face in trying to provide interesting, stimulating, productive mathematics learning experiences for their pupils.

GRAPHING



by *Ed Irmis* North School Glencoe, Illinois

The following paper was prepared and presented by the author as supplementary material for his elementary school "graphing" workshops at the forty-ninth annual meeting of the National Council of Teachers of Mathematics, in Anaheim, California, April 15-17, 1971.

The ideas presented in this paper are primarily at an introductory level and detailed development has not been provided. Further development of these initial activities can evolve from the discoveries, questions and interests of the children this material is intended to serve. Many important graphing topics (inequalities, three dimensional graphing, circular graphs, etc.) have not been included. Therefore, what has been included merely scratches the surface of the subject of graphing.

BAR GRAPHS

The following activities provide an opportunity for children to tell about themselves which they so often seem to enjoy.

CLASS ACTIVITY 1 - LIVING GRAPHS

The children in the class form a type of bar graph (in this case the proper term probably is "histogram") as an introductory experience in graphing.

PREPARATION Place the names of the twelve months across the wall or bulletin board.

DESCRIPTION Each child stands in line according to the month of his birthday. To keep a uniform starting place for each of the lines (bars), the first child in each line stands directly in front of the name indicating the month of his birthday.

SAMPLE QUESTIONS

(A) Is there anyone in your month with the same birth date?
(B) Which month has the most birthdays in our class?
(C) If you switch the order of people in your line, would some other month have the most birthdays?

(D) Which month has the fewest birthdays?
(E) What is the difference between the month with the most and the month with the fewest birthdays?
(F) Did any months end in a tie?
(G) Did anyone notice something interesting about our "living graph? that he would like to tell the class?

CLASS ACTIVITY 2 - GISMO GRAPH

Sometimes being an actual part of a graph makes it difficult for people to see clearly the results of the information collected. So, let's try a different type of bar graph (histogram).

PREPARATION Number several cards and place them in numerical order - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, across a table or across the floor.

DESCRIPTION This time we are collecting some information about the number of pets each child has at home. Each child places his math book in the pile that tells the number of pets in his home. In other words, if you have two pets at home, place your book in the stack of books labeled by card number 2.

SAMPLE QUESTIONS

(A) Did anyone else put their math book in the same pile as yours?
(B) What is the meaning of the tallest stack?
(C) What ideas can you tell the class from our "gismo" book graph?

CLASS ACTIVITY 3 - USING A GRID

It can be difficult to carry around a "living" graph or even a stack of books, especially if you want to show your graph at home. Here is another way to make a bar graph.

PREPARATION Draw a heavy dark line across the bottom of a grid marked off in one-inch squares. Below this line, number the vertical columns - 0, 1, 2, 3, 4, 5, 6, 7, etc., from left to right. Have crayons available.

DESCRIPTION The children are asked to count the number of blocks it is from their house to school. Each child colors one of the squares in the column telling the number of blocks he lives from school. Example - if you live 5 blocks from school, you color a square in the column marked with the "5." (You might have each student write his name in the square he colored to demonstrate that his square represents the number of blocks he lives from school.)

SAMPLE QUESTIONS

(A) How many other people in this class live the same distance from school as you?
(B) What can you tell from our graph?

Other topics for activities 1, 2 and 3

- height of each student in the class,
- weight of each student in the class,

- number of brothers and sisters each student has,
- temperatures during any one month,
- number of vowels in each student's full name,
- number of glasses of milk each student drinks per day,
- number of TV programs watched by each student in one week,
- number of hours each child sleeps,
- foot lengths of the children in the class.

(Also see Ida Mae Heard, "Making and Using Graphs in the Kindergarten Mathematics Program," *The Arithmetic Teacher*, October, 1968, pp.504-506.)

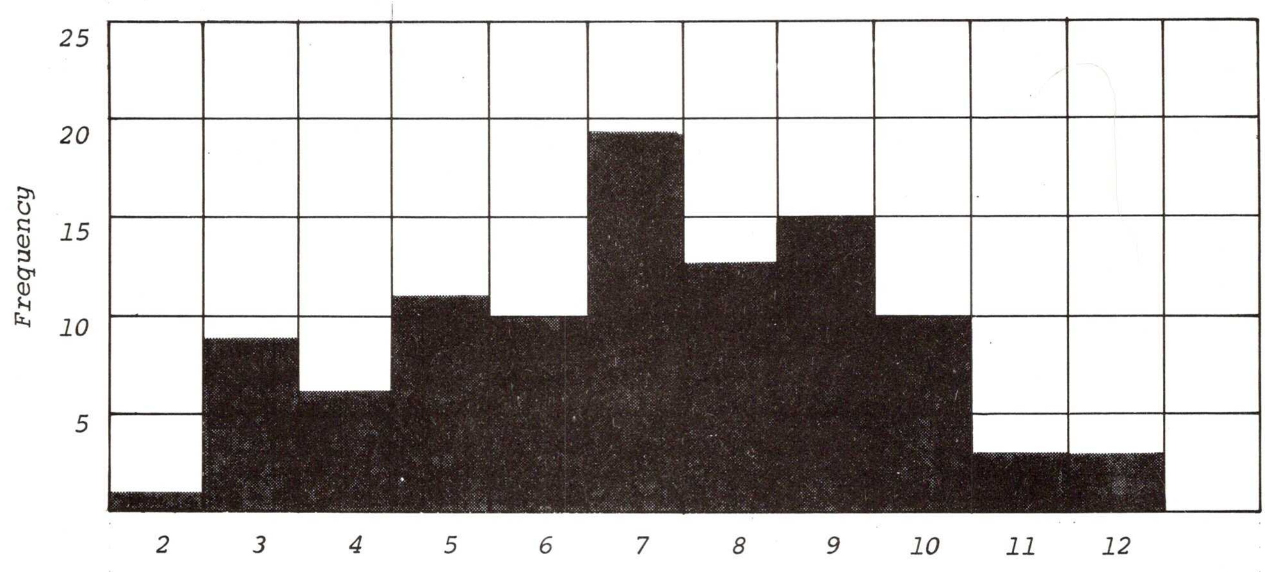
Here are some opportunities for children to work together in small groups, collect data, and exchange results with other groups by comparing graphs.

SMALL GROUP ACTIVITY A - Rolling Two Dice

MATERIALS Paper marked off in 1 or ½-inch squares, and a pair of dice for each group.

OBJECTIVE To collect data and help predict the frequency of occurrence of numbers 2 through 12 using a pair of dice.

Sample graph (100 throws)

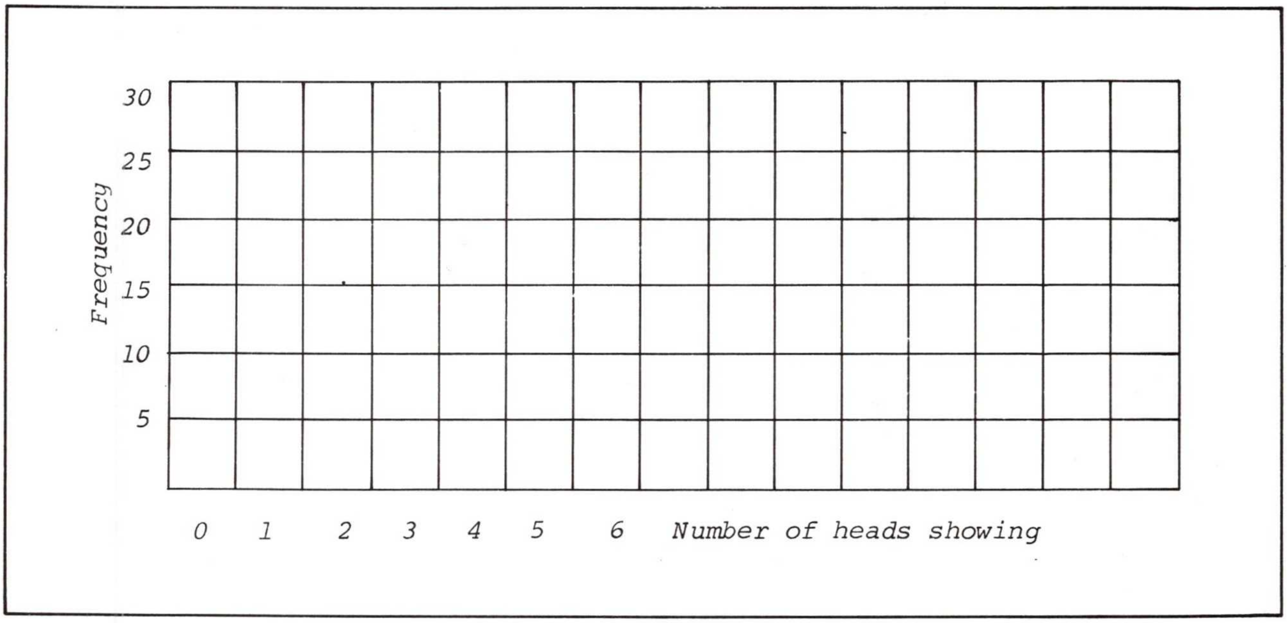


SMALL GROUP ACTIVITY B - Coin Toss

MATERIALS Paper marked off in ½ or 1-inch squares, and six coins for each group.

OBJECTIVE To collect data (by throwing six coins each toss) to determine the frequency of occurrence of the number of heads showing per toss.

Sample graph (try this one yourself on 100 tosses)



(For additional materials or activities see also *School Mathematics Project* (SMP) Book B, Chapter 9, "Statistics"; *Freedom to Learn*, Chapter 6, Sections 6-9, pp.87-93.)

GRIDS AND COORDINATES

As I was walking down the hall at school last week, John came up to me and said that he had found my five dollars. "What five dollars?" I inquired. He said, "Remember the five dollars you lost in town last October when you were teaching us about graphing?" "Oh, yes," I replied. "Where did you find it?" "Well," he said, "it must have blown around for a long time because I didn't find it where you said you lost it." We both agreed at that point that it probably was not my five dollars, so he kept his newfound treasure. But, what about the story of the lost five dollar bill that John remembered so long?

Story of the Lost Five Dollar Bill

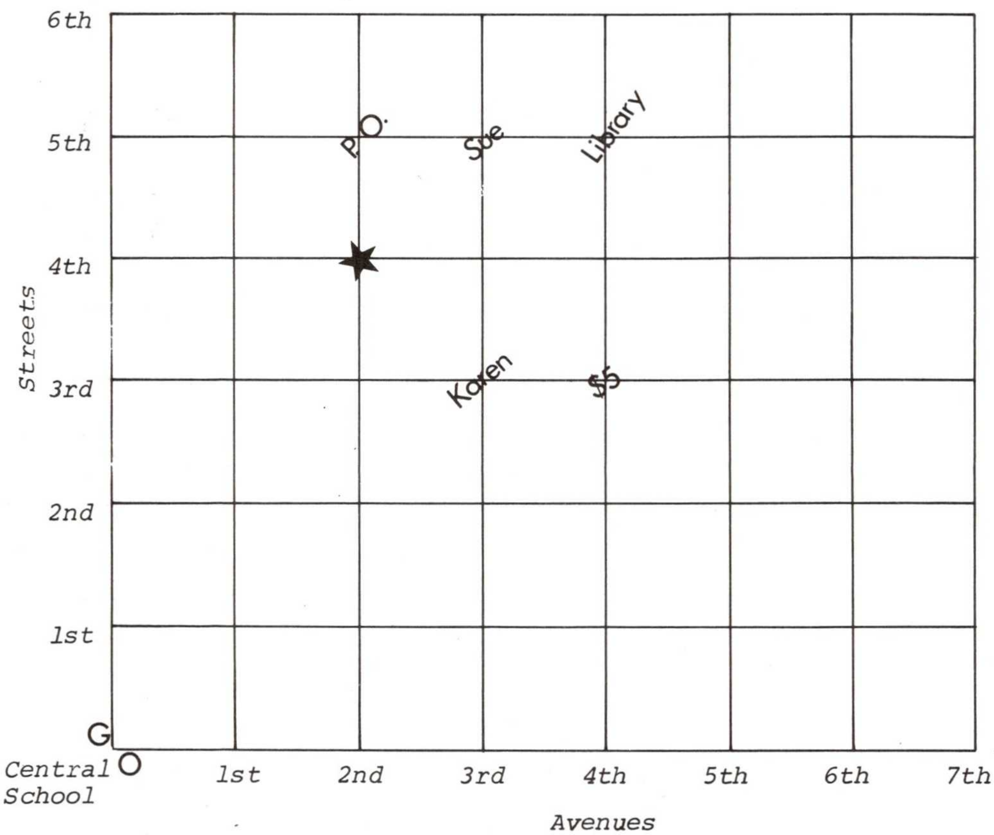
During noon hour one day last October, I left school to borrow a book at the library in town. While waiting for the light to change on the corner of Fourth Avenue and Third Street, I took out my wallet to look for my library card. It must have been on that corner when the five dollar bill fell out of my wallet.

I had planned to look for the lost bill after school but several students came in to ask some questions about their homework assignment just as I was leaving.

A good friend of mine (a mathematician, of course) was going into town and he said he would be glad to look for the money. "Where did you lose it?" asked Mr. Matt A. Maddox (he likes to be called by his full name). Not wanting to let everyone listening know where the money might be found, I used a secret code that my friend also knew.

I said, "If Central School is 'GO,' then look at four, three." My ex-friend must have found the money because I haven't heard from him since.

Map of Your Town



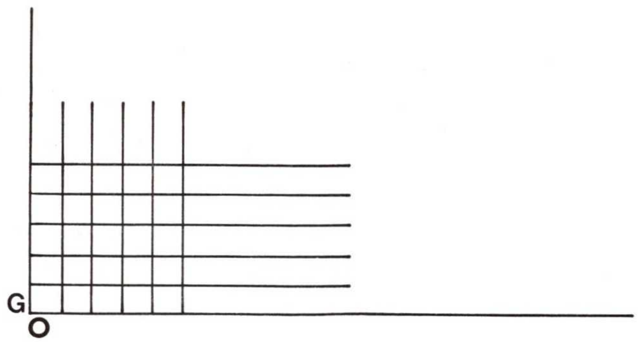
Do you know how the secret code works? The library is at (4,5). Use our secret code to tell where Karen is standing, (___,___). Put your name on the map at (2,3). Where is the star of our baseball team standing? (___,___). Dirk is standing in a straight line with Karen and our baseball star *and* he is also in a straight line with the post office, Sue and the library. Where is Dirk? (___,___). (Put Dirk's name in the correct location on the map.) Put an X on the map in a straight line with you and the library *and* also in a straight line with my $5 and the post office. What are the code numbers for the X? (__,__).

(See also *School Mathematics Project* (SMP), Book A, Part 2, "Maps," pp.15-17; *Graphs Leading to Algebra*, Chapter 1, "Introduction to Co-ordinates," pp.2-11.)

THE GAME OF FOUR-IN-A-ROW

The game of Four-In-A-Row is similar to Tic-Tac-Toe. Its primary purpose is to provide experiences in graphing ordered pairs (our "secret code") onto a grid. It also helps to develop game strategy skills. [Make a copy of the game board with large squares on the blackboard.]

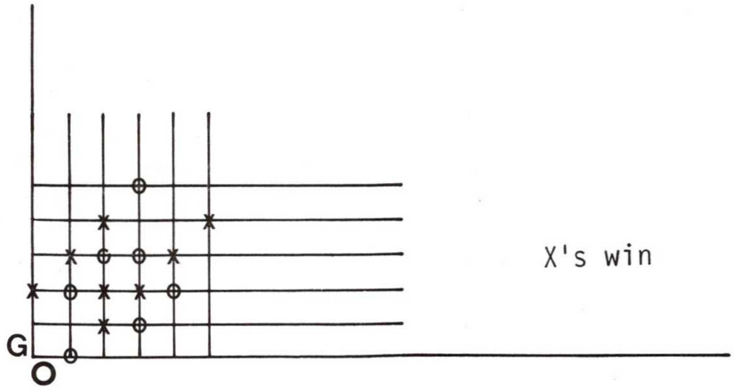
Game Board for Four-In-A-Row



HOW TO WIN Place four of your team's marks in a straight line without any of the opposing team's marks in between them.

HOW TO PLAY Divide the class into two teams: X's and O's. A member of the X's gie s the teacher an ordered pair (according to our "secret code"). The teacher places an X in that location on the game board. Then a member of the O's gives an ordered pair. The teacher places an O in that location on the game board. Teams alternate giving ordered pairs until one team wins with "four-in-a-row."

PENALTIES If a player gives a location already occupied, or a location off the game board, his team loses that turn.



Can you name the four winning locations? (___,___) (___,___)

(___,___) (___,___)

(Other games to promote skill and enthusiasm for graphing on grids are "Grid Football" (Activity 5), *Activities in Mathematics*, Graphs, Scott Foresman and Company, 1971, pp 13-14; "Hide-a-Region N< 2 Can Play," *The Arithmetic Teacher*, October 1969, pp.496-497; "Shifts," *School Mathematics Project* (SMP), Book B, p.116.)

PICTURES DRAWN ON GRIDS

Among the many activities that graphing provides, drawing pictures on grids is popular with most children. Students are generally enthusiastic about drawing pictures or designs made from a list of ordered pairs (our "secret code") to be graphed on grids. The children enjoy the "drill" and at the same time gain a proficiency for coordinate graphing.

(For coordinate picture and design drawing activities see *Activities In Mathematics*, Second Course, "Graphs," pp.15-17, Activity 6, "Making designs on a grid"; *School Mathematics Project* (SMP), Book A, Exercise B, pp.18-19.)

[*Encourage the children to make up some of their own designs or pictures for other members of the class to try. A class card file containing student-created problems, ideas, puzzles, and discoveries is a possible method of organization.*]

A spin-off from this activity provides children with some experiences with the concepts of congruency and similarity. For example, after several children have attempted the same drawing, the teacher might ask, "How does your picture compare with your neighbor's?"

[*Note - If all students use the same size grid, they should all have "matching" (congruent) figures. If the members in the class are using different size grids, some figures might be congruent while others will have exactly the same shape but will be different in size, that is, not congruent but similar.*]

(See *Freedom to Learn*, pp.153-157, for additional activities with congruency and similarity.)

Another possible activity for improving the individual's skill in graphing, is Activity 9, "Changing Size and Shape with Graph Paper," in *Activities in Mathematics*, pp.31-34, "Graphs." [If the *Activities in Mathematics* texts are not available, the teacher should draw a figure that can be fitted to a standard size grid. The children trace this prepared drawing with tracing paper, then fit their tracing to the graph paper. Using the appropriate ordered pairs, the design or picture is reproduced on larger or smaller grids. This type of activity can be quite valuable especially when the same picture is drawn on several different size grids.]

LET'S GET DOWN TO THE "FACTS"

"...So the kids have a great time drawing pictures and playing games, which is fine, but that won't help them learn their 'facts'!" This comment, made by a fifth grade teacher, is a common reaction at this point and understandably so. However, the potential dividends from introducing children to graphing in the elementary grades is almost unlimited. The real benefits of graphs often result from making graphing a vehicle for reaching the goals of your present mathematics program. Before discussing some of these "dividends," let us take time out here for a few important agreements.

5x9 = 45 is a *true* sentence.

$\frac{1}{5} \times \frac{2}{5} = \frac{3}{10}$ is a *false* sentence.

(5x□) - 3 = △ is an *open* sentence (because you can't be sure what will be substituted for the □ and the △).

Which of these sentences are *true / false / open ?*

(a) (5x6) + 3 = 5 x (6+3)

(b) □ x △ = 24

(c) 3 x (4+5) = (3x4) + (3x5)

This is a Truth Table

□	△
0	1
1	3
2	5
3	7
½	2

etc

. . . it is a Truth Table for this *open* sentence (2x □) + 1 = △

Can you guess the *open* sentence for this Truth Table?

□	△
1	19
2	18
3	17
4	16
5	15
10	10
1/4	19-3/4

etc

Each pair of numbers in the Truth Table is called an "ordered pair." Can you name several more ordered pairs that belong in the last Truth Table?

(For proper development of these ideas, see *Discovery in Mathematics,* A Text for Teachers, pp.23-24 and 75-94; *Graphs Leading to Algebra,* Chapter 2, pp.11-22; *Explorations in Mathematics,* A Text for Teachers, Chapters 25 and 26; *Activities in Mathematics,* Second Course, "Graphs," Activity 17, pp.67-70.

We will begin with a very simple activity to illustrate how information is collected, recorded in a Truth Table, graphed on a grid, and finally generalized by an *open* sentence.

GUESS MY RULE

SAMPLE ACTIVITY ONE - Rectangles and Corners

MATERIALS Unlined, rectangular (sharp cornered) paper
Graph paper
Sharp pencil

Fold the plain piece of paper down the middle and tear it along the fold so that you get two rectangles. How many corners are there altogether in the two rectangles? How many corners were in the paper before you tore it?

Now, take *one* of the rectangles. Fold it down the middle and tear along the fold to get two new rectangular pieces. How many pieces of paper do you have now? How many corners do you now have altogether?

Nancy started a table to keep a record of this information. Can you complete Nancy's table by continuing to fold and tear the paper in the same way?

Number of Rectangles □	*Total number of Corners* △
1	4
2	8
3	12
4	___
5	___
10	___
100	___
___	80

On the graph paper, locate "GO" and graph the ordered pairs (1,4) (2,8) (3,12), etc., from Nancy's table. Do all the ordered pairs fit on your graph paper? What can you say about the points you graphed? Can you use the graph to predict three more ordered pairs for Nancy's table?

Can you GUESS MY RULE? [ ]

[This can be done as a class discussion activity or a small group activity if the children have the necessary reading skills. If in written form, place on two sheets of paper.]

Another beginning activity for children to try is Activity 8, "Squaring Off," *Activities in Mathematics*, "Graphs," pp.25-26.

[It would be best to continue with the □ and △ for a long time before introducing letters such as "l" and "p." For Activity 8, use □ for "l" and △ for "p." Use a full sheet of graph paper to graph the ordered pairs from this activity so that it is not necessary to distort the graph. It is too early at this point to use a grid where the units for □ are not equal to the units for △ .]

For something a little bit different, try Activity 21, "The Dog Pen," *Activities in Mathematics*, "Graphs," pp.83-86.

ADDITIONAL INSTRUCTIONS

(1) Write □ above the word "width" and △ above the word "length" in the table on page 83.

(2) Instead of using the grid provided on page 83, use the graph on page .

(3) See if you can (a) graph 20 different points on this grid, (b) write 20 different ordered pairs in the Truth Table, and (c) Guess My Rule.

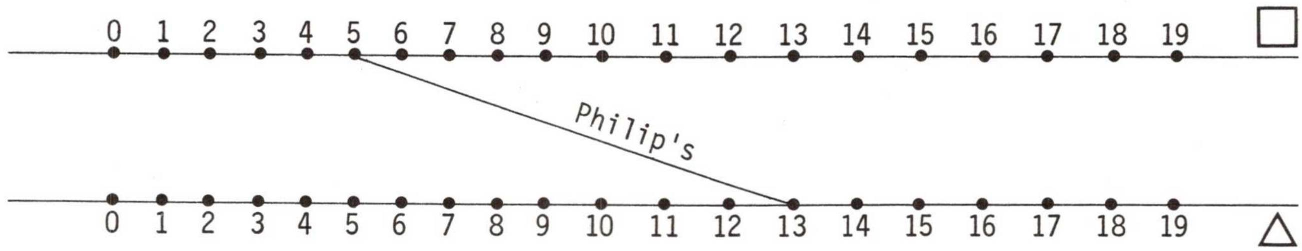
Did you include the ordered pair (0,18) or (18,0)? What would the figure described by these ordered pairs look like? Did you use a number that is *not* a whole number?

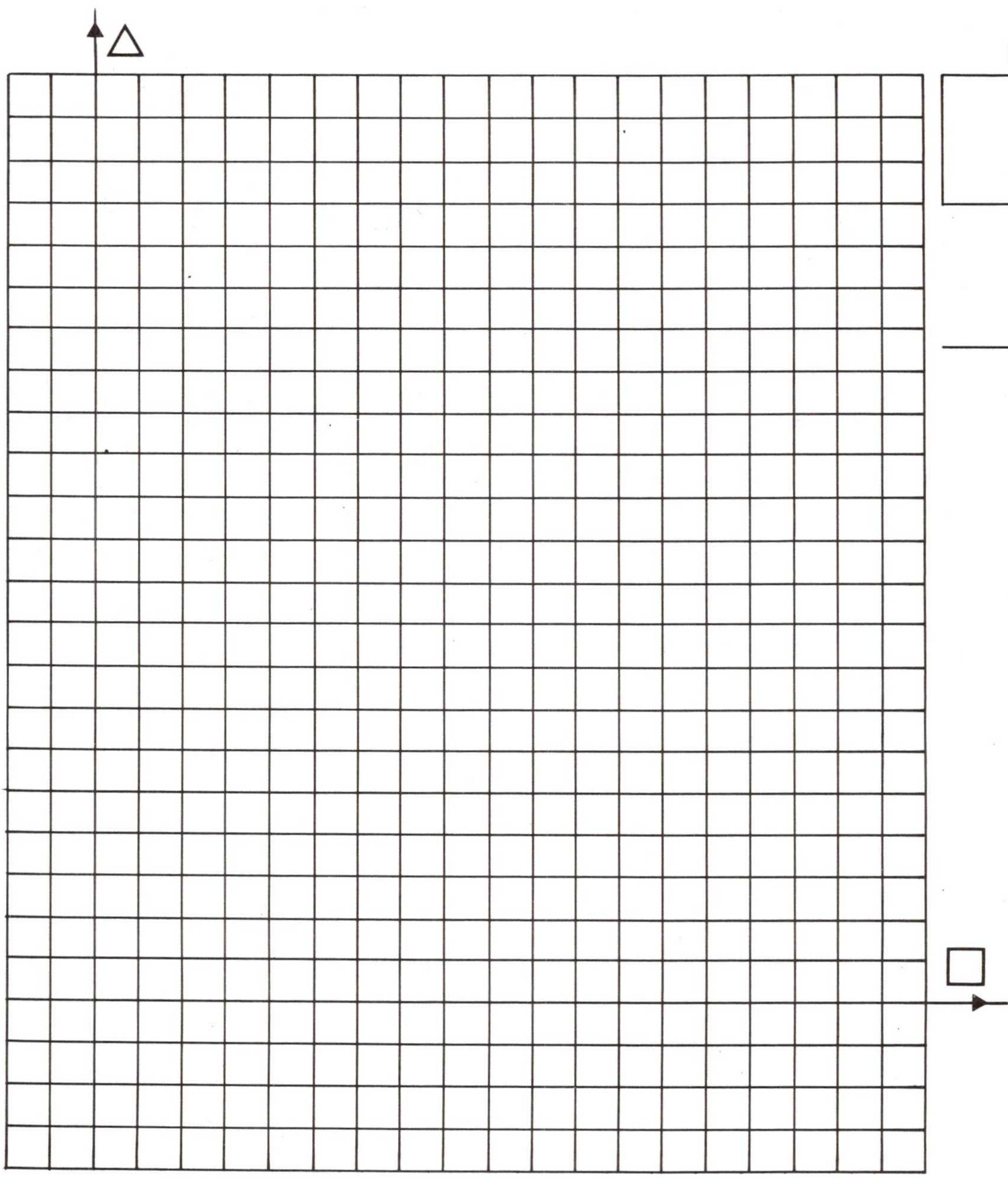
You probably noticed two parallel number lines above the grid on page 15. The top line is labeled □ while the bottom line is labeled △ . Using these number lines, it is possible to make a different type of graph.

GRAPHING ON PARALLEL NUMBER LINES

Pick any ordered pair from your Truth Table in Activity 21. Use a ruler to connect the locations suggested by your ordered pair. Example: Philip chose the ordered pair (5,13). Therefore, Philip connected the point labeled 5 on the □ number line with the point labeled 13 on the △ number line. Now, make all of the connections shown in the Truth Table. What does this graph look like?

⇨





Guess my rule

□	△

The following two situations result in graphs of a different nature, when compared to the three earlier activities.

SAMPLE ACTIVITY TWO - Inside the Rectangle

On a piece of graph paper, draw several different rectangles so that each rectangle you draw has exactly 24 squares inside. Record the length (□) and the width (△) of each of your rectangles in a Truth Table. Graph the ordered pairs from your Truth Table on both graphs (parallel and perpendicular axes).

Can you "Guess My Rule?" If not, draw more rectangles with 24 squares inside to get more information for the Truth Table. After you find the *open* sentence, see how many more ordered pairs you can get for the Truth Table, and graph the new ones you find.

[*Although the 4x6 rectangle is congruent to the 6x4 rectangle, encourage the use of both figures as "different" rectangles since each gives a unique location on the graph. For those children not aware of the Commutative Principle, this activity strongly hints at the fact that 4x6 = 6x4, 3x8 = 8x3, 2x12 = 12x2, 1x24 = 24x1, 1½x16 = 16x1½, etc. If the child uses only whole numbers, the graph will have large spaces between points. The teacher could ask how these spaces might be filled in. This suggestion is intended to promote the use of numbers other than whole numbers.*]

SAMPLE ACTIVITY THREE - Adding Odd Numbers

Suppose a nice lady down the street gave you and your best friend a bag of gumdrops to share. After you and your friend divide them up equally, it turns out that there is one gumdrop left over. Could the bag have held 16 gumdrops? How about 25 gumdrops?

Janice started a list to show how many gumdrops there could have been in the bag. Can you continue Janice's list? 1,3,5, ___, ___, ___, ___, ___, ___.

Let's try something with the numbers in Janice's list. How much do you get when you add the first *two* numbers in her list? What is the sum of the first *three* numbers in Janice's list? Now try adding the first *four* numbers in her list. What did you get? Use this table to keep a record of the totals (sums) you get each time.

Number of addends □	*Sum* △
1	___
2	4
3	9
4	___
5	___
6	___
7	___
10	___

Graph these ordered pairs on both kinds of graphs and Guess My Rule!

Can you predict how much you would get by adding the first 25 numbers in Janice's list?

[*This activity might provide a basis for discussion of odd versus even numbers. If this activity is presented in written form, place on two half sheets. See "How Squares Grow"," *Freedom to Learn, *p.129*.]

(For additional materials and activities, see *School Mathematics Project* (SMP), Book C. Chapter 3, "From Relation to Graph," pp.39-53; Madison Project Shoe Box Kits; *Activities in Mathematics*, "Graphs," Activity 7, pp.19-23, Activity 19, pp.75-76, Activity 22, pp.87-92; *Graphs Leading to Algebra*, p.18; *Mathematics Through Science* (SMSG), Part I, II, and III; *Freedom to Learn*, Chapter 7 (Section 7-4), pp.127-130.)

APPLICATIONS

The graphing situations mentioned here (and the many more like them) provide opportunities for children to discover the need for using non-whole numbers, to practice and increase their skills with all types of numbers and operations, to allow for some individual creativity, to acquaint themselves with a variety of math subjects (area, perimeter, patterns), including their interrelationships, and to experiment with making "diagrams" of their results through graphs.

There are several other important areas in mathematics where graphing is a valuable learning tool. Graphing techniques are helpful in solving "practical" and "word" problems. Here is one example.

A sixth grade class was working on a phosphate pollution project. One leading brand of detergent recommended 1/4 cup of detergent per wash load, which released 20 grams of phosphates. One of the questions in the project asked, "If a box of this detergent releases 1000 grams of phosphates, how many cups of detergent are in the box?" A frequent inquiry by the children is, "What are you supposed to do?" or "How do you do it?"

One way to answer this is to have the children make a table showing the amount of detergent (in cups- □) and the units of phosphates (in grams- △). The first item in their table, therefore, becomes 1/4 for □ , and 20 for △.

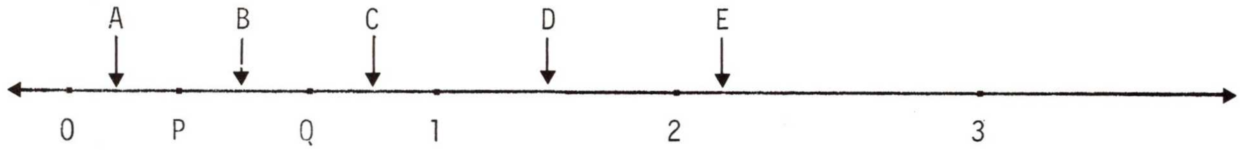
After several more pairs of numbers are placed in the table, the question becomes, "How far do we have to go before we can stop?"

As the children work on the table many will "discover" the *open* sentence 80x □ = △ , which leads them to the *open* sentence 80x □ = 1000 and, for some, the division problem, 1000÷80.

[*All children cannot be expected to discover these relationships easily. Those having difficulty might be encouraged to make* accurate *graphs (parallel and perpendicular axes) of the ordered pairs in the table, and use the graphs to help them predict the answer.*]

(Graphing activities, providing unique experiences in comparing fractions (relationships between equivalent and non-equivalent fractions through graphing), can be found in *School Mathematics Project* (SMP), Book B, Chapter 5, "Comparison of Fractions," pp.51-62.)

P and Q are numbers represented on the number line located as shown below. Which arrow (A, B, C, D, or E) points to a location that could represent the product of P and Q (PxQ)?



Frequently C or D is chosen as the answer. Children often conclude from their experiences with whole numbers that the product of two numbers (say P and Q) is *always* going to be "bigger" than the two numbers multiplied. So, when asked to give the product of 1/2 times 1/2, a common reply is "one."

Graphing can help provide "models" to illustrate what actually happens with multiplication. To begin this project, the ordered pair (2,3) was graphed on the grid in Figure A. Using this point and "GO" as opposite corners of a rectangle, we have a "model" in Figure A for the multiplication problem 2x3.

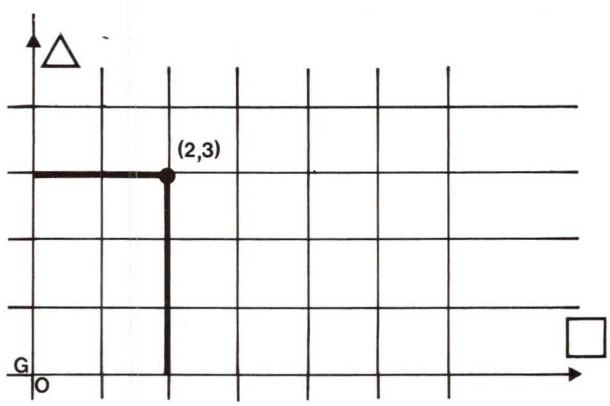


Figure A

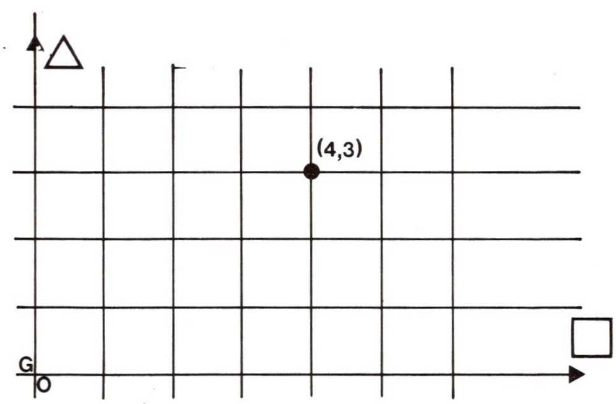


Figure B

(a) Make the rectangle that serves as a "model" to illustrate the multiplication problem 4x3 on the grid in Figure B.

(b) How can you find the answer to 4x3 from your rectangle?

[*Children require many experiences with finding the area of rectangles before being introduced to the ideas in the following activity. Geoboard activities, measuring areas of rectangular objects in the classroom, and graphing situations like those mentioned on the previous pages, for example, Sample Activity Two, are appropriate experiences to encourage children to discover the "multiplication - rectangular area" relationship.*]

Use the grid in Figure C to make a "Model" to illustrate 1/2x1/2, that is, use the ordered pairs (1/2,1/2) and (0,0) as opposite corners of a rectangle.

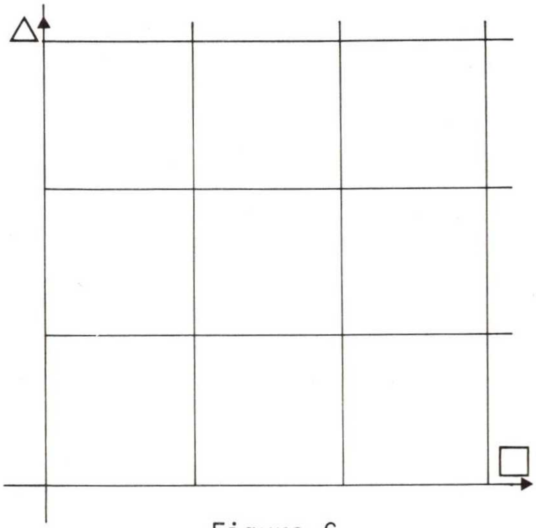


Figure C

[*Some children may argue this figure is not a rectangle. This provides the teacher with an opportunity to talk about squares as special kinds of rectangles.*]

On graph paper ruled off in 1-inch squares, try making a "model" to illustrate 1-1/2x2-1/2.

TRY THIS Cut out 1-inch squares from graph paper ruled off in 1/4-inch squares then glue them onto 1-inch ruled graph paper. Now make a "model" to illustrate 1/4x1/4.

What do you notice?

Make a "model" to illustrate 1-1/4x2-3/4. What is the product?

Try to illustrate on this same grid the multiplication problem 2-1/4x1-1/2. Do you notice anything different this time?

[*Children will discover shortcuts (algorithms) to eliminate the need to illustrate each multiplication problem when given enough opportunity to invent them.*]

Further development of these situations lead to equivalent fractions, fractions with their equivalent decimals, multiplication with decimals, division with fractions or decimals.

LIST OF MATERIALS AND SOURCES

(1) Nuffield Foundation Mathematics Project Publications

Graphs Leading to Algebra (1969)

John Wiley and Sons, Inc.
605 Third Avenue
New York, New York 10016

In Canada: Longman Canada Ltd.
55 Barber Greene Road
Don Mills, Ontario

(2) Members of the Association of Teachers of Mathematics

Notes on Mathematics in Primary Schools (1968)

Cambridge University Press
American Branch
32 East 57th Street
New York, New York 10022

/ Cuisenaire Company of America, Inc.
12 Church Street
New Rochelle, New York 10805

(3) *Freedom To Learn* (1968), Edith Biggs and James MacLean

Addison-Wesley Publishing Co.
106 West Station Road
Barrington, Illinois 60010

In Canada: Addison-Wesley (Canada) Ltd.
67 Gervais Drive
Don Mills, Ontario

(4) The Madison Project

Discovery In Mathematics, A Text for Teachers (1964), Robert B. Davis;
Explorations In Mathematics, A Text for Teachers (1967), Robert B. Davis,

(5) *School Mathematics Project* (SMP), Books A, B, C, and D (1969-70)

Cambridge University Press
and Cuisenaire Company of
America, Inc. (as above)

(6) *Activities in Mathematics,* Second Course, "Graphs" (1971),
Johnson, Hansen, Peterson, Rudnick, Cleveland, Bolster

Scott, Foresman and Company
1900 East Lake Avenue
Glenview, Illinois 60025

In Canada: Gage Educational
Publishing Ltd.
P.O. Box 5000
164 Commander Blvd.
Agincourt, Ontario

(7) Introducing Mathematics: 3

The Search for Pattern (1970), W.W. Sawyer

Penguin Books Incorporated
7110 Ambassador Road
Baltimore, Maryland 21207

(8) One-Inch (ruled graph paper) - R305

Oravisual Company Inc.
321 15th South
St. Petersburg, Florida

(9) School Mathematics Study Group (SMSG)

Mathematics Through Science, Part I "Measurement and Graphing," Part II "Graphing, Equations and Linear Functions," Part III "An Experimental Approach to Functions." (Although too difficult for Grades IV-VI, these are intended to suggest possible graphing situations.)

A.C. Vroman, Inc.
367 South Pasadena Avenue
Pasadena, California 91105

(10) *Shoe Box Kits* (Independent Exploration Material - Madison Project): (a) Geoboard, (b) Tower Puzzle, (c) Centimeter Blocks, (d) Discs, (e) Peg Game, (f) Weights and Springs.

Math Media Division
H and M Associates
Post Office Box 1107
Danbury, Connecticut 06810

REFERENCES

Bell, William R. "Cartesian Coordinates and Battleship," *The Arithmetic Teacher*, May 1974, pp.421-422.

Bruni, James V., and Helene Silverman. "Graphing as a Communication Skill," *The Arithmetic Teacher*, May 1975, pp.354-366.

____________________. "Using a Pegboard to Develop Mathematical Concepts," *The Arithmetic Teacher*, October 1975, pp.452-458.

Good, Ronald G. "Two Mathematical Games With Dice," *The Arithmetic Teacher*, January 1974, pp.45-47.

Jacobson, Herbert R. "Ecology, Rapid Transit, and Graph Theory: An Intuitive Approach for Middle and Upper Elementary Grades," *The Arithmetic Teacher*, April 1974, pp.291-293.

Rainsbury, Ron. "Where is Droopy?," *The Arithmetic Teacher*, April 1972, pp.271-272.

Ristorcelli, T. "Green Chimneys Poker," *The Arithmetic Teacher*, May 1974, p.245.

ACTIVITIES INVOLVING THE GEOBOARD

by ***Forrest Coltharp*** Kansas State College Pittsburg, Kansas

"Activities Involving the Geoboard" was prepared by Dr. Coltharp and presented by him at a workshop for elementary school teachers at the forty-ninth annual meeting of the National Council of Teachers of Mathematics, in Anaheim, California, April, 1971. The following is an edited version of that presentation.

The geoboard is a simple tool that allows children to visualize abstract mathematical concepts, develop intuitive thinking, and discover numerical relations and operations for themselves. "Modern" mathematics can be taught through the use of the geoboard in a modern manner involving students in the process of their own education.

Modern educational philosophy advocates the use of many approaches and materials to help children see mathematical relationships and to integrate rather than isolate various branches of mathematics. The geoboard has proven to be an ideal tool for this purpose. While it is widely accepted as a specific aid in explaining basic geometric concepts, its application is by no means limited to this area. We will investigate some of the possibilities in areas other than geometry.

PRESCHOOL AND KINDERGARTEN CLASSROOM ACTIVITIES

Provide each youngster with a geoboard and several rubber bands and let creativity begin. The following "Suggestions" and "Possible Questions" are quoted directly from "Geoboard Geometry for Preschool Children."[1] The article also contains sample responses from children age two to six years.

[1]W. Liedtke, and T.E. Kieren, "Geoboard Geometry for Preschool Children," *The Arithmetic Teacher,* February, 1970, pp.123-126.

FAMILIAR SHAPES

SUGGESTIONS On your geoboard, show how to make some shapes that look like something in this room. Try to make something that can be found in the kitchen, the basement, yard, grocery store, playground, garage. Show something your dad uses. Show something that is alive.

POSSIBLE QUESTIONS / INSTRUCTIONS Can you tell your friend what you have made? Look at something someone else has made and try to guess what it is. (Ask for a hint where it can be found.) Does your figure look the same if you turn the geoboard around? How many corners does your figure have? (Are there more corners or more sides?)

PLANE FIGURES

SUGGESTIONS Try to make figures with three sides that are small, large, "skinny," "fat." Try to make figures with four sides that are long, short, long and wide, long and narrow, short and wide, short and narrow, "like a square," "not like a square." Try to make figures with "many sides."

POSSIBLE QUESTIONS / INSTRUCTIONS What does the figure you have made remind you of? Does it look like anything that is familiar to you? (Where did you see something like it before?) Does the figure change if you turn your geoboard? Make two figures that (1) do not touch, (2) touch, (3) cut into each other. Look at the figures you have made.

SEGMENTS

SUGGESTIONS Try to make segments that are short, long, straight, "crooked." Try to make segments that do not touch, touch, cross each other (intersect), will never touch (parallel), are exactly on top of each other. Try to make various segments leading to two (or more) points; various numbers of segments (for example, two that are equal, two that are not equal, many different segments).

POSSIBLE QUESTIONS / INSTRUCTIONS How would you make a road? Can you make a very narrow road? Can you make one that is long and narrow? Make a railroad track. (If possible provide rubber bands of different colors.) Can you make a road and a train track that cross, do not cross, will never cross? Look at two pegs in different corners. How many different roads--crooked or straight, with few or many corners--can you build between these two pegs? Which road would you like to travel on? Why?

NINE-PIN GEOBOARD[2]

"A nine-pin geoboard is easily constructed from 3/8-inch plywood and 5/8-inch panel pins, the pins being about two inches apart (see Figure 1). The nine-pin board has the advantage of being so simple that the child believes any mathe-

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[2]Suggestions and questions from "Creating Mathematics With A Geoboard," Peter Wells, *The Arithmetic Teacher*, April 1970, pp.347-349.

matics that may arise can soon be sorted out."[3] One board and a pile of elastic bands between two children is ideal - a partner is there to encourage, correct, and argue with. During a free activity stage the teacher can watch for situations that particularly interest the children and that are suitable for development and extension.[4]

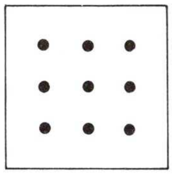

Figure 1

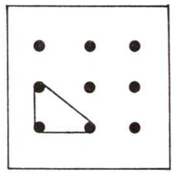

Figure 2

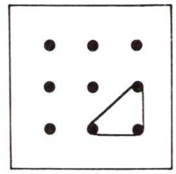

Figure 3

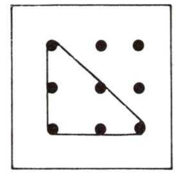

Figure 4

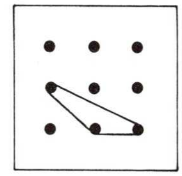

Figure 5

One activity might involve the smallest triangle possible (Figure 2). How many triangles the same as this one can you make on the board? A problem arises immediately. Is the triangle in Figure 3 the same as the one in Figure 2. Some will say yes, others no. Some will want to include Figure 4. The discussion which evolves will challenge mathematical interpretation. Definitions and classifications, equivalence relations, and ideas of sets are all involved, and the teacher should be aware of this although the children may not be.[5]

In answering the question "How many of the smallest triangles are there?" the children will normally experiment with elastic bands. Some will generalize and come up with an answer quickly. For these sharper ones, perhaps the question "How many triangles like figure 5?" will challenge them.

Some other questions which involve classification of shapes on a nine-pin board are listed. These will prove to be spring boards to further mathematical discoveries.

1. How many different squares can be made on the boards? How many of each type?

2. Consider the four-sided shapes. Which ones have an axis of symmetry? How many can you give a familiar name to?

3. Look at the triangles you can make. How many are isosceles? How many are right-angled? How many of each type are there on the board?

4. Make shapes with more than four sides. What is the largest number of sides you can get?

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[3]Wells, *op. cit.*, p.347.

[4]*Ibid.*

[5]Wells, *op. cit.*, pp.347-348.

5. How many straight line segments of different length can be made?

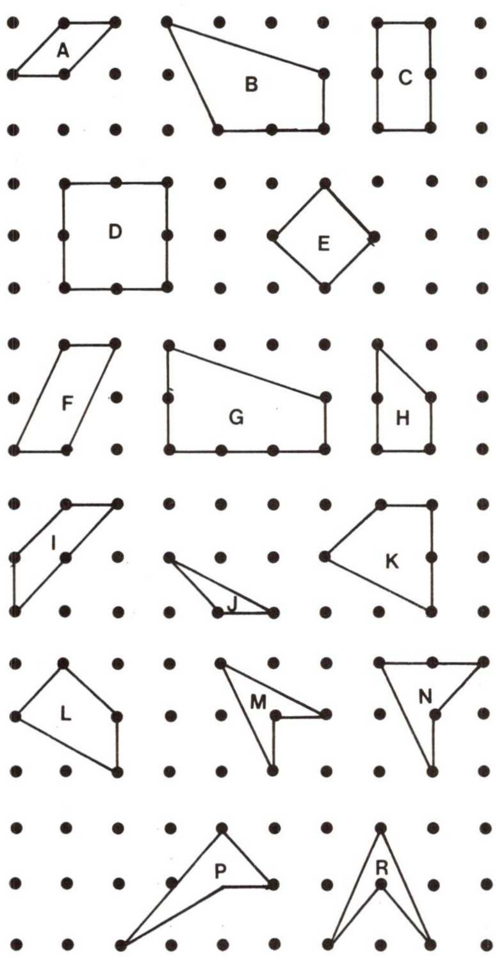
6. What shapes cannot be made on the board?

WORKSHEETS INVOLVING THE GEOBOARD

The following worksheet is an example of an activity-oriented program you may want to investigate further.

Specific behavioral objective - Identify the diagonals of various plane figures and define the idea of a diagonal.

Learning opportunity - Make the following shapes on your geoboard. Transfer each figure to dot paper and draw in all the diagonal lines using a red pencil. In which shapes are the diagonals of equal length?



Test Item - Define what you mean by the term "diagonal." Draw a shape that has no diagonal and tell why it doesn't by applying your definition.

Test Item - Does a diagonal necessarily bisect the angle at that vertex?

[*A follow-up discussion might develop the idea of whether or not the definition given would work for space figures or for a line joining two vertices which is not a side*[6]]

[6]Carol H. Kipps, "Topics in Geometry for Teachers - A New Experience in Mathematics Education," *The Arithmetic Teacher*, February, 1970, p.166.

AREA AND PERIMETER

The concepts of area and perimeter can be introduced through the use of the geoboard, pegboard, graphing paper or the T-board. The following activities and questions are adapted from the article "Geometry Via T-Board,"[7] which you may want to read to determine what constitutes a T-board.

First, enclose a rectangle two units long and one unit wide. How many square units are enclosed by the rectangle? (See Figure 6.) Find the distance around the rectangle by counting the units forming the sides. (See Figure 7.)

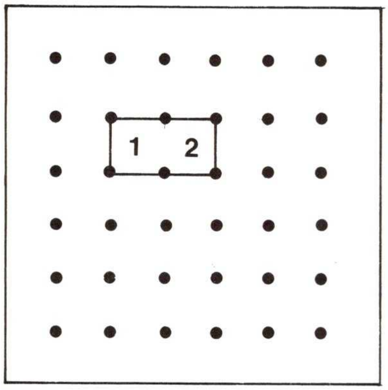


Figure 6

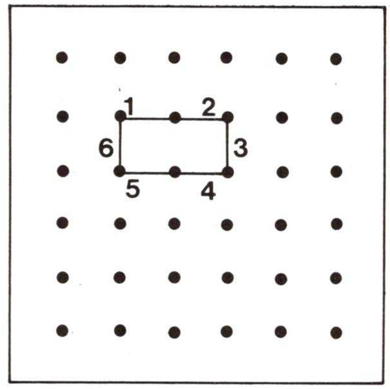


Figure 7

Next, double the length of the rectangle and maintain the same width. How many square units are enclosed by this rectangle? (See Figure 8.) Find the distance around the rectangle again by counting the units forming the sides. (See Figure 9.)

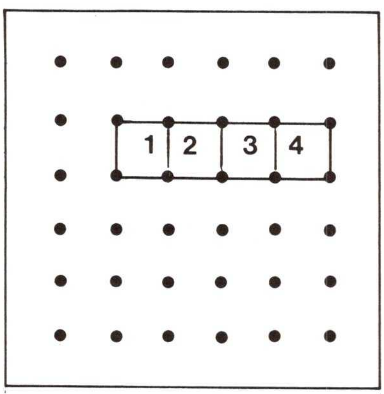


Figure 8

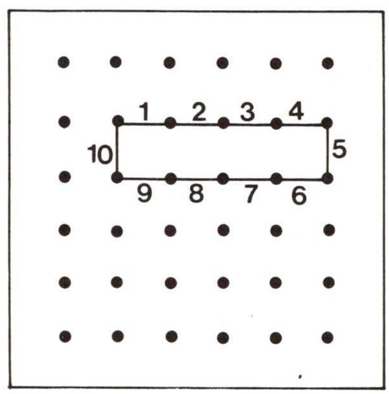


Figure 9

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[7]Donald O. Teegarden, "Geometry Via T-Board," *The Arithmetic Teacher*, October, 1969, pp.485-487.

Use your geoboard to determine the number of square units enclosed by the rectangle formed when the original length is maintained and the width is doubled. What will be the distance around such a rectangle? Consider the same questions when both the length and width are doubled. Display your results in the following table.

Length	*Width*	*No. of square units enclosed*	*Distance around*
2	1	2	6
4	1	4	10
2	2	□	△
4	2	□	△

Starting with a different rectangle, perform the same cycle of doubling the length, the width, and then both length and width. Then ask the following questions:

How could you find the number of square units enclosed by any size rectangle quickly from just knowing the length and width?

Does the distance around double when the number of units enclosed is doubled by making one dimension twice as great?

How do you make the distance around twice as large as the original?

Is there a way of figuring the distance around without counting each item?

Use the geoboard to determine the missing measures indicated in the following table.

Size of Rectangle	*No. of square units enclosed*	*Distance Around*
3 X □	△	10
3 X 3	△	○
2 X □	8	○
4 X □	△	14
4 X □	16	○
5 X 2	△	○

If the term "area" is substituted for "number of square units enclosed" and "perimeter" for "distance around" we find that many important concepts have been developed in this activity.

PRIME AND COMPOSITE NUMBERS[8]

A technique based upon the manipulation of square regions may be used to help children discover that the counting numbers may be classified as prime or composite numbers. The geoboard (or pegboard) may be used to introduce this technique.

"Considering only horizontal and vertical rectangular arrangements, in how many different ways can a given number of square regions be arranged?"[9] For example, how many different arrays can be made with only one unit region? One region may be arranged in only one way, as a 1 x 1 array (see Figure 10).

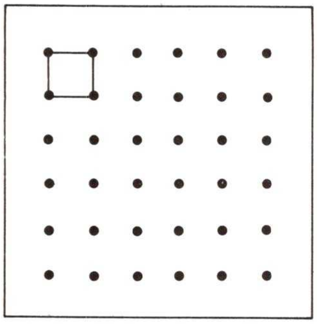

Figure 10

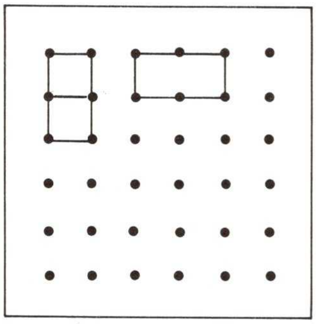

Figure 11

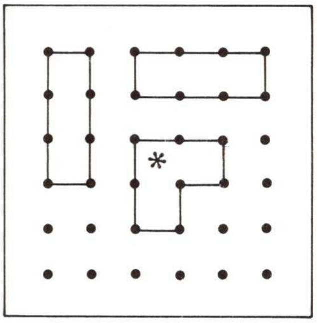

Figure 12

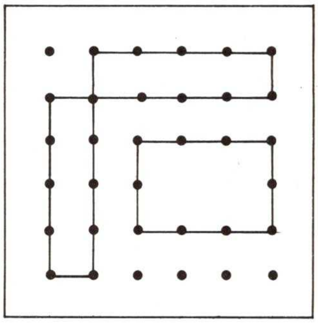

Figure 13

How many different arrays can be made with only two unit regions? Two regions may be arranged in only two different ways, as a 2 x 1 and as a 1 x 2 array (see Figure 11).

How many different arrays can be made with only three unit regions? Why isn't the array marked with * (in Figure 12) considered?

How many different rectangular arrays can be made with only four unit regions? Four regions may be arranged in three different ways, as a 4 x 1 array, as a 1 x 4 array, and as a 2 x 2 array (see Figure 13).

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[8]This activity was taken from: "Pattern for Discovery: Prime and Composite Numbers," Frances Hewitt, *The Arithmetic Teacher*, February, 1966, pp.136-138.

[9]Hewitt, *op. cit.*, p.136.

With the limitations of the typical geoboard, you may wish to resort to a pegboard with more range, coloring on graphing paper, or cut out square regions. Continue the activity for five, six, seven, eight, and possibly even more unit regions. Each time ask the question, "How many different rectangular arrays can be made with the given number of unit regions?"

The first step may be merely to determine whether the number can be represented by only one rectangular array, by only two, or by more than two different rectangular arrays.

In any discovery exercise such as this, it is important that the data be arranged in some pattern so that a generalization is possible. The information may be recorded in a table. The table below has been completed for 1, 2, 3, and 4; continue the process for several more counting numbers.

Only one array	*Only two arrays*	*More than two arrays*
1		
	2	
	3	
		4

After children are familiar with the process and have had experience with numbers, perhaps through 20, you may wish to extend the table as indicated.

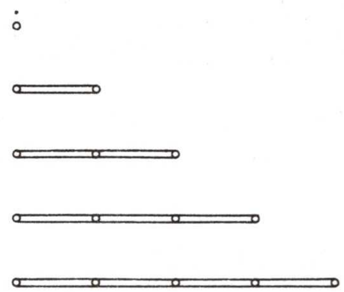
Only one array	*Only two arrays*	*More than two arrays*	*Pairs factors*	*Different factors*
1			1 x 1	1
	2		2 x 1	1, 2
			1 x 2	
	3		3 x 1	1, 3
			1 x 3	
		4	4 x 1	1, 2, 4
			2 x 2	
			1 x 4	

The final step in this discovery activity would be to find a rule that allows us to classify the counting numbers. Up to this point the children can discover everything but the names of the classes, namely *prime* and *composite*. See if you can devise the rule used to classify the counting numbers. What about the number 1?

FRACTIONS[10]

"The following exercises can be adapted to the needs of the class, and more exercises of the same sort can be easily developed by the teacher."[11]

EXERCISE 1



Prepare the geoboard in the manner above with rubber bands around two pegs, three pegs, four pegs, and five pegs. Although any of the three may be considered the unit, use a different colored rubber band around the four pegs to indicate the unit measure for this discussion. Ask questions such as the following:

How long is the unit strip? (One unit)

How long is the top strip? (One-third of a unit)

Why? (Because three of these one-third units will be as long as the unit)

How long is the next strip? (Two-thirds of a unit)

Why? (It is twice as long as the one-third strip)

How long is the bottom strip? (Four-thirds of a unit)

Why? (It is four times as long as the one-third strip, or one-third more than one unit)

---

[10]George S. Cunningham and David Raskin, "The Pegboard As A Fraction Maker," *The Arithmetic Teacher,* March, 1968, pp.224-227.

[11]Cunningham, *op. cit.,* p.224.

INSTANT GRAPHS[12]

With the aid of rubber bands, "instant graphs" can be quickly constructed on your geoboard. It may be necessary to use a larger pegboard and movable pegs but much of the data gathered can be displayed on the geoboard.

The bar graph (Figure 14) "... could be a record of the number of days of school four boys missed during the last month. The line graph (Figure 15) could show the number of pupils who were absent from the classroom for each of the five days last week.

"Pieces of cardboard could be used to label the parts of the graphs, and challenges can be added by requiring the students to calculate and construct various scales."[13]

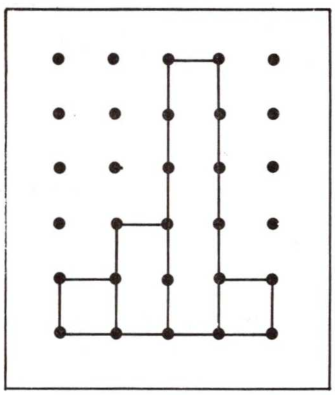


Figure 14

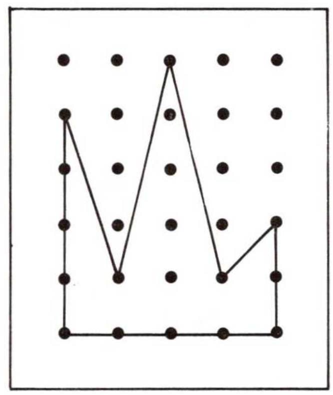


Figure 15

LIST OF MATERIALS AND SOURCES

Cuisenaire Company of America, 12 Church Street, New Rochelle, New York 10805

Plastic Geoboards. Two pamphlets also available: *Notes on Geoboards; Geoboard Geometry* by Caleb Gattegno.

Houghton-Mifflin Company, 1900 S. Batavia Avenue, Geneva, Illinois 60134.

Geoboard Kit. Each kit contains a 7" by 7" geoboard, assorted rubber bands, and 40 student activity cards.

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[12]What Can You Do With A Geoboard?" Werner Liedtke. *The Arithmetic Teacher*. October, 1969, pp.491-493.

[13]Liedtke, *op. cit.*, p.491.

Selective Educational Equipment Company, Inc., 3 Bridge Street, Newton, Massachusetts 02195.

Plastic Shapes Board.

Sigma Enterprises, Inc., Box 15485, Denver, Colorado 80215.

The Sigma Geosquare. Geosquare Classroom Kit includes 30 Geosquares, large assortment of colored rubber bands, teacher's manual, pad of Geosquare dot paper. Geosquare Teacher's Manual available separately for $1. Also available are three kinds of Geosquare dot paper: 2-inch scale, 1-inch scale, and 1/2-inch scale.

Walker Teaching Programs and Teaching Aids, 720 Fifth Ave., New York, N.Y. 10019.

Geoboards. Plastic Geoboard for overhead demonstrations. Book by Donald Cohen, *Inquiry in Mathematics Via the Geoboard.* Geo-Card Math Lab, introducing approximately 150 problems and experiments. *The Geoboard: A Manual for Teachers* by Irving Kreitzberg. Designed for instruction in Grades K-8. $2.45.

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Felder, Virginia. "Geometry Concepts in Grades K-3," *The Arithmetic Teacher,* May 1965, pp.356-358.

Hewitt, Frances. "Patterns for Discovery: Prime and Composite Numbers," *The Arithmetic Teacher,* February 1966, pp.136-138.

Jackson, Stanley B. "Congruence and Measurement," *The Arithmetic Teacher,* February 1967, pp.94-102.

Kipps, Carol H. "Topics in Geometry for Teachers--A New Experience in Mathematics Education," *The Arithmetic Teacher,* February 1970, pp.163-167.

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______________. "What Can You Do With Geoboard?", *The Arithmetic Teacher,* October 1969, pp.491-493.

______________, and T.E. Kieren. "Geoboard Geometry for Preschool Children," *The Arithmetic Teacher,* February 1970, pp.123-126.

Nelson, Diane, and Marvin N. Nelson. "Pegboard Multiplication of a Fraction by a Fraction," *The Arithmetic Teacher*, February 1969, pp.142-144.

Niman, John, and Robert D. Postman, "Probability on the geoboard," *The Arithmetic Teacher*, March 1973, pp.167-170.

Parker, Robert. "Graph Paper: A Versatile Visual Aid," *The Arithmetic Teacher*, February 1969, pp.144-148.

Skypak, Dora Helen. "Geometric Concepts in Grades 4-6," *The Arithmetic Teacher*, October 1965, pp.443-449.

Smart, James R., and John L. Marks. "Mathematics of Measurement," *The Mathematics Teacher*, April 1966, pp.283-287.

Smith, Lewis B. "Pegboard Geometry," *The Arithmetic Teacher*, April 1965, pp.271-274.

Strangman, Kathryn Besic. "Grids, Tiles, and Area," *The Arithmetic Teacher*, December 1968, pp.668-672.

Swadener, Marc. "Activity Board - the board of many uses," *The Arithmetic Teacher*, February 1972, pp.141-144.

Teegarden, Donald O. "Geometry Via T-Board," *The Arithmetic Teacher*, October 1969, pp.485-487.

Vigilante, Nicholas J. "Why Circumvent Geometry in the Primary Grades?", *The Arithmetic Teacher*, October 1965, pp.450-454.

______________________. "Geometry for Primary Children: Considerations," *The Arithmetic Teacher*, October 1967, pp.453-459.

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Wells, Peter. "Creating Mathematics With a Geoboard," *The Arithmetic Teacher*, April 1970, pp.347-349.

PART TWO

MATHEMATICS ACTIVITIES FOR THE ELEMENTARY SCHOOL

MATHEMATICS ACTIVITIES FOR THE ELEMENTARY SCHOOL

Topic headings in the following section were extracted from the 1971 Alberta "Handbook For Elementary Mathematics"[1] *and were used to group elementary school mathematics activities that were collected and developed by teachers for development-of-a-topic assignments in workshop courses conducted by the editor over the past four years. With the assistance of Mr. Penn Wong, the activities were cross-referenced according to the concepts developed, and then were edited to minimize repetition and produce some consistency in format and to ensure as far as possible that the activities would be practical and "ready-to-use." The surname beside each subheading is from the following list of teachers from whose assignments the activities were taken.*

Usha Aggarwal
Judith Bortnik
Garry Bowman
J. Burton
Brenda Chidley
R.J. Eremko
Elizabeth Foran
James Feeney
Georgina Gregg
Marilyn Grigel
James Gunn
Carlotta Hamilton
Kenneth Hampel
Sharon Inkster
Malkiat Kang
Elizabeth Kanik
Evelyn Keddie
Elizabeth Keeler
Louise Kennedy
Lenora Koleyak
Jon Kommes
Ralph Krueger
Bernice Lenz
Merlin Leyshon
Susan MacAlister
Charles Merta
Janice Millar
Anna Misselbrook
Winsome Myers
Jeanette Penner
Gail Risvold
Sally Ritcey
George Seeger
Mina Simpson
Fred Sloan
Wilfrid West
David Willis
Penn Wong
Marina Woodeye

At the end of each concept section there is a list of articles which contain teaching suggestions. Those from the 1972-75 issues of *The Arithmetic Teacher* have been cross-referenced by the co-editor; others were cross-referenced by the editor's elementary mathematics methods (EDCI 502) students in the 1971-72 academic term.

Bruce D. Harrison

[1]Alberta Department of Education, "Handbook for Elementary Mathematics," Edmonton, Alberta, 1971.

A. NUMBERS AND NUMERATION SYSTEMS

A.1 Cardinal and Ordinal Number Concepts

Boys vs. Girls *(Chidley)*

1. Give each boy in the class two blue beads and each girl two pink beads.
2. Have each member of the class put one of his or her beads on either the boys' string or the girls' string.
3. Hang the two strings side by side.
4. Are there more boys or more girls in our class?
5. How do you know this by looking at the two strings of beads?
6. How many boys are there in our class?
7. How many girls are there in our class?
8. How can you find out from your strings of beads how many people there are in our class?

Birthdays *(Chidley)*

1. Have each child place his second bead on the prong showing the month of his or her birthday. Have the boys put theirs on first and then the girls.
2. In which month are there the most birthdays in our classroom?
3. In which month are there the least birthdays in our classroom?
4. Are there any months in which no birthdays occur?
5. How many people in our classroom have birthdays before we start our summer holiday?
6. Show on a graph the number of birthdays in each month by coloring as many squares under each month as there are beads.

Calendar Activities *(Chidley)*

1. Use a calendar to help you with the following tasks.
2. How many months are there in a year?
3. We only go to school part of a year. How many months of the year do we have school?
4. Can you show me with a picture how long a year is by dividing it up into months? Also show which of these months you spend going to school. Make your picture big.
5. Using your calendar can you find out how many weeks there are in a year.
6. Similarly, can you show how many days there are in a week.

7. Find a way, with a picture, to show what you have found out about months, weeks, and days.
8. Are there more weeks, months or days in a year?
9. Are there more weeks or months in a year?
10. Are there more days or weeks in a year?

Personal Numbers *(Chidley)*

1. The date of your birthday ____________________.
2. Your age ______________.
3. The number of people in your family ___________.
4. Your address ____.
5. The number of the bus you take to school ____________.
6. Your clothing size _____________.
7. Your shoe size __________.
8. Your classroom number _________.
9. Your locker number at school _________.
10. The amount of your allowance __________.
11. Your phone number _____________.
12. The address of our school ____.
13. Your favorite T.V. channel ________________________

Base Ten Charts *(Inkster)*

Give children cards with the following columns on them:

100	10	1

Have partner place various discs under each column up to 9.

e.g.

100	10	1
o o o o	o o	ooooo

The partner must tell what the number is. Then have the partner write down certain numbers such as 14, 65, 928, etc. The other student must place the appropriate discs in the correct column.

Give them columns with the following discs already placed under the numbers:

100	10	1
o o o	oooooo	ooooo
	ooo	ooooo

Have the children draw a picture of another way of showing the same number:

e.g.

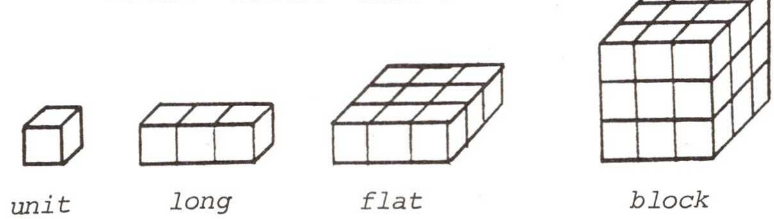
100	10	1
o o o	ooooo oooo	

Then reverse the question. For example show the bottom picture and have them expand it. This type of exercise will help the children understand the value of the columns and will make understanding easier when adding and subtracting large numbers when borrowing is involved.

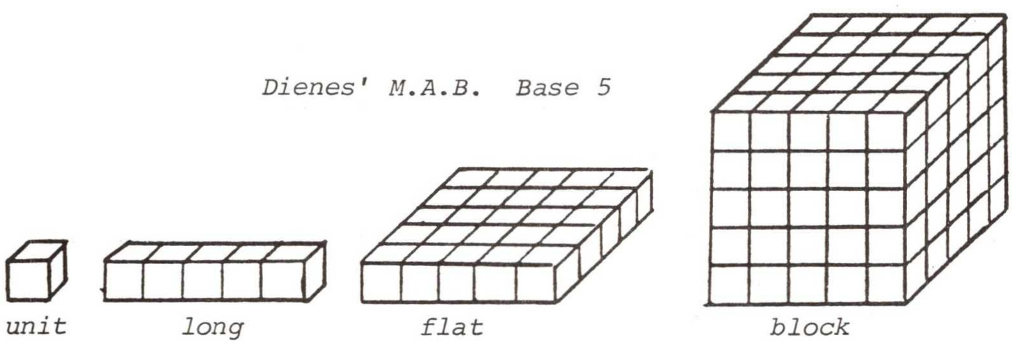
Dienes' Multibase Arithmetic Blocks *(Millar)*

A set of Dienes' Multibase Blocks, or some facsimile, would be needed for the following activities. The basic structure for the units, longs, flats and blocks are illustrated below for base three and base five.

Dienes' M.A.B. Base 3



Dienes' M.A.B. Base 5



1. What relationships can you find between the units, longs, flats and blocks? Use three different bases.
2. Make up some questions in addition and check answer by using the blocks. Try another base.

3. Pick up some blocks in each hand. Which hand has more? How many more? Try another base.

4. Take any amount of blocks and double them. Try to multiply these blocks by the number of units in a long. How does this compare to multiplication by 10 in base 10. Try other bases.

5. Give children squared paper 10 X 10. Make a number chart 1 to 100. What patterns can you discover?

6. "1000" Base 3
Mark dice 0_3, 1_3, 2_3, 10_3, 11_3, 12_3
Roll 1, 2 or 3 dice. Add the numbers and then subtract it from 1000_3.
The first to reach 0 wins. Try this with other bases.

7. "Number Bee"
Two teams - flash a card. First person to get right answer gets a point for the team.

Card Samples

Card: 793 / *Answer:* 7 hundreds, 9 tens, 3 ones

Card: 7 hundreds, 18 tens, 3 ones / *Answer:* 883

All of these activities can, and should be done with at least two bases besides base 10.

Number Patterns *(Bowman, Krueger, Leyshon)*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Circle all the even numbers on the sheet of numbers. What do you notice about the pattern you have formed? Why is this so? What number can you divide evenly into every even number?

2. Circle all the numbers on a sheet that are multiples of 4 (4, 8, 12...). What do you notice about the pattern? How is it like the first one you did? What numbers besides 4 and 1 can you divide evenly into all multiples of 4?

3. Circle all the numbers on a sheet that are multiples of 3 (3, 6, 9...). What do you notice about the pattern? How does it compare to the others you did?

4. Make patterns on other sheets using multiples of 5, 6, 7, 8, and 9. Compare the patterns.

5. Circle all the numbers on the sheet that can only be divided by themselves or by 1.
   Examples: a) 2 is divisible by 1 and 2 only so circle 2
   b) 12 is divisible by 12, 1, 3, 4, 6, and 2, so *do not circle 12.*
   The numbers you have circled are called prime numbers.

6. Devise your own pattern of prime or composite numbers, or a combination of both.

Square Numbers *(Willis)*

1. Cut out several colored one-inch squares. With these, make successively larger squares. Make a chart showing the number of unit squares in each successively bigger square. For example,

   1 x 1 = 1   2 x 2 = 4   3 x 3 = 9

   What would the eleventh square be?

2. Having now acquired some knowledge as to how squares behave, make as many statements as you can about the following pattern.

```
1   4   9   16   25
 \ / \ / \ / \  / \
  3   5   7    9
   \ / \ / \  / \
    2   2   2
     \ / \ / \
      0   0
```

Number Shapes *(Willis)*

1. On your geoboard, make a series of successively larger pentagons with two sides common. Can you find a pentagonal pattern?

2. The square numbers (which you have seen), the triangular numbers (1, 3, 6, 10, 15, etc.) and the pentagonal numbers (from above) are all related. Make a chart to study their relationship. For example -

△ 1 3 6 10 15

□ 1 4 ...

⬠ 1 5 ...

What relationships do you see?

3. What other number shapes can you grow?

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A.2 Fractional Number Concepts

Developing the Concept of Fractions *(Simpson)*

Children understand what one-half, perhaps one-third, and one-quarter are in the spoken language before they come to school. They learn this by sharing chocolate bars, pieces of cake, etc.

Here is an exercise to teach the children how to write a fraction and equate it with its meaning. Have each child take a strip of paper and fold it so the ends meet evenly. Draw a line on the fold. How many equal lengths do you have? Elicit the answer "2." Teacher writes "2" on the blackboard. Color one piece red. What part of the strip did you color? Answer "1." Write $\frac{1}{2}$. Do numerous exercises using halves and quarters. Write the fraction on the board. After some practise, have the children write on the piece of paper the representative fraction. For extra practise, use a fraction kit on flannel board, as well as many diagrams and shapes (squares, circles and rectangles). By questioning, make certain all the pupils understand that the bottom digit tells how many pieces the whole was cut into and the top digit represents how many pieces we are going to use (for example, color $\frac{2}{3}$ of the square red).

In another exercise, give each child four identical rectangles of paper. Can you fold each a different way to divide it into halves? Also, have each child take a strip of adding machine tape (about two feet long). Fold in half. Draw a line on the fold; mark $\frac{1}{2}$. Fold again. How many equal pieces do you have? Do you see any equal fractions? Equal fractions can be shown using two identical strips. Fold one in halves and one in quarters. Match the folds. What do you see? Fold again. What do you see?

Thirds are a bit harder to fold but with direction and patience they can learn to do it. Fold again and what have you got this time? (If you plan to fold thirds, halves and quarters on the same strip it is easier if you fold thirds first.)

Grade III children understand fractions quickly when they are dividing the whole into equal parts. They experience difficulty in writing fractions when they divide groups of objects (marbles, etc). They can do the manual dividing, such as dividing twelve marbles among three children, but when asked what fraction of the 12 marbles each child has, they usually answer something like, "Mary has four marbles so she has one-quarter." It takes a great deal of practise with objects, starting with two and working up until the children learn that the fraction doesn't mean the number Mary has in her hand.

Activities on Fractions: Grade IV *(Hamilton)*

This workshop is designed as an immediate follow-up procedure after the children have been introduced to the idea of fractions--what they mean, how they are written, naming numerator and denominator, etc. The idea of the workshop is to give pupils the opportunity to work with fractions using very familiar objects, to give them a better knowledge of what fractions mean, to give them the opportunity to manipulate objects in several ways so that they can see how fractions are an integral part of everyday living and, finally, to give them an opportunity to be creative in making fractions using their own ideas. In some aspects, the workshop, can be correlated with measurement and language, or even art!

1. Given a square piece of paper or a box and several one-inch squares (made from construction paper), the pupils are asked to make as many fractions as they can by fitting the inch squares into the box. Then they are asked to look for combinations of fractions that are equal to other fractions and to make shapes (using construction paper) to show these new fractions, bringing in the idea that some fractions are equivalent. The pupils are encouraged to try and make different patterns with their shapes as they fit them into the box.

2. When the first activity is completed, pupils might now be asked to repeat the first activity using larger boxes and larger or smaller squares (or even other shapes that they might come up with). For example, you might say, "What would happen if you cut the small squares in half, or divided them into three equal parts, or four equal parts, ...?"

3. Pebbles in a bag. Divide them evenly among the members of your group. What fraction of all the pebbles does each child get? 3 children? 4 children? etc.

4. Long strip of liquorice. Fold in the middle and cut into 2 halves. Write $\frac{1}{2}$ of 1 strip = $\frac{1}{2}$. Eat one piece. With the other piece, continue to fold, cut, write, and eat until the liquorice is gone.

5. Cups and jug. How would you find out what fraction of the whole jug one cup would be? 2 cups? etc.

6. Tin pie plates and construction paper. Show how mother cuts a pie so that each member of the family gets a piece.

7. Divide a strip of ribbon evenly among the members of your group. What fraction of the whole ribbon does each child get? 2 children? etc.

8. Wind two strings of different thickness around a piece of cardboard to measure one inch. Count the strands. What fraction is one string to all the strings. SURPRISE! The fraction with the larger denominator goes with the thinner string.

9. How many different fractions can you make in talking about the members of your family? Show your ideas on a chart (using cut paper, pipe cleaners, etc.).

10. What fraction represents the number of children present today? Absent? Show on a chart.

11. Show on a chart the fraction that tells the number of children in your class that are boys, girls, 9-year-old boys, 9-year-old girls, ten-year-old boys, ten-year-old girls, children 9 years old, children 10 years old, desks that are empty, desks that are filled, big desks, small desks,

windows that will open, windows that will not open, tiles that are different colors (floor), etc.

12. Cut up oranges to show the idea of improper fractions. For example, how would you cut two oranges so that each of five persons would get $\frac{1}{4}$ of an orange, or $\frac{1}{3}$ of an orange? Are any pieces left over? Which would you rather have, a third or a quarter? What fraction names all the pieces that were given to the five people?

Fractions Workshop *(Seeger)*

The objectives of the following activities are:

a) to give the student some familiarity with the concept of fractional amounts and,

b) to help the student gain an understanding of the meaning of fractional amounts.

The workshop is divided into sections. For each section, students should be divided into groups and rotated through the activities. Section A is a rather simple introductory section.

SECTION A

1. Provide the group with scissors and pieces of construction paper cut into geometric shapes such as circles, rectangles, pentagons, hexagons, etc.

   a) Can you cut each different shape into 2, 3, 4, 5, 6, 7, 8 equal pieces? What are these different pieces called? Write the name 1/2, 1/3, 1/4, etc. on the appropriate piece.

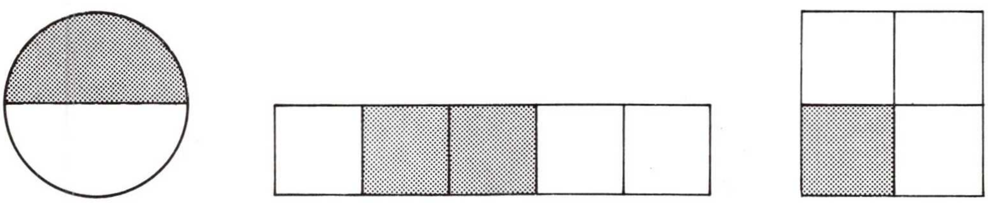
   b) Using the equal pieces of one of the shapes only, each of the halves is the same as how many of the eighths, fourths, sixths?

2. Provide each student with about two or three feet of adding machine tape.

   a) Can you fold the adding machine tape to show halves, quarters, thirds, sixths, and eighths? As you do this, mark each fold 1/2, 1/4, etc.

   b) Are there places on the tape where two or more folds are in the same place? Give several examples of this.

3. Provide the students with worksheets which have geometric figures drawn on them with portions shaded.



What fractional portion of each of the shapes has been shaded?

SECTION B

1. Provide the group with worksheets and balance beams or scales.

Worksheet

Object	*1/2 of the weight*	*estimate of 1/4 weight*

a) Collect a number of different objects in the room. How can you find $\frac{1}{2}$ of the weight of each?

b) Can you estimate what $\frac{1}{4}$ of the weight of each object might be?

2. Provide the group with worksheets and spring scales.

a) Collect several objects in the room. Can you find the weight of each object in air and in water?

b) Can you show how the weights compare by using a fraction?

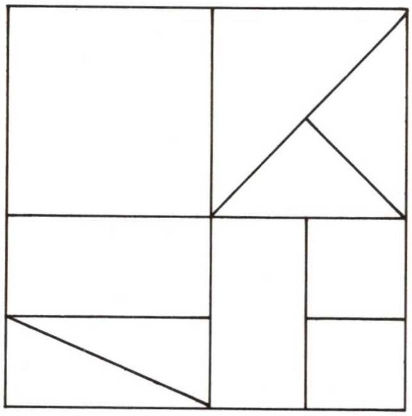
3. Provide each student with a tape measure and worksheets.

a) Find your height.

b) Can you make a fractional comparison between your height and the length of your shadow at different times of the day?

c) At what times of the day is this fraction close to being equal to one?

4. Provide each student with a sheet on which the rectangle below has been drawn.



a) Express each region in the rectangle as a fraction of the entire rectangle.

b) Combine two or more regions to form larger fractions. Shade these areas.

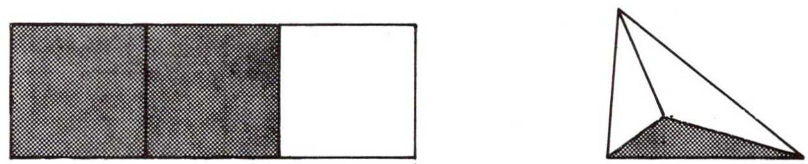
c) Can you write equations using the fractional amounts in the rectangle?

SECTION C

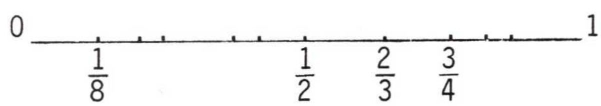
1. Provide each student with a worksheet showing geometric figures divided into fractional amounts, with portions shaded.

   a) How much of each polygon is shaded?

   b) What would you add to each shaded amount to get a value equal to one?



2. Provide each group with a fractional number line



   a) Can you complete this number line?

   b) How can you use this fractional number line to add and subtract?

3. Provide each group with a geoboard and rubber bands.

   a) Can you show fractions on 2x2 or 3x3 sections of the geoboard?

   b) Can you use your geoboard to show how to add or subtract your fractions?

Fractions *(Willis)*

This unit is intended to provide Grade V students with concrete experience in working with fractions. To be able to construct, assemble, compare, and generally manipulate the materials will help provide a more thorough understanding of the concept "fraction." The work set out is not a complete unit of study for fractions in Grade V, but it provides a good background for further work and materials. The ideas are basically - numerator and denominator, parts of a whole, equivalence, common denominators.

All these come from the S.T.A. Grade V text. The treatment is similar to what can be found in many commercial productions but the particular combination and work topics are an accumulation of many ideas. It is hoped that at the completion of this workshop the students will be able to:

1. Identify the numerator and denominator of given fractions.

2. Pick out the smallest fractions from a list of fractions with the numerator the same.

3. Pick out the largest fractions from a list of fractions with the denominator the same.

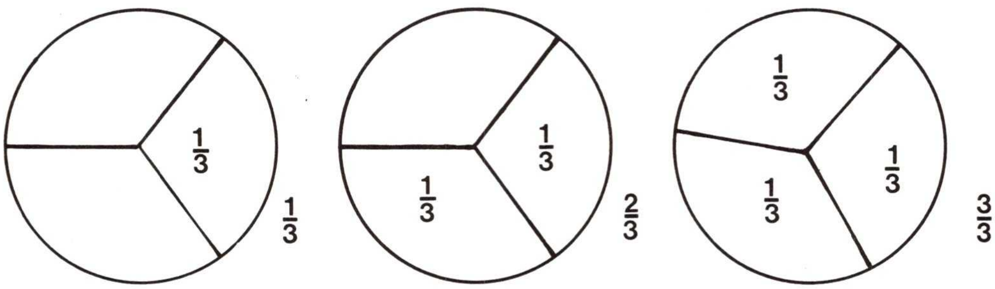
4. Identify various fractional parts of a whole object.

5. Write arithmetic sentences showing equality between fractions.

6. Figure out a common denominator for the fractions given.

TASKS

1. Take the materials and construct complete circles out of the pieces so that you are using pieces all the same size to make the circles. If your circle is being made from 3 (three) identical pieces, your notation is like this:



Can you complete a chart like the following?

Number of pieces to make whole	*Fractions you obtained as you built circle*						
1							1
2						1/2	2/2
3					1/3	2/3	3/3
4				1/4			
5					3/5		
6							6/6

2. Notice that the bottom number on each fraction in any row of your chart is the same as the number of pieces in the circle. What is the name given to the bottom number?

3. Notice that the top number of each fraction in any row changes as the number of pieces added to the circle becomes larger. What is the name of the top number?

4. Which of these fractions is the smallest? 1/3 1/5 1/7
If the top number (numerator) remains the same, how can you tell which fraction is going to be the smallest?

5. Which of these fractions is the largest? 3/4 1/4 2/4
If the bottom number (denominator) remains the same, how can you tell which fraction is going to be the smallest?

6. Manipulate the figures to show which parts of circles are equal to other parts of the same circle. For example, can you show that 1/6 + 1/6 = 2/6 = 1/3? What other equalities can you find? Write them down in the same manner as the above example.

7. If you want to add two or more fractional parts together that do not have the same denominator, what can you do? For example, 1/4 + 1/2. Try assembling the figures suggested.

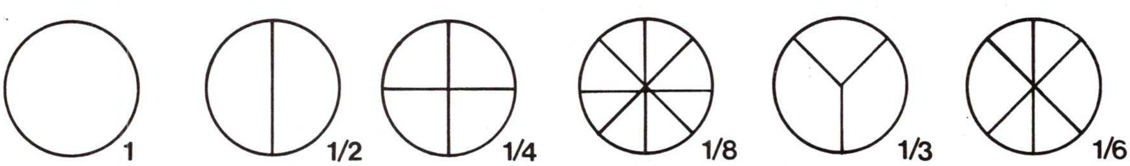
Fractions *(Feeney, Kanik)*

Fraction - one or more of equal parts into which a unit or number can be divided.

Numerator - the top number of a fraction that tells how many parts.

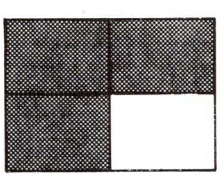
Denominator - the bottom number of a fraction that names the fractional parts.

1. Illustrate fractions beginning with -



2. Use fractions in sentences.

3. Fractions can be shown in several ways.

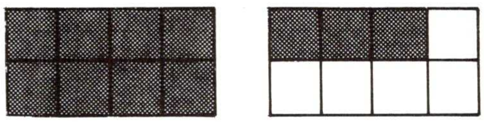


3/4 is shaded / three-quarters is shaded

4. *Improper fraction* - applies to fractions having *numerators equal to or greater than* their denominators.

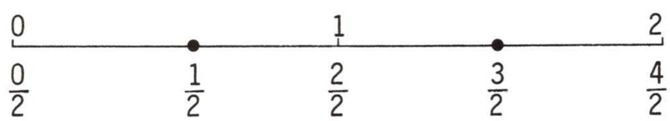
8/8 is shaded

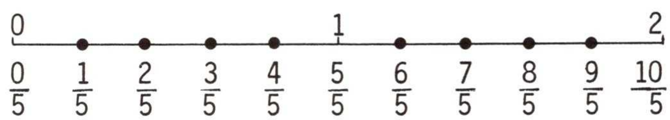
or



11/8 rectangles are shaded

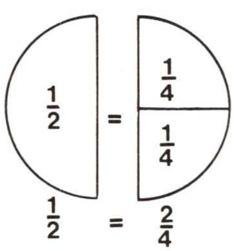
5. Fractions on number lines -

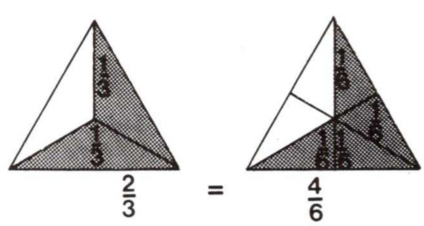




See how many different number lines you can make. Use as many different fractions as you can. Compare some of these number lines to each other. Can you find several different names for the same point on your number lines?

6. Equivalent fractions - make as many equal fractions as you can. Make them in as many different ways as you can. Label each fraction.





Here are two different sets of equivalent fractions:

1/4 2/8 3/12 4/16 5/20 6/24

5/6 10/12 15/18 20/24

- see how many sets of equivalent fractions you can make,
- compare several of these sets,
- what do you notice about the denominators of some of the fractions in different sets?

Before we can add one fraction to another we must find an equivalent fraction that has the same denominator. If you took the two sets of equivalent fractions for 1/3 and 1/5 -

1/3 2/6 3/9 4/12 5/15 6/18, and

1/5 2/10 3/15 4/20

could you find the fractions in each set that have the same denominator? Once you find the common denominator you can add the two fractions.

Write down several sets of equivalent fractions. Can you find any common denominators in these sets?

Divide several transparencies into fractions. By laying one on top of the other, show how you are able to explain to your partner how to add fractions. Take turns with your partner to make up some questions in which you add fractions. Make illustrations of your questions and answers or use the available material to show how you found your answers.

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Addition and Subtraction of Two-Digit Numbers *(Grigel)*

Level: Primary General Learning Disabilities.

Assumptions:
1. Children know the addition and subtraction number facts of ten.
2. Children know the word "digit."

Materials:
1. Match sticks, straws or paper strips to be used while doing each question.
2. Elastic bands for grouping in ten.
3. Dienes' "Multibase Arithmetic Blocks" base 10 (optional).

1. UNITS
Each of your sticks is one UNIT. Use your sticks to show all the one-digit numbers. Can you put them in order? Which number has the most sticks?

2. TENS
Take the biggest pile you had from Card One. Add one more stick. Now how many do you have? How do you write this number? We can trade one group of ten units for one TEN.

    10 units make 1 ten

    __ units make 2 tens

    __ units make 3 tens.

    Show 5 tens. How many sticks do you have? Try 7 tens. Find other groups of tens and write the number of sticks you have.

3. TENS AND UNITS
We can put tens and units together to make new numbers. If we have 5 tens and 6 units what number do we have? How do you write it? How many different two-digit numbers can you make with 5 tens? Can you put them in order? Do it with 3 tens. 7 tens. Is there an end to this?

4. ADD
Use your sticks to add 3 tens and 5 units
to 6 tens and 2 units.

    What did you get? Can you write it the short way? Now add
    4 tens and 2 units
    to 3 tens and 2 units.

    Write your answer. Try 3 tens and 7 units
    to 1 ten and 5 units.

What did you get? Can you write this answer? How is it different from the the other answers? When we have more than ten units we must REGROUP. Make up your own questions. Can you see what is happening?

5. REGROUPING TO SUBSTRACT
Subtract: 5 tens and 2 units,
1 ten and 1 unit.

Use your sticks. How many are left? Can you write the number?

Do these:	2 tens 2 units	65	79	88
	-1 ten 0 units	-33	-46	-25

Now try this one: 6 tens 2 units
-3 tens 9 units

What must you do to your units? Where could you get more units? Can you regroup? What is left? Write your answer.

Do these:	5 tens 3 units	62	74	37
	-2 tens 5 units	-29	-46	-29

Make up **some** questions for your friend. Be sure you can do the questions yourself!

Basic Facts Review *(Inkster)*

1. COW - picture of cow with paper clips on its body. Place a number underneath the cow. The children have to find as many basic facts as they can that equal this number. They then put them on the cow.

2. BLOCKS - 14 blocks having numbers up to 15 on the faces. Child shakes blocks and must fit them into the following sheet.

□ + □ = □

□ - □ = □

□ X □ = □ + □

□ ÷ □ = □ - □

[*Child's score is the value of cubes placed in true statements, minus those not placed or placed in false statements after agreed time limit. Patterned after* Heads Up!, *New York: E.S. Lowe Company, Inc., 1966.*]

3. MAKING EQUAL SETS. Child receives 100 markers. He must arrange these in as many equal sets as he can.

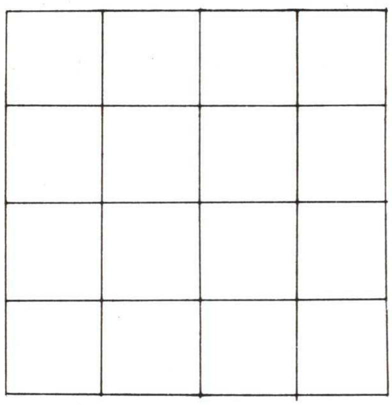
4. FIVES. Child receives sheets of squared paper. After placing 5 or 10 dots in each square, he practices counting up and down by fives or tens.

5. "SNAKES AND LADDERS." Make up a game like snakes and ladders with basic facts on the squares. Children take turns rolling a block with numbers 1 to 6 on it, and move accordingly. If they get the correct answer for the square they land on, their partner then rolls. If the answer is incorrect they miss their next turn.

6. "QUIZO." Similar to Bingo but have basic facts instead of numbers on card. Child must find expressions that equal the number called out in the column identified.

Magic Squares *(Willis)*

1. Fill in the squares below with the numbers 1 to 16 so that the sun in each row and each column equals 34. Can you do it so that the diagonals equal 34 too?



2. Use some 3 by 3 squares to make your own Magic Squares; first by repeating numbers, then without repeating numbers. Try the same procedure with some 4 by 4 squares.

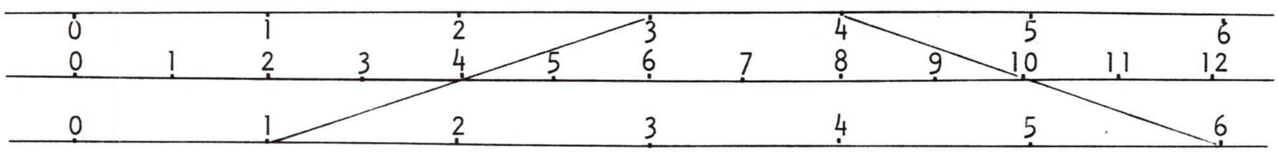
Times Table Graph *(Willis)*

1. Make a table for the two times table, such that □ = 2 △ , graph this relationship. Can you imagine what the graph of the 3,4 and 5 times table would look like.

2. Make a multiplication square using squared paper and number it from 1 to 16 along the left side and the bottom. When completed mark all the squares that have an answer of 16 in them. Draw a curve through them. Do the same for the squares that have the answer 12. What can you say about the curves?

Nomograph *(Inkster)*

Give the children the following picture:



Ask them if they can discover the purpose of the figure. Have them cut it out and use it to answer problems given to them by their partners.

Number Races *(Inkster)*

Make a large drawing on cardboard of a mountain ending at a giant's house. Problems or numbers are placed along the route. Progress is made from one point to the next by answering individual problems in sequence beginning with the first problem. Pupils may take turns in answering individual problems or in completing the entire trip. A good way to make the trip extensive is to label points with numbers. The pupil taking the trip is given a number to represent his steps. To advance he must give the answer to the sum, difference, product, or quotient of his number and the number on the trail.

Multiplication and Division *(Gregg)*

1. *Material* Colored strips of paper: 2" orange, 4" dark brown, 6" blue, 8" light brown, 10" yellow.

   Activity Use the orange strip to find the length of the other strips. If you know the orange strip is 2" long, how long are the others? Write a number sentence for each strip of paper. Show your activities in a display.

2. *Material* String, two feet long.

   Activity How many things can you find in the room that are two times longer than the string? Three times longer? Four times longer? Five times longer? Write a number sentence for each thing you measure. Display.

3. *Material* Squares of paper (two colors) about 6" x 6", pencils.

   Activity On four of the orange squares write yourfour favorite kinds of ice cream. On two of the blue squares write your two favorite kinds of cake. You are having a party - how many different kinds of desserts can you make if you use one piece of cake and one kind of ice cream for each dessert? Using three kinds of ice cream and three kinds of cake? Using five kinds of ice cream and two kinds of cake? Write a number sentence for each activity. Display.

4. *Material* Two balls (one large, one small), string, scissors.

   Activity Use the string to find the distance around each ball. If the distance around the large ball is eight and the small ball is four, how far is half way and one-quarter of the way around the balls? Write a number sentence for each. Display.

Make One *(Lenz)*

The purpose of this game is to get a sum as close to 1 as possible without going over. If you go over 1 you lose that hand.

Make a deck of cards with fractions in the upper left and lower right corners (2 cards each of all the halves, thirds, quarters, fifths, sixths, eighths, and even tenths).

To begin, you deal one card down to your partner and to yourself. If your partner wants a second card he says, "Hit me!" This and all other cards in this hand are dealt face up. He can have as many "hits" as he wants. You then play your own hand. The nearest to a sum of one is the winner.

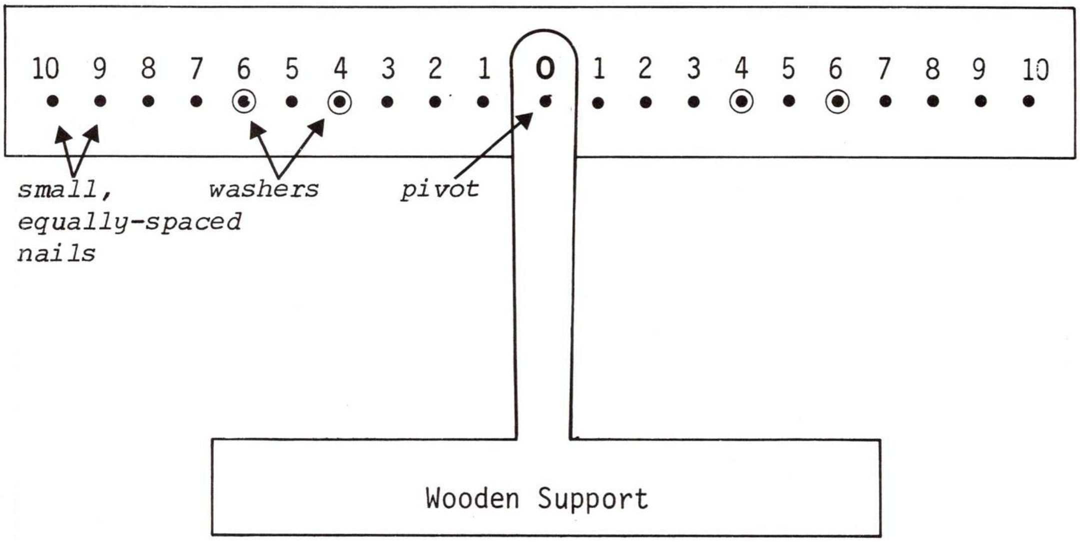
Magic Squares *(Feeney, Kanik)*

Put a fraction in each space in the "Magic Square" below so that the sum of each column and the sum of each row and the sum of each diagonal is equal to the same amount.

		$\frac{7}{8}$	$\frac{1}{4}$
$\frac{3}{4}$		$\frac{7}{16}$	
$\frac{1}{2}$	$\frac{5}{8}$		
$\frac{13}{16}$		$\frac{1}{8}$	1

Number Properties Balance[1] *(Wong, Woodeye)*

Basic number properties can be demonstrated using a simple, home-made wooden balance (as illustrated below) or any of the commercially available balances (such as the "Invicta" Mathematical Balance).



Illustrated above: 6 + 4 = 4 + 6 (commutative property of addition)

Other possibilities: 2 x 5 = 5 x 2 (commutative property of multiplication)
(*two washers on 5*) (*five washers on 2*)

(2 + 4) + 5 = 2 + (4 + 5) (associative property of addition)

9 + 3 = 8 + 4

Addition and Subtraction Slide Rules[2] *(Wong, Woodeye)*

Two wooden strips, two Bristolboard strips or two ordinary rulers placed together as shown below can be used for both addition and subtraction of whole numbers and fractions.

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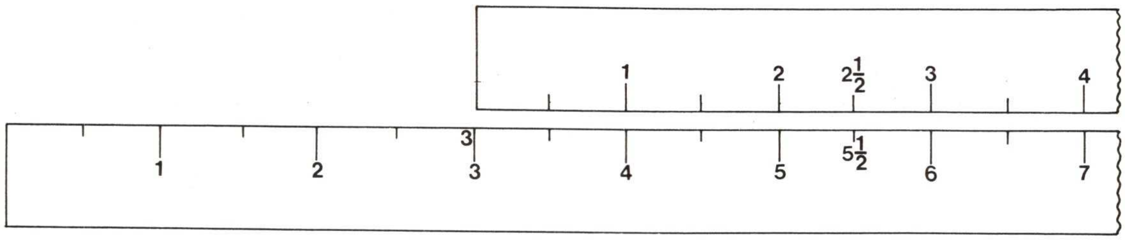
[1]R.D. Ripley and G.E. Tait, *Mathematics Enrichment*, Toronto: Copp Clark, 1966, p.43.

[2]R.D. Ripley and G.E. Tait, *op. cit.*, pp.46-47



E.g.: Addition $3 + 2\frac{1}{2} = 5\frac{1}{2}$
Subtraction $5\frac{1}{2} - 2\frac{1}{2} = 3$

Two metre sticks provide a good slide rule for adding and subtracting decimal fractions.

By extending the scales to include negative as well as positive numbers, a slide rule for adding and subtracting integers could be made.

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B. MEASUREMENT AND RELATIONSHIP

B.1 Measurement Concepts

Linear Measurement Activities : Grade II *(Bortnik)*

The assumption is made that the children will have had some previous experience with graphing and that they will have a record book in which to keep track of their findings. They will work in five groups and rotate from one activity to another so that each student can try every activity.

PART ONE

PURPOSE - TO GIVE PRACTICE WITH COMPARISON AND TO SHOW THE CHILDREN A NEED FOR STANDARD MEASURE.

Card 1 Take a ruler from the jar. How many things can you find that are shorter than it? Be sure to keep a record of these things.

Card 2 Find out how many ribbons are longer than the yardstick. How many are shorter than the yardstick? Be sure to keep a record.

Card 3 Find out which strings are shorter than you. Which strings are taller than you? Be sure to keep a record of the strings that are shorter or taller than you.

Card 4 Take a cardboard strip. Use cardboard to measure the reading table. Be sure to keep a record of what you find out.

Card 5 Measure the teacher's desk in handspans. Be sure to record what you have found out.

PART TWO

PURPOSE - TO GIVE PRACTICE WITH PRACTICAL MEASURE.

Card 6 Guess how wide our classroom is. Measure how wide it is. Be sure to record your guess and how wide you found the room to be.

Card 7 Measure some of the things you use in reading class. Be sure to keep a record of the things you have measured. Remember to record what you have found out about them.

Card 8 Ask a friend to help you, then measure (1) around your ankle (2) around your waist (3) around your wrist (4) around your knee. Be sure to keep a record of these measurements.

Card 9 Find out how tall you are. Mark how tall you are on the class graph. Find two ways of saying how tall you are. Print these in your record book.

Card 10 Guess how long the hall is. Measure it. Be sure to record your guess and how long you measure the hall to be.

PART THREE

PURPOSE - TO PROVIDE ACTIVITIES TO DISCOVER WAYS TO ESTIMATE, USING HANDSPAN AND SHOE LENGTHS (FOR THOSE WHO FINISH EARLY)

Card 11 Cut some paper strips - one inch, two inches, three inches, four inches, five inches, six inches. Mix them up. In your record book arrange them from the shortest piece to the longest. Glue them in.

Card 12 Find a classmate who is taller than you. How much taller is this classmate? Be sure to record what you find out.

Card 13 Find a classmate who is shorter than you. How much shorter is this classmate? Be sure to record what you find out.

Card 14 Measure your handspan. Use your handspan to help you to guess how high the reading table is. Compare your guess with the true measurement. Remember to keep a record.

Card 15 Measure your shoe length. Use your shoe length to help you guess how wide the hall is. Compare your guess with the true measurement. Remember to keep a record.

Getting to Know our Friends in Grade III *(Keeler)*

Since the children do not live in the area adjacent to the school in which I teach, one of our first projects is "Getting to Know our Friends in Grade Three." Working together in groups to make charts and drawings adds a personal touch that is lacking in the everyday routine of the arithmetic program.

ACTIVITIES

I. Find out where we live. Collect data.

a) How many miles from school?

b) Which direction?

c) In how many different ways do we get to school?

II. Correlate with Social Studies.

a) We learn about the Park Country so drawing a fairly large floor map (to scale if possible) is one of our tasks. City and district maps are helpful but sometimes confusing because there are so many details.

b) Find out how many buses bring the children to our classroom.

Trace the main bus routes, both city and country, and other modes of travel (bike, car, walk).

c) How many children travel by country bus, city bus, car, bike, foot?

d) In how many different ways can you show the distances travelled (blocks, paper strips, pipe cleaners, hollow straws)?

e) Would you use the same unit of measurement (for example, the block) for everyone? If not, why? What would you use?

III. Today the Indian children go to school just like we do, but in the olden times there were no schools like ours. Indian children "learned by doing." Let's pretend we are Indians and try to measure things without a ruler.

   a) In how many ways did the Indians measure (hands, feet, arrows, spears, etc.)?

   b) See how many things you can think of that Indians had to measure with to make canoes, teepees, bows, arrows, etc.

   c) Play "Find the Rock."

1. Chief hides the rock (about four spear lengths from the teepee).

2. Children measure the four lengths but have trouble finding the rocks. Why?

3. One child decides to measure in arrows instead of spears. Will the rocks be easier to find? Why?

4. Would moccasins be even easier, more accurate?

IV. Correlate with Health (for the more advanced pupils).

   a) Working with clocks
      - how long does it take each of us to get to school?

   b) Who has to ride the bus the longest? Why?
      - distance in miles from school,
      - direction bus is travelling when child picked up.

c) Safety rules
   - pedestrians, bikes
   - dangers in heavy traffic
   - giving yourself sufficient time to get to school on time.

d) Hours of sleep, bed time
   - Why is Chris tired? (On bus at 7:30 a.m. and 60 minutes on the bus is enough to tire anyone.)

MATERIALS Construction paper, chart paper, bristolboard, crayons, felt pens, scissors, pipe cleaners, drinking straws, plasticine, rulers, clock model, maps of city, district, book *Indians Knew* , by T.S. Pine and J. Levine, McGraw-Hill, New York, N.Y., 1957. Children bring toy Indians, etc (if they have them at home), data from home regarding where they live, bus route, etc., little sticks to make teepees and for measuring.

Measurement for Grade III *(Risvold)*

[*As a guide for planning, I have referred to the STA 3 text with the idea of replacing the teaching of concepts introduced in the text with the workshop approach. I have attempted to structure the questions so the children will go beyond these concepts. I have also kept in mind the aspects outlined in* Freedom To Learn *which are to be covered when working on measurement: comparison, conservation, reiteration, standard units, precisions and accuracy, metric system.*]

A preliminary whole class discussion would take about two days and would involve the following:

First Day - To establish the concepts of comparison, conservation, and reiteration, I would have the children work in groups of two or three. Each group would be given one of the following questions with 10 to 15 minutes in which to work out the answers. The questions are:

1. How many spans is the length and width of your desk?
2. How many reaches is the length of the blackboard?
3. How many reaches is the width of the room?
4. How many bodies is the length of the room?
5. How many thumbs long are these new pencils, paint brushes?
6. How many of your own feet is the length of the hall?
7. How many cubits is the length of the teacher's desk?
8. How many cubits is the length of the cupboard?
9. How many spans is the length of the bulletin board?
10. How many thumbs is the length and width of your math book?

Each group would have a few minutes to report its results to the rest of the class. In the discussion I would hope that the children come up with the idea that these are not good measuring devices because they are not consistent or accurate. After this idea has come up, we would discuss various standard units of measurement which are used every day.

Second Day - The children work in the same groups and fill out a personal inventory sheet, including height, weight, span, foot length. When these data are collected, we make graphs using square inch paper, tiles, square foot tiles, or graph paper (depending upon the understanding of the pupils). [*Previous to this I plan a unit on graphing, using the one by Ed Irmis (see Page 4) as a guide, so the second day should serve mainly as a review.*]

For the actual workshop, the class would be divided into four groups. All would receive a copy of the questions. They would be briefly discussed and then chosen by the groups. I would require that the data be presented in five or six different ways, accompanied by a verbal description. Extra activities would be available in the permanent math station for those who completed their work early. The open-ended questions would be mainly concerned with aspects of standard units, prevision and, if the children decided to use it, the metric system.

Length

1. How would you find the value of one yard of pennies, nickels, dimes, quarters?

2. Collect five objects and ask ten friends to estimate the length of each. How could you make a graph or chart to show how close the estimates are?

3. How could you measure what your pace is? Use your pace to measure the length of the gymnasium. How close were you to the exact measurement? Show this on a graph.

4. What is the height and reach of your classmates? How can you find out if they are square? How can you show this?

The group would then continue into the areas of weight, capacity and time. (Open-ended questions for these topics are incorporated in the weight, capacity and time sections following.)

Development of Standard Units of Length: Grade III *(Gregg)*

Arbitrary Measurement

MATERIALS Blocks, books, string, boxes, etc.

Pick an object from the table. Measure your partner with this object. How are you the same? Different? Who is bigger? Find something smaller than both of you and guess the measurement of it. Measure it.

Using Body Measurements for Measuring

MATERIALS Charts showing and naming body measurements (digit, palm, span, cubit, fathom, foot, pace).

1. Measure an object in the room using at least three body measurements to measure it.
   a) Are your results the same as your partner's?
   b) Which body measurement gave you the best measurement?
   c) Which took you the longest to do?

2. Find the distance around the areas of the playground. Use a body measurement to measure. Guess before you measure.
   a) Whose play area is the largest? Smallest? Are there any the same?

3. Use a body measurement to measure three or more of cupboards, floors, doors, hallways, windows, boards, desks, papers, books.
   a) Compare your measurements with those of others who measured the same thing. Are your results different? How can you explain this difference?

Development of the Need for Standard Units

Using string, find the length of your body measurements. Have your partner help you. Place your body measurements on the chart beside your name. (Class discussion of the differences in each person's body measurements leads to the need for something standardized. Introduce each child to three measuring sticks: inch, foot, yard. Each child receives three measuring sticks, unmarked.)

Relation of Body Measurements to Standard Measurements

1. Compare your inch, foot and yard to your body measurements. Are there any the same?

2. Measure a small, medium and a big object in the room. Use a body measurement first. Guess the number of standard units. Check, using standard units. How good was your guess?

3. Measure your reach and height. Are you a square?

Relation Within Standard Units Using Standard Units

1. Mark your yard measure stick using the other two measure sticks. Mark your foot measure stick. Now answer these questions:
   a) How many feet and inches are in three yards?
   b) How many inches are in two feet?
   c) How many inches and yards are in three feet?

2. Find two things that are 1 inch, 12 inches, 24 inches, 36 inches. Which measure stick will you use?

3. Find the length of the boards, room, doors, hall, windows, desk. Guess, then measure. Which measure stick will you use for each measurement?

Units of Length: Grade IV *(Merta)*

OBJECTIVES (a) To direct students of Grade IV level in discovering and experiencing concepts pertaining to distance, length, width, perimeter, circumference, broken lines, and comparison of units of length. (b) To introduce appropriate units for measuring length (inch, foot, yard, mile, and others of student discovery).

MATERIALS Ruler, yardstick, tape measure, string, discs or cans, tacks, stick of non-standard length, squared paper.

Experiences with Distances or Lengths in the Classroom

1. Walking heel to toe, measure the length of your classroom. Get three friends to do the same thing. Make a table of your results. Do you all have the same answer? Why do the answers differ?

2. How many steps do you and your friends get when you measure the length of the classroom? Why do the answers differ?

3. How could we find the exact length of the room? Measure the length and compare your answers with your other measurements.

4. Suppose we wanted to measure the distance from here to Mexico. Would we use these methods of measuring? Why?

5. Can you give other units that we could use to measure lengths or distances?

6. What would be a good unit of measurement to find -
   a) the distance to the moon
   b) the distance to the office
   c) the length of a thumb tack
   d) the length of my finger
   e) the length of my pencil
   f) how far I am from my home.

Estimating and Experimenting with Height

1. How high do you think the door is?

2. Use three ideas that may help you to find the height of the door. Compare the results.

3. Could we use these methods to find the height of a flagpole? The Husky Tower? A tall building?

4. Take two rulers and bolt them together at one end. Using squared paper and other items along with these rulers, how could you find the height of a tree?

5. Have four groups find the height of the flagpole. Do you all have the same answer? Why? Estimate the height first.

Perimeter (Estimating and Measuring Distances Around Things)

1. In measuring the distance around the classroom, is it necessary to measure the length of all four sides?

2. Can you arrive at a method that will help you find the perimeter of a rectangle? What is the secret?

3. Try this secret to find the perimeter of your desk, the teacher's desk, the blackboard, the table. Does it work?

4. Draw six triangles on the squared paper. Can you see the secret you used before?

5. Are there some special triangles that have a secret method for finding their perimeters? What kinds are these?

Circles (Diameter/Circumference Relationships)

1. Use the discs and measure the diameter and the circumference of each and record your findings on a chart. Draw a graph that shows the relationship.

2. What is the secret or the formula that you could use to find any answer?

3. Can you predict what the circumference would be if the diameter is 6 inches? Four and a half miles? Eleven yards? Sixty feet?

4. Can you predict from the graph what the diameter would be if the circumference is 18 inches? Four blocks? Eight feet?

Perimeters of Other Four-Sided Figures

1. Draw a neat picture of a trapezoid, a parallelogram, a quadrilateral, a rhombus. What do you notice about the sides of these figures?

2. What is the secret about finding the perimeter of the parallelogram?

3. Is there any special way of finding the perimeter of a quadrilateral?

4. What is the secret for finding the perimeter of a rhombus?

5. Why does the trapezoid have no two sides the same length?

6. Draw a neat diagram for each of the figures and pass it to your friend. Have him find the perimeter of each of the figures. Do you agree with his answer?

7. Can you name some things in the school, at home, or up town that are in the form of a rhombus? A parallelogram? A trapezoid? A quadrilateral?

8. How do you think an engineer would find the perimeter for these figures?

Distances for Broken Lines

1. The road map says that it is 560 miles to Vancouver but the pilot says that it is only 480 miles. Who is right? Why?

2. Why can we not always go in a straight line when we travel?

3. Can you find some rule that would help us find the distance between two towns if there were obstructions in between? Tell the class about your method.

4. Find the distance from the door to the top of the flagpole in feet, yards, steps, inches, sticks. Do you get the same answer each time? Why, or why not?

Making Graphs for Units of Length

Complete each of the following tables, then plot each on a graph. Make a large one for display.

Inches	Feet	Feet	Yards	Yards	Miles	Feet	Miles
12	1	0	0	1760	1	5280	1
	2	3	1		2		2
	3		2		3		3
	4		3		4		4
	5		4		5		5
	6		5		6		6
	.		6		.		7
	.		.		.		.

What is the secret for each relationship?

By using the graphs find the answers for the following.

38 inches =	feet
7 yards =	feet
80 feet =	inches
9 miles =	yards
40 yards =	feet
90 feet =	yards
900 yards =	miles

Make up some to test your friends.

Workshop in Measurement: Grade IV *(Koleyak, Misselbrook, Ritcey)*

OBJECTIVE To give the children experience in using various units of measurement. This is solely an introduction to buidiing readiness for more extensive experiences in measurement.

PART ONE

Open-ended questions

Try three different methods for each of the following.

1. What would you use to measure your wrist?
2. What would you use to measure your neck and your waist?
3. What do you guess is your wrist measurement?
4. What is it after you measure it?
5. What do you guess your neck measurement to be?
6. What is it really?
7. How many wrist measurements will you need for your neck measurement?
8. What do you guess your waist measurement to be?
9. What is the right measurement?
10. How many neck measurements will go around your waist?
11. Are your measurements the same as your partner's?
12. Where did you differ?

PART TWO (Accompanied by drawings of a giant and a dwarf)

Here is a team of acrobats who are giants and dwarfs. All the giants are the same height. All the dwarfs are the same height. All the heights are given in feet.

1. What is the height of the giant and the dwarf together?
2. What is the height of the giant less the height of the dwarf?
3. What is the height of the giant?
4. What is the height of the dwarf?
5. What is the height of a giant and two dwarfs together?
6. What is the height of a giant less the height of two dwarfs?

PART THREE

1. Have you a shadow?
2. Is it always the same length?
3. What is your height at 10:15 a.m., 11:15 a.m., 12:15 p.m., 1:15 p.m., 2:15 p.m.?
4. What is your shadow length at 10:15 a.m., 11:15 a.m., 12:15 p.m., 1:15 p.m., 2:15 p.m.?
5. How do you compare your shadow lengths to your height?
6. At what time would you expect your shadow to be the same length as your height?
7. At what time would you expect your shadows to be near the same length?
8. Do you and your shadow ever make a square?
9. Do you and your shadow ever make a triangle?

Group Work on Linear Measurement: Grades IV, V, VI *(Burton)*

Pacing

For 1 to 2 pupils

1. Is it useful to know how long your pace is? To find out, mark a chalk line in the yard and walk ten paces at your usual speed and comfortable stride.

2. When you have walked ten paces, mark the yard with chalk again.

3. Measure the distance between the starting chalk mark and the chalk mark where you finished.

4. What must you do to find the length of one of your paces? Discribe what you did and record the length of your pace.

5. Do the same thing for your running stride.

Trundle Wheel (Division 2)

For 2 to 3 pupils

1. Measure out 110 yards.

2. Walk the 110 yards you measured at your normal speed and find out how long it takes you to walk this distance. Use a stop watch.

3. Find out how many yards are in a mile. What fraction of a mile is 110 yards?

4. Can you find out an easy way to discover how long you would take to walk a mile at your normal speed?

Early Measures of Length (Division 2)

For 1 to 5 pupils

1. Use your pencil as a unit of measure and estimate how long your desk is in "pencils."

2. Measure how long your desk is in "pencils."

3. Estimate how wide your desk is in "pencils."

4. Measure how wide your desk is in "pencils."

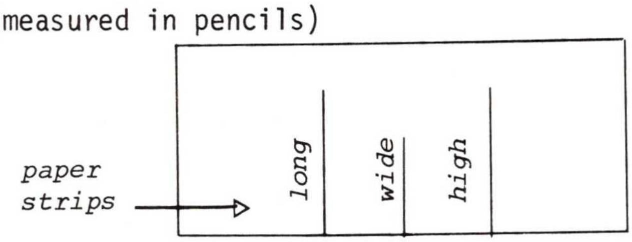
5. Estimate how high your desk top is in "pencils."

6. Measure how high your desk top is in "pencils."

Linear Measurement

Name: ____________________

My Desk (measured in pencils)



Spans

For 3 to 5 pupils

1. Each member of the group guesses the height, in spans, of each of the other persons in the group. Write it down.

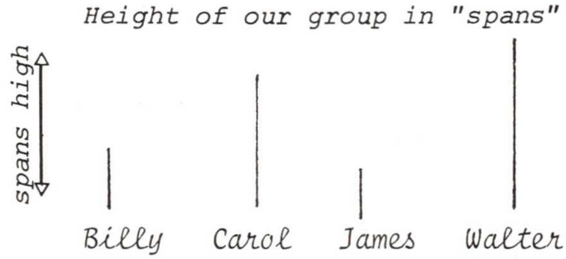
   - A span (hand span) is

2. Each person measures the others in the group and writes it down.

3. Did all in the group get the same answer for each person's height?

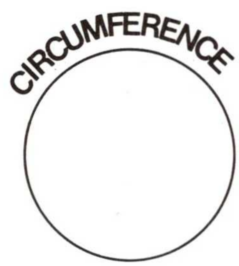
4. Why didn't they get the same answer?

5. Draw a bar graph to show the "spans high" of each member of the group.



Circumference I

1. Collect some round objects and measure all around them. This measurement is called the *circumference*



2. Can you find a way to do this?

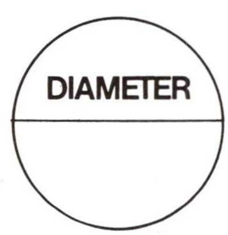
3. Draw the objects and write down each circumference to make a bar-type graph showing the circumferences.

4. Tell how you measured all around the objects you collected.

Circumference and Diameter II

For 1 to 5 pupils

1. Measure the distance across the middle of circles you want to draw (this is called the *diameter*).



2. Write down the diameter and the circumference of each circle in a table like the one below.

	Circumference (Distance around)			Diameter (Distance across)			
	Object	*Estimate*	*Measurement*	*Difference*	*Estimate*	*Measurement*	*Difference*
1							
2							
3							
4							
5							

3. Can you find a relationship between the circumference and the diameter of a circle? Make a chart like the one below and see if you can find the relationship.

	Round object	*Circumference (C)*	*Diameter (D)*	*(C+d)*	*(C-d)*	*C×d)*	*(C÷d)*
1							
2							
3							
4							
5							

4. Which column shows the special relationship between circumference (C) and diameter (d)?

5. What is the relationship between circumference and diameter?

Perimeter

For 2 pupils

1. The *perimeter* is the *distance around* something.

2. On a piece of graph paper each person draw a square, a rectangle, a triangle, and one or two other figures. Change papers.

3. Estimate the perimeter of each of the figures and write it down by the drawing.

4. Measure the perimeters of the figures and write them down by the drawings.

5. What was the difference between your estimate and measurement?

6. Are you a good estimater? If not, have your partner draw more squares, rectangles and other figures for you.

For 1 pupil

1. Obtain a new pencil and find out how many pencils, placed end to end, would go all around the room.

2. Do you have to crawl all around the room measuring with your pencil? What is a quick way of doing this? (Record your answers as follows.)

   ____ pencils placed end to end would go all around the room.
   An easy way to find this answer is to ______________________________

For 2 or 3 pupils

1. Estimate, then measure the perimeter (distance around) of places and things in the school, for example, the room, the yard, the chalk box, the S.R.A. box. Write down your estimates and measurements on a chart.

2. What was the difference between your estimate and the measurement? Are you good at guessing perimeters?

3. If you are measuring the perimeter of a square, do you have to measure all the sides? Show what sides you must measure on a diagram.

4. If you are measuring a rectangle, do you have to measure all the sides? Show, on a diagram, what sides you must measure.

How long would it take to walk a mile?

For 2 to 5 pupils

1. Measure 1/4 mile on the field or the running track. Do you have a 1/4 mile tape measure? If not, what other way can you measure 1/4 mile?

2. Write down how long it takes you to walk 1/4 mile. Use a stop watch.

3. What must you do to your answer to find out how long it would take you to walk one mile?

4. How many miles is it to your house from school? Tell how long it would take to walk from home to school.

5. How far do you live from Calgary? Okotoks? High River? Black Diamond? Cayley? Longview? Turner Valley? Millarville?

6. Figure out how long it would take you to walk to some of these places. Give the exact times and then round your times off to the nearest quarter hour. Use the table below to record your answers.

Length of time it would take me to walk to the following towns

Town	*Distance*	*Exact time to walk*	*Approximate time to walk*

Linear Measurement, Daily Use

For 1 pupil

Make up a book showing how people use measurements in their work. Use pictures and a short story to tell what people are doing. Some people in your town whom you might ask for information are people who make things, sell, test and build things.

Standards of Length (inches)

For 1 pupil

1. Collect some long objects and measure them in inches. Draw them and write down their sizes.

2. Collect containers (garbage can, plant pot, juice can) and measure how high they are inside. Is the height the same as the depth in every case? Can you measure in inches and parts of an inch?

For 2 or more pupils

1. Cut strips of construction paper into the following sizes: 1 inch, 2 inches, 3 inches, 4 inches, 5 inches, 6 inches, 7 inches, 8 inches, 9 inches, 10 inches, 11 inches, 12 inches. Mix them up.

2. Arrange them from the shortest piece to the longest piece on a table.

3. Ask a friend to pick out the 6-inch length. Did he pick up the right one? Ask him to pick out another length. How many did he get right out of five lengths you asked him to pick out?

4. Ask other friends to pick out lengths you call out.

5. To make it more difficult you can mix up all the lengths and then lay them out on the desk for your friends to pick the sizes you ask them to.

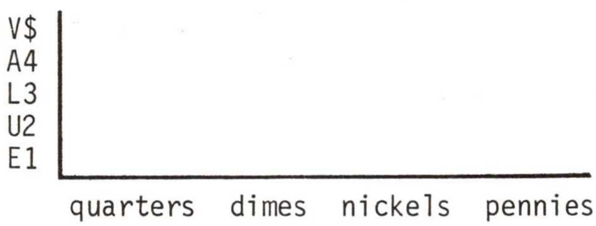
6. Ask your friend to call out lengths and see how many you get right out of five calls.

Money

For 1 to 5 pupils

1. Find the value of 1 yard of quarters, dimes, nickels, pennies.

2. Make a bar graph of your results.



Standards of Length (inches, feet, yards)

For 1 to 5 pupils

1. Each person in the group guess the length of the room and write it down. (Pick the most suitable units of length.)

2. Each person measure the length of the room. Did you have to use more than one unit of measure?

3. Were all your guesses the same?

4. Were all your measurements the same?

5. Each person work out if their guess was too long or too short.

6. How much too long or too short was your guess?

7. Pick a parner and measure the width of the room and write it down. (Pick the most suitable units of measure.)

8. Could you draw the room to scale on a piece of paper showing the length and width?

9. Use a scale where 1/4 inch stands for 1 foot.

	Things I Measured	*Guessed Length*	*Measured Length*	*Difference**
1				
2				
3				

**Put a check if you feel your guess was good.*

Estimating Distances

For 2 to 3 pupils

1. Draw 3 or 4 lines of different lengths in the yard with chalk.

2. Ask your partner to guess the lengths of the lines. (This is called *estimating* their lengths.)

3. Measure the lines to see how closely you estimated.

4. Have your partner draw lines. Try to estimate their lengths before you measure them.

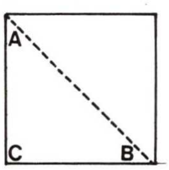
5. Do your estimates get better with practice?

Estimating Height

For 2 or 3 pupils

There are many ways of estimating height. One way is described below.

1. Make a 4-inch square in a thin card. Draw a line from one corner to the opposite corner. Cut along this line (A,B). Make the other corner "C." This triangle will help you to estimate the height of the school, telephone poles, etc.



2. To measure the height of the school, put point "B" at the end of your nose. Shut one eye and move back from the school until point "A" seems to be at the top of the building. Side AC must be upright.

3. Stand still and have your partner place a mark where you are standing. Measure the distance from the school to where your partner placed the mark. The height of the school is about the same as the distance from the school to your mark. To be more exact, add the height of your eye above the ground to the distance from the school.

Time Measurement *(Grigel)*

(Ages 6 to 9; M.A. 4 to 6)

OBJECTIVES (Primary General Learning Disabilities)

1. To promote in the children a general idea of time and how various times relate to various activities during the day.

2. To have the children recognize a need for standardized time measurements.

3. To teach the children to tell time--hours and half hours.

4. To have the children recognize the relative length of a minute, second, hour.

EXPLANATION In working with the cards, care must be taken to have at least one "reader" in each group to interpret the questions to the nonreaders. Groups of three or four would prove best at this level. The previous knowledge of the children is very limited. Only a few would have any concept of time and some will not know their numbers. I would have introduced some very simple pictographs and possible block graphs prior to this series of cards. On some of the cards, questions are asked that only a few of the children will be able to answer. These can act as "teachers" for the others of the group. The cards are quite structured in keeping with our theory of a structured (but not rigid) approach to the teaching of retarded children.

Card 1: How many handclaps does it take for your friend to walk around the table? Let him clap his hands to time you. Using heartbeats see how many it takes for each person to walk around the table. What other ways can you think of to time things. Are these good ways to time things? Why?

Materials - no special materials for this card.

Card 2: Look at Clock 1 and put the numbers on the face. Can you put them on Clock 2 without looking at the big clock? What can you say about the numbers on the clock?

Materials - mimeographed sheets of two clock faces without the numbers on identified as Clock 1 and Clock 2.

Card 3: The two black shapes are hands. What is different about them? Find the minute hand, and hour hand. Put them on your big clock. Show on your big clock a. when you get to bed, b. when you get up, c. recess, d. noon hour, e. home time, f. supper time.

Materials - clock faces approximately 12" diameter; 2 hands for each and fastener to put on hands.

Card 4: Find out when everyone goes to bed. Make a graph to show this. What time do most children go to bed? Why? Can you do the same thing with the time that each person gets up? Can you tell how many hours of sleep each child has? The teacher? Why is it different? (This is a "hard" chart. Only a few of the children will be be able to complete it.)

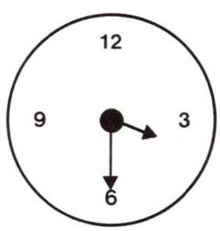
Materials - block graph paper.

Card 5: On the page of clocks show:
8:00 2:00
5:00 9:00
3:00 12:00

What do you see about the minute hand? Take your big clocks and show other times that the minute hand does the same thing. Tell what time you have.

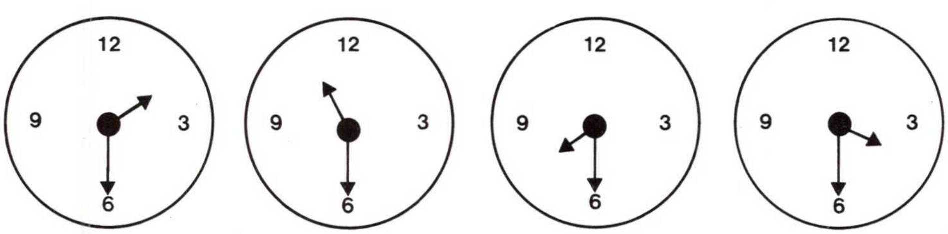
Materials - mimeograph sheet with six clock faces. Clock faces with hands from card 3.

Card 6: On your big clock put the minute hand at 6. Now move the hour hand. Can you tell what time you show in each case? We write this:



3:30

How would you write:



Show on your big clock:
6:30   9:30
5:30 10:30
1:30 12:30

Materials - clock faces from card 3 or small individual clock faces from a watch company.

Card 7: Can you hold your breath for 10 seconds? 10 minutes? 10 hours? Why? How can you find out who can hold their breath the longest? Why are there differences?

Materials - have a watch with a second hand available. Hopefully someone will think to ask for it!

Card 8: Can you find pictures of other ways people use to tell time? Ask your father or mother to help you. You can tell us about what you find out. Maybe you could make a model for the class. An attempt to involve parents!

Materials - vary with the class.

Time Unit: Grades II, III *(Penner)*

1. Can you tell how much time you spend playing in one day? One week? One month? How can you show this?

2. How much time do you sleep? Compare this with others. How can you show this?

3. Make a graph to show how many hours you spend on each activity during your day. What other way can you show your results?

4. In how many different ways can you measure time? Make a chart to show as many of these ways as you can.

5. How much time does it take to walk the length of the hall? hop? skip? move backwards? with eyes shut? downstairs? to school? with a heavy box? Guess your answers first. Compare with time taken by others. Show your results.

6. Using a TV guide, find the time between two specific programs. Find the time you would spend if you watched all the programs you like. Show this in some way.

7. Using a calendar, can you find out what a leap year is and why we have it? Report on what you find. Do all the months have the same number of days? Why or why not? How can you show this?

8. What do you do before breakfast? Between breakfast and school time? Before dinner time? After dinner time? After school? Before bedtime? Show this in some way.

9. What time do you get up? Who at your house gets up earlier? Later? Same time? Do the children in your class get up at the same time? How many get up earlier? Later? Show your results. Repeat this for bed times.

10. What things travel fast? Slowly? Find or draw pictures.

11. How long does it take you to travel across the playground? Your friend? Try this more than once. Show your results.

12. What things can you do for a longer time than your friend? Show this in any way you wish.

13. Make a sand timer with 2 bottles, sand, plasticine. Use it to measure the time taken to do various activities. Compare with other children. Record the results.

14. Make a water timer with a pan of water and tin can. Do similar activities to those with the sand timer.

15. String timer (pendulum). Make it using plasticine and string. Count swings. Time various activities by counting swings. Vary length of string. What happens? Record your results.

16. How many different clocks have you seen at home? At school? Outside? Show as many different kinds as you can.

17. Use the clock. Show a time on it. Write the time in as many different ways as you can. Repeat for at least six different times.

Time Questions: Grade III *(Risvold)*

1. Using a clock with a second hand, find out:

    a) How many times the second hand goes around in 1 minute, 5 minutes, 10 minutes. Is there a pattern?

    b) How many times the second hand will go around in 1 hour, 10 hours, 1 day.

    c) How many times the minute hand goes around in 1 hour, 2 hours, 5 hours, 1 day, . . .

    d) How many times the hour hand will go around the clock in 1 day, 2 days, . . .

    Make a table showing what you have learned.

2. a) Using a pendulum made from string and a weight, find out what happens as the length of the string is changed. How can you make a pendulum speed up? slow down?

b) How much time does it take for your pendulum to make 10, 20, 30 swings? Try different pendulums. Would a pendulum be good for measuring time?

3. How do you spend the number of hours in a week? Make a chart to show how 5 classmates spend the time in a week.

4. Could you use your pulse to measure time? How can you find out how many times your pulse beats in one minute? Does your pulse stay beating at the same speed? Why? Does everyone have the same pulse rate? Would pulses be good for measuring time?

Pendulum: Grades IV, V, VI *(Eremko)*

MATERIALS Stop watch (or watch), string, weights (balls, lead, etc.), tape measure, balance scale.

1. Measure how far a pendulum travels in each swing.

2. Does the swing of a pendulum vary with the length of a string?

3. Using different lengths, how many strokes in one minute? Half a minute?

4. Use different weights. Does a heavy "bob" swing faster or slower than a light "bob"?

5. Measure the size of the "bob." How does the size of the"bob" affect the swing of the pendulum?

6. Graph the relationship between the swing of a pendulum and time interval.

7. What every-day uses has a pendulum?

8. Using a stop watch can you estimate the length of a pendulum with 30 strokes per second? Sixty strokes per second?

9. How can a pendulum be used to measure time?

10. Using a pendulum, calculate how long it takes to walk 10 feet, 15 feet, 20 feet.

11. How long would it take to walk 50 feet, 100 yards, 400 yards, 1 mile?

12. Using a pendulum, discover how long it takes your classmate to run a dpecified distance.

13. Using a pendulum, determine how long it takes someone to complete a task. What is the fastest time? The slowest time?

14. How else can a pendulum be used to measure pace and time intervals?

15. If a pendulum made one swing every second, how many swings would it make in 30 seconds, 60 seconds, 10 hours, 24 hours?

Money Activities *(Inkster)*

1. Write out a list of sums, such as 25¢, $.33, $1.05, $.08, and so on. Using play money have the children illustrate how many ways they can show those amounts of money.

2. Cut out some pictures (baseball glove, chocolate bar, etc.) and make up questions concerning change. Have the children show in how many ways they can give you change for those items.

3. Write down some amounts of money. With coins, have the children illustrate how many ways they can show you this amount using the fewest coins, most coins.

4. Set up a "store" with various items priced. Ask in how many ways one can pay for them. Have your partner overpay you. Can you give him the correct change in more than one way?

5. Your partner gives you a certain amount of money and asks you questions such as, "If you had this much money what items could you buy?" "Would you get change?" "How much?"

Money Assignment Cards: Grade III *(Keddie)*

MATERIALS Real money (amount will depend upon the cards used), collection of toys for each group, foot ruler for each group, apparatus for weighing, masking tape for price tags. [A letter to the parents explaining your activity will usually provide much of the material.]

IMPLEMENTATION Vary the procedures to suit the topic, the children and yourself. Set up five or six activity centers all on the same assignment and five or six activity centers each on a different topic or different aspect of the same topic. Pairs of children can work together, either on an individual basis or with a friend. [This could be your week's work since the groups rotate each day.]

Activity alone is not enough. There must be discussion with an understanding teacher who can help pupils to verbalize their experiences and clarify their ideas. This can be done during the activity period when the teacher is free to circulate from group to group, talking with children individually.

The reporting session at the end of an activity period is also extremely important. It is then that children discuss their problems, listen to other groups describe how they worked together, and generally consolidate their learning, extend their vocabulary, improve their thinking skills, and develop their concepts of social cooperation.

You
If you had as many pennies as your age, how much would you be worth? What would be the least number of coins you could use to find your worth? Compare your results with those of your partner.

Coins
Use your box of coins to find out how few coins can be used to make each of these amounts: 19¢ 41¢ 75¢ 91¢ 97¢

Coins
Can you arrange your coins in patterns to show relationships between the coins? Tell about your results.

Coins
Count all the money in your box. Share the money. How much is your share? Sell the toys to each other using your share of the money. One person will be the clerk and the other person will be the customer. Change the price of the toys as often as you wish.

Money
Arrange your coins in piles. How much money is in each pile? Now make up an addition sum to find out how much money there is in all the piles. Can you make up any subtraction sums to prove that your answer is correct? Do this as many times as you like.

By the Foot
Estimate, then find the value of one foot of each kind of coin that we use. Compare your results.

$1
Estimate, then find the weight of $1 worth of each of the coins we have used. Can you manage with just a few of each kind of coin? Compare your results.

5. Have four groups find the height of the flagpole. Do you all have the same answer? Why? Estimate the height first.

Perimeter (Estimating and Measuring Distances Around Things)

1. In measuring the distance around the classroom, is it necessary to measure the length of all four sides?

2. Can you arrive at a method that will help you find the perimeter of a rectangle? What is the secret?

3. Try this secret to find the perimeter of your desk, the teacher's desk, the blackboard, the table. Does it work?

4. Draw six triangles on the squared paper. Can you see the secret you used before?

5. Are there some special triangles that have a secret method for finding their perimeters? What kinds are these?

Circles (Diameter/Circumference Relationships)

1. Use the discs and measure the diameter and the circumference of each and record your findings on a chart. Draw a graph that shows the relationship.

2. What is the secret or the formula that you could use to find any answer?

3. Can you predict what the circumference would be if the diameter is 6 inches? Four and a half miles? Eleven yards? Sixty feet?

4. Can you predict from the graph what the diameter would be if the circumference is 18 inches? Four blocks? Eight feet?

Perimeters of Other Four-Sided Figures

1. Draw a neat picture of a trapezoid, a parallelogram, a quadrilateral, a rhombus. What do you notice about the sides of these figures?

2. What is the secret about finding the perimeter of the parallelogram?

3. Is there any special way of finding the perimeter of a quadrilateral?

4. What is the secret for finding the perimeter of a rhombus?

5. Why does the trapezoid have no two sides the same length?

6. Draw a neat diagram for each of the figures and pass it to your friend. Have him find the perimeter of each of the figures. Do you agree with his answer?

7. Can you name some things in the school, at home, or up town that are in the form of a rhombus? A parallelogram? A trapezoid? A quadrilateral?

8. How do you think an engineer would find the perimeter for these figures?

Distances for Broken Lines

1. The road map says that it is 560 miles to Vancouver but the pilot says that it is only 480 miles. Who is right? Why?

2. Why can we not always go in a straight line when we travel?

3. Can you find some rule that would help us find the distance between two towns if there were obstructions in between? Tell the class about your method.

4. Find the distance from the door to the top of the flagpole in feet, yards, steps, inches, sticks. Do you get the same answer each time? Why, or why not?

Making Graphs for Units of Length

Complete each of the following tables, then plot each on a graph. Make a large one for display.

Inches	Feet	Feet	Yards	Yards	Miles	Feet	Miles
12	1	0	0	1760	1	5280	1
	2	3	1		2		2
	3		2		3		3
	4		3		4		4
	5		4		5		5
	6		5		6		6
	.		6		.		7
	.		.		.		.

What is the secret for each relationship?

By using the graphs find the answers for the following.

38 inches =	feet
7 yards =	feet
80 feet =	inches
9 miles =	yards
40 yards =	feet
90 feet =	yards
900 yards =	miles

Make up some to test your friends.

Workshop in Measurement: Grade IV *(Koleyak, Misselbrook, Ritcey)*

OBJECTIVE To give the children experience in using various units of measurement. This is solely an introduction to buidiing readiness for more extensive experiences in measurement.

PART ONE

Open-ended questions

Try three different methods for each of the following.

1. What would you use to measure your wrist?
2. What would you use to measure your neck and your waist?
3. What do you guess is your wrist measurement?
4. What is it after you measure it?
5. What do you guess your neck measurement to be?
6. What is it really?
7. How many wrist measurements will you need for your neck measurement?
8. What do you guess your waist measurement to be?
9. What is the right measurement?
10. How many neck measurements will go around your waist?
11. Are your measurements the same as your partner's?
12. Where did you differ?

PART TWO (Accompanied by drawings of a giant and a dwarf)

Here is a team of acrobats who are giants and dwarfs. All the giants are the same height. All the dwarfs are the same height. All the heights are given in feet.

1. What is the height of the giant and the dwarf together?
2. What is the height of the giant less the height of the dwarf?
3. What is the height of the giant?
4. What is the height of the dwarf?
5. What is the height of a giant and two dwarfs together?
6. What is the height of a giant less the height of two dwarfs?

PART THREE

1. Have you a shadow?
2. Is it always the same length?
3. What is your height at 10:15 a.m., 11:15 a.m., 12:15 p.m., 1:15 p.m., 2:15 p.m.?
4. What is your shadow length at 10:15 a.m., 11:15 a.m., 12:15 p.m., 1:15 p.m., 2:15 p.m.?
5. How do you compare your shadow lengths to your height?
6. At what time would you expect your shadow to be the same length as your height?
7. At what time would you expect your shadows to be near the same length?
8. Do you and your shadow ever make a square?
9. Do you and your shadow ever make a triangle?

Group Work on Linear Measurement: Grades IV, V, VI *(Burton)*

Pacing

For 1 to 2 pupils

1. Is it useful to know how long your pace is? To find out, mark a chalk line in the yard and walk ten paces at your usual speed and comfortable stride.

2. When you have walked ten paces, mark the yard with chalk again.

3. Measure the distance between the starting chalk mark and the chalk mark where you finished.

4. What must you do to find the length of one of your paces? Discribe what you did and record the length of your pace.

5. Do the same thing for your running stride.

Trundle Wheel (Division 2)

For 2 to 3 pupils

1. Measure out 110 yards.

2. Walk the 110 yards you measured at your normal speed and find out how long it takes you to walk this distance. Use a stop watch.

3. Find out how many yards are in a mile. What fraction of a mile is 110 yards?

4. Can you find out an easy way to discover how long you would take to walk a mile at your normal speed?

Early Measures of Length (Division 2)

For 1 to 5 pupils

1. Use your pencil as a unit of measure and estimate how long your desk is in "pencils."

2. Measure how long your desk is in "pencils."

3. Estimate how wide your desk is in "pencils."

4. Measure how wide your desk is in "pencils."

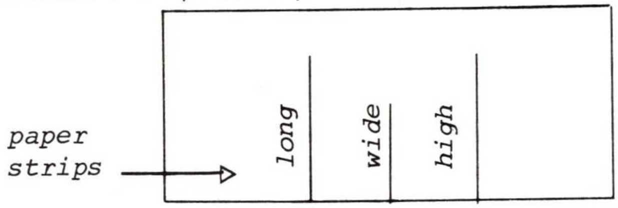
5. Estimate how high your desk top is in "pencils."

6. Measure how high your desk top is in "pencils."

Linear Measurement

Name: ____________________

My Desk (measured in pencils)



Spans

For 3 to 5 pupils

1. Each member of the group guesses the height, in spans, of each of the other persons in the group. Write it down.

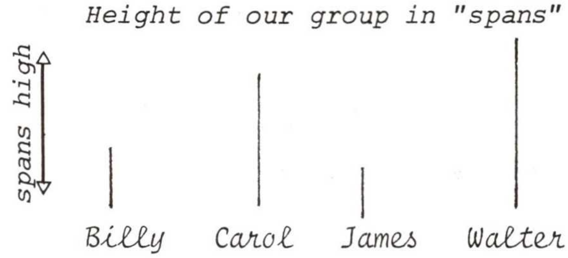
   - A span (hand span) is

2. Each person measures the others in the group and writes it down.

3. Did all in the group get the same answer for each person's height?

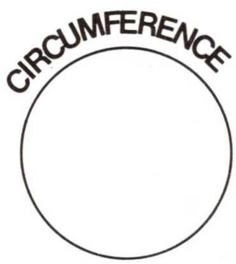
4. Why didn't they get the same answer?

5. Draw a bar graph to show the "spans high" of each member of the group.



Circumference I

1. Collect some round objects and measure all around them. This measurement is called the *circumference*



2. Can you find a way to do this?

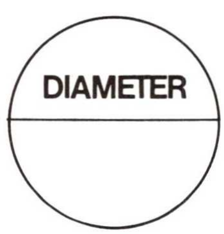
3. Draw the objects and write down each circumference to make a bar-type graph showing the circumferences.

4. Tell how you measured all around the objects you collected.

Circumference and Diameter II

For 1 to 5 pupils

1. Measure the distance across the middle of circles you want to draw (this is called the *diameter*).



2. Write down the diameter and the circumference of each circle in a table like the one below.

	Circumference (Distance around)				Diameter (Distance across)		
	Object	Estimate	Measurement	Difference	Estimate	Measurement	Difference
1							
2							
3							
4							
5							

3. Can you find a relationship between the circumference and the diameter of a circle? Make a chart like the one below and see if you can find the relationship.

	Round object	Circumference (C)	Diameter (D)	(C+d)	(C-d)	C×d)	(C÷d)
1							
2							
3							
4							
5							

4. Which column shows the special relationship between circumference (C) and diameter (d)?

5. What is the relationship between circumference and diameter?

Perimeter

For 2 pupils

1. The *perimeter* is the *distance around* something.

2. On a piece of graph paper each person draw a square, a rectangle, a triangle, and one or two other figures. Change papers.

3. Estimate the perimeter of each of the figures and write it down by the drawing.

4. Measure the perimeters of the figures and write them down by the drawings.

5. What was the difference between your estimate and measurement?

6. Are you a good estimater? If not, have your partner draw more squares, rectangles and other figures for you.

For 1 pupil

1. Obtain a new pencil and find out how many pencils, placed end to end, would go all around the room.

2. Do you have to crawl all around the room measuring with your pencil? What is a quick way of doing this? (Record your answers as follows.)

   ____ pencils placed end to end would go all around the room.
   An easy way to find this answer is to ______________________________

For 2 or 3 pupils

1. Estimate, then measure the perimeter (distance around) of places and things in the school, for example, the room, the yard, the chalk box, the S.R.A. box. Write down your estimates and measurements on a chart.

2. What was the difference between your estimate and the measurement? Are you good at guessing perimeters?

3. If you are measuring the perimeter of a square, do you have to measure all the sides? Show what sides you must measure on a diagram.

4. If you are measuring a rectangle, do you have to measure all the sides? Show, on a diagram, what sides you must measure.

How long would it take to walk a mile?

For 2 to 5 pupils

1. Measure 1/4 mile on the field or the running track. Do you have a 1/4 mile tape measure? If not, what other way can you measure 1/4 mile?

2. Write down how long it takes you to walk 1/4 mile. Use a stop watch.

3. What must you do to your answer to find out how long it would take you to walk one mile?

4. How many miles is it to your house from school? Tell how long it would take to walk from home to school.

5. How far do you live from Calgary? Okotoks? High River? Black Diamond? Cayley? Longview? Turner Valley? Millarville?

6. Figure out how long it would take you to walk to some of these places. Give the exact times and then round your times off to the nearest quarter hour. Use the table below to record your answers.

Length of time it would take me to walk to the following towns

Town	*Distance*	*Exact time to walk*	*Approximate time to walk*

Linear Measurement, Daily Use

For 1 pupil

Make up a book showing how people use measurements in their work. Use pictures and a short story to tell what people are doing. Some people in your town whom you might ask for information are people who make things, sell, test and build things.

Standards of Length (inches)

For 1 pupil

1. Collect some long objects and measure them in inches. Draw them and write down their sizes.

2. Collect containers (garbage can, plant pot, juice can) and measure how high they are inside. Is the height the same as the depth in every case? Can you measure in inches and parts of an inch?

For 2 or more pupils

1. Cut strips of construction paper into the following sizes: 1 inch, 2 inches, 3 inches, 4 inches, 5 inches, 6 inches, 7 inches, 8 inches, 9 inches, 10 inches, 11 inches, 12 inches. Mix them up.

2. Arrange them from the shortest piece to the longest piece on a table.

3. Ask a friend to pick out the 6-inch length. Did he pick up the right one? Ask him to pick out another length. How many did he get right out of five lengths you asked him to pick out?

4. Ask other friends to pick out lengths you call out.

5. To make it more difficult you can mix up all the lengths and then lay them out on the desk for your friends to pick the sizes you ask them to.

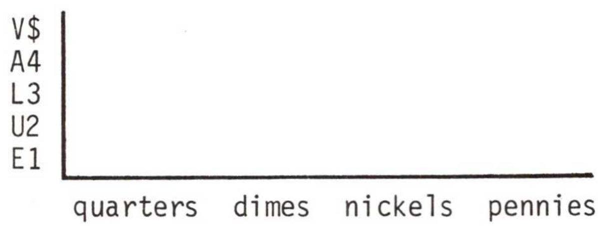
6. Ask your friend to call out lengths and see how many you get right out of five calls.

Money

For 1 to 5 pupils

1. Find the value of 1 yard of quarters, dimes, nickels, pennies.

2. Make a bar graph of your results.



Standards of Length (inches, feet, yards)

For 1 to 5 pupils

1. Each person in the group guess the length of the room and write it down. (Pick the most suitable units of length.)

2. Each person measure the length of the room. Did you have to use more than one unit of measure?

3. Were all your guesses the same?

4. Were all your measurements the same?

5. Each person work out if their guess was too long or too short.

6. How much too long or too short was your guess?

7. Pick a parner and measure the width of the room and write it down. (Pick the most suitable units of measure.)

8. Could you draw the room to scale on a piece of paper showing the length and width?

9. Use a scale where 1/4 inch stands for 1 foot.

	Things I Measured	*Guessed Length*	*Measured Length*	*Difference**
1				
2				
3				

**Put a check if you feel your guess was good.*

Estimating Distances

For 2 to 3 pupils

1. Draw 3 or 4 lines of different lengths in the yard with chalk.

2. Ask your partner to guess the lengths of the lines. (This is called *estimating* their lengths.)

3. Measure the lines to see how closely you estimated.

4. Have your partner draw lines. Try to estimate their lengths before you measure them.

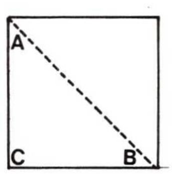
5. Do your estimates get better with practice?

Estimating Height

For 2 or 3 pupils

There are many ways of estimating height. One way is described below.

1. Make a 4-inch square in a thin card. Draw a line from one corner to the opposite corner. Cut along this line (A,B). Make the other corner "C." This triangle will help you to estimate the height of the school, telephone poles, etc.



2. To measure the height of the school, put point "B" at the end of your nose. Shut one eye and move back from the school until point "A" seems to be at the top of the building. Side AC must be upright.

3. Stand still and have your partner place a mark where you are standing. Measure the distance from the school to where your partner placed the mark. The height of the school is about the same as the distance from the school to your mark. To be more exact, add the height of your eye above the ground to the distance from the school.

Time Measurement *(Grigel)*

(Ages 6 to 9; M.A. 4 to 6)

OBJECTIVES (Primary General Learning Disabilities)

1. To promote in the children a general idea of time and how various times relate to various activities during the day.

2. To have the children recognize a need for standardized time measurements.

3. To teach the children to tell time--hours and half hours.

4. To have the children recognize the relative length of a minute, second, hour.

EXPLANATION In working with the cards, care must be taken to have at least one "reader" in each group to interpret the questions to the nonreaders. Groups of three or four would prove best at this level. The previous knowledge of the children is very limited. Only a few would have any concept of time and some will not know their numbers. I would have introduced some very simple pictographs and possible block graphs prior to this series of cards. On some of the cards, questions are asked that only a few of the children will be able to answer. These can act as "teachers" for the others of the group. The cards are quite structured in keeping with our theory of a structured (but not rigid) approach to the teaching of retarded children.

Card 1: How many handclaps does it take for your friend to walk around the table? Let him clap his hands to time you. Using heartbeats see how many it takes for each person to walk around the table. What other ways can you think of to time things. Are these good ways to time things? Why?

Materials - no special materials for this card.

Card 2: Look at Clock 1 and put the numbers on the face. Can you put them on Clock 2 without looking at the big clock? What can you say about the numbers on the clock?

Materials - mimeographed sheets of two clock faces without the numbers on identified as Clock 1 and Clock 2.

Card 3: The two black shapes are hands. What is different about them? Find the minute hand, and hour hand. Put them on your big clock. Show on your big clock a. when you get to bed, b. when you get up, c. recess, d. noon hour, e. home time, f. supper time.

Materials - clock faces approximately 12" diameter; 2 hands for each and fastener to put on hands.

Card 4: Find out when everyone goes to bed. Make a graph to show this. What time do most children go to bed? Why? Can you do the same thing with the time that each person gets up? Can you tell how many hours of sleep each child has? The teacher? Why is it different? (This is a "hard" chart. Only a few of the children will be be able to complete it.)

Materials - block graph paper.

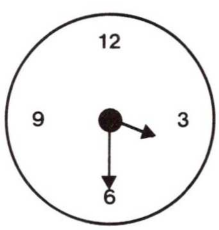
Card 5: On the page of clocks show:

8:00   2:00
5:00   9:00
3:00   12:00

What do you see about the minute hand? Take your big clocks and show other times that the minute hand does the same thing. Tell what time you have.

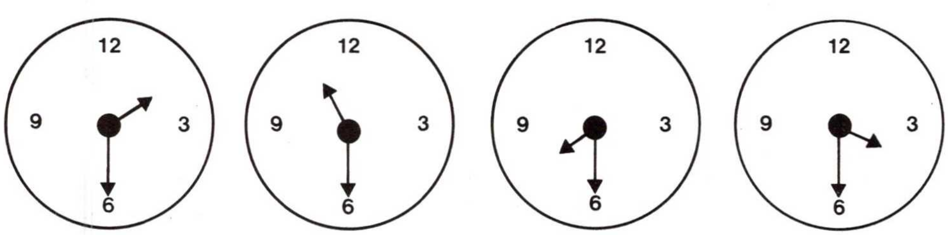
Materials - mimeograph sheet with six clock faces. Clock faces with hands from card 3.

Card 6: On your big clock put the minute hand at 6. Now move the hour hand. Can you tell what time you show in each case? We write this:



3:30

How would you write:



Show on your big clock:
6:30   9:30
5:30 10:30
1:30 12:30

Materials - clock faces from card 3 or small individual clock faces from a watch company.

Card 7: Can you hold your breath for 10 seconds? 10 minutes? 10 hours? Why? How can you find out who can hold their breath the longest? Why are there differences?

Materials - have a watch with a second hand available. Hopefully someone will think to ask for it!

Card 8: Can you find pictures of other ways people use to tell time? Ask your father or mother to help you. You can tell us about what you find out. Maybe you could make a model for the class. An attempt to involve parents!

Materials - vary with the class.

Time Unit: Grades II, III *(Penner)*

1. Can you tell how much time you spend playing in one day? One week? One month? How can you show this?

2. How much time do you sleep? Compare this with others. How can you show this?

3. Make a graph to show how many hours you spend on each activity during your day. What other way can you show your results?

4. In how many different ways can you measure time? Make a chart to show as many of these ways as you can.

5. How much time does it take to walk the length of the hall? hop? skip? move backwards? with eyes shut? downstairs? to school? with a heavy box? Guess your answers first. Compare with time taken by others. Show your results.

6. Using a TV guide, find the time between two specific programs. Find the time you would spend if you watched all the programs you like. Show this in some way.

7. Using a calendar, can you find out what a leap year is and why we have it? Report on what you find. Do all the months have the same number of days? Why or why not? How can you show this?

8. What do you do before breakfast? Between breakfast and school time? Before dinner time? After dinner time? After school? Before bedtime? Show this in some way.

9. What time do you get up? Who at your house gets up earlier? Later? Same time? Do the children in your class get up at the same time? How many get up earlier? Later? Show your results. Repeat this for bed times.

10. What things travel fast? Slowly? Find or draw pictures.

11. How long does it take you to travel across the playground? Your friend? Try this more than once. Show your results.

12. What things can you do for a longer time than your friend? Show this in any way you wish.

13. Make a sand timer with 2 bottles, sand, plasticine. Use it to measure the time taken to do various activities. Compare with other children. Record the results.

14. Make a water timer with a pan of water and tin can. Do similar activities to those with the sand timer.

15. String timer (pendulum). Make it using plasticine and string. Count swings. Time various activities by counting swings. Vary length of string. What happens? Record your results.

16. How many different clocks have you seen at home? At school? Outside? Show as many different kinds as you can.

17. Use the clock. Show a time on it. Write the time in as many different ways as you can. Repeat for at least six different times.

Time Questions: Grade III *(Risvold)*

1. Using a clock with a second hand, find out:

   a) How many times the second hand goes around in 1 minute, 5 minutes, 10 minutes. Is there a pattern?

   b) How many times the second hand will go around in 1 hour, 10 hours, 1 day.

   c) How many times the minute hand goes around in 1 hour, 2 hours, 5 hours, 1 day, . . .

   d) How many times the hour hand will go around the clock in 1 day, 2 days, . . .

   Make a table showing what you have learned.

2. a) Using a pendulum made from string and a weight, find out what happens as the length of the string is changed. How can you make a pendulum speed up? slow down?

b) How much time does it take for your pendulum to make 10, 20, 30 swings? Try different pendulums. Would a pendulum be good for measuring time?

3. How do you spend the number of hours in a week? Make a chart to show how 5 classmates spend the time in a week.

4. Could you use your pulse to measure time? How can you find out how many times your pulse beats in one minute? Does your pulse stay beating at the same speed? Why? Does everyone have the same pulse rate? Would pulses be good for measuring time?

Pendulum: Grades IV, V, VI *(Eremko)*

MATERIALS Stop watch (or watch), string, weights (balls, lead, etc.), tape measure, balance scale.

1. Measure how far a pendulum travels in each swing.
2. Does the swing of a pendulum vary with the length of a string?
3. Using different lengths, how many strokes in one minute? Half a minute?
4. Use different weights. Does a heavy "bob" swing faster or slower than a light "bob"?
5. Measure the size of the "bob." How does the size of the"bob" affect the swing of the pendulum?
6. Graph the relationship between the swing of a pendulum and time interval.
7. What every-day uses has a pendulum?
8. Using a stop watch can you estimate the length of a pendulum with 30 strokes per second? Sixty strokes per second?
9. How can a pendulum be used to measure time?
10. Using a pendulum, calculate how long it takes to walk 10 feet, 15 feet, 20 feet.
11. How long would it take to walk 50 feet, 100 yards, 400 yards, 1 mile?
12. Using a pendulum, discover how long it takes your classmate to run a dpecified distance.
13. Using a pendulum, determine how long it takes someone to complete a task. What is the fastest time? The slowest time?
14. How else can a pendulum be used to measure pace and time intervals?

15. If a pendulum made one swing every second, how many swings would it make in 30 seconds, 60 seconds, 10 hours, 24 hours?

Money Activities *(Inkster)*

1. Write out a list of sums, such as 25¢, $.33, $1.05, $.08, and so on. Using play money have the children illustrate how many ways they can show those amounts of money.

2. Cut out some pictures (baseball glove, chocolate bar, etc.) and make up questions concerning change. Have the children show in how many ways they can give you change for those items.

3. Write down some amounts of money. With coins, have the children illustrate how many ways they can show you this amount using the fewest coins, most coins.

4. Set up a "store" with various items priced. Ask in how many ways one can pay for them. Have your partner overpay you. Can you give him the correct change in more than one way?

5. Your partner gives you a certain amount of money and asks you questions such as, "If you had this much money what items could you buy?" "Would you get change?" "How much?"

Money Assignment Cards: Grade III *(Keddie)*

MATERIALS Real money (amount will depend upon the cards used), collection of toys for each group, foot ruler for each group, apparatus for weighing, masking tape for price tags. [A letter to the parents explaining your activity will usually provide much of the material.]

IMPLEMENTATION Vary the procedures to suit the topic, the children and yourself. Set up five or six activity centers all on the same assignment and five or six activity centers each on a different topic or different aspect of the same topic. Pairs of children can work together, either on an individual basis or with a friend. [This could be your week's work since the groups rotate each day.]

Activity alone is not enough. There must be discussion with an understanding teacher who can help pupils to verbalize their experiences and clarify their ideas. This can be done during the activity period when the teacher is free to circulate from group to group, talking with children individually.

The reporting session at the end of an activity period is also extremely important. It is then that children discuss their problems, listen to other groups describe how they worked together, and generally consolidate their learning, extend their vocabulary, improve their thinking skills, and develop their concepts of social cooperation.

ASSIGNMENT CARDS

You

If you had as many pennies as your age, how much would you be worth? What would be the least number of coins you could use to find your worth? Compare your results with those of your partner.

Coins

Use your box of coins to find out how few coins can be used to make each of these amounts:

19¢ 41¢ 75¢ 91¢ 97¢

Coins

Can you arrange your coins in patterns to show relationships between the coins? Tell about your results.

Coins

Count all the money in your box.
Share the money. How much is your share?
Sell the toys to each other using your share of the money.
One person will be the clerk and the other person will be the customer.
Change the price of the toys as often as you wish.

Money

Arrange your coins in piles.
How much money is in each pile?
Now make up an addition sum to find out how much money there is in all the piles.
Can you make up any subtraction sums to prove that your answer is correct? Do this as many times as you like.

By the Foot

Estimate, then find the value of one foot of each kind of coin that we use. Compare your results.

$1

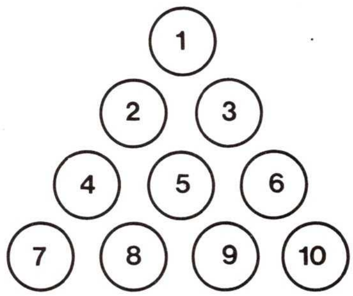
Estimate, then find the weight of $1 worth of each of the coins we have used. Can you manage with just a few of each kind of coin? Compare your results.

Providing for Individual Differences

Assignment cards provide challenging material for the advanced pupil, without frustrating the child who needs more time to absorb and personalize learning. Activity centers using ability grouping can be used from time to time when it is evident that some children have failed to grasp the basic understanding. Often it is useful to use an advanced pupil as an "assistant teacher" with a weaker group. Games and puzzles put on cards may be used by those groups finished before the others.

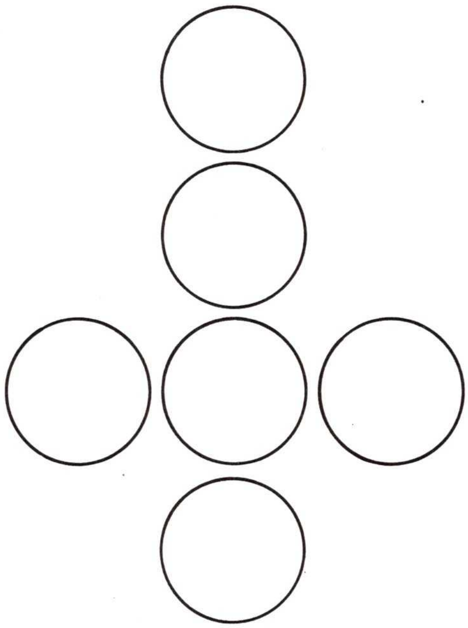
Sample Games and Puzzles

1. A triangle of ten pennies points away from you. Moving only three pennies, make the triangle point toward you.



2. I purchased some drawing supplies and spent 25 cents for 25 articles. I bought four kinds of articles: paper at two sheets for a cent, pens at one cent apiece, pencils at two for a nickel, and erasers at a nickel each. How many of each kind did I buy?

3. Six coins are arranged like this:



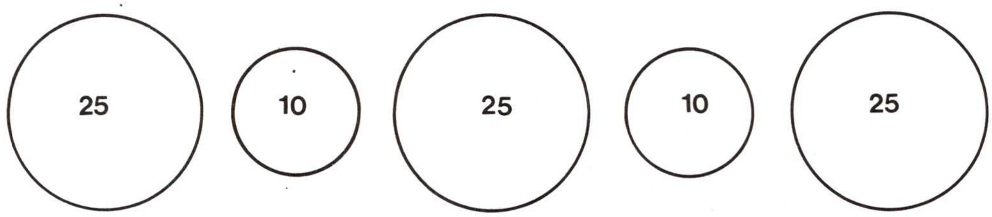
Move just one coin to another position so that the two rows (horizontal and vertical) contain four coins each.

4. A college student, Mr. Kantstand Prospeaurity, wishing to be subtle about how much money he needed, sent the following telegram to his father:

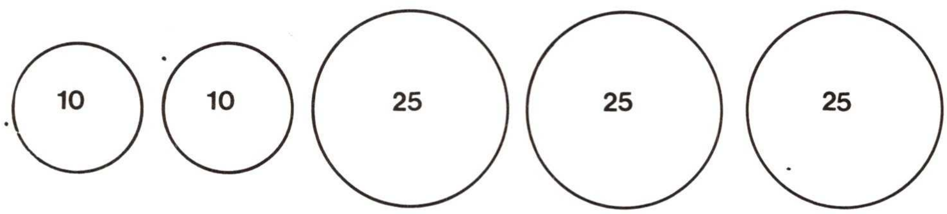
```
 $SE.ND
  MO.RE
$MON.EY
```

If each of these letters stands for a digit, how much money did Kantstand want?

5.



In four moves, and moving two coins at a time (always a dime and a quarter), position the coins as follows (without reversing the order or moving the coins to fill up spaces).



Solutions

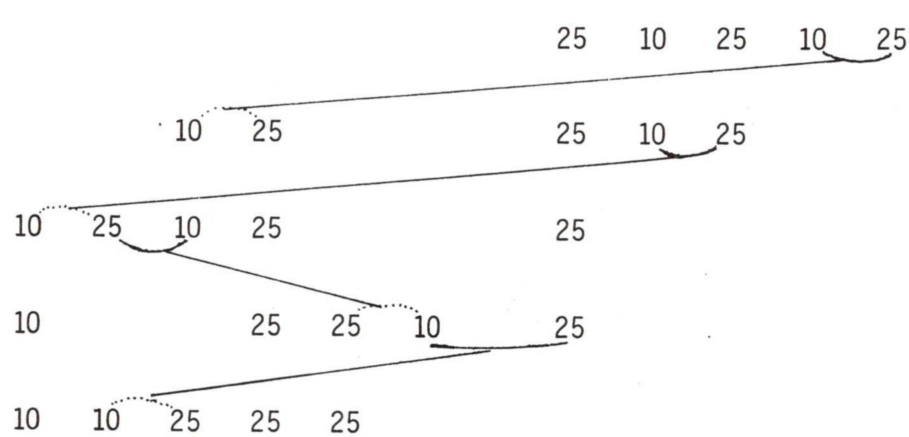
1. Move 7 to the left of 2, 10 to the right of 3, then 1 below and between 8 and 9.

2. Fourteen sheets of paper (7¢), 8 pens (8¢), 2 pencils (5¢), and 1 eraser (5¢).

3. Place the top coin on top of the center coin.

4. S = 9, E = 5, N = 6, D = 7, M = 1, O = 0, R = 8, Y = 2.

5.



Capacity: Grades I, II, III *(Kennedy)*

PURPOSE To introduce children to liquid measurement.

MATERIALS Several different sized containers--egg cup, tea cup, bowl, juice container, milk bottle, pan, jar, one-half gallon jug, pail, measuring cups, pint containers, quart containers, gallon containers, tablespoon, teaspoon, one-half pint containers, materials for making charts and graphs.

WORKSHOP A (rotating groups)

1. Using several different sized containers, find out which one holds the most water. Find out which one holds the least water. Make a picture showing the containers in order of capacity.

2. Using several different sized containers, estimate how many cups in each. Measure the actual cups in each container. Record your results.

3. Using several containers (different from those in group 1), which do you *think* holds the most? Which do you *think* holds the least? How can you find out if you are right? Record your findings.

4. Can you use spoons to measure water? How can you find out how many tablespoons in a cup? How can you find out how many teaspoons in a cup? Illustrate your findings.

WORKSHOP B (rotating groups)

1. Special measures: cup, pint, 1/2 pint, quart. Find out which holds the most. Find out which holds the least. In more than one way measure a pint, a quart. Illustrate your findings. Make a chart to show how much milk your mother buys each day and for a week.

2. Use several containers. Estimate number of pints in each. Find out the correct number of pints in each. How close did you guess the correct number of pints? How many things can you think of that are sold by the pint?

3. Estimate, then find a relationship among the three-containers (pint, quart, gallon) at your station. How many different ways can you measure one gallon? Do you know what things are sold by the gallon? How far do you think a car would go on a gallon of gas?

4. How many cups in one pint? Find out the number of cups in one quart without counting the cups. Find out the number of pints in one gallon without counting the number of pints. Illustrate how you found out.

Can You Find Volume? *(Gunn)*

A closed container, such as a can or box, divides space into three sets of points--the points forming the simple closed container itself, the points forming the interior of the container, and the points forming the exterior.

The volume of the inside of a large hollow object can be measured by comparing it with a smaller object and counting the number of times the smaller object is contained in the larger.

1. Use box A and a number of one-cubic-inch blocks (blocks measuring 1" high, 1" long, 1" wide). How many blocks are required to fill the box? This is the volume of the box in cubic inches.

2. Measure length, width and height of box B. What is the area of the base in square inches? Cover the base area with one-inch cubic blocks. How many did you need? Now finish filling the box with one-inch cubic blocks and complete the chart below.

Measurement of Volume

1. Number of cubes in bottom layer ____
2. Number of layers ____
3. Volume of box in cubic inches ____

3. Using the information gathered from exercises 1 and 2, can you arrive at a mathematical shorthand for finding the volume of a rectangular box?

4. Using the pieces of wood provided, construct several solids and estimate and then measure their volume. Can you use mathematical shorthand to determine or verify your findings in every case?

5. a) Estimate the volume of the triangular prism D in cubic centimeters.
   b) Measure the area of the base and the height. Calculate the volume to the nearest cubic centimeter.

6. Find the volume of cylinder C. Show your work. (Remember when using blocks to find area, 1/2 or greater counts one, <1/2 not counted.)

7. a) Construct a cube with a base of 4 inches by 4 inches, and a height of 4 inches.
   b) Construct a square-base pyramid 4 inches high, with a base 4 inches by 4 inches.
   c) Compare the volumes of their interiors. (This may be found by filling the pyramid with sand and pouring into the cube until the cube is full.)
   d) Graph the results and devise a mathematical shorthand for calculating the volume of a square base pyramid.

8. a) Construct a triangular pyramid with the same size base and height.
   b) Compare their volumes. (This may be done as in 7(c)).
   c) Graph your findings and devise a mathematical shorthand for finding the volume of regular triangular pyramids.

9. How would you find the volume of a stone?

Introduction to Area: Grade III *(Kennedy)*

1. *Materials* 3 x 5 inch index cards.

   Activity Estimate how many index cards are needed to completely cover your desk. Count the number of cards actually needed to cover your desk. In the same way, estimate and measure the number of cards needed to cover the front of your work book, a wide textbook. Show these findings on a pictograph.

2. *Materials* Equilateral triangle tiles with three-inch sides (tagboard).

   Activity Estimate how many triangular tiles are needed to completely cover your desk. Count the number of triangular tiles actually needed to cover your desk. In the same way, estimate and measure, in triangular tiles, the front of your work book, a wide textbook. Show these findings on a pictograph.

3. *Materials* One-inch square tiles (tagboard).

   Activity Estimate number of square inches in a given piece of paper. Count the number of square inches actually needed to cover the paper. Estimate and measure, in square-inch tiles, the front cover of your reader, your work book. Show these findings on a pictograph.

4. *Materials* Area kit including several area cards and a grid (8x8" ruled).

   Activity Estimate the number of square inches in each area card. Measure the number of square inches in each area card. Use the grid. Find out which one has the largest area, the smallest area.

Area Card	*My Guess*	*Measured Length*

5. *Materials* Geoboards and elastic bands.

   Activity Make your own geometric figures and find the areas. On the given graph paper draw your geometric figures and mark the area. Can you make any geometric figures that you can't find the area of?

POSSIBLE OUTCOMES

1. These activities may heighten students' concepts of "greater than" and "less than." For example, at stations 1, 2, and 3, students may discover which object measured needs the most cards or tiles to cover it and thus find out which is the largest. Similarly they discover the smallest.

2. By comparing results from the three stations, they may discover which is the largest and smallest unit of measure.

3. They may also point out that, using a grid (station 4), is easier for small surfaces and that it is a more accurate measure.

4. These activities will give practice in counting when measuring and counting out the number of cards or dishes.

5. The concept of area and a unit measure will be developed.

6. Students will gain practice in estimating.

Perimeter and Area *(Myers)*

OBJECTIVES

1. To explore more areas of measurements relating to perimeter and area.

2. To develop a continuous activity program in area and perimeter through different shapes.

3. To foster the interest that the group already has from past experiences in measurement and fractions.

4. To enable the class to discover, through activity, the relationships between area and perimeter in rectangles and squares.

Open-ended Questions

1. a) Use a part of your body to measure. Give this unit of measurement a name. Use it to find the perimeter of a rectangle and a square. Make two models of each shape on paper.

   b) Here are a number of rectangles and squares. Arrange them in order of size, first the rectangles, then the squares.

   c) Measure their perimeter and area and record your answers in a table. (Remember that to find area you count the number of small squares used to cover the big rectangle or square.) Make a graph to show the relationship between area and perimeter.

2. a) How many words are used in a newspaper column? Find out the actual number of words after you have calculated it. Explain on a chart how you did this.

b) All newspapers have advertising space. What fraction of this paper is in advertising?

c) Some advertising space is used for clothes, cars, etc. What percentages of advertising space is used for these?

d) If advertising costs $1,050 per page, how much does the newspaper collect for advertisements?

3. a) Measure the shadow of a piece of squared paper two inches in length when you hold it one foot, two feet (and so on) from a wall.

b) How large would the shadow be when it is 12 feet from the wall?

c) Find the relationship between the size of the actual square and the shadow.

d) Compare the shadows in terms of length of sides.

e) What can you say about perimeter and area of the shadows?

4. a) There are some circular objects in the classroom. Can you measure their circumference, diameter, radius?

b) Can you find the relationship between these measurements?

c) How could you find the area of any of these objects?

Weight Measurement: Grades I, II *(Bortnik)*

The following activities may serve as introductory mathematics corner activities for the development of interest in the measurement of weight. The only special equipment needed for these activities is a balance scale and a box of plasticine.

1. How many things can you find that are heavier than a reader? Record your findings.

2. How many things can you find that are lighter than your shoe? Record your findings.

3. Try to make two balls of plasticine balance each other. Change the two balls into two boats. What happens? Be sure to record your findings.

The following activities should give further practice with the concepts of heavier and lighter, and introduce the idea of a standard measure. Each group of two or three children will make a simple balance from a coat hanger, two paper cups and some strings. They will be reminded to check that the two sides are evenly balanced or be encouraged to make it balance by using a piece of plasticine. Other materials required are nails of the same size, sawdust, sand, dried peas or beans, a box of cereal, and some marbles. Each group of two or three will do the two following activities.

4. Choose two materials from the measuring table. Fill one paper cup with one of the materials. Fill the other paper cup with the other material you have chosen. When both cups are full do they balance? Be ready to tell the class what you have found out.

5. Choose either a box of marbles or a box of nails. Fill one of your paper cups with one of the materials from the measuring table. Fill your other paper cup with only as many nails or marbles as you need to make the two cups balance. Be ready to tell the class what you have found out.

The following activities should provide the children with experiences with standard units of measure. Most groups of two or three would use the balances previously constructed from clothes hangers and weights of less than five ounces. Some groups would use commercially built balance scales or more sturdily built wooden balance scales, and pound or half-pound weights. The children would rotate so as to have practice with different weights.

6. Find things that are heavier than the weight you have. Find things that are lighter than the weight you have. Record which weight you have used as well as the things that are lighter and heavier than it.

For the following, groups of four or five would rotate so as to do each of the activities. A variety of scales (bathroom scales, commercially built balance scales, spring scales) and a set of weights are required for these activities. Activities should be matched to particular scales.

7. How many ways can you find to balance a pound weight by using different sets of ounce weights. Record your findings. Can you see a pattern in the kinds of weights used to balance a pound? (Could be matched with commercially built balance scale.)

8. Guess the weight of six different things. Weigh them carefully. Record your results. (A balance scale.)

9. Weigh your partner. Add your weight to your partner's weight. Try to prove that your answer is right. By how many pounds has your weight increased since you were born? Be sure to record your findings. (Bathroom scale or the scale in the nurse's room.)

10. Count out 30 pennies and weigh them. How heavy are 60 pennies? How heavy are 5 pennies? Find out how heavy 20 pennies are in two different ways. (A balance scale or a spring scale and a bag with a string around the top.)

11. You have a pint carton, a set of scales and some weights. Use these to help you to find out the weight of a two-quart carton filled with water. The two-quart carton weighs five ounces. Describe how you found the answer. (Balance scale.)

12. Weigh equal amounts of six different things. Show the results by using a graph. (Balance scale or a spring scale.)

The following activities may be done by those children who finish early.

13. Make a record to show what your mother buys by the pound.

14. Can you find things that are sold in ounces? Make a record that shows what you have found.

15. Make a record that shows where you see people using scales.

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B.2 Ratio and Rate Concepts

Experience With Ratio: Grades V, VI *(Woodeye, Wong)*

OBJECTIVE To give students some experiences related to the concept of ratio. An overhead projector is used to project an image of an object. The length of the object and the length of its image on the wall form a pair of numbers which illustrate the concept of ratio. Students are asked to tabulate pairs of measurements (for object and image) in a two-column table.

Experience 1 - Find the Image

This experience serves to introduce the idea of a ratio by the use of an overhead projector. Students are asked to match different colored sticks, or straws, with their images for a given ratio. Ratios such as 1:2, 1:3, and 1:4, can be used.

Materials Envelopes, four colors of sticks (or straws), overhead projector, worksheets.

Experience 2 - Sketch the Image

Students are asked to construct images of specific objects rather than to identify the image. A ratio of 1:2-1/2 can be used. The stations should each have a box of several small objects.

Materials Unlined paper, small boxes, small objects.

Experience 3 - Measuring the Image

Students use a centimeter ruler to measure line segments and their images. In some cases, either the length of the object or the length of the image cannot be determined. Students record information in ratio tables and determine the ratio.

Materials Overhead projector, transparency, metersticks, centimeter rulers, worksheets.

Experience 4 - Recognize Like Shapes

Students work with regions rather than with one-dimensional segments. Six stations are established. Students are required to match objects with their images.

Materials Graph paper, centimeter rulers, worksheets.

Experience 5 - Enlarge a Printed Circuit

Students construct drawings, to scale, of printed circuits.

Materials Graph paper, worksheets.

Experience 6 - Shrink a Head

Students construct an image that is smaller than the object. A scale of one centimeter to 1/4 inch is used.

Materials Graph paper, centimeter rulers, worksheets.

(Source: "Experience in Mathematical Ideas," *NCTM* 1970, Vol.2, pp.69-95.)

Concept of Ratio Proportion: Grades V, VI *(Woodeye, Wong)*

OBJECTIVE To introduce ratio and proportion to students through a discovery situation. (This approach can also be used for junior and senior high general mathematics.) The discovery method will, hopefully, help students find patterns that will enable them to verbalize algorithms for the solution of proportion.

PROCEDURE As a starting point, have two students each take an arbitrary number of objects from a container. A comparison is then made of the number of objects drawn by each of the students. A table is used to keep a record of the number of objects selected.

Example

Girl	3	6	9	...	15
Boy	4	8	12	...	20

Each time the children come to the container, they are instructed to withdraw the same number of objects that they had drawn on their first turn. The objects drawn are displayed where they can be seen by the entire group. The girl in the example cited above drew three objects while the boy drew four. Thereafter, and with each successive drawing, the total number of objects is recorded in the table. (If the same number of objects are drawn by both, these numbers should be recorded, but another withdrawal is made to get different numbers. After three or four selections, the students should be able to supply new values without actually drawing objects from the container. If the children have trouble supplying the values, they should return to the container for additional drawings.)

After completion of the first table, a second table is constructed without having the children draw objects from the container. At this point the word "rate" is introduced. The students are asked to think of the table as a record of the rate at which these objects were selected from the container.

Example

Girl	3	6	12	15	...	*(G,B)*
Boy	5	10	20	25	...	(3,5; (6,10); (12,20); (15,25)

In this example, the girl drew three to the boy's five (expressed as a ratio of 3 to 5). The children are then asked to fill in the table. Once again it should be stressed that the table can be verified by having the children actually draw the number of objects specified.

At this stage, the notation of ordered pairs is introduced. The tables are constructed vertically to make this transition easy. The values listed in the second table are written using ordered pairs which were referred to as "rate pairs." Once the concept of rate pairs is established, the next step is to show equivalence between rate pairs. At this point, a student is asked to explain to his classmates how the objects were drawn from the container and how the tables were constructed. The children are asked to volunteer other rate pairs that show the same rate of drawing.

Example $(3,5) \rightarrow (9,15)$

$(3,5) \rightarrow (21,35)$

$(3,5) \rightarrow (6,10)$

The students should see that equivalence exists between other rate pairs from the same table. They are asked to find a rate pair that is equivalent to (21,35), but they are instructed not to use (3,5). A second rate pair is introduced and the students supply rate pairs. The term "equivalent rate pairs" is used to express the same rate. The pair (3,5) express the same rate as (6,10) even though different numbers are used.

As a further exercise on finding equivalent rate pairs, examples can be devised which call for replacements supplied by the students.

Example $(2,\square) = (6,9)$

$(12,16) = (\square,4)$

The symbol of replacement should be used in all positions.

A game based on equivalent pairs can help the children see the relationship between the products of the means and the extremes.

The rules of the game are as follows. The girls pick a pair of numbers that would yield equivalent rate pairs. They give one value and let the boys find the other. In a similar fashion the boys could pick a pair of replacement values.

Hopefully, the children will soon realize that the product of the replacements listed in the table is the same as the product of the numbers in the examples. This might set the stage for a verbalization of the rule. It is possible to see that replacement could also include fractions.

[*This brief development of proportion, as outlined above, would, with extension, lend itself to problems involving percent, scale drawing, conversion, similar figures, area, and volume.*]

C. GEOMETRY

C.1 Three- and Two-Dimensional Shapes

CLASSROOM ACTIVITIES[1]
(Preschool and Kindergarten)

Provide each youngster with a geoboard and several rubber bands and let creativity begin.

FAMILIAR SHAPES

On your geoboard, show how to make some shapes that look like something in this room. Try to make something that can be found in the kitchen, basement, yard, grocery store, playground, garage. Show something your dad uses. Show something that is alive.

Possible Questions

Can you tell your friend what you have made?
Look at something someone else has made and try to guess what it is.
Does your figure look the same if you turn the geoboard around?
How many sides does your figure have?
How many corners does your figure have?
Are there more corners or more sides?

PLANE FIGURES

Try to make figures with three sides that are small, large, "skinny," "fat." Try to make figures with four sides that are long, short, long and wide, long and narrow, short and narrow, "like a square," "not like a square."

Possible Questions

What does the figure you have made remind you of?
Does it look like anything that is familiar to you?
Where did you see something like it before?
Does the figure change if you turn your geoboard?

Make two figures that (1) do not touch, (2) touch, (3) cut into each other. Look at the figures you have made.

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[1]"Geoboard Geometry for Preschool Children," *The Arithmetic Teacher*, February 1970, pp.123-126.

SEGMENTS

Try to make segments that are short, long, straight, "crooked." Try to make segments that do not touch, touch, cross each other (intersect), will *never* touch (parallel), are exactly on top of each other. Try to make various segments leading to two (or more) points, various numbers of segments (for example, two that are equal), two that are not equal, many different segments.

Possible Questions

How would you make a road?
Can you make a very narrow road?
Can you make one that is long and narrow?
Make a railroad track.
Can you make a road and a train track that cross? Do not cross?
Will never cross?

Look at two pegs in different corners. How many different roads (crooked or straight, few or many corners) can you build between these two pegs? Which road would you like to travel on? Why?

Three-Dimensional Shapes: Grades I, II, III *(Hamel)*

We need to question continually the values of certain forms of geometry for our students, especially at the primary grade level. Below are some statements to substantiate the inclusion of geometry at this level.

- Children can see position, slope, and size as something they can understand, use and manipulate to explore their environment.

- Children are able to use geometric insights to facilitate and develop creativity and the spirit of inquiry.

Geometry is indispensable in our way of life. From their very structure, form and beauty, all objects are governed by properties of geometry. This is what the children at the primary level should realize - first structure and secondly form.

The students are asked to collect objects from their environment. These objects (flowers, leaves, boxes) will help them in studying two-dimensional shapes. The students can be presented with a series of open-ended questions which will help them in the development of learning shapes

Developmental Questions (for discovering shapes - three dimensional)

1. Take all the small boxes and see how many different shapes you can make. How do your shapes compare? Are some smaller than others? [*When a child has spent time making different shapes, he begins to think about two-dimensional shapes for himself.*]

2. Take balls, cylinders and boxes and notice how different they are from each other. How different does a ball roll from a cylinder? [*When a child manipulates certain shapes, he will quickly realize and be able to think in a three-dimensional pattern.*]

3. Sort boxes as to height and see how many other ways you can sort them (color, width, etc.).

Distribute popsickle sticks and thumbtacks to the students and ask

4. How many different shapes can you make with the thumbtacks and popsickle sticks? Do some of them move? Which ones do not move?

Give the students certain objects and ask the students what other objects they can make from them (a square can be made from two triangles). Then give them some other objects (triangles, squares, cubes, rectangles) and ask them the same question.

Students must first learn how to work with two-dimensional shapes before being introduced to three-dimensional shapes. Let them manipulate objects and get a feel for them. Later they can be given three-dimensional work (as in developmental question 1) and explore different ways of using objects.

Geometry *(Risvold)*

WORKSHOP #1

Assignment 1

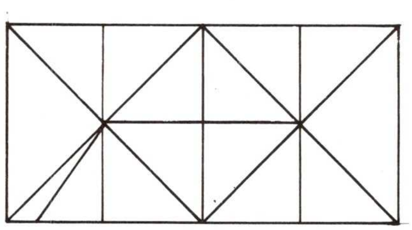
How many different shapes can you see in the classroom, outside and at home? Draw these shapes. Use all the shapes you have made and prepare a chart. It could look like this.

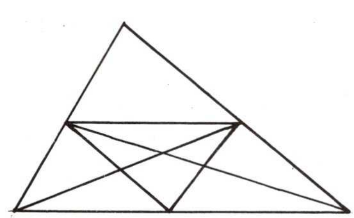
Shape	*Number - Vertices*	*Number - Sides*	*Number - Angles*
△	3	3	

Assignment 2

a) How could you design a one-room house of the future? Try to build your house.

b) How many triangles are there.





c) How would you name these triangles?
Make puzzles like these for your friends to solve.

Assignment 3

a) Using strips of paper and paper fasteners, make as many different figures as you can.

b) Using one or more of the shapes you have made (or the cardboard ones which are already made) make an interesting design.

Assignment 4

a) With two parallel lines, what are the most points of intersection using
- two intersecting lines?
- three intersecting lines?
- four intersecting lines?

How would you make a chart of this?

b) Draw some pairs and sets of parallel lines in your book - some straight, some curved.

c) Write down examples of some parallel lines you know.

[*These assignments should be discussed with the class to clarify any difficulties.*]

WORKSHOP #2
(Each pupil should have a typed copy)

Circles

a) Using the round objects provided (tins, bottles, colored discs), guess the diameter and circumference. How many guessed correctly? How could you show that on a graph?

b) Is any part of you round? Can you find objects the same circumference? How could you make a chart of this?

c) Circles can be used to make interesting designs. How could you make a design?

Perimeter and Area

a) How many different shapes can you cut from one-inch graph paper using one square, two squares, three squares, four squares, five squares? What is the perimeter and area of each shape? Is there a pattern?

b) Who in the class has the smallest footprint? How can you find the area and perimeter of the footprint?

c) Using the geoboard, make a shape. See if a friend can find the area.

Angles

a) Draw ten angles on a piece of paper. How can you find out if these angles are bigger or smaller than a right angle?

b) How many total degrees are there in the angles of a triangle? Does this change with different shapes and sizes of triangles? Can you find out how many degrees there are in other polygons?

c) Make a circle and divide it into five parts. Name the angles you made and find the total number of degrees in the whole circle. How many degrees are in bigger circles, smaller circles?

Squares and Rectangles

a) On a sheet of graph paper, cut out squares with sides of one inch, two inches, three inches, and four inches. In how many ways can you measure these squares? How could you show this?

b) You are planning to keep chickens in your garden. You have 20 feet of wire fence and you wish to make the pen as large in area as you can. What would be the length and width which will enclose the largest area?

c) Using strips of paper and paper fasteners, make some rectangles. By pushing the sides, change your rectangles into parallelograms. Is the area the same?

General Introduction to Shape: Grade III *(Foran)*

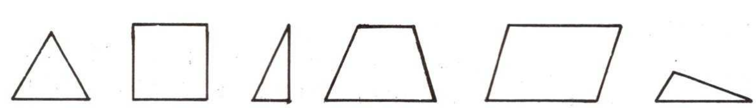
OBJECTIVES To introduce various shapes and how to make them; to name the various shapes and show how they can be put together to form other similar or different shapes; to observe and compare the differences between shapes; to gain background for work on perimeter and area.

MATERIALS Geoboards, plastics of various colors, cut-out shapes, 1" squares of paper, paper and other equipment (for recording results).

1. How many different shapes can you make using your geoboard? How many of these can you name? Show us your shapes in diagram.

2. How many shapes can you make using 2, 3, 4, 5, pegs on your geoboard? Can you see any similarities in any of the shapes?

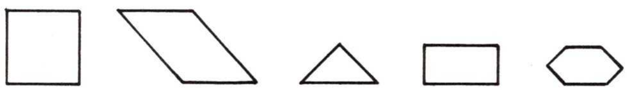
3. Given these shapes:



[These shapes should be cut of different colors of construction paper and given to the students.]

How many ways can you group them? Show us your results.

4. Using one of these shapes as often as you wish, what other shapes can you make?



[Sets of these should be cut of construction paper and given to the students. They will need more paper to make more of the shapes they choose.]

What other shapes can you see inside the shapes you made?

5. Cut out a square piece of paper. How many shapes can you find by folding it in various ways?

6. How many shapes can you make from 2 units; 3, 4, 5, units? Compare the shapes you make. One unit = 1" square of paper.

7. How many objects in the room are similar in shape? See how many different groups you can form.

8. Stretch an elastic band into a circle shape on your geoboard. Use other bands to make regular polygons inside the circle. Increase the number of sides. How many sides can you get in a figure? How many sides does a circle have?

FOLLOW-UP GAMES AND ACTIVITIES

- Curve stitching to see different types of curves.
- Working with tangrams to see how shapes can be combined.

FUTURE WORKSHOPS:

- Work on perimeters.
- Work on area.
- Work on symmetry.
- Work on tessellations.

[Many of these ideas, started in workshops, can be expanded later.]

Symmetry in Shapes: Grade IV *(Hamilton)*

1. Look around you. Make a list of as many shapes as you can see in the classroom. Do any of these shapes appear to be similar? Try grouping all the similar shapes together. Your heading might be, "Shapes I See That Are Similar To The Floor Tiles," or "Shapes I See That Are Similar To The Windows," etc.

2. Here are some pieces of manilla paper and also some pieces of carbon paper. Fold each piece of manilla paper in half and place one side down on the carbon paper. Write your name in large letters on the side facing you, making sure that each letter comes all the way down to the fold in the paper. Press very hard with your pencil. Now look carefully at what you have done. Do both sides appear to be the same? Now, with your crayons, try making shapes on each side to form an interesting pattern so that both sides will look the same. Examine all the shapes you have made. Write a few sentences about them. Does your drawing remind you of something you've seen before?

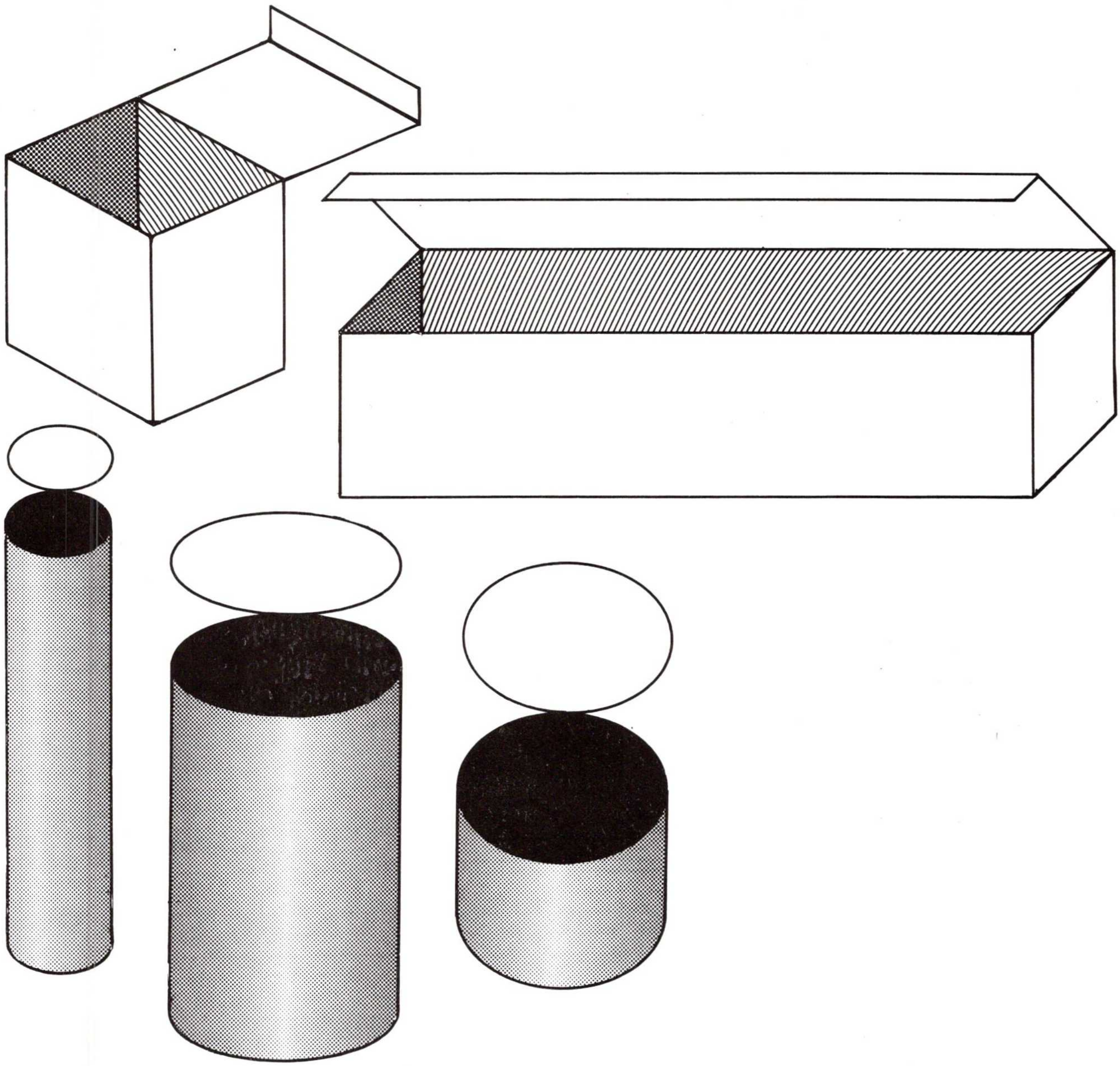
3. Here are some scissors and pieces of colored tissue paper. Try folding your piece of paper once, and cut out shapes with your scissors to make an interesting pattern. Try and get something completely different from the others in your group and be careful not to cut away all of the fold or you will have *two* pieces of paper instead of one. When everyone is finished, look carefully at the designs that were made on each side of the fold. Discuss them together. How many different shapes do you see? Are any of these shapes similar to other shapes you are familiar with? Note carefully what you see on each side of the fold. Write one or two sentences about what you see. Now, do the same as you just did only this time try folding the paper twice, then three times, then four times. Can anybody fold five times for some really interesting patterns?

4. Think about nature for a moment. Does nature provide for us any shapes that show symmetry? Go out to the playground. How many different things in nature can you find that are good examples of symmetry? Collect some and bring them back to the work table. Try reproducing some of these shapes on paper. What do you notice about each shape? Can you write a sentence or two to describe what you see? Do any of these shapes remind you of some of the more regular shapes on the list you made of classroom shapes? (Here the children will probably say some of the flowers look like circles, some leaves like triangles, etc.)

5. The idea similar to that brought out in item 3 could also be done using ink blots, or, better still, for a more colorful effect, using drops of different colored paints. Again, the children could discuss their patterns and write a few sentences about each. You would probably not have each group do *both* questions unless you have extra time.

6. Collect an assortment of different shaped boxes (shoe boxes, rectangular, square, round boxes; hexagonal boxes that felt-tipped pens come in would be excellent for this). Make sure that each box has a lid. How many different ways can the lids be replaced on these boxes? Record them. What does this tell you about the symmetries of the boxes and the lids? Are any boxes more symmetrical than others?

7. Symmetry by reflection - here is an assortment of rectangular and square shapes. Trace around the edges of some of them on a sheet of paper. Cut out these paper shapes. How many ways can you fold these shapes so that one half matches the other half? Each fold is called an axis of "symmetry." How many axes of symmetry does each figure have?



8. Try writing down all the capital letters that appear balanced about an "up and down" axis of symmetry. (A is one example.) Show the axis by a dotted line on each letter. Do the same with all the letters which appear balanced about an "across the page" axis of symmetry. (K is one example.) Show the axis by a dotted line on each letter.

A K N

9. Some letters look the same if you turn them through half a complete turn about a point marked. Here is one example: N. (Dot shows the point marked.) Find all the other letters that look the same when turned in this way. Now make lists showing everything that you discovered. Would any letters be included in two lists? Make an interesting display showing your findings.

10. You can find examples of symmetry in human faces, pictures, advertisements, etc. Find as many examples as you can and make an interesting display with them.

[*A very interesting art lesson and one which children really enjoy is to collect many pictures of human faces and cut them in half. Then have the children choose one, mount it on a piece of paper and try to draw the other half of the face. A good art lesson on symmetry!*]

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Developing Concepts of Perimeter and Area: Grade IV *(Misselbrook, Koleyak)*

OBJECTIVE To build a readiness for the learning of the perimeter and area concepts as applied to geometric figures of varied shapes and sizes.

INTRODUCTION Today we will do some experimenting. You will find on the table materials such as a foot ruler, yardstick, tape-measure, geoboard, a tile and some circles which you may use to carry out your experiment. First we will divide into groups for the various activities. Each group will take an assignment card on which it will place the actual measurements.

Open-ended Question

1. Can you guess the distance around your desk?
2. Can you guess the distance around your book?
3. Can you guess the distance around the classroom?
4. What would you use to measure the distance around the classroom?
5. Can you guess the distance around our piano?
6. How would you measure around our piano?
7. Can you guess the distance around the cement patio?
8. How would you measure it?
9. Can you give the word which means distance around objects?
10. Who can find out what that word is?
11. If you find it, don't tell the secret. Tomorrow you may write it on the board for all to see.

While we are measuring distances, let's see if we can find out how many squares or long shapes it will require to cover these flat areas.

1. Can you guess how many squares it will take to cover your book?
2. Can you estimate how many squares will cover your desk?
3. How many squares will it take to cover the classroom floor?
4. Can you guess how many squares it will take to cover the space where our piano stands?
5. How would you find the number of squares it would take?
6. What would you use to cover the patio space?
7. Can you give the name to the surface space cover with the squares?
8. Can you find out for tomorrow what that name would be?
9. If YOU know, DON'T tell the secret!

[In order to give the class experience in measuring perimeter and area, in the next lesson they will be asked to measure circles, balls, the soles of their feet, palms of their hands, their bodies, cylindrical and triangular shapes. We find that, in order to teach perimeter and area effectively, we must have several sessions in measuring in order to instill the correct concepts. This is followed by another set of open-ended questions and to conclude the workshop, a display of completed assignment cards, graphs or other suitable materials.]

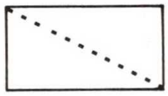
Parallelograms and Triangles: Grade IV *(Leyshon)*

ASSUMPTION The students know that area = base x height for rectangles.

MATERIALS Scissors, pencil, ruler, prepared rectangles, parallelograms, and triangles (right angle, isosceles, scalene).

1. What is the area of this rectangle?

2. Fold the rectangle in half diagonally. What will the area of the resulting triangle be? Is it half the area of the rectangle? Can you make an equation to find the area of the right angled triangle?

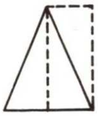


3. Can you find the area of the right angled triangle?

4. Take one of the parallelograms. Can you find its area? By making one cut with the scissors, can you make the parallelogram into a rectangle? Is the area the same? What is the base of the new rectangle? What is its height? Can you find the area now?

   Take another parallelogram. What is its base? What is its height? Where do you measure the height? What is the area of this parallelogram?

5. Take a triangle labelled "isosceles". How would you describe it? Let the side that is not equal to the others be the base. Can you make one cut with the scissors to make a rectangle? What is its base? What happened to the base of the triangle? What is its height? Can you find the area? Can you apply the same equation that you used for the right angled triangle to find the area of an isosceles triangle? Take another isosceles triangle and apply the equation. How do you measure the height? Cut the triangle to make a rectangle. Were you right about the area?



6. Take a scalene triangle. How are you going to measure the height? What is its area? What equation did you use?

7. Can you use your equation to find the area of all triangles? Try some other triangles and see.

8. What is the important thing to remember about measuring the height of a triangle or a parallelogram?

9. Does it matter which side of a triangle you use for the base? Check your answer with some of the triangles.

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PART THREE

MATHEMATICS ACTIVITIES FOR JUNIOR AND SENIOR HIGH SCHOOL

MATHEMATICS ACTIVITIES FOR JUNIOR & SENIOR HIGH SCHOOL

As in the elementary school section of this monograph, topic headings for the following section were extracted from Province of Alberta mathematics curriculum guides. These headings were used by teachers to group secondary school mathematics activities which were collected and prepared for development-of-a-topic assignments in workshop courses conducted by Dr. Harrison over the past four years. With the assistance of Mr. Penn Wong, the activities were cross-referenced according to the concepts developed. They were edited to minimize repetition and to produce some consistency in format, and also to ensure, as far as possible, that the activities would be practical and "ready-to-use." At the end of each concept there is a list of articles which contain teaching suggestions. Those from the 1972-75 issues of The Mathematics Teacher *have been cross-referenced by the co-editor; others were cross-referenced by the editor's EDCI 470 students in the 1971-72 academic term.*

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JUNIOR HIGH

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D.N. Fisher
Marion Florence
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JUNIOR HIGH

A. NUMBERS AND NUMERATION SYSTEMS

A.1 Whole Numbers

Patterns in Numbers: Grade VII *(Falk)*

OUTLINE Various lists of numbers are given: the natural numbers, the counting numbers, various arithmetic series, various geometric series, triangular numbers, square numbers.

Students are asked to discover the way in which the lists are made up and to continue the lists. They are shown, by examples, how to find successive differences, successive sums, successive products, moving sums, series of the form an + b. It is envisioned to extend the ideas and activities in order to lead students into graphing the linear relationships that the activities contain, then to further extend the ideas into quadratic and hyperbolic relationships. Some examples of the activities follow.

Card 2 Use the method of successive differences to find the pattern in this list:

B: 2, 5, 8, 11, 14, 17 ...

Can you continue the list? Make up a list of numbers according to a pattern. See if your partner can continue your list.

Card 5 Sometimes you cannot tell from the first few numbers in a list, how the list was made. For example, continue this list in two or more different ways.

G: 2, 4, 8 ...

Explain to your partner how you continued the list. Make up a list which could be continued in more than one way.

Card 8 Triangular numbers are introduced.

Card 9 See if you and your partner can agree on what a *square* number should be. Make a list of square numbers. Arrange them in order and see if there are any "gaps" in the list. If so, try to fill these gaps. Continue the list as far as you wish. Can you find another way to get square numbers?

Card 15 Here is a new list of numbers

F: 1, 1, 2, 3, 5, 8, 13, 21 ...

Can you continue the list? See if you and your partner can discover some interesting properties of list F.

Cards 17 to 19
In cards 17 to 19, "___ times n" and "n plus ___" rules are used to generate linear functions.

Card 20 A "___ n" rule, and an "n plus ___" rule could be combined to get a new rule. [A list M is given and it is shown that it was made up by a "2 times n plus 4" rule.] Find the 7th and 8th members of M. Continue list M as far as you wish. Make up some lists using different "___times n plus ___" rules. See if your partner can figure out what rule you have used.

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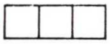
A.2 Rational Number Concepts

Fractions *(Keeler)*

1. Into how many parts must we divide things equally so that each get a similar share?

 share an apple- each get one half

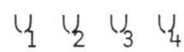
 share a chocolate bar  - each get one third

 share an orange  - each get one fourth

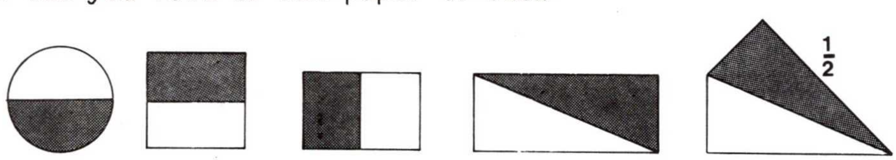
[The idea that one half is one of two equal parts, and one third is one of three equal parts, etc. is important.]

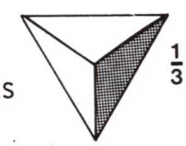
2. Pour a jar of sand into smaller jars. In how many ways can you divide it equally?

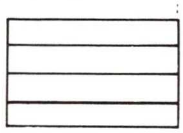
3. Pour koolaid into a measuring cup. How many equal parts make one cup?

fourths <u>4</u>  thirds <u>3</u> 

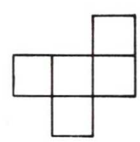
4. In how many ways can you fold or cut paper to show

halves 

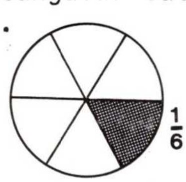
thirds 

fourths 

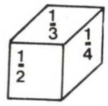
How many parts in each picture?
How many fourths make one half?

Fold your paper to make five equal parts
In how many ways can you fold it and make  open boxes?

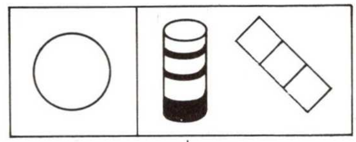
Fold circular and rectangular cut-outs to make the smaller fractions.



5. Play "Conquer." Dice are marked with simple fractions The game board has figures shaped to match the various fractions on the dice.



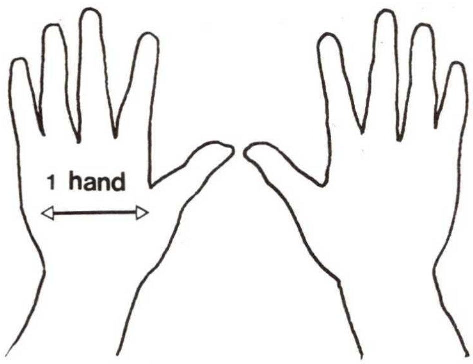
6. Geoboard Fraction Exercises
   a) How much of each unit is shaded?

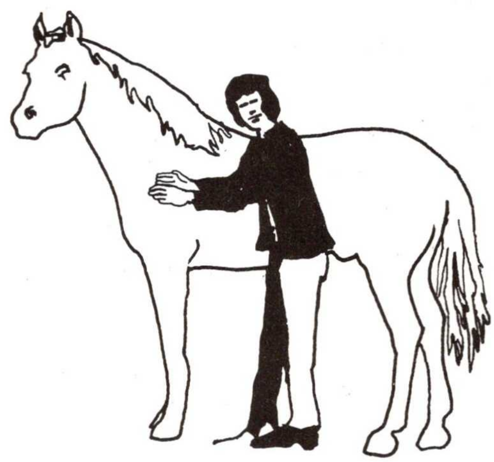


   b) Join dots to make each unit show the fraction. Indicate color.

[*Tasks 3 to 6 above can be rotated so that each group tries two or three different ones. A check-up here would be a good idea.*]

7. People do not walk around carrying a ruler with them. The height of a horse is measured in *hands*.

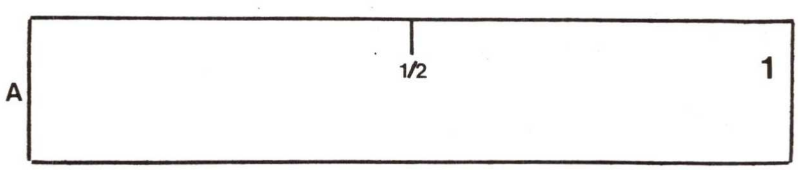




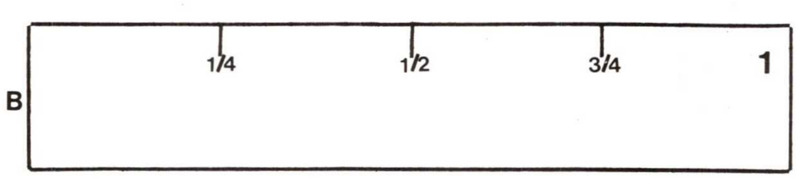
How many things can you measure with your hands (for example, books, desk top, skipping rope)? Compare your measurements with those of your classmates. Make a graph to show the differences.

8. Estimate your height in hands.

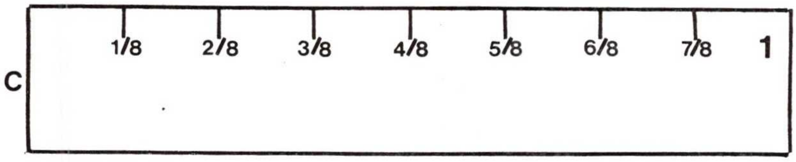
Each person's hand is a little different so a standard unit of measurement should be used. Four inches is an average hand so make hand rulers that size. A hand ruler can be divided into fractions of hands in order to measure precisely.



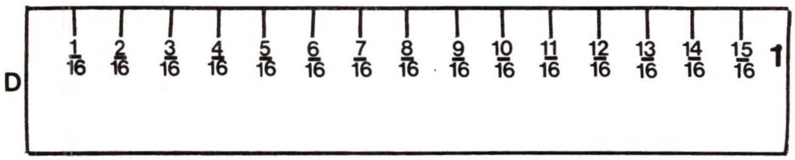
a) Divide ruler A into two equal parts (mark and label it). Each half is considered half a hand.



b) Divide ruler B into four equal parts. The length of each part is one quarter hand.

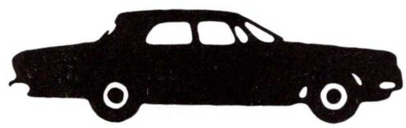


c) Divide ruler C into eight equal parts. Each part is (__) of a hand. Label your ruler.



d) Divide ruler D into 16 equal parts. Each part is (__) of a hand. Label your ruler.

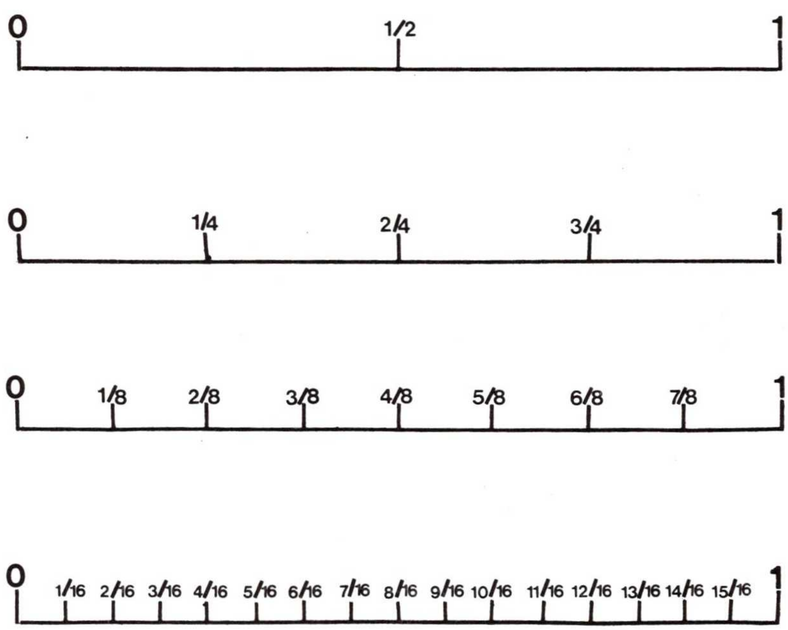
e) These rulers can be made of bristol board and used to measure different objects. Measure the car below.



Compare results with different rulers: Ruler A 1/2, Ruler B 2/4, Ruler C 4/8, Ruler D 8/16.

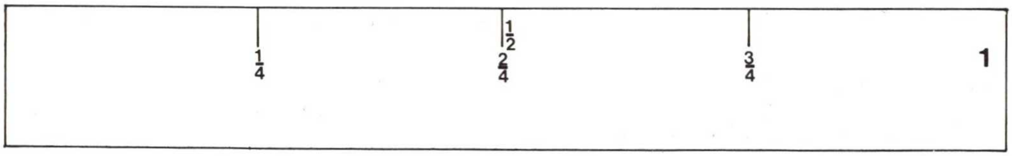
f) Do the measures show the same lengths? These fractions are equal since they are all measuring the same car.

9. Refer to your rulers to see which fractions are equal.

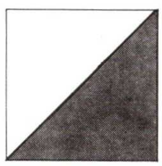


10. With adding machine tape, about 30" long, the students work in groups. By folding, mark 1/2, 1/4, 1/3, 1/6, 2/3, 2/6, 3/4, etc. Look for equivalent fractions. This works best if students work first with one denomination and then refold for others.

With the tape it is easy to see that (__) is larger than 1/4, and smaller than 1/2, but not half way between *(from workshop with Dr. Harrison)*.

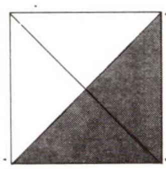


11. Sandwich fractions - equal fractions can be shown by "sandwich" pictures. Think of each square as one sandwich. Cut it into two equal parts



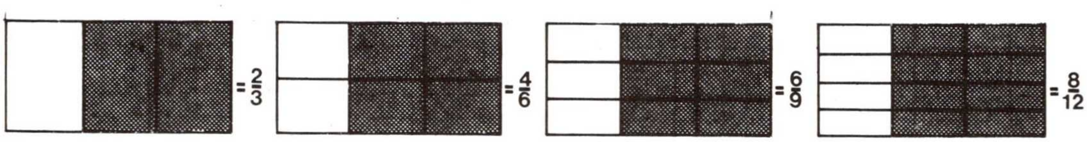
How many halves in 1?
1 = 2/2
Each part is 1/2 sandwich

Cut it into four equal parts



It takes two of these parts to make 1/2 of one sandwich
2/4 = 1/2

12. Shade (__) of each picture and then have the students tell about each. How many names for the same colored area? The fractions are equal because they all show the same amount.



13. Play a simple game of one-a-part. [*Use only the fractions the students are used to and at first do not convert the fraction to the same denominator or expect the children to add 3/6 and 4/8 to get 1.*] Add 2/8 and 6/8, or 2/3 and 1/3, etc.

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B. MEASUREMENT AND RELATIONSHIP

B.1 Probability

Probability *(Florence)*

PURPOSE To reinforce and refine students' basic concepts of probability. These activities will provide practice in sampling and graphing. They will enable students to give meaning to number facts upon which we base enquiries, and make interpretations (since they know where the data were collected and how) to investigate the effects of the various types of bias that could have been introduced.

MATERIALS One red and one white dice; a coin; a bag containing six yellow and four red marbles.

WORKSHOP 1. Flip a coin and move one pace to the right if it comes up heads or one pace to the left if it comes up tails. Flip the coin ten times following the directions it gives you each time. How far away are you from the starting point? Do this whole procedure several times. Do you always come out at the same place? Pool your results with several others and make a graph showing (vertically) the total number of times that any of you come to a given final position, against (horizontally) the final position measured from the starting point. What is your average distance in paces in ten flips? What other comments do you care to make?

WORKSHOP 2. Place ten marbles (6 yellow, 4 red) in a bag. Draw a marble; replace it and record the result. Do this 30 times. Now draw three marbles without replacement and record the results, perhaps in a table similar to the one below. Repeat 30 times.

single marbles		*three marbles*	
draw	*color*	*draw*	*colors*
1		*1*	
2		*2*	
3		*3*	
etc		*etc*	

WORKSHOP 3. Make a table showing the various ways numbers can come up when you throw a pair of dice (one red, one white). How many different ways do they come up? In how many different ways do the numbers add up to 7? Throw the dice 25 times. Record and graph the results. From your record compute the fraction

$$\frac{\text{number of throws with 7}}{\text{total number of throws}}$$

Do this for all possible sums. Compute the values you would expect to get for these fractions and compare them with the answer you got in (3).

[*Obviously this work could be preceded or followed by more work in probability and statistics involving ideas such as frequency, modes, median, mean, average, etc. The workshops used are really based on game situations themselves so a game to be used as reinforcement was not included. However, there are many games using dice which could be incorporated: spiners, snakes and ladders, roulette, probability in baseball, football probability.*]

Probability *(Robinson)*

I. We are given a set of red discs numbered 1, 2, 3, and 4, and a set of blue discs 1, 2, 3, and 4. Each set is placed in a bag and one disc is drawn from each bag to produce an ordered pair.

1) Generate the set of all possible ordered pairs that might be obtained in this way.

2) Do the same thing for sets of three, numbered 1, 2, and 3, and sets of five, numbered 1, 2, 3, 4, and 5.

3) Suppose that we have the same number of members in each of the red and blue sets. Can you find the total possible number of different ordered pairs that you could draw from any size of set?

4) If the red and blue sets have different numbers of members, how could we determine the total possible number of different ordered pairs that we could produce? (Hint - try generating ordered pairs from sets with different numbers of members.)

II. We call the total possible number of different ordered pairs that we can generate from two sets the sample space.

1) If we have a set of three red discs and a set of three blue discs, each set numbered 1, 2, and 3, what fraction of the sample space will be ordered pairs that have the same number for both components?

2) Do part (1) again for pairs of sets numbered 1, 2, 3, and 4, and for pairs of sets numbered 1, 2, 3, 4, and 5.

3) If we have red and blue sets of the same size, how can we find the fraction of the sample space which has ordered pairs with the same components, for any size of set?

4) If the red and blue sets have different numbers of members, how can we find the fraction of the sample space which has ordered pairs made of the same components? (Hint - try generating some sets with different numbers of members.)

III. The fraction that any set of ordered pairs is of the sample space is called the probability of drawing a member of that set from the sample space.

1) Take a set of red discs and a set of blue discs, each numbered 1, 2, and 3, and generate a sample space from the two sets. If we add the first component of any ordered pair to its second component, what is the largest sum we could obtain? What is the smallest sum we could obtain? What is the probability of drawing the smallest sum?

2) Try part (1) again for a pair of sets each numbered 1, 2, 3, and 4, and a pair of sets numbered 1, 2, 3, 4, and 5.

3) For the sets in Part I and Part II, what would be the probability on any one draw of drawing either the largest or the smallest sum of components?

4) For the sets in Part I and Part II, what would be the probability of not drawing either the largest or the smallest sum of components on any one draw?

5) Can you find any patterns in the probabilities you have determined?

IV. 1) Generate a sample space from a set of red discs numbered 1, 2, and 3, and a set of blue discs numbered 1, 2, and 3. For each ordered pair, add its first component to its second component. What is the probability of choosing each of the sums of the components on any one draw?

2) Do part (1) again for sets of discs numbered 1, 2, 3, and 4, and sets of discs numbered 1, 2, 3, 4, and 5.

3) Using the sets from parts one and two, can you find a pattern in the possible sums?

4) Using the sets from parts one and two can you find a pattern in the probabilities of each sum being chosen?

V. A Coin-Flipping Experiment - Please *guess* the answer to each question before you carry out the experiment. If heads comes up one time in a row we say that we have a run of one. If heads comes up two times in a row we say that we have a run of two, and so on.

If you flipped a coin 400 times -

1) How many heads would you expect to get?

2) How many tails would you expect to get?

3) How many runs of one, either heads or tails, would you expect to get?

4) Answer question #3 for runs of 2, 3, 4, 5, 6, 7, 8, 9, 10, and more than 10.

Carry out the experiment by flipping a coin 400 times and carefully recording your results in order.

5) How many heads did you get?

6) How many tails did you get?

7) How many runs, either heads or tails, did you get of: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and more than 10?

8) How do the results of the experiment compare with your guesses?

9) Can you find any pattern in the results of your experiment?

VI. Flipping Coins - If we flip one coin once, its sample space has two members, heads and tails. The probability of getting heads is ½. If we flip two coins at once, the sample space has four members, (H,H) (H,T), and (T,T).

1) What is the probability of getting two heads when we flip two coins at the same time?

2) Can you make a sample space for flipping three coins at the same time? What is the probability of getting three heads if we flip three coins at the same time?

3) Do #2 again for four coins.

4) Can you use the number of coins flipped at one time to determine the probability of getting all heads, or all tails?

5) What can you say about the probability of getting any number of heads from 0 to the number of coins that are flipped?

VII. Another Coin Flipping Experience - Guess about the answers before you do the experience.

1) If you flip two coins at the same time, how many times would you expect to get both heads if you flip them 100 times, 200 times, 300 times, 400 times, 500 times?

2) Carry out the experiment. Flip two coins 500 times and carefully record, in order, your results.

3) How many times did you get two heads in the first 100 flips, first 200 flips, first 300 flips, first 400 flips, first 500 flips?

4) How does what you actually got compare with what you guessed?

5) Does the number of time that you flipped have any effect on the accuracy of your guesses?

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B.2 Ratio Concept

Ratio and Proportion *(Steiert)*

The project is designed for a Grade VII class of about 25 students in groups of five. The entire workshop will take only two or three classes, with the final class being a presentation of results. Using objects of their own experience will, I hope, stimulate and keep the interest of the pupils throughout. They are free to make observations beyond those specified.

MATERIALS Measuring tape, rulers, scale models of aircraft, cars or boats, (opportunity to compare models with originals, e.g., cars) squared paper, globes, atlas, road maps, envelopes of Christmas cards or letters received during summer holidays from out of town, solid shapes, string, cubes, books to look up statistics, paper, scissors, (11 results must be recorded).

ACTIVITIES

I. a) Using one large piece of paper, cut it into parts and compare each smaller part to the whole and each smaller part to other parts of the whole.

b) Compare the measurement of your waist to your neck. Is there a general tie within your group? Compare your height to the measurement of your arms from fingertip to fingertip. Why common ratios?

II. Check the length of your shadow several times during the day. Make a record of the shadow length and the time. The group can agree on variables which are to be kept constant. Compare your height to the length of your shadow using ratios. At what time would you expect your shadow to be the same length as your height?

III. Choose some models from a set of solids. Find the perimeter of each face. Make a chart showing how these perimeters compare.

a) Compare the perimeter of each face of a model to the perimeter of each of the other faces.

b) Compare the perimeters of the faces of the model to those of another model.

c) If a model were twice as large how would this affect the perimeters of its faces?

IV. Use unit cubes to build larger cubes. Record your findings on a chart with:

a) edge distance
b) face area
c) total surface area
d) volume
e) ratio volume/surface area.

What happens to the ratios as cubes get larger? Find other ratios from the chart.

V. Measure and record lengths of the sides of various rectangles in the room.

a) Estimate the ratio of the sides of the classroom door.

b) Using that ratio, how long would the chalkboard be if its sides were in the same ratio?

c) What would the width of the table be if its sides were in the same ratio? (Ratios of 2:1 or 4:1 could be tried first if your ratio is difficult to find.)

VI. Choose one of the following

a) Using scale models of airplanes, cars or boats, calculate dimensions of actual objects and draw life size plans. Also discover the scale of some models by comparison with the originals.

b) Using about ten cards or letters received from out of town, calculate the distances they have travelled on both a globe and a flat map. (The distance around the earth is about 250,000 miles at the equator. Use this distance and a piece of string to find the above distances.)

c) Compare distances on a globe and a flat map.

d) Use road maps to compare "crow's flight" distances to road distances.

VII. An added activity for those interested.

a) Compare the sizes of ten major cities to the number of people in them.

b) Compare the average size of the young of five animals to the size of a full-grown adult.

[*As a follow-up and perhaps reinforcement exercise, there are two sheets on ratio and proportion from "The World Book Encyclopedia Cycle-Teacher Learning Aid." This device, if available, gives the student an opportunity to do questions and correct himself.*]

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Simple Machines: The Wheel and Axle *(Eremko)*

MATERIALS (approximately one set per station). Wooden circular discs with comparable circumferences (to keep calculations simple) or rubber-tired LEGO wheels of different sizes (or LEGO gears), tape measure, assortment of wooden dowels, strip of paper or tape, stop watch or a watch with a second hand.

WHEEL COMPARISONS

1. Measure two different wheels. What distance would each travel if they made one revolution? two? five?

2. How many turns would it take each wheel to go 5 feet? 10 feet? 50 feet?

3. If each wheel made 10 revolutions a minute, how long would each of them take to go 100 feet?

FIXED AXLES

1. Affix a dowel (or axle) solidly to a wheel. Compare the diameters of the axle and wheel. How many times does the axle turn for each revolution of the wheel? Attach a string to the axle and one to the wheel. What length of string is taken up in one revolution? two? five?

2. Connect another wheel solidly to the other end of the axle. Attach a string to this wheel. Does it take up string at the same speed as the first wheel? Demonstrate and record various uses of such a combination. How can this combination be used in machines?

A wheel and axle can be used to gain force or speed. To gain speed, apply force to the smaller wheel or axle. To gain force apply power to the larger wheel. When dealing with gears or wheels and axles, they can be arranged to drive other wheels or gears in a straight line or at right angles. Belts and chains are used to transfer speed and force in different directions. Basically, combinations of wheels and axles are the most common form of mechanical apparatus.

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C. GEOMETRY

C.1 Concept of Size

Summary of Shapes in the World Around You *(West)*

CLASSIFICATION OF TWO-DIMENSIONAL SHAPES

For each card:

a) Find a solid with a face like that shown on the card.

b) Examine the face carefully.

c) Answer the questions.

d) Record the answers.

e) Make a chart to display the information you have gathered.

Questions:

a) How many sides (faces) does the shape have?

b) How many corners (vertices) does the shape have?

c) How many curves does the shape have?

d) What are some other properties of the shape?

e) Make a list of the things around you that make use of the shape.

f) Is there anything else you might say about all the shapes?

CLASSIFICATION OF THREE-DIMENSIONAL SHAPES

Use the same directions as in two-dimensional shapes.

Questions:

a) How many faces does the solid have?

b) How many corners (vertices) does the solid have?

c) How many edges does the solid have?

d) Make a list of the things (every-day objects) that make use of the solid.

e) Can you find relationships among the variables (edges, faces, vertices) you have worked with in the above questions?

f) Is there anything else you might say about all the solids you have worked with?

FIVE REGULAR POLYHEDRA

The hexahedron, tetrahedron, octahedron, dodecahedron, and icosahedron belong to a special class of solids. Find out why these five solids belong to a special class. You might find the necessary information in the library.

MATERIALS Set of cards with two-dimensional shapes, set of cards with three-dimensional shapes, various models of solids.

INVESTIGATION OF TWO- AND THREE-DIMENSIONAL SHAPES

DIRECTIONS Make successively larger models of the unit cube or pyramid.

a) What things are changing?

b) What remains the same?

c) Find the patterns of the sequences you obtain.

d) Either draw a graph or make a table and continue the patterns as far as you can.

e) Compare the lengths of successive edges, successive volumes, successive surface areas.

f) What do you notice about these patterns?

g) What is the sequence of the ratios of the surface area to the volume?

MATERIALS Sets models.

LAKE AND ISLANDS

Questions

a) Can you arrange the islands in order according to their perimeters?

b) Can you arrange the islands in order according to their areas?

c) Can you find a relationship between areas and perimeters of the islands?

d) Have you considered the lake in your investigation?

e) Can you display in some way the information you have gathered?

MATERIALS Graph paper, lake and islands board. [Make the island irregular in shape but keep the perimeter constant. Perhaps the island perimeter could be a fractional part of the lake perimeter.]

A PROBLEM FOR YOU

You are planning to keep chickens in your garden. You have fifty feet of wire fence and you wish to make the pen as large in area as your fence will allow. Work out the length and width that will enclose the largest space. Use graph paper to prove that you are right.

MATERIALS Graph paper.

Density, Specific Gravity: Grades VIII, IX *(Fisher)*

[Based on research with this type of activity, it should not be taught below the Grade VIII level. The following is designed to lead the student to the ability to predict whether an object will float or sink in water. This depends on two concepts: density and specific gravity. As a criteria to help determine whether your students are ready for this kind of activity, you might use Piagetian-type tests based on conservation of weight and conservation of volume. As a follow-up to these activities, introduce ideas and activities dealing with the principle of flotation, and Archimedes' Law.]

PROBLEM How do you know when an object will float or sink in water?

Make an hypothesis or guess. (An hypothesis is sometimes called an educated guess.) Proceed with an investigation to test your hypothesis. If it is correct, well done. If not, you can make another guess, or rule.

STATION 1 Examine the four blocks and the hollow plastic cube. What do you discover? You should notice that the four blocks are all of equal size or volume and that this volume is equal to the inside volume of the plastic cube.

Now make a guess as to which of the four blocks will float or sink. Record your guesses. Put each block in water and find out how many of your guesses are correct. How did you do?

You might like to discover a better way of determining whether an object is going to float or sink in water.

a) Weigh each of the four blocks and record (grams).

b) Weigh the cube filled with water and determine the weight of the water.

c) Compare the weight of each block with the weight of the water, remembering which blocks floated or sank.

Do you notice any relationships or patterns? Discover anything? Can you make a rule which tells you when an object will float or sink? Try.

STATION 2 Complete the data table (next page) by -

a) checking the weights of the blocks and the water (nearest 1/10g),

b) finding the volume of the blocks and water (use graduated cylinder and record your answer to the nearest ml),

(do you need to measure five different volumes?)

c) compare weight to volume for each of the blocks and the water by dividing the weight by the volume. What units will your answers be in?

BLOCK	*WEIGHT (g)*	*VOLUME*	*WEIGHT/VOLUME*
A			
B			
C			
D			
WATER			

The figures you have just calculated have a special name. Do you know it?

Weight/Volume = Density

Density is quite useful in predicting whether an object will float or sink. Can you see a relationship between the densities of the objects that floated or sank and the density of water? Can you state a rule using the term density and whether or not an object will float or sink?

STATION 3 Let us check to see if you really do know the rule. Examine the six bottles in front of you. They all have different weights, but equal volume. Find their weights, volume and density and make a guess as to which will float or sink. Record your answers in the data table below. Did you use your rule to help make your guess or prediction?

BOTTLE	*WEIGHT*	*VOLUME*	*DENSITY*	*GUESS*	*OBSERVATION*
R					
S					
T					
W					
X					
Y					

Now put each bottle in the water to see if it floats or sinks. Record your observations. How many did you get right? Was your rule correct? Can you make a better rule to predict without fail when a bottle will float or sink? State your rule once more as best you can.

STATION 4 Can you find the density of each of the objects in front of you? Record your calculations in the following data table.

OBJECT	*WEIGHT (g)*	*VOLUME (c.c.)*	*WEIGHT OF AN EQUAL VOLUME OF WATER*	*DENSITY*	*SPECIFIC GRAVITY*	*FLOAT OR SINK*

If one cubic centimeter (C.C.) of water weighs 1 gram, what would an equal volume of water of each of the above objects weigh?

Using a reference book, look up the definition of specific gravity. Does the definition agree with or resemble any of your rules?

STATION 5 Try and figure the specific gravity of each of the objects. Does it agree with the numbers you have for density? Can you make a rule about when an object will float or sink using the definition or idea of specific gravity? Try.

FOLLOW-UP ACTIVITIES

A. *What is your volume in cubic feet?* - You need to find the weight of a cubic foot of water. (Hint - remember that you nearly float in water.)

B. *Sand and You* - Remembering that you almost float in water, try to work out the comparison in weight between a bucket of sand and a bucket of you.

C. *Eureka!* - What is the famous principle that Archimedes, the Greek mathemathician, discovered, which excited him so that he ran around with no clothes on hollering Eureka! Eureka!

D. *Hydrometers* - Find out what a hydrometer is and how it works. How can you use a hydrometer to test the strength of the antifreeze in your car radiator?

E. *Would you float in a tank of gasoline or mercury?*

F. A *Float* - Weigh a large float or piece of cork. What do you notice about its weight in relation to its size? Think of a way by which you can find out how much water would occupy the same amount of space as the float occupies. When you have discovered the answer, weigh the water. From your answers, find out how many times heavier water is than the substance from which the float is made. (What is the scientific name for this term?)

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C.2 Regular Polygon Relationships

Perimeter, Area, Volume: Grades VII, VIII *(Gajdos)*

[*The following workshops have been structured to prevent oonfusion for students who may not have been exposed to the workshop approach previously.*]

WORKSHOP 1   INTRODUCTION TO MEASUREMENT

How many students are there in your class?
How many brothers and sisters do you have?
How many players on a baseball team?

To answer these questions you count the number of elements in each set - you match the natural number with each element of the set in the counting squares. But, suppose we ask the questions -

How tall are you?
How far do you live from school?

These questions couldn't be answered by counting elements as we did in the first series of questions. In order to answer questions like "how long," you need to be able to make measurements.

TASK Students experiment with different forms of measurement.

Open-ended questions

1. How many different ways can you measure objects such as the length of our classroom, your table, the blackboard? Make a list.
2. First estimate your answer.
3. Using your new units that you inverted, measure the length of your classroom, table, blackboard.
4. Make a chart.

	Your estimate	First unit of measure	Second unit of measure	Third unit of measure	Actual measure
Length of classroom					
Length of table					
Length of black board					

WORKSHOP 2 PRIMITIVE FORM OF MEASUREMENT

TASK Collecting data in experiments involving measuring: fathom, cubit, digit, hand, foot, span. The results will be put in a chart.

PURPOSE To help students discover the need for standard measurements.

UNIFYING IDEAS Functions and relations, measurement, addition.

MATERIALS Roll of adding machine paper, scissors, yardstick or tape measure, glue.

PROCEDURE For each person in your group cut a strip of paper equal in length to one fathom, one cubit, one digit, one hand, one foot, one span. Is there any relationship between one fathom belonging to one person and one fathom belonging to another? Measure the length of each measure and record your answers.

	Person 1	*Person 2*	*Person 3*	*Person 4*	*Average*
Fathom					
Cubit					
Span					
Hand					
Foot					
Digit					

1. Do you see any difficulties that could occur using these units of measure?

2. Can you relate experiences involving non-standard units of measure?

3. How can we overcome the problems of non-standard units of measure?

WORKSHOP 3 PERIMETER OF TRIANGLES .

TASK Collecting data in experiments involving rolling cardboard triangles along a yardstick, winding a thread around the triangle and measuring the thread. For each object or triangle, students record in a table and graph the distance traveled around.

PURPOSE To help students discover the meaning of perimeter; how to use it.

MATERIALS Cardboard triangle, thread, scissors, tape measure, yardstick.

Open-ended questions

1. How many things can go around our triangle?

2. How many different ways can we measure around the triangle?

3. Estimate the distance around.

Triangles	*Measure side 1*	*Measure side 2*	*Measure side 3*	*Distance around*	*Your estimate*
1					
2					
3					
4					
5					
6					

4. What conclusion do you come to about the perimeter of a triangle?

5. Can you suggest a formula that will describe the perimeter of the triangles?

WORKSHOP 4 PERIMETER OF A RECTANGLE

TASK Collecting data in experiments involving measurement of the sides of different rectangles. For each rectangle, students record in a table the sides and the distance around.

PURPOSE To help students discover the perimeter of a rectangle

MATERIALS Rectangles of varying size, ruler, tape measure, thread, paper, scissors.

Open-ended questions

1. How many things can you go around that are rectangular in shape?

2. How many different ways can we measure around? Is there a better way?

3. Estimate the perimeters of the rectangles. How close were you?

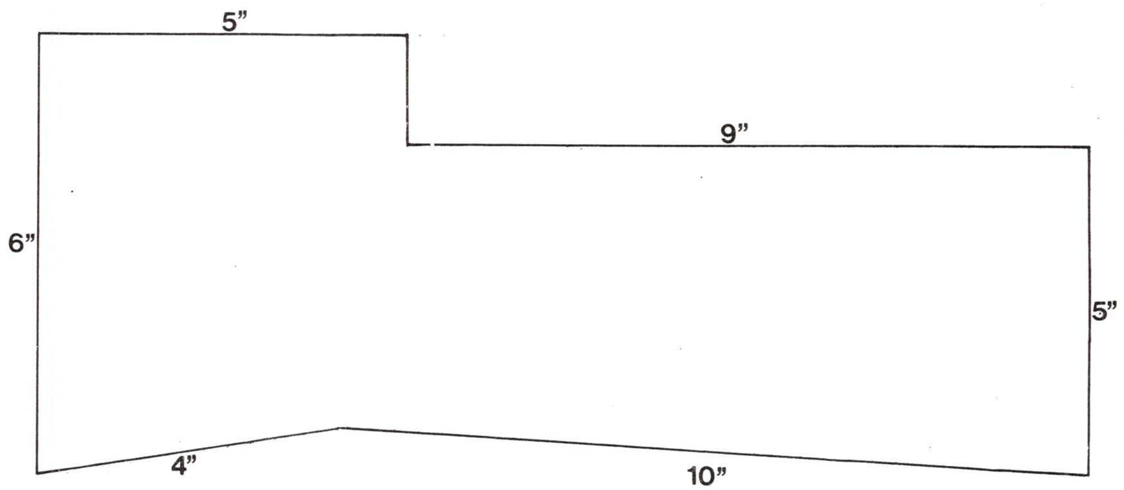
4. Can you find the perimeter of a rectangle without measuring all the way around?

5. Can you determine a formula for the perimeter of a rectangle that will work in all cases?

	Length 1	*Width 1*	*Length 2*	*Width 2*	*Perimeter*	*Your estimate*
1						
2						
3						
4						
5						
6						
7						

Open-ended questions

1. Can you find the perimeter of a 5 sided polygon?
2. Can you find the perimeter of a 12 sided polygon?
3. Find the perimeter of the following:



WORKSHOP 5 CIRCUMFERENCE OF A CIRCLE

TASK Collecting data in experiments involving rolling tin cans or other circular objects along a ruler or yardstick; on winding the thread around the round object and measuring the thread. For each object, students record in a table and graph the distance traveled in one turn.

PURPOSE To help students discover that the rate of the circumference of a circle to its diameter is a constant (7); to determine a formula for the circumference.

UNIFYING IDEAS Multiplication and division, functions and relations, measurement. [It is intuitively clear that the longer the diameter of a circle, the longer the circumference will be.]

MATERIALS Yardstick, thread, scissors, round objects of varying diameters, graphing paper.

Open-ended questions

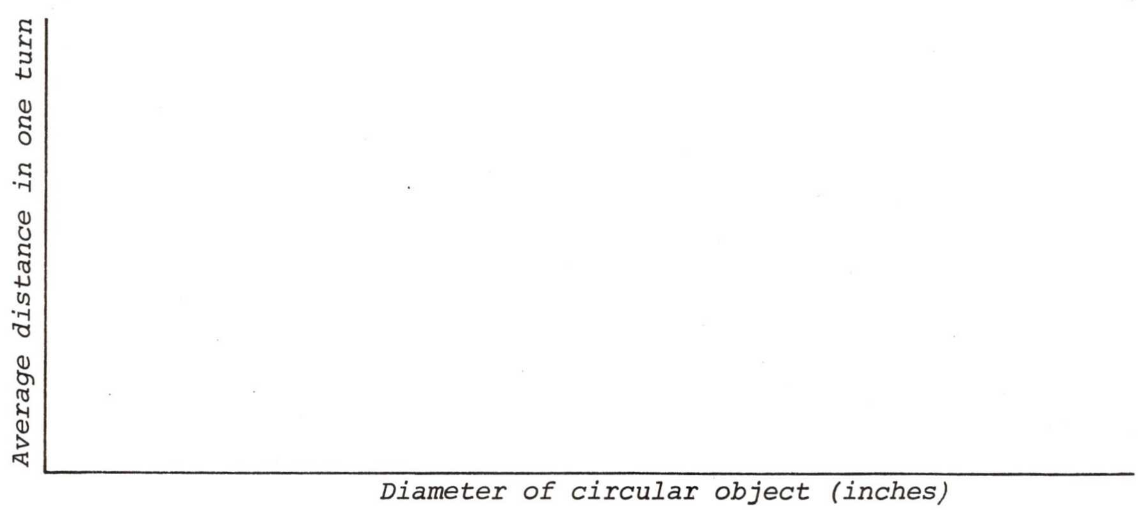
1. How many ways can we measure the distance around the object?

2. Use your method to find the circumference; make more than one measurement and average your results.

Distance in one turn

Diameter	1st	2nd	3rd	4th	Average

Graph Your Results



Open-ended questions

1. Describe what you notice about the graph.

2. Do you see any relationship between the circumference and the diameter?

3. Is it a constant ratio? If so, what is it?

4. If you were given this ratio and the diameter, could you then find the circumference?

5. Can you suggest a way of finding out what the circumference of a circle is without measuring all the way around?

6. If the ratio of circumference is π can you suggest a formula for the circumference of a circle?

WORKSHOP 6 AREA OF RECTANGLES

TASK Collecting data in experiments involving placing square units on a cardboard rectangle. For each rectangle, students record the number of square units needed to cover each.

PURPOSE To help students discover the meaning of area of rectangles.

UNIFYING IDEAS Multiplication, functions and relations, measurement. [The student will realize intuitively that area means the amount of surface exposed.]

MATERIALS Ruler or tape measure, 50 squares, (exactly 1 inch by 1 inch), rectangles of varying sizes, scissors.

Open-ended questions

1. Estimate the number of squares needed to cover each rectangle.

2. Record your answers.

Rectangle	*Length (inch)*	*Width (inch)*	*No of squares to cover*	*Your estimate*
1				
2				
3				
4				
5				
6				
7				

3. Do you see any relationship among the length, width and the area of a rectangle?

4. Graph your results.

5. Can you think of an easier way of finding the area of a rectangle?

6. Explain your finding in a formula that will work for all rectangles.

WORKSHOP 7 RELATIONSHIP BETWEEN AREA AND PERIMETER

TASK Collecting data in experiments involving the comparison of constant area of a rectangle with perimeter, and comparison of constant perimeter with area.

PURPOSE To help students discover the relationship between area and perimeter.

UNIFYING IDEAS Relations and functions, measurement.

MATERIALS Piece of string 19" (ends tied together to form a band), one-inch square units.

Open-ended questions

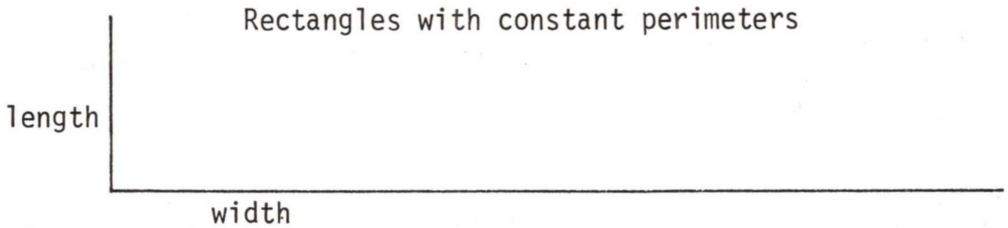
A. Constant Perimeter - find the largest area.

1) What are the measurements of the figure that produces the largest surface area?

2) What is the shape called?

3) Make a table showing how the length changes with the width.

4) How does the area change with the width?



B. Constant Area - find the largest perimeter.

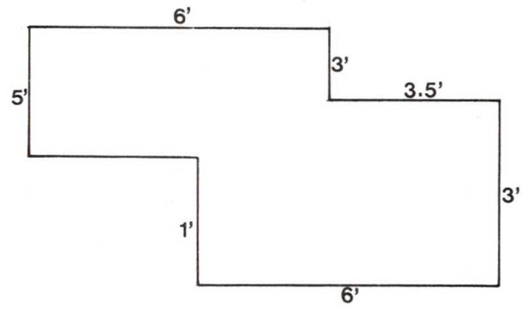
1) Given an rectangle with a constant area of 16 sq. inches, find the largest perimeters available.

2) Make a chart of the possible formations.

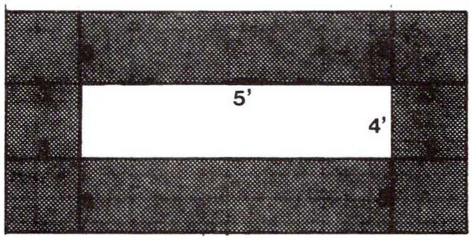
3) Draw a graph to show the relationship between the constant areas ordered by the widths.

Puzzles involving area and perimeter

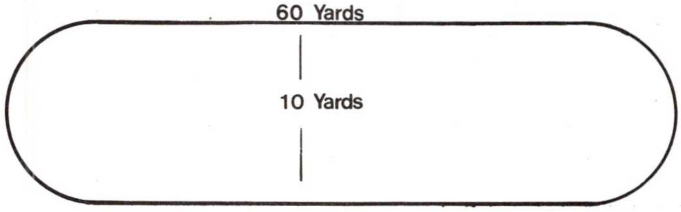
1. Find the area of the figure below.



2. Find the area of the shaded part.



3. Find the perimeter of the figure below.



WORKSHOP 8 VOLUME OF RECTANGULAR BOX

TASK Collecting data in experiments involving filling a cardboard box with salt measured with a one-cubic-inch container. For each box students record the measurements of the box, the surface area of the box, and the volume of the box.

PURPOSE To help students discover the meaning of volume.

UNIFYING IDEAS Multiplication, relations and functions, measurement. [It is intuitively clear that volume means capacity. The student will know that, to determine the volume of a container, he has to measure how much material will go into it.]

MATERIALS Various rectangular containers, salt, one-cubic-inch container.

Open-ended question

1. What is the meaning of volume?

2. Is there a relationship between the measurement of the container and the volume?

3. Is there any relationship between the surface area and volume?

Estimate the number of inches in each box. Fill the boxes with salt. Were your estimates close?

	Width	*Length*	*Height*	*Volume*	*Volume estimate*
1					
2					
3					
4					
5					
6					

4. Can you see any relationship among length, width, height and volume?

5. Can you suggest a formula that will give the volume for all rectangular containers?

Length	*Width*	*Height*	*Surface area*	*Volume*

6. Can you see any relationship between the surface area and the volume?

Regular Polygons: Grades VIII, IX *(Klopp)*

OBJECTIVES To increase familiarity with polygons, to introduce the concept of a limit, to make the construction of regular polygons more meaningful, to provide a problem in which the students can analyze and evaluate data and generalize from them, to introduce a problem with more than two variables, to discover their own formulas for the area of a regular polygon, and to provide an opportunity for creative thinking.

MATERIALS Glassheaded colored pins, roll of thread, protractors, colored felt pens, supply of duplicated sheets with the circle r=12.

1. Find in as many ways as you can the triangle, then the quadrilateral, the hexagon, the octagon, with the maximum area that can be inscribed in the circle given.

2. How are the area and the perimeter of a polygon related to each other?

3. How does the area of a polygon change as you increase the number of sides of the polygon?

4. Find the approximate area of the polygon with 99 sides.

First, students will concentrate on the maximum area of the triangle because of its simplicity. I would expect at least three different approaches: Keeping the base of the triangle constant and

1. changing the measure of the angle:
   a. at one of two endpoints of the base or
   b. (as displayed) the angle at the midpoint of the base,

2. changing the measure of the altitude,

3. changing the measure of the perimeter.

From all three cases, students will conclude that an isosceles triangle will give the maximum area.

Next, they will change the length of the base of an isosceles triangle. From their data the students will see that the area is maximized when the base is congruent to the other two sides. An equilateral triangle is a regular polygon, and it is hoped that the students will generalize that the required quadrilateral is a square, etc.

When finding the area of a hexagon (later an octagon), students are encouraged to develop their own area formulas by dividing the hexagon into convenient areas (triangle, rectangle, etc.) which can be easily computed.

By making a graph of the relationship between the number of sides of a polygon and its area, students will soon discover that the area reaches a limit, the area of the circle. Therefore, the area of the circle is the best and easiest estimate of a 99-sided polygon.

Mapping the School Yard *(Herman)*

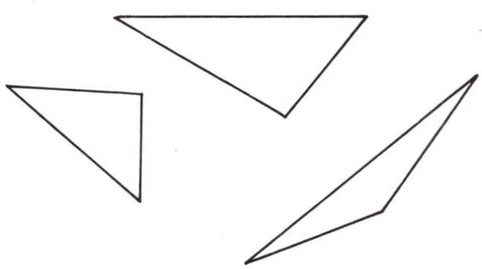
Pupils are always asking of what use is this to us? By mapping their school yard, and building a model to scale, they will find use for scales, fractions, proportions, accuracy of measurement error, probable error, percent in the possible error, decimals, some simple geometry and trigonometry and, above all, the need for careful and accurate work in all computations of basic math.

First the pupils would be taught to use the protractor. Then they would be given time to discover that the interior angle sum of all triangles = 180°; isosceles triangles have two angles equal and two equal sides, whereas an equilateral triangle has all three sides equal and the interior angles all equal 60°. They would also be encouraged to find ratio relationships among the measures of sides of similar triangles.

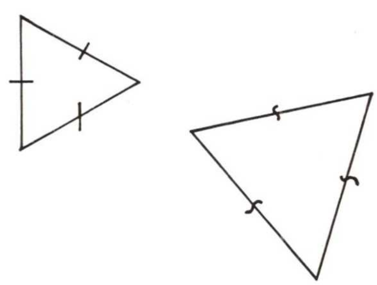
The pupils could then make instruments for measuring angles with these, along with tape measures and by using triangulation, they could map their school yard. By using shadows and ratios they could find heights of any vertical objects. With their measurements they could build a scale model of the school yard.

This project could be correlated with the social studies course, giving stronger reinforcement for the value of math as a useful tool in every-day life.

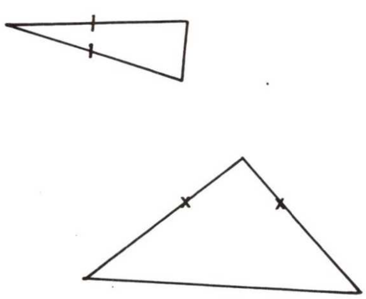
I. Σ interior $\angle$ measures = 180°



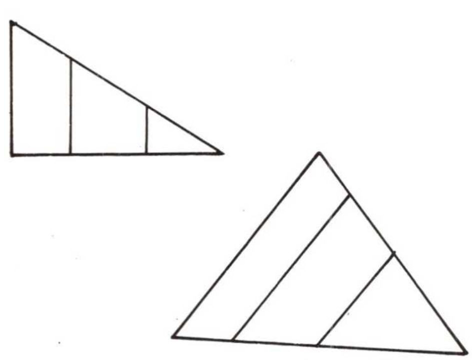
II. Equilateral Δ's



III. Isosceles Δ's



IV. Ratio in similar Δ's



Similarity of Triangles *(Gunn)*

PURPOSE Students acquire the idea of a similarity correspondence between triangles. Hence the idea of "similar." They must be led to the concept that two triangles are similar if and only if there is a one-to-one correspondence between their vertices such that corresponding pairs of angles are congruent and corresponding pairs of sides are proportional. This is similarity.

BACKGROUND Students must already have covered the concepts of ratio and proportion and congruence of triangles (SAS, ASA, AAA Correspondences).

MATERIALS Protractor, ruler, pencil, paper.

ACTIVITY Similar triangles.

A. Using a protractor, draw $\triangle$ ABC and $\triangle$ KMN so that $\angle A \simeq \angle K$ and $\angle B \simeq \angle M$. Measure the sides of the triangles and $\angle C$ and $\angle N$.

1. Is $\angle C \simeq \angle N$?

2. Is $\frac{AB}{KM} = \frac{BC}{MN}$?

3. Is $\frac{KN}{AC} = \frac{KM}{AB}$?

4. If two angles of one triangle are congruent to corresponding angles of a second triangle, are the third angles congruent? Why?

B. Draw a $\triangle$ ABC which has sides with the following lengths. AB=4, BC=5, AC=6. Construct a $\triangle$ A'B'C' with A'B'=8, B'C'=10, A'B'=12. Measure very carefully the angles of both triangles. Within the limits of accuracy you will notice that $\angle A \simeq \angle A'$, $\angle B \simeq \angle B'$, and $\angle C \simeq \angle C'$.

1. Since these congruences exist and since $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$ are the two triangles included similar?

C. From A and B above, does it seem that the congruence of two pairs of corresponding angles is sufficient for the similarity of two triangles? Is an SSS correspondence pattern sufficient to guarantee similarity of two triangles?

D. Investigate other possibilities.

correspondence pattern sufficient to guarantee similarity of two triangles?

D. Investigate other possibilities.

E. Investigation of pairs of similar triangles reveals several possible relations. The angles of one triangle are congruent to the corresponding angles of the other. There also seems to be a relation between the length of the pairs of sides. The angle congruences and relations between lengths of sides do not occur at random. There is a clear cut correspondence between similar triangles which maintains an order of correspondence of vertices, and, therefore, of angles and sides.

F. Can you arrive at a reasonable definition of similar triangles?

G. If two triangles are congruent, does it necessarily follow that they are similar? Are two similar triangles necessarily congruent? Why?

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SENIOR HIGH

A. ALGEBRA

Guess a Function *(Dyrholm)*

USES This is designed for Grades X, XI, and XII but much of it is also appropriate for junior high. It is felt that these games are more useful if the student is familiar with the various types of "conditions" and with the process of taking differences. Their uses would be as an interest captivator, a teaching device, a review technique, and as an enrichment.

MATERIALS

1. Horseshoe Tale - Based on counting the cost of each successive nail (nails vs. cost).
2. Tower Puzzle - Puzzle which generates the same function as the horseshoe tale (disc vs. moves); available from materials center.
3. Peg Game - Available from materials center - can be constructed easier with checkers (pegs vs. moves).
4. Forming Squares - 1" squares used to form successive squares (edge vs. 1" squares).
5. Cube Construction - Unit cube used to form successive cubes (edge vs. unit cubes).
6. Checkers - Eight squares in a row, home square empty (checkers vs. moves).

   a) Game 1, jump any number or move one square.
   b) Game 2, move one square only.
7. Elevator - Eight squares in a row. Checker on square indicates full load. Black checker is elevator which must transport the loads from each floor to the basement (load vs. time).

[*Plotted points have been joined on all graphs although it may be hard to justify as all these games deal with discreet points and not a continuum - however, the points were joined to better demonstrate graph shapes of the various functions.*]

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B. GEOMETRY

B.1 Lines

Parallel Lines and Perpendicular Lines *(Dufresne)*

In order to create an atmosphere of active learning with today's students, they must first be "conditioned." The majority of our senior high students will suffer the same restrictions that I feel. That is, they feel inferior when compared with their teacher and fellow students. That fear, that their way is not the correct way or at least not the expected way to attack a given problem, restricts creative work on their part. Since active learning requires creative work on the part of the student, time must first be spent on changing students' attitudes. I feel this is very important and as this is the developement-of-a-topic that I, personally, will find useful, I shall state how I overcome these difficulties.

From the start of the year, I think of the students as "people." I treat them with respect, listen to everything they have to say and value what they have said. If a student knows that you feel this way, he will respond by giving more and more of himself and not just give expected answers or someone else's answers. Since students are more intelligent than we give them credit for, their suggestions and view points are full of merit, and with the correct handling they gradually gain confidence which, in turn, generates more ideas from their own thinking.

In class, wrong and right answers must be handled in such a way as not to work the student's participation. The students must be taught that a wrong answer is valuable. All answers should be proved. When a wrong answer has been disproved, generally the student who gave that answer, and the others, have learned valuable information and perhaps this understanding will lead to a correct answer. A student must be made to feel that his answer is a good one, although not necessarily correct, and that he had grounds to give such an answer. As such, it must be treated with respect and given the same treatment as other answers in proving or disproving them. Above all, students should not be graded on the quality of their responses, but, rather, on the quality of their participation.

The above sounds simple, but I have found that it works. For the lessons, the students have already learned the following mathematical skills. They know what is meant by the slope of a straight line, and how to find the slope and equation of a line. They know what is meant by parallel and perpendicular lines.

Different methods suit different teachers. I have tried the formal group method but have not found it to work satisfactorily and, therefore, I prefer to have students work individually. Through past experiences the students know that they are allowed to confer on their work so, in reality, the situation is one of group work as they do compare answers, argue and agree. The advantage of their method is that each student must record his own results and can't rely on the work of others. Also, if a student doesn't agree on the method used by others around him he can gracefully withdraw and do it the way he wants.

Each student has several sheets of graph paper, a ruler and a hand-out sheet. I would prefer to ask only one question and from there let the students take it on their own. The question would be similar to the following one. "Knowing what you know about the slopes and equations of straight lines, find out everything you can about the slopes and equations of lines that are (i) parallel, and (ii) perpendicular." The student would be given the following instructions: he is not allowed to use his text book, only his graph book, and try to discover the relationships on his own. This simple question is enough for some students, but for others it is frustrating as they don't know how to begin. Therefore, the following sheet of instructions has been prepared.

1. On your graph paper draw several lines that are parallel.

2. Find the equations and slopes of these parallel lines. (Hint - if you draw your parallel lines to intersect a few lattice parts, you can use these points to find the slopes and equations of these lines. This will give a greater degree of accuracy.)

3. Can you find any similarities or relationships in the slopes and equations of these parallel lines?

4. Draw a different sets of parallel lines to check your results.

5. What conclusions can you draw?

6. Repeat 1-5, using perpendicular lines.

   (If you cannot see any relationships try drawing another set of lines.)

This active learning lesson is relatively easy for Math 20 students. A few students found that their curiosity led them further into the slope concept. For example, one student came to me with the following question. "What about the slope of $y = x^2$? I don't think it is constant." I agreed, gave him a quick explanation of how to find the slope by drawing a tangent to the curve and explained that the slope of $y = x^2$ could be found by using a certain formula, and could he find out what the formula was? The student drew the graph to a large scale taking many points to ensure greater accuracy. He took the slope at many points recording both the point and slope, and came to the conclusion that the slope was equal to 2x for any pt. with coordinates (x,y). He also ventured that the slope of any curve $y = x^n$, would be nx. I drew the graph of y x and pointed out that the slope is always positive, even when x is negative. He was able to come up immediately with the answer that the slope of $y = x^3$ must be 3x and then the slope of $y = x^n$ must be nx^{x-1}. He must have done a lot of work on his first answer that he could switch his answer so readily because he did seem to have a clear explanation of it. I was greatly pleased and gave him a pat on the back, explaining that he had developed a "formula" in calculus for the derivative of y, when $y = x^n$. I believe that the best results come from students' own questions and if a teacher can encourage them to investigate for their own answers, the results will be greater.

A student is truly interested to question in the first place and even if the question isn't entirely related to the topic under discussion he should be encouraged to pursue it if there is any mathematical merit in it.

After all of the students had a chance to find the relationships, we discussed them in class and then tried some of the questions in the textbook that used this new information. I find it is necessary to have a discussion period at the end of the lesson to help those students who are unable to formulate all their conclusions until they are told that they are correct.

In concluding I will say that the lesson was satisfactory. The students found it enjoyable and it provided for all levels of students in that the quicker students had questions to ask that led them into further discovery of their own. There was no student that couldn't complete at least a part of the assignment.

Parallelism *(Moynihan)*

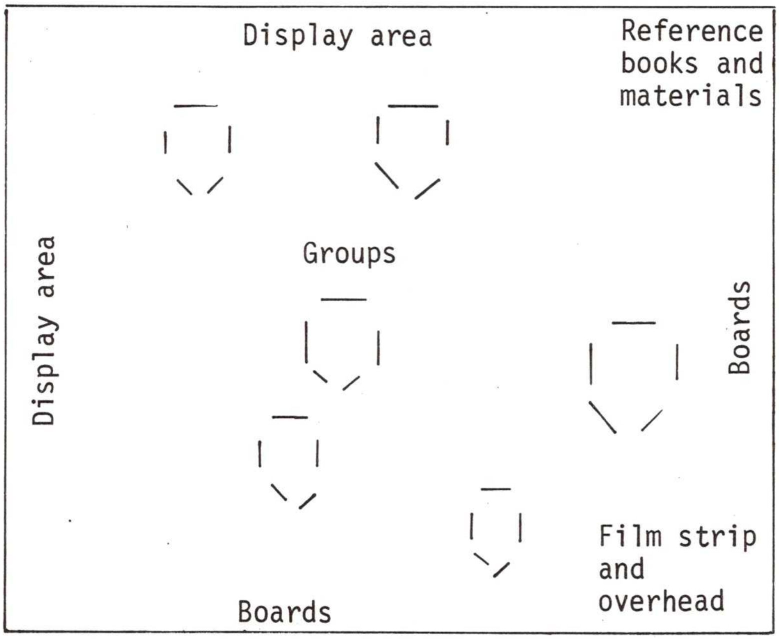
A Workshop Approach to Teaching the Concept of Parallelism

PURPOSE *Activity I* - To discover the conditions which make two or more lines parallel; to apply the knowledge gained to the solution of some problems.

Activity II - To discover the results when two or more parallel lines are crossed by a transversal.

Activity III - To discover the kinds of parallelograms, their relation to each other and to quadrilaterals in general.

CLASSROOM SET-UP



MATERIALS Text: *Geometry - A Modern Approach.* Wilcox. Transparencies to accompany text. Reference Texts: *Geometry, Plane and Solid.* Brown and Montgomery; *Modern School Mathematics.* Jurgensen, Donnelly and Dolcianni; *Glossary of Mathematical Terms.* Gundlach; *Universal Exercises in Geometry.* Charles E. Merrill Books, Inc. Film strip projector. Film strips:

S.V.E. - Modern Elementary Geometry

1. Points, Lines and Planes
2. Angles
3. Parallelism (used as a review after activities completed).

Overhead projector, colored mounting paper, paste, brushes, scissors, colored pencils, magazines (for pictures), graph paper, mimeographing services (for copies of instructions so that each pupil has own copy to work with), geoboards, punched cardboard strips, pins, and elastic bands.

[*For this project, 30 Grade X students enrolled in the Math 10 course were randomly chosen and divided into six groups of five students each. When an activity was finished, each student was required to write up the findings. Before beginning this activity, you may wish to review. There are a number of film strips in the film strip corner. Run them if you wish. Please rewind them carefully when you have finished. Note - the strip on Parallelism was not made available at this time; it was used for an overview when the project was finished.*]

ACTIVITY I

Draw AB. At A in AB draw a co-planar line perpendicular to AB. Call it AC. At B in AB draw BF perpendicular to AB.

Take any point D in AC. What is the measure of AD? Record it on your data sheet.

Measure a distance BE on BF such that AD = BE. Record it on your data sheet.

Join DE and extend it to form DE. Select a number of points on AB. Call them X_1, X_2, X_3. Measure the perpendicular distance between these points and points on DE. Call the point on DE Y_1, opposite X_1, Y_2 opposite X_2, etc. Record your data. What do you notice about your results? What conclusions have you reached concerning AB and DE?

Such lines are said to be parallel. Can you write a definition of parallel lines?

EXERCISES

1. Given the co-planar AB — MN and CD — MN, prove that AB is parallel to CD. Draw conclusions from this exercise and state it as a theorem.

2. Construct a line parallel to a given line at a given distance from it.

3. Given a triangle ABC with the angle B bisected by BD, can you construct a line through A parallel to BD?

ACTIVITY II, Transversals

Draw MN and PQ parallel to each other. Now draw XY intersecting MN at the point R and PQ at S. XY is called a transversal on MN and PQ.

How many angles have been formed? Identify them by numbering them 1, 2, 3, 4, etc. Measure the angles and record your data. What do you notice about your results?

Do you get the same kind of results with other parallel lines crossed by transversals having different slopes? Try several pairs and record your data.

What happens if you use a set of more than two parallel lines? Try it and record your data. What conclusions regarding the angles have you drawn? Discuss them and write them down.

There is an interesting problem involving parallels on page 142 of *Geometry, Plane and Solid*, by Brown and Montgomery.

Alternate Angles - Angles 1, 7 and 2 and 8 are called alternate angles; angles 4 and 6, and 3 and 5 are interior alternate angles.

Exterior Angles - Angles 1, 2, 8, 7, are called exterior angles.

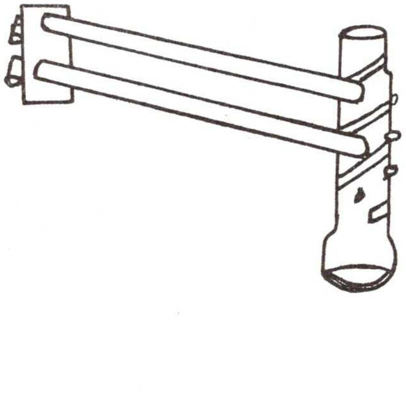
Interior Angles - Angles 3, 4, 5, 6, are called interior angles. Note that alternate angles lie on opposite sides of the transversal.

Corresponding Angles - are those angles which lie in the same relative position with regard to the transversal. Angle 2 and angle 6 are corresponding angles. How many other pairs of corresponding angles are there? List them.

Are there any supplementary angles in Figure 1? If so, list them. Can you prove that the angles you have named are supplementary? If so, write out your proof.

AN INTERESTING PROBLEM

Dave's hobby is photography. At times he needs a dim light. In his dark room, which is unlighted, he can see no convenient way to extend wiring for an electric circuit. From some flat curtain rods, a discarded but usable hinge, tape, and rubber band, Dave devised a flashlight holder as shown below. Now he has a light whose height is adjustable yet the flashlight will remain perpendicular to the floor at any height. What parallel relations did Dave control in his arrangement of materials?



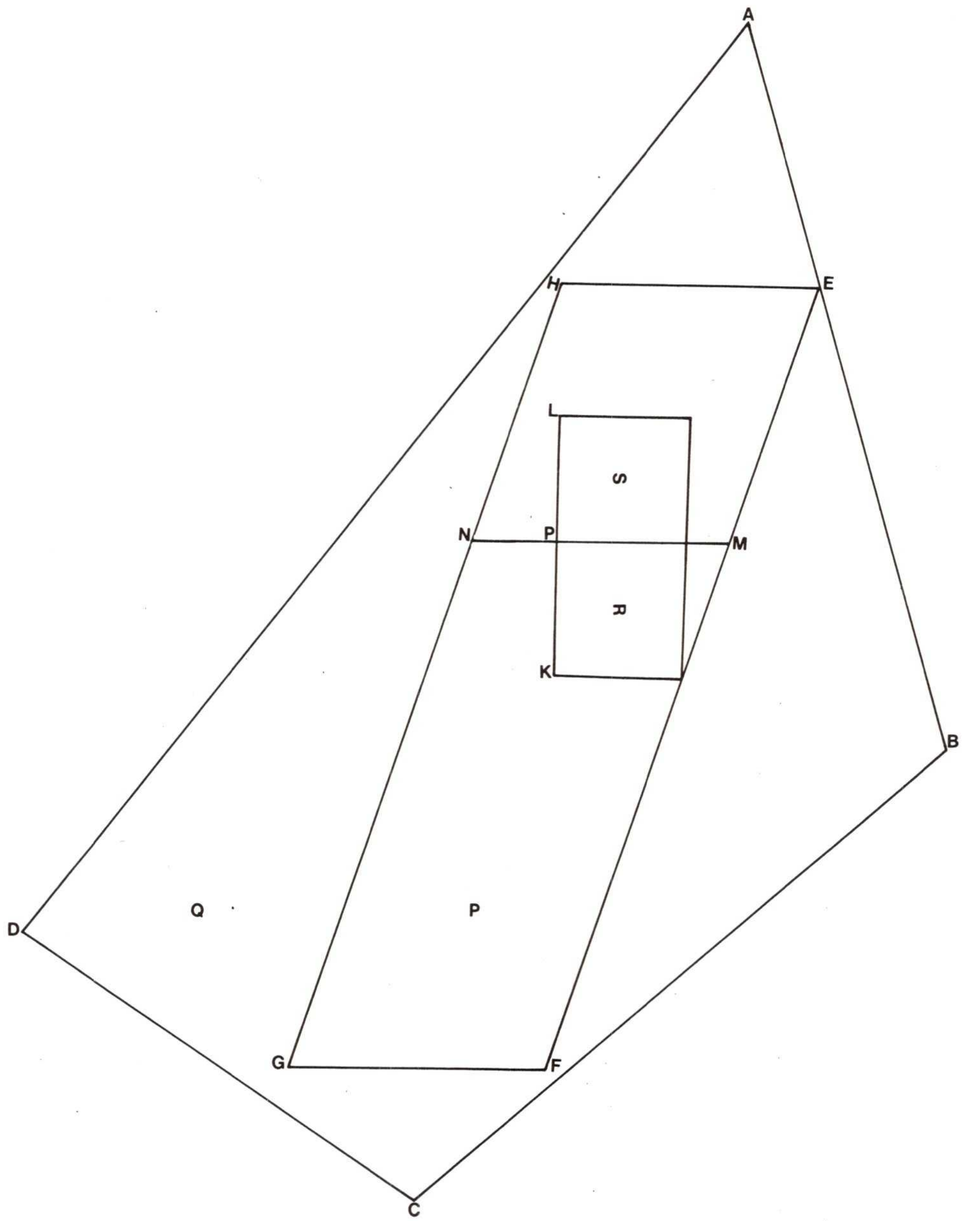
ACTIVITY III, *Parallelograms*

DEFINITION A parallelogram is a quadrilateral having its opposite sides parallel.

Using the cardboard strips, how many kinds of parallelograms can you make? Try the same thing with the geoboard. Compare your results. Record the results on your data sheet.

What do all the parallelograms have in common?

Draw a large quadrilateral on a sheet of graph paper. Can you fit the different kinds of parallelograms you found into it?

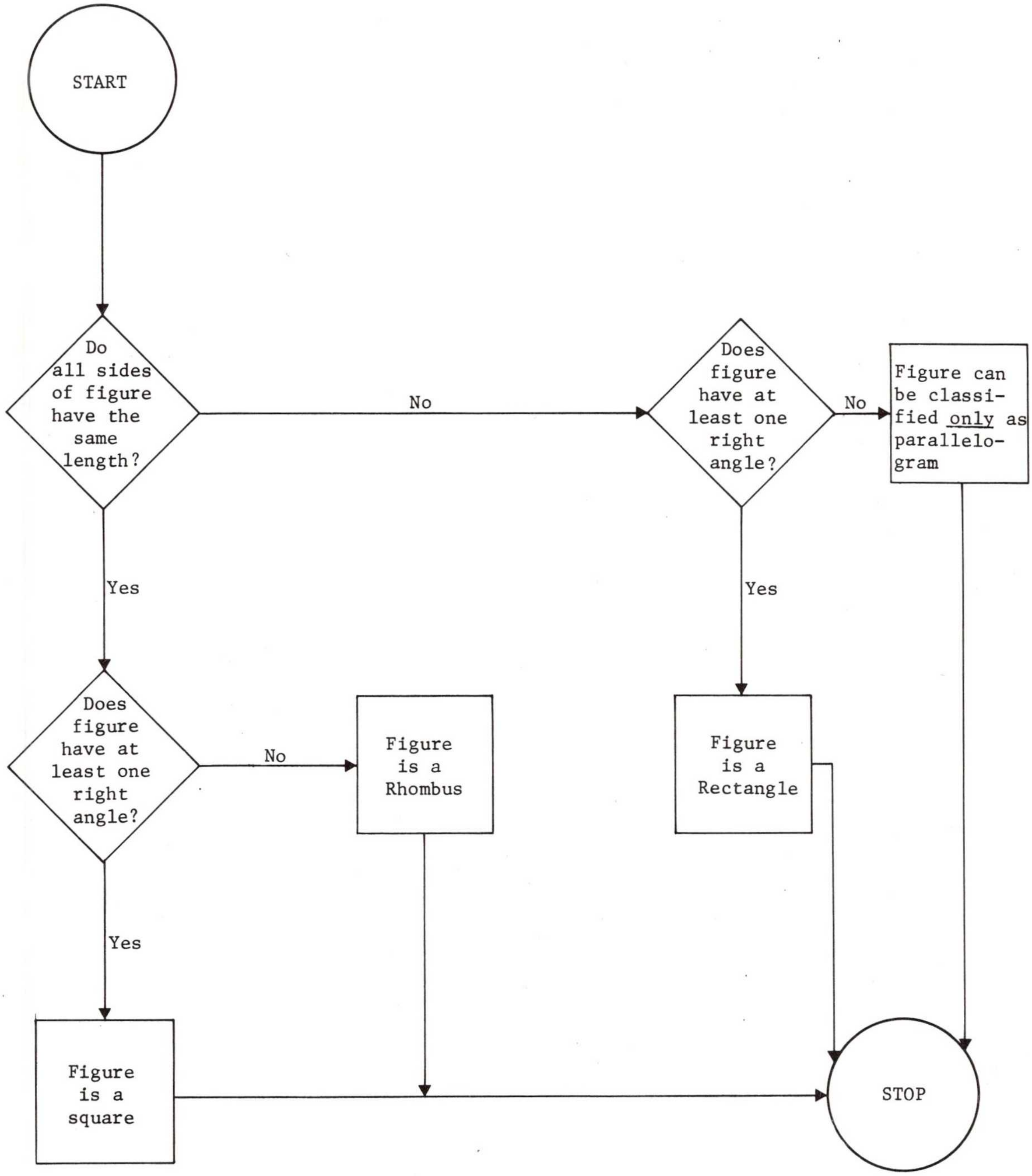


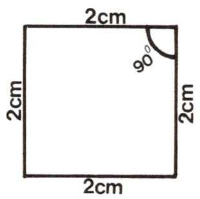
Can you fit any of the parallelograms into any of the others? Can you express your findings in terms of sets? Are any of the parallelograms subsets of any other kinds?

Find some pictures illustrating the use of parallelograms in such fields as architecture, fabric design, furniture design, or others. Mount them. See slide for one of them.

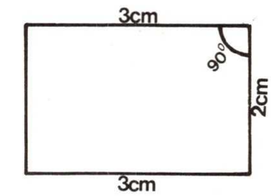
NAMING PARALLELOGRAMS

Using the flow chart below, name each of the figures on the next page.

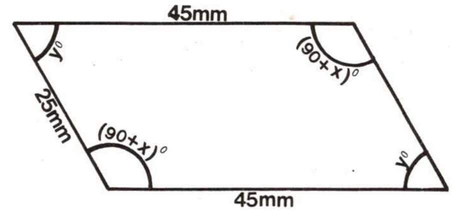




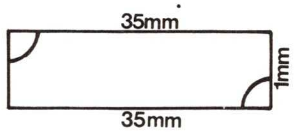
1 ________



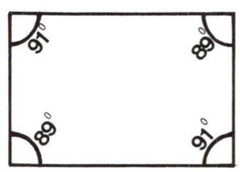
2 ________



3 ________



4 ________



5 ________

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B.2 Regular Polygons

1. Square - Use the unit squares to find the area of several squares whose sides are: 1 unit, 2 units, 3 units, etc. long. See if you can find an equation that gives the area of any size square.

2. Rectangle - Use the unit squares to find the area of several rectangles whose dimensions are:

Width	Length
1 unit	2 units
1 unit	3 units
2 units	3 units
2 units	4 units
3 units	3 units
3 units	6 units

You can make up a few more rectangles of your own if you like. Can you find an equation that gives the area of any size rectangle? Is there a relationship between the equations for the area of a square and a rectangle?

3. Parallelogram - Use the unit squares to find the area of several parallelograms whose dimensions are:

Base	Altitude
2 units	1 unit
3 units	2 units
3 units	3 units
4 units	3 units
5 units	3 units
6 units	3 units

Draw the parallelogram first if necessary. Can you find an equation that gives the area of any parallelogram? Is there a relationship between the equations for areas of a rectangle and a parallelogram?

4. Triangle - Use the unit squares to find the area of several triangles whose dimensions are:

Type	Base	Altitude
Right angle	1 unit	1 unit
Right angle	2 units	1 unit
Isosceles	2 units	4 units
Isosceles	3 units	2 units
Right angle	4 units	3 units
Scalene	5 units	3 units

Draw the triangles first if necessary. Can you find an equation that gives the area of any triangle? Does the type of triangle make any difference? Is there a relationship between the equations for areas of a triangle and a rectangle?

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B.3 Circles

Properties of a Circle *(Gordon)*

PURPOSES

1) To develop an awareness of some of the properties of the circle to the extent that pupils can solve numerical exercises which require no rigorous proofs,

2) to develop an interest or desire to look beyond the obvious in a diagram,

3) to develop a desire to have some interest in going further than the given question requires and finding why certain conditions exist,

4) to review previous knowledge of geometry.

PROCEDURE Divide the class into small groups (four has seemed to be the most workable number). Each group is given a series of questions which require conditions. Not only are they asked to discover a conclusion to each but also to explain their conclusions. At the end of each activity--which in most cases requires an hour--the groups report their conclusions - some of them obvious to all in the class, others requiring explanations by the group with ensuing discussion.

[*At the beginning of each class, necessary definitions of new terms and a review of basic constructions need to be discussed. This can be done in a class previous to the workshop. However, if it is done at the beginning of each day, it allows the teacher to make comments about the previous day's work and tie up loose ends when there is no heated discussion with which to compete. Also, if done at the beginning of the class, the terms and constructions are new for the day and ready to be worked with. At the end of some activities the groups are assigned numerical exercises, if appropriate, and at the end of the five activity sessions the class is given several numerical exercises which involve the* *use of all properties. (If a more rigorous approach were the aim, at the end of each activity the class could be assigned deductions rather than numerical exercises.) At this point it is the decision of the students to work independently or remain in groups. At the end of the workshops, one class period can be spent discussing the properties of the circle. This serves to answer questions that arise from numerical exercises and gives time for new discoveries to be discussed. This is a fun class, allowing moments of glory for some who find answers to questions that they ask themselves and sorting out troubles for others who need help.]*

MATERIALS Compasses, ruler, protractor, question sheets.

[In the existing Math 20 course time does not allow for development of theorems and application of them to rigorous deductive proofs. Therefore, the workshops are set up using only compasses, ruler and protractor. Nevertheless, in a situation where the prupose is to develop deductive reasoning, the same workshops can be used only requiring rigorous proofs rather than accepting conclusions based on measurement alone.]

Each student is given the following series of instructions. The same instructions are given each day, that is: for each condition, construct two or three diagrams of different size and draw a conclusion about the additional information you find. Why is your conclusion correct?

Activity One - Chord Property Theorems

1. Construct the right bisector of any chord of a circle.
   Construct the right bisector of two or more chords of the same circle.

2. Construct a line through the center of a circle meeting the mid-point of a chord.

3. Construct a line through the center of a circle and perpendicular to a chord.

4. Construct two chords of a circle equidistant (perpendicular distances) from the center of the circle.

5. Construct two equal chords of a circle.

6. Construct two chords of unequal lengths.

Activity Two - Sector Angle and Sector Arc Theorems

1. In the same circle, construct equal sector angles.
   In equal circles, construct equal sector angles.

2. In the same circle, construct unequal sector arcs.
   In equal circles, construct unequal sector angles.

Activity Three - Inscribed Angles

1. Construct an angle at the center of the circle (sector angle).
   Construct an angle in the circle (touching the circle) on the same arc as the sector angle.

2. Construct two different angles in the circle subtended by the same arc.

3. Construct an angle subtended by the diameter of the circle.

4. Inscribe a quadrilateral in a circle. What can you say about the angles?

5. Inscribe a quadrilateral. Produce one of the sides. What can you say about the angle so formed and the angles of the quadrilateral?

Activity Four - The Tangent

1. Construct a line from the center of a circle perpendicular to a tangent to a circle at the point of contact.

2. Construct two tangents to a circle from a common point outside the circle. Join the center of the circle and the external point.

3. Construct a chord and a tangent from the same point on the circle.

4. Construct a tangent to a circle. From the same exterior point construct a secant. What can be said about the lengths of the secant segments and the tangent length?

Activity Five

1. Construct two equal angles subtended by the same line on the same side of the line. What can you say about the end points of the line and the vertices of the angles?

2. Construct a circle on the hypotenuse of a right triangle as diameter.

3. Construct a quadrilateral having interior and opposite angles supplementary. What can be said about the vertices of the quadrilateral?

4. Construct a quadrilateral having the exterior angle equal to the interior and opposite angle. What can be said about the vertices of the quadrilateral?

[*The workshops on the circle were designed specifically for the Math 20 course and developed the suggested topics in the course. In comparison to the usual lecture approach, the class seemed to have a better "feeling" for the properties of the circle. When it came to applying the properties to exercises, there was much more enthusiasm about attacking the problems. It was not just another assignment to be completed, but, rather, another time to find new ideas. There was more desire to find* <u>*why*</u> *conditions existed rather than merely completing an exercise.*]

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B.4 Circles and Polygons

Inscribed Angles, Sector Angles and Inscribed Quadrilaterals *(Connolly)*

[*Prior to this activity students have been introduced to circles and some of the basic definitions. In particular, they know the meaning of chord, arc, sector, segment, inscribed angles, sectors angles and inscribed figures. It can be used at whatever level students are introduced to a study of circles. Presently this is at the Math 20 level. The activity is best performed in pairs.*]

MATERIALS (per pair of students) A set of geometry instruments, scissors, paper from which circles may be cut or already prepared circles of various radii.

Activity I

Cut from the paper several congruent circles. Draw in each equals chords AB (Figure 1a) and cut each into congruent segments as illustrated in Figure 1b,, keeping segment AXB for first activity.

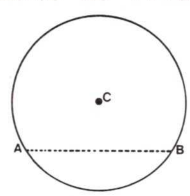

Figure 1a

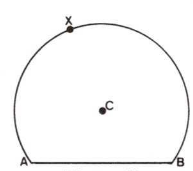

Figure 1b

From segment AXB cut various inscribed angles ACB, Figure 2a and 2b

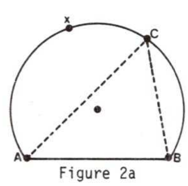

Figure 2a

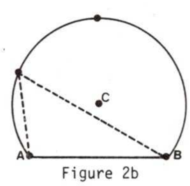

Figure 2b

Measure each angle at C and record your results.
What do you discover?
How would you increase the measure of angle C?
How would you decrease the measure of angle C?
What happens if chord AB is a diameter?

Cut a sector angle AOB from one of the segments used above as illustrated in Figure 3.

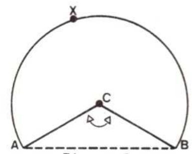

Figure 3

Measure angle AOB.

How does it compare with angle ACB?

Can you make a general statement about the inscribed angle ACB and the sector angle AOB?

Could you develop a formal proof for your conclusion?

Activity II

Using several congruent circles, cut them as illustrated in Figure 4, varying the positions of X and Y.

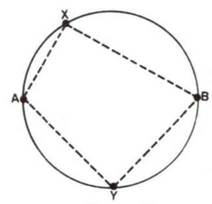

Figure 4

Measure the angles AXB and AYB and record your data.

What do you discover?

What do you think is true of angles at A and B?

What conclusion can you draw about an inscribed quadrilateral?

Is this true for all quadrilaterals?

Make a general statement for your conclusions.

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C. STATISTICS

Permutations and Combinations *(Loose)*

[This is a topic in the Mathematics 30 text so Grade XII would be the intended grade level. However, as these lessons cover only basic permutations and combinations, they could, in my opinion, be introduced at, say Grade X.]

Discovery Lesson 1 - Patterns in Arrangements

[This first lesson could commence after a review of sets and of Cartesian products of sets. It should not be necessary to express the relationship between sets and permutations as many students will likely grasp it after some activity. This work on sets would be the only lead-up necessary, and even this could be optional.]

OBJECTIVE It is hoped that, by working through the activities, the students will find patterns in arrangements and, using the "blank" method, will be able to solve such problems. It is not necessary for the students to know any general formula, but the better students may discover a law for 'n' objects to be arranged 'r' at a time. However, at the end of the lesson one might mention the n! law to them. In general, then, this lesson will introduce basic permutations.

MATERIALS
- three books (encyclopedias, for example - numbered preferably),
- a number of different colored chips,
- a typed menu with a choice of 2 soups, 3 mix courses, 2 desserts,
- a set of problems on numbered file cards.

PROCEDURE Divide the class into stations by grouping two or three desks together for each. Assign two or three students to each station. They should progress through the cards *in order*, performing the activity suggested, then working through the set of questions which follow it.

FILE CARDS

Card 1 Arrange three books in as many different orders as possible.

Card 2 Suppose there are three slots in which to place books.

___ ___ ___
 1 2 3

a) How many books have you available to put in slot 1?
b) How many choices of books do you have left for slot 2?
c) How many for slot 3?

Do you notice anything? Is there any relationship between the numbers in the blanks and your experimental result?

Card 3 In how many different ways can you and your partner sit in two desks?

Card 4 Represent the desks by two blanks. How many people do you have available to sit in the first desk? How many people are left to fill the second desk? Do you notice anything? How do these blanks compare with your actual result?

Card 5 Take four different colored chips. Arrange them in a pile in as many different color combinations as possible.

Card 6 For the first chip, how many chips do you have to choose from? How many are left to choose from, to put on top of this first one? How many are left now? Have you any choice for the chip on the top? Are there any similarities between these answers and your experimental result?

Card 7 How many different numbers could you form from the digits 4, 8, 9, if no digit is to be repeated (three-digit numbers)?

Card 8 If $\underset{1}{\_\_}\ \underset{2}{\_\_}\ \underset{3}{\_\_}$ represents the different numbers, how many ways can you fill in blank 1? blank 2? blank 3? Do you notice anything? Do any similarities exist?

Card 9 Arrange the material you have collected from your activities in a meaningful way and see if you can discover any pattern.

[*Students will find, hopefully, that one can arrange two objects in 2.1 = 2 ways, three objects in 3.2.1 = 6 ways, four objects in 4.3.2.1 = 24 ways. From this, students should be able to readily say how many ways nine objects can be arranged. If the students have grasped the idea, one could mention n! This simply means, for example, 3! = 3.2.1 = 6 or n! = (n) (n-1) (n-2) down to 1. The students should continue on with the next file cards. They also deal with permutations but the problems are somewhat different and lead to a different discovery.*]

Card 10 Given two digits, 4 and 9, how many two-digit numbers could you make from these if digits can be repeated?

Card 11 Using two blanks to represent the number $\underset{1}{\_\_}\ \underset{2}{\_\_}$, how many digits do you have available to put in blank 1? blank 2? Do you see any relationship between two blanks and the result of your experiment?

Card 12 How many two-digit numbers can you make which are less than 70 but which are also multiples of 5?

Card 13 How many numbers can form the first space in the number? How many in the second space? Do you see any relationship between the numbers and the blanks?

Card 14 You have a menu on your desk. How many different three-course meals could you order from this menu if you were in a restaurant?

Card 15 Let the three courses of the meal be represented by three blanks. $\underset{1}{\_\_}\ \underset{2}{\_\_}\ \underset{3}{\_\_}$. From your previous activities, can you fill in these blanks? Do you see any relationship here?

Card 16 Suppose you are taking a trip to Hawaii. You can make this trip by different routes. You have a choice of two train routes to Vancouver and of three routes to the island: by ship, by plane, by helicopter. How many different types of trips can you make?

Card 17 How many ways can you go to Vancouver? Put this in blank 1. $\underset{1}{\_\_\_}$ $\underset{2}{\_\_\_}$ How many to Hawaii? Put this in blank 2. Is there any similarity between your answers and the blanks?

Card 18 You have performed a number of activities now, all of which have been different. Are they similar in any way? Could you solve an arrangement problem now without doing the experiment? Can you describe the "blank method" which you were using in the previous problem?

[*The students could be given a few problems on permutations at this point. By looking at their work, the teacher can determine whether the students have actually discovered the blank method and are using it, or whether they are still solving the problems by experiment. A short review on permutations could follow these problems if desired. Students could proceed immediately to the next lesson or it could be postponed for a couple of days. However, Discovery Lesson 2 should only be used after Discovery Lesson 1 has been covered.*]

Discovery Lesson 2 - Patterns in Selections

[*This lesson is a continuation of lesson 1, therefore, the same materials are needed. However, one more book should be added to the collection. Problems are put on file cards in the same manner as in the previous lesson and students progress through the cards, in order, as before.*]

OBJECTIVE The objective of this lesson is for students to find patterns in selections, and perhaps more important than this, to see the relationship between permutations and combinations.

FILE CARDS

Card 1 How many different selections of two books at a time can you make from a group of four books?

Card 2 In making the first selection, how many choices do you have? In making the second selection how many books are available? Put these in blanks one and two $\underset{1}{\_\_\_}$ $\underset{2}{\_\_\_}$. Are any of these combinations of books the same set of books? How many times do the same groups occur together? What effect does this have on the result of your first two steps, that is, the numbers which you put into the blanks? How do both of these compare to your experiment? Can you describe the relationship? Can you explain why the relationship exists?

Card 3 Take three desks and four people. How many different selections of people can sit down? (To make a group different, one or more people must be different.)

Card 4 How many ways are there of filling the first seat? of filling the second seat? of filling the third? $\underline{\quad}_1\ \underline{\quad}_2\ \underline{\quad}_3$. But are all of these different groups of people? How many times does the same group of people occur together? What effect does this number of times have on the result you obtained in the blanks? Describe what you find.

Card 5 You have three books - how many different ways can you make two selections? How many choices of different books have you?

Card 6 Let two blanks represent the two selections. $\underline{\quad}_1\ \underline{\quad}_2$. Can you fill in these blanks? What is the relationship between this result and the result of your actual experiment with the books? Can you explain why the relationship exists?

Card 7 A basketball coach has three good guards - John, Bill and Terry. How many ways can he pick two guards for tonight's game?

Card 8 How many people has he to choose from first? Now, how many has he to choose from? $\underline{\quad}_1\ \underline{\quad}_2$. Has he chosen the same people more than once? Is this reasonable? Why is there a discrepancy between this answer and your actual result?

Card 9 You have five coins (dime, nickel, quarter, penny, half-dollar). In how many ways can you make a selection of two different coins?

Card 10 You have how many coins to choose from first? Put this in the first blank $\underline{\quad}_1\ \underline{\quad}_2$. How many do you have to choose from now, for your second choice? Put it in the second blank. Are all of these selections different? If not, how many times does the same pair of coins appear? How does this number and the number obtained in the blanks compare to your experimental result?

Card 11 Can you summarize your findings from the preceding activities? Does there seem to be a pattern? Can you relate it to a general problem?

[*It is not crucial that the students discover a general law (this is mainly a question for the better students). However, all students should be able to solve any of this type of problem. A formula is not necessary. I think, however, that the good students may come up with a general law for selecting 'n' things 'r' at a time. The main thing it is hoped the student will discover is that, for each selection problem he chooses the objects in order, after which he divides by the number of objects which appear together more than one time. It is also hoped that he will discover the difference between permutations and combinations as he works through the different activities. That is, permutations involve order; combinations involve picking things regardless of order. Discovering combinations and permutations by active learning, rather than by abstract formulas, may result in more meaningful concepts for students.*]

Probability *(Webber)*

DICE

Part A - Individual Experiment

Roll a die 25 times and record the outcomes in the appropriate columns

Tabulation Sheet

Roll	*1*	*2*	*3*	*4*	*5*	*6*
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						
22						
23						
24						
25						
Total						

How many times did a 3 turn up? ____ (This is called the frequency of the event and is represented by $f_n A$ for an event A and n trials of the experiment.)

1. Find: (a) $f_{25}3$ (b) $f_{25}5$ (c) f_{25}(an odd number) (d) f_{25}(a number greater than 3).

2. The relative frequency of an event A is $f_n A/n$. Find the relative frequencies for the events in question 1, and
   (a) an even number or a number greater than 3,
   (b) an even number and a number greater than 3.

Part B -

Combine your results with those of others in the group. Compile the data on the chart provided.

3. Find the following for the statistics obtained by the group:

(a) $f_n 3$ (b) $f_n 5$ (c) $f_n 3/n$ (d) $f_n 5/n$

(e) $f_n \frac{\text{(an odd number)}}{n}$ (f) $f_n \frac{\text{(a number greater than 3)}}{n}$

4. On the group chart, sketch a bar graph to illustrate the frequency of each of the possible outcomes against the number of tosses. Predict the result if the number of trials is 1000.

Part C - Group Experiment

Appoint one group member to act as statistician in recording the results of the chart provided. Take turns rolling the pair of dice until 100 trials have been completed.

5. Calculate (a) the frequency of a sum of seven
   (b) the frequency of an even sum
   (c) the frequency of an odd sum
   (d) the frequency of a sum of two.

6. Calculate (a) the relative frequency of each of the above
   (b) the relative frequency of a sum which is even
   (c) the relative frequency of a sum which is odd.

7. The relative frequency of an event A is an expression of the probability of event A so that $P(A) = \frac{f_n A}{n}$

   From the group data collected, compute
   (a) the probability of a sum of 6
   (b) the probability of an even sum
   (c) the probability of a sum greater than 6
   (d) the probability of a sum which is 2 or odd.

POKER CHIPS

Part A - Individual Experiment

Draw a chip from the bag. Record its color in the appropriate column on the tabulation sheet below. Return it to the bag. Draw again until 25 draws have been recorded.

Draw	Red	White	Blue
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
Total			

How many times was a red chip drawn? ____ (This is called the frequency of the event A and n trials of the experiment.)

1. Find: (a) f_{25}red (b) f_{25}red (c) f_{25}white

2. The relative frequency of an event A is $f_n A/n$. Find the relative frequencies of the events in question 1, and
   (a) drawing a red or blue,
   (b) drawing a red or white.

Part B -

Combine your results with those of others in the group. Compile your data on the chart provided.

3. Find the following using the statistics obtained by your group:

   (a) $f_n$red (b) $f_n$blue (c) $f_n$white (d) $\frac{f_n\text{red}}{n}$

   (e) $\frac{f_n\text{white}}{n}$ (f) $\frac{f_n\text{blue}}{n}$

4. On the group chart sketch a bar graph to illustrate the frequencies of each color against the colors. From the graph predict the ratio of the three colors in the bag. If the same ratio is maintained and the bag contains 1000 chips, predict the frequencies (relative) of each outcome.

Part C - Group Experiment

Take turns drawing a chip, recording its color and returning it to the bag until 100 draws have been recorded. (Appoint one group member to act as statistician in recording the results on the chart.)

5. Calculate the frequency of drawing
   (a) a red chip
   (b) a blue chip
   (c) a white chip
   (d) a red or blue chip
   (e) a red or white chip.

6. Calculate the relative frequency of each of the events in question 5.

Repeat the above experiment in Part A but do not return the chip drawn to the bag.

7. Calculate the frequency and relative frequency of the probability as listed in question 5 under the conditions outlined in this project.

8. The relative frequency of an event is an expression of the probability of the event so that $P(A) = \frac{f_nA}{n}$

   From the data, compute:
   (a) the probability of drawing a red chip if the chips are returned after each draw; if the chips are not returned after each draw,
   (b) the probability of a red or blue chip if the chips are returned after each draw; not returned after each draw.

CARDS

Part A - Individual Experiment

Draw a card from the deck. Record its suit and kind on the tabulation sheet below. Return the card to the deck before drawing a second card and repeat the experiment until 25 cards have been drawn and the statistics recorded.

	Heart	*Spade*	*Diamond*	*Club*	*Total*
2					
3					
4					
5					
6					
7					
8					
9					
10					
Jack					
Queen					
King					
Ace					
Total					

How many times was a ten drawn? ____ How many times was a heart drawn? ____ (This is called the frequency of the event and is represented by $f_n A$ for an event A and n trials of the experiment.

1. Find: (a) $f_{25}6$ (b) $f_{25}\text{ace}$ (c) $f_{25}\text{club}$ (d) $f_{25}\text{spade}$.

2. The relative frequency of an event A is $f_n A/n$. Find: (a) $f_{25}\text{ace}/25$ (b) $f_{25}\text{diamond}/25$ (c) $f_{25}\text{face card}/25$ (d) $f_{25}\text{black card}/25$.

Part B -

Combine your results with those of others in the group. Compile your data on the chart provided.

3. Find the following from the group statistics: (a) $f_n 6$ (b) $f_n\text{ace}$ (c) $f_n\text{club}$ (d) $f_n\text{spade}$ (e) $f_n\text{queen}$ (f) $f_n\text{diamond}$ (g) $\frac{f_n\text{face card}}{n}$ (h) $\frac{f_n\text{red card}}{n}$.

4. On the group chart sketch a bar graph to illustrate the frequency of the suit against the number drawn in that suit.

   (a) What is another way of plotting a graph to illustrate the statistics from your experiment? Is this the only alternative?

   (b) As the number of trials increases indefinitely how does the complexion of the graph change?

   (c) What is the relative frequency of getting the ace of clubs in 10,000 trials?

Part C - Group Experiment

Take turns drawing a card but do not return it to the deck until 25 draws have been made. Total the tallies for the 25 draws without returning the cards. (Appoint one student to act as group statistician in recording results.) Repeat the experiment four times until a total of 100 draws is completed.

5. Find: (a) $f_{100}6$ (b) $f_{100}\text{ace}$ (c) $f_{100}\text{club}$ (d) $f_{100}\text{spade}$

   (e) $\frac{f_{100}\text{queen}}{100}$ (f) $\frac{f_{100}\text{diamond}}{100}$ (g) $\frac{f_{100}\text{face card}}{100}$

   (h) $\frac{f_{100}\text{red card}}{100}$

   Compare these with the frequencies of the same events obtained in question 3.

6. The relative frequency of an event A is an expression of the probability of event A so that $P(A) = \frac{f_nA}{n}$

   From the data, compute:
   (a) the probability of drawing a red card if the card is not replaced after each draw,
   (b) the probability of drawing a 6 without replacement; with replacement,
   (c) the probability of drawing a heart without replacement; with replacement.

COINS

Part A - Individual Experiment

Toss a coin 25 times and record the outcomes in the appropriate columns on the tablulation sheet provided.

Tabulation Sheet

	Toss	Heads	Tails
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
Total			

How many times did a head turn up? _____

(Remember, this is called the frequency of the event and is represented by f_nA for an event A and n trials of the experiment.)

1. The relative frequency of an event A is f_nA/n. Find: (a) $f_{25}H$ (b) $f_{25}T$ (c) $f_{25}H/25$ (d) $f_{25}T/25$.

Part B -

Combine your results with those of others in the group. Compile your data on the chart provided

2. Find the following for the statistics obtained by the group:

   (a) $f_n H$ (b) $f_n T$ (c) $f_n H/n$ (d) $f_n T/n$.

3. On the group chart sketch a bar graph to illustrate the frequency of heads and tails against the number of tosses. (Use the statistics obtained by the whole group.) Predict the result if the number of trials is 1000.

Part C - Group Experiment

Using three coins of the same kind, have each member toss the coins (all at once). Appoint one student to act as statistician to record 100 trials on the chart provided.

4. Calculate:

   (a) the frequency of 3 H
   (b) the frequency of tails
   (c) the frequency of all three alike
   (d) the frequency of 2H, 1T
   (e) the frequency of 2T, 1H
   (f) the frequency of at least one H
   (g) the frequency of 1H or 1T.

5. Calculate the relative frequency of each of the above.

6. The relative frequency of an event A is an expression of the probability of event A, so that $P(A) = \frac{f_n(A)}{n}$. From the data collected by the group compute:

   (a) probability of a head
   (b) probability of a tail
   (c) probability of a head or tail
   (d) probability of at least one tail
   (e) probability of all three alike.

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Johnson, Donovan A. *Probability and Chance.* New York: McGraw-Hill, 1966(o.p.).

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National Council of Teachers of Mathematics. *Arrangements and Selections.* Washington, D.C., 1966.

D. CALCULUS

D.1 Symmetry

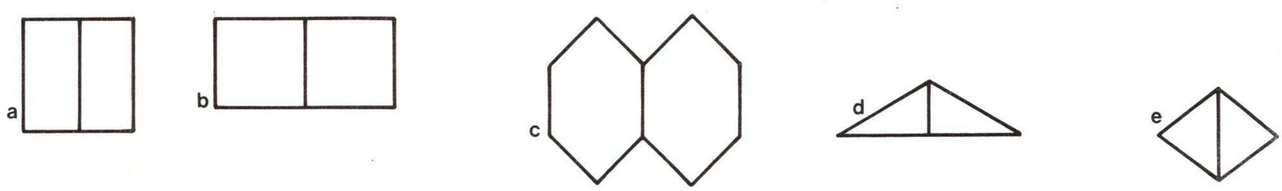
Rediscovery and extension of knowledge of symmetry *(Dyrholm)*

[For individual or small group work. Criterion from Bates, Irwin and Hamilton Teacher's Guide. *"We maintain that open-ended activities are appropriate in the initial stages of developing new concepts. . ." As most high school concepts are not new but extensions of previous concepts, partially-directed activities may be preferred to open-ended activities.]*

MATERIAL Paper and scissors

Cut shapes which you believe to be symmetrical and indicate the line or point with respect to which they are symmetrical. Good for decorating bare bulletin boards. In discussion of figures produced and introduced the terms "bilateral" (line) symmetry and "rotational" (point) symmetry.

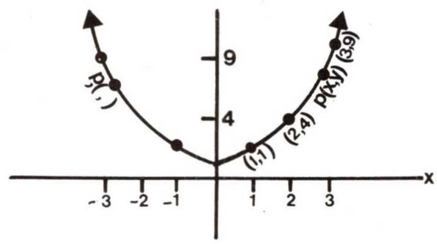
Identify the axis of symmetry for each of the regions below. Compare the areas and perimeters of the regions on each side of the axis. Which figures also possess rotational or point symmetry?



Explore the printed letters of the alphabet to find out which are symmetrical. Can you complete the following chart? (Some letters may fit in more than one category.)

(1) No symmetry	*(2) Symmetrical about vert. axis only*	*(3) Symmetrical about horiz. axis only*	*(4) Symmetrical about both horiz. and vert. axis*	*(5) Symmetrical about a point (rotational sym.)*
Q	T	B	H	H S

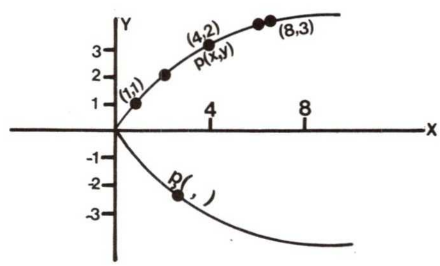
Compare results in columns 4 and 5 and try to develop a rule to explain the results.



The above graph is symmetrical with respect to ______________________.
Indicate the coordinates of the point symmetrical to each of the given points.
[(x,y) is a general point.]

Given	*Symmetrical Counterpart*
(1,1)	
(2,4)	
(3,9)	

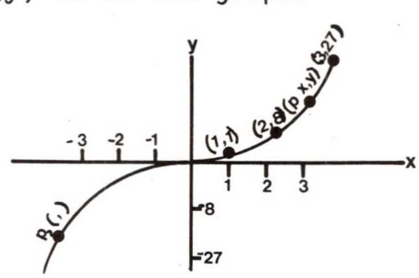
A graph is symmetric with respect to the y-axis if the point P,(,) is on the graph whenever the point p(x,y) is on the graph.



The above graph is symmetrical with respect to ______________________.
Indicate the coordinates of the point symmetrical to each of the given points.
[(x,y) is a general point.]

Given	*Symmetrical Counterpart*
(1,1)	
(4,2)	
(9,3)	
P,(x,y)	

A graph is symmetrical with respect to the y-axis if the point P (,) is on the graph whenever the point p(x,y) is on the graph.

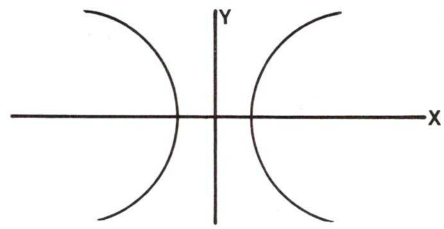


The above graph is symmetrical with respect to ______________________________.
Indicate the coordinates of the point symmetrical to each of the given points. [(x,y) is a general point.]

Given	*Symmetrical Counterpart*
(0,1)	
(2,8)	
(3,27)	
P(x,y)	

A graph is symmetrical with respect to the origin if the point P (,) is on the graph whenever the point p(x,y) is on the graph.

What type of symmetry does the following graph have?



Does it obey the rule you make when working with the alphabet?

You have worked with the rotational symmetry (respect to a point) and bilateral symmetry (respect to a line). What other types of symmetry do you believe there to be? What types of objects would you use? Can you set up a display?

REFERENCES from *The Mathematics Teacher*

Atneosen, Gail H. "The Schwarz Paradox: An Interesting Problem for the First-Year Calculus Student," March 1972, pp.281-84.
Fabricant, Mona. "A Classroom Discovery in High School Calculus," December 1972.
Shilgalis, Thomas W. "A Theorem on Lines of Symmetry," January 1972, pp.69-72.

D.2 Functions

Derivative of a Function *(Webber)*

Sketch a diagram as you read the following paragraph.

Let us consider the motion of a particle along a straight line. Let 0 be a point of reference of the line. If we say the particle is moving with a constant speed of 4 ft/sec we know that the object moves 4 feet every second but we don't know the direction of its motion. Therefore, let us consider one direction as the negative direction. If the particle moves with a constant speed of 4 ft/sec in the positive direction, then we say the velocity of the particle is 4 ft/sec or +4 ft/sec. If the speed is 4 ft/sec in the negative direction, we say the velocity is -4 ft/sec. We can also consider the motion of particles whose velocities are not constant.

Note - Velocity is the *rate of change* of distance with respect to time in a specified direction.

Acceleration is the *rate of change* of velocity with respect to time.

Please answer the following questions using graphs, tables, charts, or any other method you wish.

1. Suppose a particle moves along a line such that the distance, s, from 0 is given by $s = f(t) = 3t + 2$.

(a) What does the graph of $f(t) = 3t + 2$ look like?

(b) What is the slope of the line joining the points on the graph where $t_1 = 1$, $t_2 = 3$?, where $t_1 = 2$, $t_2 = 4$? where $t_1 = -4$, $t_2 = -4.5$?

(c) What do you notice about the slopes?

(d) Can you write a general expression for the slope of the line joining the points on the graph where time equals t, and time equals t_2?

(e) What is the meaning of the slope on the distance-time graph of the above function?

2. If a particle moves along a straight line such that the distance, s, from 0 is given by $s = f(t) = 144 - 16t^2$,

(a) what does the graph of $f(t) = 144 = 26t^2$ look like?

(b) What is the slope of the line joining the points of the graph where $t_1 = 0$, $t_2 = -2$?, where $t_1 = 1$, $t_2 = 2$?, where $t_1 = -2$, $t_2 = 2$?, (try any other values you wish.)

(c) What is the meaning of the slope on the distance-time graph of the above function?

(d) What is the slope of the line joining the points on the graph where $t_1 = 1$, $t_2 = 4$?, where $t_1 = 1$, $t_2 = 3$?, where $t_1 = 1$, $t_2 = 2$?, where $t_1 = 1$, $t_2 = 3/2$?, where $t_1 = 1$, $t_2 = 5/4$?, where $t_1 = 1$, $t_2 = 9/8$?

(e) Indicate what happens to the slope as t_2 approaches t_1.

(f) What is the value of $\lim_{t_2 \to t_1} \frac{f(t_2) - f(t_1)}{t_2 - t_1}$ where $t_1 = 1$? $t_1 = 0$?

(g) What is the meaning of $\lim_{t_2 \to t_1} \frac{f(t_2) - f(t_1)}{t_2 - t_1}$?

(h) Can you write a general expression for $\lim_{t_2 \to t} \frac{f(t_2) - f(t)}{t_2 - t}$?

(i) Letting $t_2 = t + h$ how would you write $\lim\limits_{t_2 \to t} \dfrac{f(t_2) - f(t)}{t_2 - t}$?

(j) Suppose we write $y - f(x) = 144 - 16x^2$. What is the meaning of $\lim\limits_{h \to 0} \dfrac{f(t+h) - f(t)}{h}$?

3. (a) Using simple polynomial functions such as $y = x^2$, $y = x^3$ can you determine $\lim\limits_{h \to 0} \dfrac{f(t+h) - f(t)}{h}$ for each function?

(b) Can you now conjecture (guess intelligently) how you might determine $\lim\limits_{h \to 0} \dfrac{f(t+h) - f(t)}{h}$ for polynomial functions?

4. (a) Using the ideas you obtained from the preceding questions and references such as your text, can you explain the meaning of the symbols $f'(t)$, $Df(t)$, $\dfrac{dy}{dt}$ and y'?

(b) How would you define the <u>tangent to a graph at a point x</u>?

REFERENCES from *The Mathematics Teacher*

Atneosen, Gail H. "The Schwarz Paradox: An Interesting Problem for the First-Year Calculus Student," March 1972, pp.281-284.

Fabricant, Mona. "A Classroom Discovery in High School Calculus," December 1972.

Lipsey, Sally Irene, and Wolfe Snow. "The Appreciation of Radian Measure in Elementary Calculus," January 1973, pp.31-2.

D.3 Conics

Construction of Conics and Their Properties *(Farndale)*

1. If you sliced a solid cone into two parts and looked at the bared surfaces, what shapes night you see?

2. (a) Given two pins, a length of string and a pencil, draw as many different curves as possible. Can you make an ellipse?

(b) Draw a large circle on paper and a long straight line through its center. If one vertex A of a wooden triangle or set square is made to move along the line, what shape will the third vertex trace out if AB = radius of the circle?

3. Attach a length of string to one end A of a rod AB and pivot the other end at B. Fix the free end of the string at C and discover the curve produced by moving a pencil P along AB in such a way that the string is kept taut and passes between the pencil and the rod.

4. Devise apparatus so that a rod AB can move at right angles to a line L. Fix a length of string to one end A of the rod, and the other end of the string to a fixed point C. What will the curve be that is traced out by a pencil which runs along AB, trapping the string between itself and the rod so that the string is kept taut?

5. (a) Construct any conic. Take any six points A, B, C, D, E, F, in that order, on the conic and join up A to D, B to E, and C to F. What do you notice? Repeat for other conics.

(b) Construct any conic. Take six points A, B, C, D, E, F, in that order on the conic. Let AE and FB meet at P, BD and CE meet at Q, and AD and CF meet at **R**. What do you notice about P,Q, R?

Conics - Division IV *(McIntyre)*

Although much of the following may be familiar to some students, these definitions should renew old memories and bring the student body to a uniform starting point.

Circular cone - A surface generated by a straight line moving so to intersect a given circle and to pass through a given fixed point not in the plane of the circle.

Element of a cone - The line in each of its positions as it moves around the circle.

Vertex of a cone - The fixed point.

Nappes - Each of two cones formed on either side of the vertex.

The following materials should be placed at student stations in reasonably large quantities: paper cones of various sizes and shapes, plastic models of cones of one or two nappes, a sharp cutting tool (preferably a knife), paper clips, glue, graph paper, pencils, rulers, construction paper of various colors, scissors.

Developmental questions

1. Experiment with the paper cones. Imagine a plane intersecting your cones in different ways. How many different "cuts" can you make? How many cuts go through only one nappe? Both? Are the cuts really different or only different kinds of the same cut? How are they different? How are they the same?

2. How many of the cuts are like a circle? How many are not? If the cut only cuts one nappe and is not like a circle, what is peculiar about how you cut? If the cut cuts both nappes, how do the cuts compare? Does it depend upon the shape of the cones?

3. On the given graph paper, plot a graph of values which satisfy the equation. (Circle). $x^2 + y^2 + 12x = 0$.

How could you cut a cone to form a figure the same shape as your graph?

4. Plot a graph for $3x^2 + 9y^2 = 45$ (ellipse). How would you cut a cone in this shape?

5. Plot $y^2 = 8x$ (parabola). How could you cut this one?

6. Plot $4x^2 - 9y^2 = 36$ (hperbola). How could you cut this?

[*Following a thorough study of the above experiments a classroom study session would come. Students would offer suggestions for finding the equation. Later, the terms will be introduced for the curves, the idea of "e" and so on. I think this type of exercise will be much more meaningful for the students.*]

Quadratic Relations *(Burton)*

I. *Circle* - centered anywhere

II. *Ellipse* - center the origin, axes of symmetry the x-axis and the y-axis

III. *Hyperbola* - center the origin, axes of symmetry the x-axis and the y-axis.

[*The Grade XII students have graphed some circles and ellipses in previous years. A few are familiar with hyperbolae. Most of their previous work started with an equation and ended with a graph. Each individual works on his own. Students are free to consult with friends or fellow geniuses at any time. They may even consult with the teacher. For difficult problems they switch to small groups. To develop general equations, one "neat-writing" student at the board follows class suggestions and directions.*]

I. CIRCLE

Given the line segment AB and the fixed point C -

(1) Find as many points as you can whose distance from C is of length AB.

(2) Is it possible to join these points with a smooth curve? Is it logical to join them? Is it a closed curve?

(3) Do you know the name of this curve? Could you cut a cone to form this curve? What is the name usually used for the fixed length and for the fixed point?

(4) On a sheet of graph paper draw Cartesian axes and label your scales. Do #1 again making AB any integral length and putting C on any lattice point (please don't all of you put C at the origin!)

(5) If $P(x,y)$ is any point on your curve can you find the relationship that must exist between x and y?

(6) Simplify your equation, collecting any like terms.

(7) Show your equation to several others to see if they can decide what your center and your radius were. Can you calculate the radius and center they used without seeing their graphs?

CLASS AS A WHOLE - CIRCLE

Before attempting exercises in the text the entire class working together should be able to develop

standard form - $(x-y)^2 + (y-k)^2 = r^2$

general form - $x^2 = y^2 + 2gx + 2fy + c = 0$.

II. ELLIPSE

Given the line segment AB and the fixed points F_1 and F_2 -

(1) Find as many points as you can so that the distance from F_1 plus the distance from F_2 equals the length AB.

(2) Is it possible to join these points with a smooth curve? Is it logical to join them? Is it a closed curve?

(3) Do you know the name of this curve? Could you cut a cone to form this curve? What is the name of the fixed points?

(4) Do (1) again, putting F_1 and F_2 the same distance apart as before but choosing your own fixed length. Is the question ever impossible? Why? Is the curve ever a circle?

(5) Working in groups of three or four and using the neatest graph in the group [from the first time you did (1)] -

(a) draw a line through F_1 F_2 and use this as the x-axis. Draw the right bisector of F_1 F_2 and use this as the y-axis. The given length AB was 10 cms and F_1 F_2 were 8 cms apart. Label the coordinates of the focal points and of all the intercepts of the curve on the axes. Does the fixed length appear in one or more places on your graph?

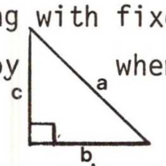
(b) If $P(x,y)$ is any point on your graph, can you find the relationship that must exist between x and y? (How did we solve radical equations last year that had two radical terms?)

(c) If you can't arrive at a simple relationship, use inductive reasoning to arrive at a possible answer. (You do know the x and y intercepts and can see the necessary symmetry from your graph.)

(6) If the fixed length had been 26 units and the focal points 24 units apart, make a sketch placing the axes in position as before in (5). (Join one y-intercept to one focal point.) Calculate a simple relationship between x and y for any point (x,y) on the graph.

CLASS AS A WHOLE - ELLIPSE

The entire class working together should be able to develop $\frac{x^2}{a^2} + \frac{y^2}{b} = +1$ starting with fixed length 2a and focal points 2 c units apart with a, b, c, related by



when foci are on x-axis and also $\frac{x^2}{b^2} + \frac{y^2}{a^2} = +1$ when foci are on y-axis

The names major axis and minor axis must be given before attempting exercises in text.

III. HYPERBOLA

Given the line segment AB and the fixed points F_1 and F_2 -

(1) Find as many points as you can so that distance from F_1 - distance from F_2 equals the length AB.

(2) Is it possible to join these points with a smooth curve? Is it logical to join them? Is it a closed curve?

(3) Do you know the name of this curve? Could you cut a cone to form this curve? What is the name of the fixed points?

(4) Work in small groups or by yourself. Draw a line through F_1 F_2 and use this as your x-axis. Draw the right bisector of F_1 F_2 and use this as y-axis. The given length AB was 8 cms and F_1 F_2 were 10 cms apart. Label the co-ordinates of the focal points and all the intercepts of the curve of the axes. Does the fixed length appear in one or more places on your graph? If P(x,y) is any point on your graph can you find the relationship that must exist between x and y?

(5) If the fixed length had been 2a and the distance between fixed points had been 2c, make a rough sketch. Can you calculate the relationship that must exist between x and y for any point P(x,y) on the graph.

GLASS AS A WHOLE - HYPERBOLA

Develop $\frac{x^2}{a^2} - \frac{y^2}{b^2} = +1$, foci on x-axis

$\frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$, foci on y-axis

Discuss asymptotes and their use as a help in sketching a graph if given the equation. The names transverse axis, conjugate axis, and focal radius, must be given before attempting exercises in text.

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