SOME USES OF PROGRAMMABLE CALCULATORS IN MATHEMATICS TEACHING

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The purposes of this article are to suggest uses of programmable calculators in mathematics teaching, and to disseminate certain programs I have developed that may be useful in mathematics teaching. It has been written because of a conviction that programmable calculators are now, or soon will be, available to many classrooms, and, for many purposes, are viable alternatives to computers (LaBar).

KEYBOARD

Most people like to "fiddle" with calculator keyboards. A little fiddling by an inquisitive student, or a little directed fiddling by one that isn't so inquisitive, will often teach in minutes what lecture and blackboard example may not teach in hours. Flashing displays and/or printed tapes are a powerful reinforcement.

Many mathematical operations are directly available on programmable machines. The Texas Instruments SR-56, one of the smallest programmable calculators, has natural and base 10 logs, powers and roots, trigonometric functions and inverses, polar to rectangular conversion and the reverse, mean, variance and standard deviation directly accessible, as well as other operations such as degrees to radians and the reverse and factorials, available through programming. Most of these operations, of course, are not unique to programmable calculators (NCTM).

PROGRAMMING

Programming itself is a valuable learning experience. Having a student develop a flow chart will probably teach him or her a great deal about logical thinking and about mathematics. As examples, developing Program A (programs are indicated by letters throughout, which are briefly outlined in the final section of this paper) taught me some things about primes, and developing Program K forced me to apply the quadratic equation for the first time ever outside of a mathematics class.

Assigning students to program on a calculator is also training for computer programming. The principles are the same, without the necessity of learning a language.

ITERATION, LIMITS AND PROBABILITY

Besides their use in teaching programming, PCs may be used to repeat a calculation over and over. This is useful for at least three reasons.

The first of these is that the calculator will do the same calculation, or a similar one, over and over, automatically. Determining prime numbers is an example of this. If you wish to determine whether 1 000 001 is prime, a simple and exact method would be to divide 1 000 001 consecutively by 1, 2, 3, 4, ... 1 000 000, determining, in each case, whether there was a fractional remainder following division. If at any division there is no such remainder, then 1 000 001 is not prime. This operation is the basis of Program A.

Secondly, the iterative process is also useful in determining limits (Johnsonbough). Thus for example, $\sum_{k=1}^{\infty}$ 1/2k = 1/2 + 1/4 + 1/8 + 1/16 ... is an expression for Zeno's paradox of Achilles and the tortoise. Adding this sum for K of 1 through 10 versus K of 1 through 100 should satisfy students that this is an expression for a finite sum. (Warning - all calculators are subject to rounding errors. Also, they will print only a finite number of digits. In dealing with very small numbers, the effects of rounding errors will be reflected in answers. Significant rounding errors may occur in limit problems.) Programs H, J, M find approximate limits by iterative processes.

A third usefulness of iterative processes is for Monte Carlo simulations. That is, simulations wherein some quantity is found by random means. An example is Program L. I have called this the random archer program because of the following analogy: Assume an archer shooting randomly at a square 2 units on a side, within which a circle of radius 1 is inscribed. The ratio of the times he hits within the circle to the total number of shots (assuming he always hits somewhere on the square) should be $\pi/4$. A development of this method, using another analogy for the generation of random hits, is given by Vervoort. Another random approach to determine π is Program K (Schroeder).

Random simulations may involve game theory. A highly modified application of game theory is Program G. A non-random simulation is Program F (Gardner).

PROGRAMS

Each of the programs in this section is available in the form of an annotated listing of steps for the program.[1] To program a Monroe 1880 calculator, it is only necessary to repeat these steps in sequence. For other machines, including computers, the annotation attempts to be specific enough that necessary changes can be made. The annotation includes mathematical explanations, references, and examples, where necessary. Programs requiring indirect memory addressing are

[1]Program lists and annotations are available from the author and will be sent on receipt of 40¢ per program to cover handling and postage.

marked with an asterisk. Where the number of memories required is more than 10, or the number of steps more than 250, this is indicated.

A. PRIME NUMBER GENERATOR

Generates primes, starting with 1, or may be made to generate primes starting from any desired number. Useful as an algorithm example.

B. FACTORS

Factors any number, printing out all factors. (If number is prime, prints zero.) Useful as an algorithm example.

C. ITERATIVE SQUARE ROOT

Determines square root of any positive number to the ninth place by approaching it as a limit, starting with the number itself and 1 as outer bounds. Can be modified to determine any integral root, or to determine more or less places. Useful as an algorithm example.

D. RATIONAL NUMBER GENERATOR

Generates rational numbers which are ratios of integers between 0 and 100, randomly. Will print the integers or not, as desired.

*E. NUMERICAL SORTER

Accepts input numbers (depends on number of memories available) and ranks them from algebraic lowest to algebraic highest. Can be simply modified to determine quartiles, deciles, and so on, and to give mean and standard deviation. Illustrates principles used in automatic alphabetizing.

*F. LIFE

Simulation, or game, with simple rules for play which are used by one "generation" of "cells" to replicate the next. Has been a great hit with "terminal freaks." Size of field depends on number of memories. At least 25 are suggested. About 450 steps.

*G. FOOTBALL

A game is played, between the machine and the operator, complete with downs and touchdowns. The game is based on a matrix of rewards and losses which can be input by the instructor, or randomly generated by the machine. Operator chooses a play, with or without knowing this matrix. Machine responds randomly, but is programmed to change its responses based on the matrix and the play of the operator (that is, it "learns"). About 350 steps. Useful as an illustration of game theory, and as an amusement.

H. DETERMINATION OF $y = x^2 - 5$

Approximates limit of the area under the curve between any specified xs. Useful in illustration of methodology of calculus, as you can use Δxs of different sizes, and thus approximate the limit as closely as desired.

I. STRAIGHT LINE

Operator punches in coordinates of any two points. Program determines slope and *y* intercept, and will then output *y* for any *x*.

J. *Pi* BY ARCHIMEDES' METHOD

Pi is approximated, based on simple geometric considerations, between an upper and lower bound. Method based on perimeters of inscribed and circumscribed regular polygons of greater and greater number of sizes, as circumference of circle is approached more and more closely. Annotation uses knowledge of simple geometry, gives references.

K. BUFFON'S NEEDLE

Monte Carlo determination of *Pi* by simulating random dropping of a needle onto a background of parallel lines. Two versions, one using polar coordinates, the other the quadratic equation. Annotation gives references.

L. RANDOM ARCHER

Monte Carlo determination of *Pi* by simulating random shooting of an arrow into a circle inscribed in a square. Annotation uses the formula for a circle, cartesian coordinates.

M. LIMIT APPROXIMATION OF *Pi*

Approaches *Pi* by establishing a square matrix of paired coordinates and determining whether or not each pair is in a circle. Approximation may be made better or worse, depending on size of Δx and Δy. Annotation uses the formula for a circle.

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