EXCITING EXCURSIONS IN NUMBER THEORY WITH AN ELECTRONIC CALCULATOR

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The activities reprinted here have been selected from a booklet which is available from the author.

PALINDROMES

DEFINITION

Palindromes are positive integers such as 4735374 or 461164 that read the same forward as backward.

PROJECT 1

Take a positive integer N.	N = 139
Add N to its reverse.	139 + 931 = 1070
If sum is a palindrome, STOP.	
If not, continue process.	1070 + 0701 = 1771
	1771 is a palindrome in two cycles.

PROJECTS FOR YOU TO DO

1. Investigate whether or not N = 5, 6, 7, ... 99 all produce palindromes and, if so, how long are the cycles.
2. If you wish, extend your investigation to more than two digits.
3. Can you determine several sets of starting values that will always produce palindromes in one cycle? Two cycles?
4. What else can you discover about starting values that produce palindromes?

HAPPY NUMBERS

Take a number, say 13. 13

Square each digit and add. $1^2 + 3^2 = 1 + 9 = 10$

Repeat the above with 10. $1^2 + 0^2 = 1$

A number for which this pattern yields finally a 1 is a HAPPY NUMBER. The number 13 is therefore a happy number.

Take the number 2. 2

Square it and add. $2^2 = 4$

$4^2 = 16$

$1^2 + 6^2 = 1 + 36 = 37$

$3^2 + 7^2 = 9 + 49 = 58$

$5^2 + 8^2 = 25 + 64 = 89$

. =

. =

. =

. = 4

You are back to the number you started with.

This pattern will cycle forever.

The numbers 2 and 4 are not happy numbers.

EXERCISE

Find all happy numbers less than 100.

FIBONACCI SERIES

QUESTION

How many pairs of rabbits can be produced from a single pair in a year if each pair begets a new pair every month, each new pair produces from the second month on, and no rabbit dies?

ANSWER

In an effort to answer the above question, the following series of numbers evolved:

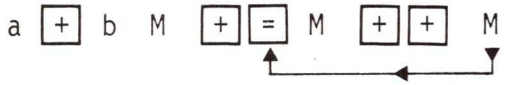
1, 1, 2, 3, 5, 8, 13, 21, ...

This series is called the Fibonacci Series, after the man who discovered it.

QUESTIONS FOR YOU TO ANSWER

1. Can you write the next five numbers in the series?
2. Study the question about the rabbits and see if you could generate the series.
3. Using your calculator, write the first 50 terms of the Fibonacci series.
4. Calculate the sum of the n terms of a Fibonacci series.
5. Calculate the sum of the squares of the terms.
6. Study your findings for possible patterns. Can you make any generalizations?

ALGORITHM to help you generate a series of Fibonacci numbers. General algorithm:
a = 1st term, b = 2nd term



EXTRAS FOR EXPERTS

FOR YOU TO INVESTIGATE

Let N_0 be any 4-digit positive integer.

Let L_k = the largest integer obtainable by rearranging the digits of N_k.

Let S_k = the smallest integer obtainable by rearranging the digits of N_k.

THEN, $N_{k+1} = L_k - S_k$

EXAMPLE

$N_0 = 7162$

$N_1 = 7621 - 1267 = 6354$

$N_2 = 6543 - 3456 = 3087$

N_3 = 8730 - 0378 = 8352

N_4 = 8532 - 2358 = 6174

N_5 = 7641 - 1467 = 6174 which clearly repeats forever.

Your problem is to investigate this recurrence.

The results may surprise you!

NOTE

It is not necessary to investigate all 9,000 possible 4-digit numbers to completely solve this problem. Can you tell why?

How many possible values for N_1 are there?