CALCULATING MACHINES IN SCHOOLS

Scottish Central Committee on Mathematics

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The continuing rapid development of technology has brought about widespread changes in the everyday lives of all of us. Since the end of the second world war, our lives have been transformed by jet aircraft, the spread of television, transistor radios, the computer and many other less obvious technological advances. All of these, especially the computer, have influenced curricular thinking in some measure, but the sudden appearance on the market of cheap pocket-sized electronic calculators has had a powerful direct impact on education, an impact so sudden that a majority of senior school pupils in Scotland (and not a few younger children) may well be in possession of a calculating machine by the time this paper appears. The purpose of this paper is to review the priorities in arithmetical and mathematical education in the light of the new situation, to make recommendations regarding the extent to which the use of calculating machines should be encouraged at various stages in education, and to consider the types of machines that are most appropriate for school use.

1. It is probably fair to say that the instinctive initial reaction of many teachers of mathematics is to be somewhat alarmed by the widespread availability of calculating machines. People have been saying for many years (probably since the adoption of the Arabic numeral system in Europe some five centuries ago) that arithmetical skills are on the decline, and whether they are right or wrong, there is good reason to suppose that skill in arithmetical computation will indeed be eroded by the habitual use of machines. On the other hand, skill in computation is not an end in itself, and if it can be shown that the skill is now irrelevant, then there is little point in bewailing its disappearance. What we must do is to make as rational an assessment as possible of the new priorities in arithmetical education, accepting that these priorities will be materially affected by the availability of calculating machines.

2. Computational aids, in the shape of logarithmic tables and slide rules, have of course been used in schools for many years. Both are open to the objection that they erode the more elementary computational skills, and it is doubtless the case that some slide-rule-using Sixth Year pupils would be more than a little shaky on long division. That this has never been a cause for serious concern is probably attributable to the fact that logarithms and

slide rules are used only by relatively senior school pupils who have already acquired a fair measure of arithmetical competence and understanding. This delay in use arises not because of any arbitrary edict from the mathematical/educational establishment, but because both these computational aids require a considerable measure of mathematical sophistication before they can be used at all. The child who attempts to work out 7 x 9 by logarithms needs to be able to look up tables twice, to add 0.845 and 0.954, to look up a table yet again, and finally to use the characteristic of the logarithm to sort out that the answer is 63 rather than 6.3 or 630. It is fairly easy to persuade such a child that it is easier simply to remember that 7 x 9 = 63. In many ways the slide rule is still more treacherous to the unsophisticated, since it makes no distinction between numbers whose ratio is a power of 10 and so requires a considerable feeling for numbers on the part of its user. The essentially new educational issue introduced by the advent of the pocket electronic calculator arises from the fact that a calculator is usable by quite young children. It may indeed be hard to persuade the child in primary school who presses 7 $\boxed{x}$ 9 $\boxed{=}$ ___ and instantaneously reads off 63 that there is some point in committing to memory the 36 non-trivial multiplication facts that constitute the traditional multiplication tables and the comparable addition and subtraction facts that are the basis of the standard arithmetical algorithms. Should we care? We believe that we should, and offer justification for our belief in the next few paragraphs.

3. First, there is the sheer matter of time. Properly used, calculators can save huge amounts of time, but if one happens to *remember* that 7 + 9 = 16 or that 7 x 9 = 63, then one has the answer before the instructions can even be fed into a calculator. It is quite simply more efficient to remember some arithmetical facts of this type. There is, no doubt, room for difference of opinion regarding which facts should be remembered and how many, but surely they should include the previously mentioned multiplication tables up to 10 together with addition and subtraction facts to an extent indicated by the examples 8 + 9 = 17 and 15 - 7 = 8.

4. Secondly, though it is interesting and useful that 7 x 5 = 35, it is also of interest that 35 = 7 x 5, and a calculator is not nearly so useful in establishing a factorization statement of this sort. The arithmetical competence that is the basis of numeracy consists of knowing facts like this *both ways round*, and it may help to persuade primary school children of the importance of basic arithmetical knowledge if more emphasis is placed on partitions and factorizations.

5. Thirdly, it is unrealistic to suppose that one will always have a calculator within reach when an arithmetical problem arises, any more than one always has a dictionary within reach when one is in doubt about a meaning or spelling. If pupils are to be properly equipped for life, they must be able to do some simple arithmetic without a calculator.

6. We are arguing that the importance of 'tables' is unaffected by calculators and that competence in this area should continue to be a major aim of primary school arithmetic. We are not, however, arguing that calculators create no change at all in teaching priorities. Let us examine some of the traditional arithmetical skills and attempt to reassess their usefulness in the

new situation. The algorithms for addition and subtraction of large numbers continue to be useful, and many people in suitable cases still find it quicker and more reliable to do such calculations without a machine. On the other hand, when the numbers get larger or the column of figures gets longer, there comes a point where even the most dedicated arithmetician is glad to have a calculator to relieve him of the drudgery of computation. It seems reasonable to suppose that the adults of the future will reach this 'cut-off' point sooner than those of the present time. This surely does not matter a great deal. The conclusion is that, while the algorithms of addition and subtraction should continue to be taught and practiced, there need not be the same emphasis as formerly on the development of speed and accuracy in the extended written application of these algorithms. Speed and accuracy in mental arithmetic remain important, as we shall see later.

7. The same conclusion applies to the more difficult algorithms for 'long' multiplication and division. The fact that algorithms exist is a point of some mathematical importance, and we believe that children should continue to be able to apply them in simple cases. However, these skills are nowadays very little applied in real adult life, and we must expect that they will tend to fade away.

8. The question of when to use a calculator is an important one, on which pupils will require guidance. The 'cut-off' point referred to above even now among adults varies widely from individual to individual, and one must expect that variation will continue in the future. If one takes a graded series of arithmetical tasks such as

$$5 + 7,\ 8 \times 6,\ 17 + 15,\ 23 + 48,\ 17 \times 15,\ 23 \times 48,$$

$$668 \div 23,\ 27.48 \times 133.72,\ 2876.52 \div 346.9,$$

people will vary in their judgment as to when a calculator is necessary. We would not regard it as important if a generation were to grow up with only a general idea of how to tackle the last two of these without a calculator, but we would regard it as quite disastrous if the first two could not be answered from memory, and extremely disappointing if the third and fourth could not be done mentally.

9. The matter of mental arithmetic is a very important one. It is a feature common to all calculating aids that errors, when they occur, tend to be of a gross nature, leading to an answer that is not even approximately correct. It is an essential aspect of numeracy to be able to know whether an answer is of the right order of magnitude, and the unwillingness of pupils to perform even the most rudimentary checks on their calculations is a constant source of disappointment. Anyone with a calculating machine handy will use it to multiply 19.35 by 21.77; the numerate person knows in advance that the answer will not differ very much from 400. Mental arithmetic, unlike paper and pencil arithmetic, remains an indispensable skill, but possibly the emphasis ought to change. We would suggest increased emphasis on oral questions in class such as "About how much is 53.1 x 32.7?," to which there is no one answer, but to which the first answer might be "a bit more than 50 x 30, that is, a bit more than 1500," while a more refined attempt might

be "about the same as 50 x 35, that is, about the same as 1750." (The exact answer is 1736.37) There is a vast scope for activity of this sort, and one could even make a game of comparing approximate methods offered by different class members, using a calculator as an adjudicator. Other examples:

> About what percentage of 178 is 57? (About the same as 60/180, that is, about 1/3, that is, about 33%)
>
> What approximately is 17/43 as a decimal? (About the same as 16/40, that is, about 2/5, that is, about 0.4)
>
> Would you believe that 1/13 is approximately equal to 0.187? (No, because $\frac{1}{13} < \frac{1}{10} = 0.1$)

The ability to answer questions like these is the quintessence of numeracy and is totally unaffected by calculators. The skills involved are complex and take time to acquire. Perhaps one may hope that calculators will release more time for the development of these skills.

10. It is probably fair to assert that the unease felt by many mathematics teachers regarding the use of calculators is in inverse proportion to the age of the pupils involved. Their worry is that children will begin to use calculators at too early an age and will fail to develop the kind of numeracy we have described. By contrast, pupils in Higher or Sixth Year Mathematics can concentrate all the better on genuinely mathematical ideas if they are released from the drudgery of repetitive arithmetic. In any event such pupils have all for many years used logarithmic tables or slide rules as aids to calculation, and it would be excessively puritanical to object to this new computational aid solely on the grounds that it is better and easier to use. Calculators also enable mathematics to be applied (whether in statistics, science, or geography) to real rather than artificial data, for the arithmetical drudgery formerly involved in handling real data is reduced to manageable proportions. This, too, is surely a change for the better. We would in fact see it as an important part of mathematical education in secondary schools that pupils should learn how to make intelligent use of calculators. This learning should not consist merely of finding out which buttons to press. It should emphasize the wisdom of checking a calculation either (as previously described) by making an approximate mental calculation to verify that the order of magnitude is correct or by performing the calculation twice. It should also emphasize the necessity of giving answers with an appropriate level of accuracy. For example, if the sides of a rectangular piece of paper are given as 10.1 cm and 8.3 cm respectively, a calculator readily gives that the diagonal has length 13.07287268 cm, but it is essential in any real-life situation to realize that the sensible answer is 13.1 cm, that is, that the answer should not be given to a higher level of accuracy than the original data justify. This, of course, is a general point but is especially important in the present context since calculators give so many decimal places.

11. As regards primary schools, we would not go so far as to suggest that calculators should never be seen in the classroom. We did suggest above that a calculator could be used by a teacher as an 'adjudicator' in an approximation game. Additionally, the children might use a classroom machine

themselves as a check on sums done by paper-and-pencil methods. It might be also that children could be stimulated into greater interest in arithmetic by handling and using what is in fact a very remarkable and highly entertaining toy. Nonetheless, we see a very limited place for calculators in the primary school and reiterate that they must not be allowed to prevent the development of the kind of numeracy we have described. We would emphasize also that a certain level of mathematical sophistication is required before a calculator can be used with any confidence. Many of the simpler and less expensive calculators assert, for example, that $(35 \div 9) \times 9 = 34.99999999$, and some arithmetical understanding is required if one is not to be disturbed by this.

12. The first years of the secondary school are a time of consolidation and extension of arithmetical understanding. We see danger in too much reliance on calculators at this stage, though with growing understanding the danger would diminish, and we certainly would not advocate a situation whereby pupils might be using calculators in every classroom except the mathematics one.

13. It is, of course, a corollary to the use of calculating machines that there is now no point in teaching logarithms in the arithmetic syllabus as an aid to computation. This does not, of course, mean that logarithms have no place in the curriculum. They remain an immensely important idea in mathematics and their position in the S5 syllabus is quite unaffected by the irrelevance of common logarithms as a computational aid.

14. It is time now to consider the social issue of whether the pupil who owns a calculator has an unfair advantage over the pupil who does not. It is, of course, possible to exaggerate the importance of this point. Success in examinations, whether in mathematics or in other numerate disciplines, depends far more on understanding than on computation - depends, that is, far more on knowing which sum to do than on being able to do the sum. Nonetheless, it must be conceded that a time advantage does lie with the pupil who has a calculator. He also has an advantage stemming from the greater confidence he can have in the results of computations, for he has time to check them. This may be an argument for setting out to provide a calculator for each pupil in S3 or above. It is not an argument for banning calculators altogether.

15. Is it possible to provide calculators out of public funds? The simplest calculators (which are certainly capable of doing everything that logarithms previously did) seem to cost between £6 and £10. This is in fact not a great deal of money: schools currently buy books for their pupils, and at the present rate of progress books will soon cost more than calculators. The total sum that would be required to provide every pupil in S3 to S6 with a calculator sounds quite large, but viewed as a fraction of the total cost of education it does not appear so significant. At this time of restraint on public expenditure it is probably not feasible to enter into a specific new commitment to purchase calculators in the large numbers that would be necessary, but it is important to measure the financial problem against an appropriate scale and to realize that in the long term such a policy is by no means out of the question.

16. The low cost of calculators is also a relevant consideration if one considers how many parents are likely to be able and willing to buy a machine for their children. There must be comparatively few families in Scotland who would be seriously inconvenienced by the outlay of £6, and the evidence is that huge numbers of parents have already demonstrated their willingness in this respect. Indeed, many have spent considerably greater sums by purchasing more expensive and sophisticated machines for their children.

17. The most likely outcome, unless we specifically make plans for something different to happen, is that schools will provide an inexpensive basic calculator to the minority of children whose parents are unable or unwilling to buy one. This will certainly keep public expenditure to a minimum and will minimize the disadvantage to children not in possession of their own machines. For the advantages at school level to be gained from having a more expensive and sophisticated machine are slight. It does sometimes save a little time to be able to read off trigonometric functions from a machine, or to be able to change a number instantaneously into scientific notation ($a \times 10^n$, $1 \leq a < 10$), but at school level the main benefit of a calculator is in performing the basic arithmetical operations, and even the simplest machine does that. Indeed there is probably an actual disadvantage in having too sophisticated a machine, for in the hands of an unsophisticated user the very complication of the machine presents perils. One of the disturbing aspects of the present situation is that parents are in many instances buying over-sophisticated machines for their children, in the mistaken belief that the advantage to their child is in direct proportion to the cost of the machine.

18. Reference has been made to basic 'four-function' machines, and the opinion has been expressed that such machines are adequate for school purposes. It is time now to consider the specifications of a machine that would be ideal (as opposed to merely adequate) for school purposes. Here we shall limit ourselves to the years S3 to S5. For Sixth Year Studies work a full 'scientific' calculator could be useful. It should be emphasized that at the present time there seems to be no calculator on the market specifically designed for school use and that the specification given below does not correspond exactly to any machine currently available. It comes closest to the 'semi-scientific' machines which presently retail at about £20.

   a. The machine should have a clear, bright display. Its keyboard should be easy to read and use, with firm, positive action switches.

   b. It should have a fully rechargeable battery. Experience in schools indicates that machines are frequently left switched on.

   c. It should employ algebraic rather than reverse Polish logic. As an example of the difference, on a machine employing algebraic logic the routine for determining 5 - 3 is

   $$5 \ \boxed{-} \ 3 \ \boxed{=} \ \_\_,$$

   While on a machine using reverse Polish logic the routine might be something like

   $$5 \ \boxed{+} \ 3 \ \boxed{-} \ \_\_$$

We feel that this latter system can confuse a beginner.

d. The machine should have a square root key.

e. The machine should have at least one memory.

f. Sums of squares should be easy to handle. This is often achieved by having an $\boxed{x^2}$ key, a single memory, and a facility for adding to the number in the memory.

g. It should include the trigonometric functions and their inverses.

h. It should have a 'clear last entry' key.

i. It need not have a percentage key.

j. It should not include an automatic constant facility. Such a facility enables (for example) 8 x 5, 8 x 6, 8 x 7 to be computed by a routing such as:

$$\begin{array}{rl} 8 \ \boxed{\times} \ 5 \ \boxed{=} & (40) \\ 6 \ \boxed{=} & (48) \\ 7 \ \boxed{=} & (56) \end{array}$$

The constant multiplier 8 is 'held' in the machine and need not be re-entered each time. This device can be useful for repeated calculations, but can certainly be confusing to a beginner, and for school use the disadvantages probably outweigh the advantages.

19. If 'semi-scientific' or 'scientific' machines come into widespread use in in the senior classes in secondary schools, then trigonometric tables will virtually disappear. While we see some advantage in showing pupils a page of tables so that the general behavior of (say) the sine function in the range 0° - 90° can be seen, we do not believe that it would matter greatly if the use of tables ceased to be taught in schools. It is a skill that has little virtue in its own right and can very easily be acquired if necessary at a more advanced stage in education.

20. Summary of conclusions:

a. Calculators are valuable to senior school pupils, and their use should be encouraged.

b. They should be used by pupils only when a substantial degree of arithmetical understanding and competence has been achieved.