THE INFLUENCE OF CALCULATORS ON MATHEMATICS CURRICULUM

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The combination of technological advancement and price reduction has made the electronic calculator available to virtually anyone in our society who wants one. Pocket-size calculators and desk models alike today have capabilities beyond the wildest dreams of 15 or 20 years ago. And the price of these machines has gone down while prices of most items have gone up. The curriculum has already been affected, and the changes have only begun.

The curriculum has already been changed by the availability of these machines. An ever-widening segment of the student population has access to machine computation. Students will use machines to save labor whenever they can. Teachers can and should prevent calculator use at those times when calculator use would interfere with specific learning objectives. But at other times teachers should not, and many times teachers can not prevent student use of calculators.

Changes in the curriculum can result from sheer pressure, or they can come out of thoughtful work by educators. Pressure has resolved the issue of whether students should be allowed to use calculators. Careful consideration by educators is needed to determine what curriculum changes will be positive and therefore should be encouraged, and how to minimize the negative influences. Educators in many fields are facing the need for such thoughtful planning, but those in mathematics education have a special interest and responsibility in guiding the influence of electronic calculators in positive directions.

The calculator is a powerful thinking tool in its own right, and students must be taught to use calculators effectively. We need new experiences in the curriculum which are designed for student mastery of the calculator. Other changes may be more difficult, but perhaps even more important. The calculator must be considered in the design of student exercises on other topics. Most exercise material for present mathematics curricula is designed to be done by a student without the aid of a calculator. Many of those exercises are well designed to help a student master a particular educational objective or set of objectives. Students who use calculators to do those exercises probably will not master the objectives for which the exercises were designed, because the calculator will prevent the student from gaining the experience the exercises were designed to provide. Student experiences must be designed to enable students to master the same objectives in the presence of a calculator.

That part of the curriculum related to the teaching of computation may be influenced most. The activities designed to teach computation to a student without a calculator may be inadequate to teach computation to a student with a calculator. But the availability of calculators is no reason to neglect the teaching of computational skills. The availability of the machines will require educators to modify the student activities used to teach such skills. Some work in this direction has already begun.

Some of the programmable calculators have been programmed to provide drill and practice routines to help students learn computation. Research on the use of these and other immediate feedback devices seems to indicate a positive effect. Creative teachers are also designing various activities for using calculators to help teach computation. Among these activities, games are becoming popular. One game is called Target and requires one calculator to be used by two players. The name comes from a target interval that is established before the game starts. The first player enters any number as one factor into the machine. The second player enters a number as a second factor and obtains the product. If the product is in the target interval, the second player wins. If not, the product stays in the machine as one factor; the first player enters a number as another factor and obtains a new product. The game continues until a player wins by obtaining a product inside the target interval.

This particular game provides practice in computational estimation in multiplication (or perhaps division). The example is presented here not to show teachers how to play the game but as an example of how a calculator which is or is not programmable can be used to teach computational estimation. The key to the game lies in the fact that students who can estimate computations well are more likely to win. The desire to win may provide motivation to learn to make better estimates. The teacher can control the difficulty level for individual students by judicious pairing of students and corresponding selection of the target interval. Difficulty increases as the length of the interval decreases. Practice in computational estimation can thus be provided in a game setting. Other games and activities can be used to teach other computational skills, but games will not be the only innovations.

Fractions may actually be easier to teach with a calculator than without one. The idea that the fraction 3/8 means 3 ÷ 8 becomes immediately useful to the learner since the calculator is restricted to decimal fractions. Learning decimal equivalents can be a shortcut to calculating them each time that they are needed. Fractions may be introduced at a much earlier age for most students of the future. Decimal fractions will certainly become of greater relative importance while the value of common fractions may diminish somewhat.

Students may need to be convinced that learning to compute has value now that the calculator can compute for them. Perhaps they have even heard adults take the absurd position that students no longer need to learn to compute. Not only are calculators often unavailable when needed, but many calculations are easier to do without a calculator. Some examples of exercises that are easier without a calculator are 300 x 60, 48 - 2, 10% of $367.50, and 3/8 x 24. Of course, one must have some computational skills before these can be done more easily without a calculator. People who fail to learn to compute are handicapped, and the calculator may provide some help, but it cannot overcome that handicap. Whether "Modern Mathematics" deserves the criticism it is receiving for a de-emphasis in the teaching of computational skills is another issue, but similar criticism would certainly be appropriate for a program that would attempt to substitute the teaching of machine skill for the teaching of computational skills.

The point is that the activities used to teach computational skills in the presence of calculators probably will differ from those activities used to teach the skills without calculators, but the skills can still be taught and must still be taught. As the computational skills are taught, the calculator can become a powerful tool in the teaching of other mathematics. Electronic calculators may help teachers put computation into its proper perspective with respect to the rest of mathematics.

Computation has always been and will continue to be a necessary component of the elementary and secondary school mathematics curriculum, but it has never been a sufficient component. Perhaps the advent of the inexpensive calculator will help to point out that fact. Indeed, students have a legitimate complaint if everything they learn to do in mathematics can be done faster and more accurately by an inexpensive machine. As teachers of mathematics we need and have always needed to teach computation, but we need and have always needed to teach more than that. The calculator may very well serve to make both teachers and laypeople more aware of the needs in mathematics above and beyond computational skills, and such awareness may prove to be the greatest contribution of the calculator.

Many mathematical ideas are not computational ideas in themselves but are based on computations. Students who follow the computations may comprehend the idea better, but teachers often find students unwilling or unable to do enough computation to understand the ideas. Even students who have mastered computational skills are reluctant to do excessive computation as a part of other learning. The calculator places a whole new perspective on what computation is excessive.

An old activity for teaching π involves having students measure the circumference and diameter of a variety of circular objects using a variety of measuring units. For each object the student is asked to compute the quotient: circumference divided by diameter. The activity involves much computation and, due to computational error, teachers often find it difficult to even convince students that the quotient is a constant, let alone a particular constant. The objective itself, "the understanding of π," is often lost due to the focus on the computation. With electronic calculators, students can concentrate on the objective, check and re-check measurements, and the activity has a much higher probability of success.

Similar examples abound in the studies of sequences and series. Who wants to sum $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} \ldots$ by hand? With a calculator it can be rather enjoyable, and the experience can be powerful in helping a student understand convergence.

Electronic calculators with programming capabilities lend themselves to a whole variety of mathematical learning situations. Programming itself involves logical thought and mathematical reasoning. Functions can be programmed and studied. Students can literally put in X and get out f (X).

Iterative processes are worth studying and yet the computation can be overwhelming without machine help. One example of iterative thinking is a process for computing square root usually attributed to Newton. For the initial step in the computation of the square root of 17 one might guess 5 as a first approximation. (Any guess will work, but close guesses are more efficient.) Divide 17 by 5 and the quotient is 3.4. Average 5 and 3.4 to get 4.2 as the second approximation, which is better than the first. To repeat the process 17 ÷ 4.2 = 4.0476. The average of 4.2 and 4.0476 is 4.1238 which is the third approximation, and better than the second. The process can be continued indefinitely and requires a decision to terminate when closer approximations are no longer desirable. (When a calculator is used, limits are imposed on the process by the capability of the calculator used and the ingenuity of the person operating it.) Any process is said to be iterative if it involves the repetition of an algorithm where the output of one cycle is used for the input of the next cycle in order to obtain successively better approximations. Newton's method applies not only to square root, but can be applied to a root of any polynomial. Iterative thinking is very productive in a variety of applications, some of which involve complex algorithms and computation that would be overwhelming. But with a programmable calculator, all steps of the algorithm can be programmed once, and then repeated easily and accurately. Thus iterative thinking becomes accessible to study at the high school level.

Compound interest is a topic that has been hard to teach because of the volume of computation involved. Typically, we assign a few exercises to show a student that interest is computed by multiplying principal by rate, and that the sum of the principal and the interest becomes the principal for the next period of time, repeating the process for the total time of the investment. Then we show the student that the value of "one plus the rate" can be used as the base of an exponential function which simplifies the whole process. Too often the exponential simplification is given to the student before the student has experienced enough computation to know just what is being simplified. With the calculator more computations can be done prior to the simplification, and perhaps the whole experience will be more meaningful.

Many textbook examples of mathematics applications involve carefully selected numbers to keep the students from being overwhelmed by the calculations. Students often recognize such problems as contrived, and approach them with little enthusiasm. With calculators available, the exercises can be more realistic. Perhaps we will see more problems in which students collect their own data from the world around them. Making applications more realistic could have a positive effect on students' desire to study mathematics in the first place.

In summarizing, then, educators will not be allowed to decide the issue of whether students will or will not use calculators. Students will use them. If the students use calculators to complete student activities which were designed for students without calculators, the effect on student learning will probably be negative. But if curricular materials are designed to take full advantage of the power of the calculator as an educational tool, then perhaps student use of calculators will lead to increased mathematical achievement. Students may even find mathematics more interesting and more useful.