A COMPARISON OF ACHIEVEMENT AND ATTITUDES OF STUDENTS USING CONVENTIONAL OR CALCULATOR-BASED ALGORITHMS FOR OPERATIONS ON POSITIVE RATIONAL NUMBERS IN NINTH-GRADE GENERAL MATHEMATICS

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A growing number of mathematics educators consider the electronic calculator to be a useful and practical aid in the teaching/learning process. Its value as a motivating device has been suggested by teachers and researchers (Mastbaum, 1969). Its utility in reducing computational drudgery is apparent; its applicability in many instructional settings with children of divergent skills and abilities attests to its utility in the mathematics laboratory (Johnson, 1970). Most children are easily able to master operation of calculators.

Since many variations of strategies and methodologies within the framework of the conventional algorithmic processes for performing operations on positive rational numbers have been proposed and tried with apparently little success in Grade IX general mathematics, the efficacy of the conventional algorithms for teaching low-ability or low-achieving students is questionable. Accordingly, an algorithm set that is calculator-dependent suggests a reasonable alternative to the conventional algorithm set and provided the basis for the investigation.

The purpose of this study was to assess the differential effects on achievement and attitude of conventional or calculator-dependent algorithms for performing the four fundamental operations on positive rational numbers in ninth-grade general mathematics. No research has been reported that relates directly to the problem under investigation. However, the alternative algorithmic set is dependent on the use of the calculator. Therefore, selected research relative to machine use in the classroom is examined. The literature was also surveyed to study effects on student achievement of variations within the conventional algorithm set.

Fehr, McMeen, and Sobel (1956), Schott (1955), and Findley (1966) report studies suggesting that student use of calculators improves student achievement in arithmetic fundamentals when machines are used over extended periods of time. These findings differed from those observed in a later study by Johnson (1970), who found that comparison of a group using activity-oriented lessons without the calculator to a group using activity-oriented lessons with the calculator in a rational numbers unit resulted in the former group scoring significantly higher. However, Johnson also reported that low- and middle-ability students who had used machines regularly displayed more positive attitudes toward mathematics than did similar groups that did not use the calculator.

Mastbaum (1969) reported an extensive calculator-oriented study which showed that use of machines as a teaching aid with slow learners in seventh- and eighth-grade mathematics did not significantly improve attitude, increase mathematical achievement, noncalculator computational skill, mastery of mathematical concepts, or ability to solve mathematical problems. Mastbaum also found that seventh- and eighth-grade students could learn to use the calculator in solving one-step problems, could solve problems faster, and could work more accurately than students not using machines. It was also reported that ability to solve problems with machines did not transfer to noncalculator situations.

Research on algorithms for performing rational number operations has generally been limited to examining the effects on achievement of making variations within the conventional algorithm sets. The findings of the following studies suggested the strategies to be used for performing operations on positive rational numbers according to the conventional algorithm set.

Brownell (1933), Anderson (1965), and Capps (1962) reported studies showing that alternative strategies for presenting the conventional algorithms of addition and multiplication of fractions do not significantly alter achievement. Brownell also found that students who used the least-common-denominator method of adding fractions achieved as well as students who used the process whereby the denominators of the fractions were multiplied to obtain "a" common denominator. Anderson reported no significant difference in achievement resulting from adding fractions by finding the least common denominator by (1) setting up rows of equivalent fractions or (2) finding different prime factors of each denominator and using their product as the least common denominator.

Brueckner (1928) found that major difficulty with all four operations resulted from three sources: lack of comprehension of the processes involved, difficulty in reducing fractions to lowest terms, and difficulty in changing improper fractions to mixed numbers. Scott (1962) found that children make more errors in subtracting common fractions where regrouping is necessary than in subtracting whole numbers involving regrouping. Capps (1962) found no significant difference in the achievement of students dividing fractions by the common-denominator method as compared to the inversion method. Dutton and Stephens (1960) obtained similar results with respect to retention.

METHOD

The study was conducted during fall, 1971 in three ninth-grade general mathematics classes at each of two schools - Marshall-University High School

(School A) and Coon Rapids Junior High School (School B). The former institution, a secondary school containing Grades VII through XII, is a Minneapolis public school and the laboratory school for the University of Minnesota. The latter school is part of the suburban Anoka-Hennepin Independent School District #11 and contains Grades VII through IX.

The investigator, assisted by a student teacher, taught all three classes at School A. A volunteering teacher, assisted by a paid teacher aide, instructed all three classes at School B. Fifty-three students at School A and 48 students at School B were randomly assigned to classes during spring, 1971. Table 1 summarizes the numbers of students involved in the treatment groups (classes) at the two schools. The duration of the study, including all pretesting, remediation, and retention period, was 10 weeks.

TABLE 1
NUMBERS OF STUDENTS IN TREATMENT GROUPS AT SCHOOL A AND SCHOOL B

School	Treatment 1	Treatment 2	Treatment 3
A (Marshall-University)	18	16	19
B (Coon Rapids)	20	16	12

Three different treatments (developed in a pilot study during the fall of 1970) for performing operations on positive rational numbers were used in the experiment. Treatment differences resulted from the use of the electronic calculator or from the use of two different algorithm sets. The two algorithm sets may be briefly described as follows:

CONVENTIONAL ALGORITHM SET (CAS) - Operations on positive rational numbers are performed according to the "usual" text approaches.

ALTERNATIVE ALGORITHM SET (AAS) - Each fractional operand is converted to a decimal on the calculator (truncated to thousandths). The indicated operation is then performed on the decimals using the calculator. The three treatments for performing operations on positive rational numbers are defined as follows:

T1 - CAS without calculators
T2 - CAS with calculators
T3 - AAS with calculators

Students in the T2 and T3 groups were allowed to use machines throughout the unit on rational numbers and on the post-test and retention test. Each student had his own calculator.

The units of instruction used by the T1 and T2 groups were adapted by the investigator from several current junior high mathematics texts, including recent publications of Houghton-Mifflin Co.; Allyn and Bacon; Holt, Rinehart, and Winston; and SRA. The T1 and T2 units were constructed to adhere strictly to the conventional algorithmic techniques for performing operations on positive rational numbers and were identical but for the following exception: Instructions for use of the electronic calculator in performing the four fundamental operations on positive rational numbers were incorporated into the T2 instructional materials. The T1 group was not exposed to machines. The instructional materials used by the T3 group were constructed in their entirety by the investigator and presented rational numbers according to the AAS.

All students participating in the study were pre-tested for (1) possession of certain skills prerequisite to the study of operations on positive rational numbers, (2) ability to perform operations on positive rational numbers, (3) reading level (Metropolitan Reading Test), and (4) IQ level (Lorge-Thorndike Verbal Battery). Remediation was provided to students showing deficiencies in prerequisite skills. Students scoring above the 80-percent level on the pre-test on operations with positive rational numbers were excluded from the study.

All treatments were conducted within a mastery-learning model as advocated by Bloom (1969). The criterion tests of the mastery of operations on positive rational numbers consisted of a set of five test worksheets, one for each of the five units of instruction. Eight parallel forms of each of the five test worksheets were constructed to conform to the specific objectives of the units of instruction. Parallel forms made it possible for two students in a given treatment group never to work on the same form of a test worksheet at the same time. All completed test worksheets were kept on file by the teacher. Mastery criterion was set at 80 percent (12 of the 15 items on the test worksheets).

The five units of instruction were (1) adding fractions, (2) subtracting fractions, (3) multiplying fractions, (4) dividing fractions, and (5) operating on fractions. The fifth unit, operating on fractions, was a combination of the previous four units, and the first test worksheet a student attempted in this unit served as the post-test for the unit's objectives. If the student did not score at the 80 percent level on any test worksheet, he completed additional forms until mastery was achieved.

The T2 and T3 groups were given instruction in calculator operation. In addition, the T3 group was instructed in changing fractions to decimals on the electronic calculator prior to commencing the unit, adding fractions. Table 2 summarizes the experimental sequences for the three treatment groups.

TABLE 2
UNITS COMPRISING THE EXPERIMENTAL SEQUENCES FOR THE THREE TREATMENTS

	Treatment		
Unit	1	2	3
Operating the Calculator		X	X
Changing Fractions to Decimals			X
Adding Fractions	X	X	X
Subtracting Fractions	X	X	X
Multiplying Fractions	X	X	X
Dividing Fractions	X	X	X
Operating on Fractions	X	X	X

NOTE: The X's denote the units studied by a particular treatment group. The order in which the units are listed in the table corresponds to the order in which they were presented to the students.

The calculators used in the study, provided by the Minneapolis office of the Singer Company, Friden Division, were Friden 1115, 1116, and 1117. The three models were operated identically with respect to performing the four fundamental operations.

The criterion measures used in the investigation were either constructed by the investigator or computer generated via the Honeywell Arithmetic Test Generator (ATG) using a wide range of common rational number skill forms selected by the investigator. The ATG system is an operational example of what measurement theorists term domain-referenced testing. It is made available through remote teletype terminals and is capable of generating large numbers of parallel forms of arithmetic tests. In the present investigation, all test worksheets, the fractions pre-test, and the fractions retention test were generated through the ATG system. A study by Hively, Patterson, and Page (1968) established that reliability estimates for tests generated through the ATG system ranged from .80 to .90 when a variety of skill forms and multiple items within skill forms were used.

The transfer-oriented post-test was constructed by the investigator to measure the performance of the three treatment groups with respect to selected tasks not taught as part of the students' present mathematics course. The test consisted of six subtests concentrating on the following areas: (1) general rational number concepts (not operations), (2) estimation of fractional values, (3) ordering rational numbers, (4) combining rational numbers involving more than two operands or more than one operation, (5) solving open sentences involving rational numbers, and (6) selecting the appropriate operation to use in solving verbally stated problems involving rational numbers. Reliability estimates for the transfer-oriented post-test ranged from .70 to .92.

Students' attitude toward mathematics was measured through a semantic differential (SD) instrument containing 17 sets of bipolar adjectives arranged on seven-point scales. The SD was administered prior to the onset of differentiated instruction and immediately following the students' completion of the unit on operations on positive rational numbers. The SD pre-test carried the stimulus "Mathematics and Me," and the post-test carried the stimulus "Studying Fractions."

Students completed all units of instruction and all post-treatment tests on an individualized basis. When an individual student achieved mastery of the unit operating on fractions, he immediately completed the SD post-test followed by the transfer-oriented post-test.

Upon completion of the post-treatment tests, the student began a two-week retention period during which he completed units related to topology, symmetry, area, spatial representation, and volume. No rational number operations were used during the retention period. At the end of the retention period, the student completed the fractions retention test, a parallel form of the fractions pre-test.

The investigation was concerned with testing the following general null hypotheses:

H1: There is no difference with respect to the achievement of computational skill with rational numbers among the three treatment groups.

H2: There is no difference with respect to the mean performance on selected transfer-oriented, rational number related tasks among the three treatment groups.

H3: There is no difference with respect to the retention of skills for performing operations on positive rational numbers among the three treatment groups.

H4: There is no difference with respect to the attitude toward studying operations on positive rational numbers among the three treatment groups.

H5: There is no difference with respect to the rate of the mastery of operations on positive rational numbers among the three treatment groups.

H6: There is no interaction effect when the treatment groups are blocked on the basis of high or low reading ability with respect to the six subtests of the transfer-oriented post-test, the fractions retention test, and the SD attitude instrument.

To test the null hypotheses of the investigation, the following statistical techniques were employed: (1) one-way ANOVA (unequal frequencies), (2) two-way ANOVA (unweighted means analysis), and (3) two-way ANCOVA (unweighted means analysis). Significant F-ratios ($p < .05$) from the one-way ANOVA prompted further examination of all possible ordered pairwise contrasts on the treatment group means using the Newman-Keuls procedure (unequal frequencies). Figure 1 shows the variables being examined in a given contrast. In discussing ordered contrasts on treatment group means, the treatment group having the higher mean is listed first. For example, T3 versus T2 indicates that the T3 group had the higher mean in the comparison of the T3 group mean with the T2 group mean.

Each school was considered a separate experiment and results from each school are reported separately for each criterion measure.

Treatment Description	T1 CAS	T2 CAS Calculators	T3 AAS Calculators
T1 CAS	—	compares effect of calculator under CAS	compares effect of calculator and AAS to CAS without calculator
T2 CAS Calculators	compares effect of calculator under CAS	—	compares effect of AAS v. CAS under calculator
T3 AAS Calculators	compares effect of calculator and AAS to CAS without calculator	compares effect of AAS v. CAS under calculator	—

Fig. 1. Interpretation of Ordered Pairwise Contrasts

RESULTS

Tables 3 and 4 summarize treatment group means and standard deviations for all criterion measures of the investigation.

TABLE 3

TREATMENT GROUP MEANS AND STANDARD DEVIATIONS FOR THE CRITERION MEASURES AT SCHOOL A

Criterion Measure	Possible Score	T1 M	T1 SD	T2 M	T2 SD	T3 M	T3 SD
Operating on Fractions	15	12.30	1.70	12.10	1.60	13.80	1.10
Transfer-Oriented Posttest							
Pt. 1 (general)	5	2.88	1.28	3.19	1.56	2.95	1.27
Pt. 2 (estimation)	5	1.89	1.08	2.69	1.20	2.63	1.01
Pt. 3 (order)	7	2.78	1.70	3.62	1.54	4.79	1.40
Pt. 4 (combining)	5	2.22	1.11	1.94	1.53	3.32	1.45
Pt. 5 (open sent.)	4	2.50	1.10	2.31	1.35	2.32	1.29
Pt. 6 (select. oper.)	5	2.39	0.98	2.62	1.31	2.53	1.43
Fractions Retention Test	20	11.11	4.56	11.75	5.21	17.89	1.97
SD—Studying Fractions	119	75.70	20.40	79.40	23.30	84.20	19.80

TABLE 4

TREATMENT GROUP MEANS AND STANDARD DEVIATIONS FOR THE CRITERION MEASURES AT SCHOOL B

Criterion Measure	Possible Score	T1 M	T1 SD	T2 M	T2 SD	T3 M	T3 SD
Operating on Fractions	15	12.90	2.00	11.70	2.10	13.10	3.1
Transfer-Oriented Posttest							
Pt. 1 (general)	5	3.45	1.23	2.94	1.12	2.75	1.16
Pt. 2 (estimation)	5	2.40	1.05	2.13	1.22	2.42	1.11
Pt. 3 (order)	7	3.70	1.13	3.38	1.54	3.67	2.01
Pt. 4 (combining)	5	2.80	1.70	2.63	1.71	2.75	1.54
Pt. 5 (open sent.)	4	2.65	0.59	3.13	0.81	1.75	1.05
Pt. 6 (select. oper.)	5	2.95	1.10	2.88	0.62	2.33	1.15
Fractions Retention Test	20	14.15	3.96	13.50	5.14	17.25	1.82
SD—Studying Fractions	119	63.50	28.40	77.60	14.70	78.30	25.90

Tables 5 and 6 summarize the results of the statistical analyses of the experiments conducted at the two schools. The tables include F-ratios, the p-value associated with each F-ratio, the significant contrasts associated with a significant F-ratio ($p < .05$), the p-value associated with a given contrast, and the p-value associated with the test for interaction effects.

Findings Pertaining to H1.

Mean treatment group performance with respect to the individual student's first-attempted form of the test-worksheet for the unit, operating on fractions, was analyzed to provide evidence of achievement of computational skill with positive rational numbers. Results obtained at School A supported rejection ($p < .01$) of the null hypothesis of no difference in treatment group means. The results favored the T3 group. Consequently, the contrasts T3 v. T2 and T3 v. T1 were significant at the .01 level.

TABLE 5

Summary of Achievement and Attitude Test Results at School A

	Treatment Effects				Interaction	
Criterion Measure	F	p	Sig. Contr.	p	F	p
Operating on Fractions	7.08	$p < .01$	T3 v. T2	$p < .01$		
				$p < .01$		
Transfer-Oriented Posttest						
Pt. 1 (general)	0.70	$p > .25$	—	—	0.64	$p > .25$
Pt. 2 (estimation)	3.27	$p < .05$	T2 v. T1	$p < .05$	0.26	$p > .25$
Pt. 3 (order)	7.57	$p < .01$	T3 v. T1	$p < .01$	0.49	$p > .25$
			T3 v. T2	$p < .05$		
Pt. 4 (combining)	5.06	$p < .05$	T3 v. T2	$p < .05$	0.28	$p > .25$
			T3 v. T1	$p < .05$		
Pt. 5 (open sent.)	0.82	$p > .25$	—	—	1.12	$p > .25$
Pt. 6 (select oper.)	0.15	$p > .25$	—	—	0.95	$p > .25$
Fractions Retention Test	15.52	$p < .01$	T3 v. T1	$p < .01$	0.21	$p > .25$
			T3 v. T2	$p < .01$		
SD—Studying Fractions	0.23	$p > .25$	—	—	0.65	$p > .25$

TABLE 6

Summary of Achievement and Attitude Test Results at School B

	Treatment Effects				Interaction	
Criterion Measure	F	p	Sig. Contr.	p	F	p
Operating on Fractions	1.59	$p < .25$	—	—	N/A	
Transfer-Oriented Posttest						
Pt. 1 (general)	1.52	$p < .25$	—	—	2.35	$p < .25$
Pt. 2 (estimation)	0.56	$p > .25$	—	—	0.39	$p > .25$
Pt. 3 (order)	0.23	$p > .25$	—	—	0.08	$p > .25$
Pt. 4 (combining)	0.05	$p > .25$	—	—	0.03	$p > .25$
Pt. 5 (open sent.)	10.34	$p < .01$	T2 v. T3	$p < .01$	0.59	$p > .25$
			T1 v. T3	$p < .01$		
Pt. 6 (select oper.)	1.61	$p < .25$	—	—	0.24	$p > .25$
Fractions Retention Test	3.30	$p < .05$	T3 v. T2	$p < .05$	0.09	$p > .25$
			T3 v. T1	$p < .05$		
SD—Studying Fractions	0.88	$p < .25$	—	—	1.70	$p < .25$

Results obtained at School B with respect to the individual student's first-attempted form of the test/worksheet for the unit, operating on fractions, did not support rejection of the null hypothesis of no difference in treatment group means.

Findings Pertaining to H2 and H6.

Results obtained at School A on the transfer-oriented post-test supported rejection of the null hypothesis of no difference in treatment group means with respect to Part 2 (estimation of fractional values), Part 3 (ordering rational numbers), and Part 4 (combining rational numbers involving more than two operands or more than one operation). In all instances, the results favored the T3 group. The only significant ($p < .05$) contrast on Part 2 was T3 v. T1. The contrasts T3 v. T1 and T3 v. T2 were significant at the .01 level on Part 3. Part 4 yielded treatment-group means that were significantly different at the .05 level for the contrasts T3 v. T2 and T3 v. T1. The tests for reading ability level by treatment interaction effects showed no significant effects on any of the six parts of the transfer-oriented post-test at School A.

Results obtained at School B on the transfer-oriented post-test supported rejection of the null hypothesis of no difference in treatment group means only for Part 5 (solving open sentences involving rational numbers). The T2 group was favored at the .01 level of significance. Pairwise comparisons of the treatment group means showed the contrasts T2 v. T3, T1 v. T3, and T2 v. T1 all significant at the .01 level. The tests for reading ability level by treatment interaction effects indicated no significant effects at School B.

Findings Pertaining to H3 and H6.

The results obtained at School A on the fractions retention test supported rejection ($p < .01$) of the null hypothesis of no difference in treatment group means. The results favored the T3 group. The contrasts T3 v. T2 and T3 v. T1 were significant at the .01 level. Interaction effects were not significant. An additional result of the fractions retention test was the proportion of students in each treatment group at School A who retained the mastery level (80 percent correct) performance following the two-week retention period. The proportions were 5/18 for the T1 group, 5/16 for the T2 group, and 16/19 for the T3 group. These results were subjected to no further statistical analysis, but clearly favor the T3 group.

Results obtained at School B with respect to the fractions retention test supported rejection of the null hypothesis ($p < .05$) of no difference in treatment group means. The differences favored the T3 group. The contrasts T3 v. T2 and T3 v. T1 were significant at the .05 level. The data did not support rejection of the null hypothesis of no reading ability level by treatment interaction effects. An additional result of the fractions retention test was the proportion of students in each treatment group at School B who had retained mastery level performance (80 percent correct) following the two-week retention period. The proportions were 9/20 in the T1 group, 7/16 in the T2 group, and 11/12 in the T3 group. These results were subjected to no further statistical analysis.

Findings Pertaining to H4 and H6.

Results on the semantic differential attitude instrument at School A did not support rejection of the null hypothesis of no difference in treatment group mean scores. However, according to test results all three treatment groups exhibited a positive attitude toward mathematics. The data at School A also did not support rejection of the null hypothesis of no reading ability level by treatment interaction effect with respect to the SD instrument.

Results obtained from attitude testing at School B indicated no significant difference in treatment group mean scores of attitude. In addition, the data did not support rejection of the null hypothesis of no reading ability level by treatment interaction effects.

Findings Pertaining to H5.

As one means of determining the relative efficiency of the three treatments, the rate of mastery of operations on positive rational numbers by each of the three treatment groups was examined descriptively for School A. The results showed that the mean time required by the T1 group to complete the experimental treatment was 6.2 days; the mean time required by the T2 group to complete the experimental treatment was 7.7 days; and the mean time required by the T3 group to complete the experimental treatment was 6.3 days. These mean numbers of days include instructional time for calculator operation in the T2 and T3 groups, and a unit on changing fractions and mixed numbers to decimals in the T3 group. Calculation of the mean number of days minus calculator-related instruction revealed that the T1 group required 6.3 days, the T2 group required 6.3 days, and the T3 group required 4.8 days.

No data on time to complete the experimental treatments was available from School B.

LIMITATIONS OF THE INVESTIGATION

Certain limitations more or less specific to this investigation should be recognized:

1. Fifteen calculators were used at each school. This number was probably appreciably larger than the number available at many schools.

2. The six subtests of the transfer-oriented post-test contained relatively small numbers of items (from four to seven), with no reliability estimates determined for the separate subtests.

3. Because the Newman-Keuls procedure keeps a constant level of significance for all ordered pairs of contrasts, the power of the collection of all tests conducted is less than for a single F-test, thus increasing the likelihood of committing a Type II error.

CONCLUSIONS

The experimental results at the two schools involved in the investigation suggest the following conclusions subject to the limitations just identified:

1. When development of computational skill with positive rational numbers is a goal of instruction, the alternative algorithm set with the calculator appears to be a viable alternative to the conventional algorithm set with or without the calculator for low-ability or low-achieving students in the ninth-grade general mathematics.

2. When development of computational skill with positive rational numbers via the conventional algorithm set is a goal of instruction, use of a calculator does not significantly and consistently affect performance.

3. The alternative algorithm set with the electronic calculator can produce success for students in dealing with semi-novel problem situations such as estimation of fractional values, ordering rational numbers, and simplifying rational numbers involving more than two operands or more than one operation.

4. When a goal of instruction is retention of skill for performing operations on positive rational numbers in ninth-grade general mathematics, use of the alternative algorithm set with the electronic calculator can promote superior performance when compared to use of the conventional algorithm set either with or without the electronic calculator.

5. When development of a positive attitude toward mathematics is an instructional goal, no one of these is more effective than the other: use of the conventional algorithm set without the electronic calculator, use of the conventional algorithm set with the electronic calculator, or use of the alternative algorithm set with the electronic calculator.

6. When deployment of the conventional algorithm set with or without the electronic calculator or the alternative algorithm set with the electronic calculator depends on the relative efficiency of the two algorithm sets in promoting computational skill and retention of skill, the alternative algorithm set with the calculator is apparently more efficient in terms of the rate of mastery, the performance on the individual student's first-attempted form of the test/worksheets for each unit, and the proportion of students retaining mastery-level performance two weeks after termination of instruction.

DISCUSSION

In today's technological society, where sophisticated electronic processing media are becoming available to the "average" American citizen, teachers with low-ability or low-achieving children should consider a machine-based curriculum for students unable to master the skills necessary for learning practical mathematical concepts by conventional procedures. The opinion is held by some mathematics educators that the advent of a machine-based curriculum would lead to the creation of machine-dependent learners. However, the students to whom this investigation was directed were, typically, youngsters who had repeatedly demonstrated their inability to attain and retain the skills that the experimental treatments were attempting to develop.

Therefore, one implication of the study is clearly indicated: the alternative algorithm set tested in this investigation provides a means by which low-ability and low-achieving children can compute with rational numbers. Students are able to apply their learned skills in semi-novel situations and they are able to retain the skill to a significantly greater degree than students of like ability using conventional algorithms.

The results of the investigation indicated that the groups using the alternative algorithm set with the electronic calculator exhibited transfer of skill for operating on positive rational numbers to such areas as (1) ordering rational numbers and (2) combining rational numbers involving more than two operands or more than one operation to a significantly greater extent than did the groups using the conventional algorithm set. This suggests a step in the direction of making the study of rational number operations applicable to further study of mathematics appropriate for slow-learning children. If further research shows the alternative algorithm set to indeed have applicability to a wide range of areas of mathematics appropriate to low-ability or low achieving children, then the likelihood of its acceptance by mathematics educators will be increased.

It must be remembered that the alternative algorithmic approach with the electronic calculator was directed only toward low-ability or low-achieving children. Mathematics teachers must consider the future educational plans of their students before adopting such a machine-oriented approach to performing operations on positive rational numbers. If a particular group of students is capable of studying high school algebra to a relatively high degree of sophistication, then the alternative algorithmic approach should not be stressed (or possibly not used at all) in light of its impracticality in solving equations and in combining or simplifying algebraic fractions. But, if the study of computation-related or computation-dependent topics represent the future mathematics of the children, the alternative algorithm set and the electronic calculator are worthy course inclusions.

Hopefully, additional uses of the electronic calculator in the general mathematics classroom will be developed. It is logical to propose that machine use could allow some aspects of many topics to be included in the curriculum that currently are omitted from the general mathematics course because of arduous computation. Selected material from topics such as estimation, area, volume, maxima-minima, ratio-proportion, probability and statistics, conversion, trigonometry (numerical), linear interpolation, sequences and series, and evaluation of polynominal expressions might be included in courses for students classified as low-ability or low-achieving.

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