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calculators in the classroom

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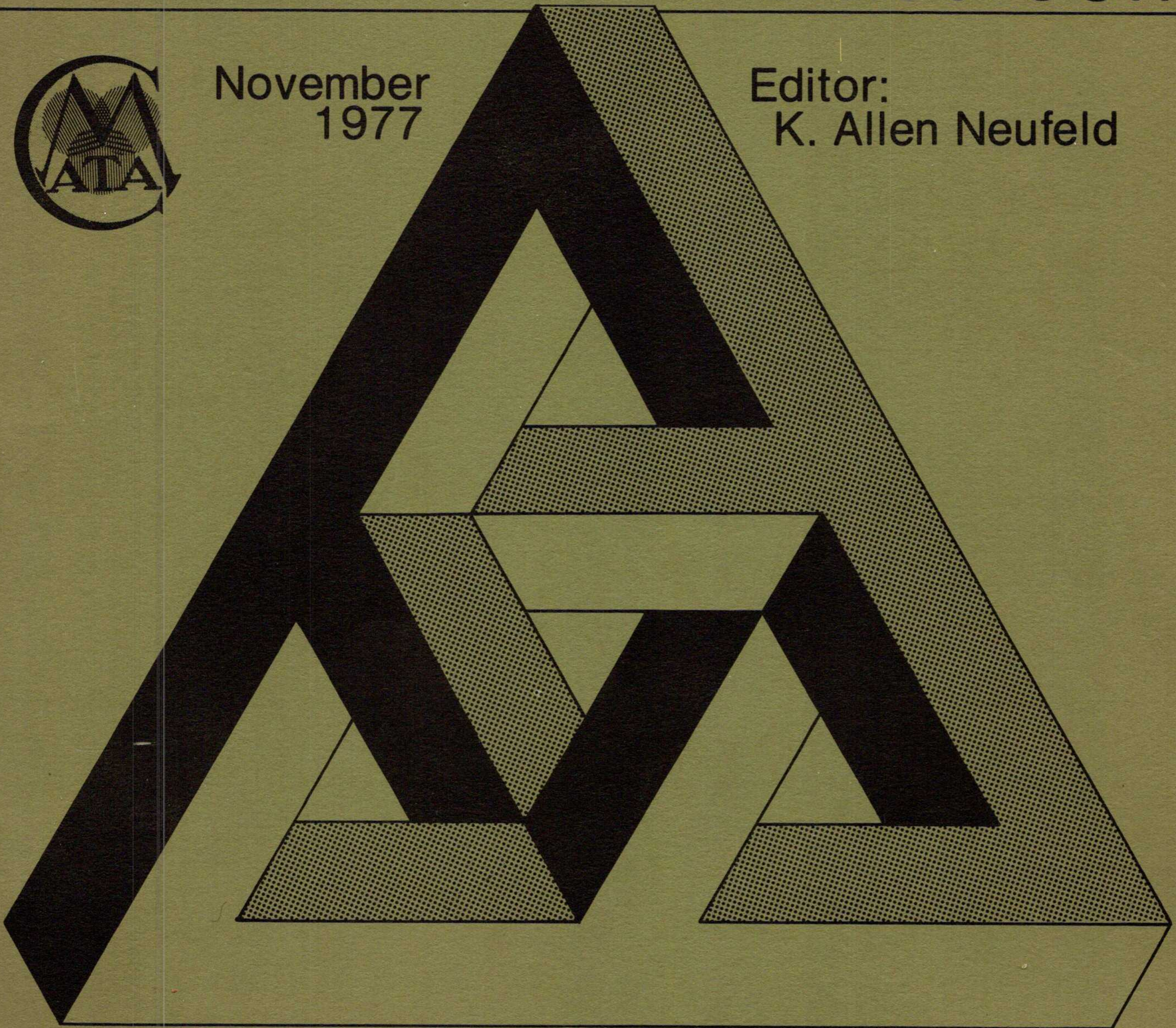

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calculators in the classroom

NOVEMBER 1977

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FOREWORD

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K. ALLEN NEUFELD

Calculators are here to stay! To be more specific, electronic hand calculators are being increasingly used as an instructional aid in the scnool and as a convenience in the home. This is not to say that calculators will always receive such high attention. The slide rule, abacus, and even the ordinary pencil and paper have all "had their day" and continue to be used in varying degrees.

The purpose of this *Monograph* is to present a variety of papers addressed to the use of calculators in the schools. Educators in Britain, U.S.A., and Canada were invited to submit articles either for initial publication or in reprint form from other journals. Articles from the November 1976 issue of *THE ARITHMETIC TEACHER* devoted entirely to calculators are not used in this publication because of their availability to most schools at present.

You are encouraged to sample the opinions and make up your own mind, peruse the specifications and buy appropriately, study the research and consider implications for your classroom, and select activities which will supplement and enrich the mathematics curriculum for your students.

PART ONE
Opinions

CALCULATORS IN THE CLASSROOM:
PROCEEDINGS OF A SYMPOSIUM SPONSORED BY ROCKWELL INTERNATIONAL

The proceedings of the symposium held in Chicago, December 1974, have been reprinted with the permission of Ronald J. Baron of Rockwell International.

In December, 1974, an audience of Chicago-area teachers and student teachers participated in a unique symposium entitled: *Calculators In The Classroom,* sponsored by Rockwell International.

For nearly four hours, the invited teachers listened, questioned, and commented as a panel of university-affiliated educators talked about the potentials and promises of electronic calculators as teaching aids in middle-grades (VI through IX) arithmetic classes.

The symposium's moderator was Dr. Max Bell (University of Chicago); the three other panelists were Prof. George Immerzeel (University of Northern Iowa) and Drs. Joy Rogers and Jack Kavanagh (both of Loyola University).

All of the panelists are faculty members of the schools of education at their respective universities. Further, Dr. Bell is affiliated with the Laboratory School of the University of Chicago, and Prof. Immerzeel is affiliated with the Price Laboratory School of the University of Northern Iowa. Dr. Bell and Prof. Immerzeel hold degrees in education; Dr. Rogers is a psychologist; Dr. Kavanagh is a statistician.

This report on the symposium contains synopses of the comments made by the panelists, and of the question-and-answer session that followed the formal presentations.

DR. BELL:

"We are here this morning to announce and usher in a new revolution in pedagogy.

"To any old hands in teaching here today, this will sound depressingly familiar, because we've heard the rhetoric of revolution before. We've been told in the past that new techniques are going to revolutionize the way that we teach ... that we must all prepare for a new way of doing things. We've all lived through films, and closed-circuit TV, and teaching machines, and programmed learning, and language labs, and computer-assisted instruction, and so on ...

"Nevertheless, in the case of electronic calculators, there is a sound basis for believing that something really new is about to happen to the way arithmetic is done - and the way arithmetic is considered - in the classroom.

"I think that there is a good reason to believe that calculators are not simply a minor advance in technology, but are, instead, products that will have a substantial effect on the world at large, and, possibly, on the classroom too.

"This morning, we will explore this possibility."

(After his opening remarks, Dr. Bell introduced Prof. Immerzeel to the audience, and noted that "George Immerzeel has been using calculators of one sort or another as teaching aids for over 10 years ... he will try to 'turn you on' to the use of calculators, and will show you how they can be used as aids to the teaching of basic arithmetic and mathematical concepts."

Prof. Immerzeel distributed copies of a student workbook he developed that contains scores of unique calculator-based exercises; Rockwell 20R calculators were distributed for audience use.)

PROF. IMMERZEEL:

"One of the first things I have learned through my experience with calculators is that you first give a calculator to someone (be he a child or adult), and then you provide the student with 'experiences' built around the use of the calculator. A teacher must resist the temptation to tell the student how to work through a problem ... to 'flow chart' the operation for him."

(Prof. Immerzeel led the audience through a series of exercises that demonstrated basic calculator operations and fundamental arithmetic principles.)

"The point I continually want to make is that the calculator will do more than just calculate. It will put a focus on ideas if we, as teachers, learn to use it properly. Initially, the calculator helps you get right answers, and it allows you to do problems that are more complex than you've done in the past. But, it can do a lot more than this if we (as teachers) learn how to design material that makes full use of a calculator's capabilities."

(A member of the audience asked: "How can a calculator help other than by doing the computation?" Prof. Immerzeel replied:)

"Because it gives you the facility to build basic arithmetic concepts as well as perform computations readily.

"Some people say a calculator just calculates. I believe it is going to increase the teacher's ability to expose children to concepts and ideas. You can take almost any subject in a middle-grade curriculum, and if you use your imagination, you can find ways of using a calculator to reinforce it.

I don't believe that the calculator will replace the other things you do in class ... I think it will support them.

"I think one of the things we must emphasize to students is that there are many things you can do better in your head than on a machine. I think we must make sure that students understand this. I don't think that the calculator will ever replace a student's knowledge of basic mathematical facts, but I do think it is going to change the level at which we operate. I think it would be silly to spend considerable time in the future teaching children how to divide three digits into five digits, but, I think it is equally silly not to teach them how to divide by two digits. They need this kind of skill, but they aren't going to get it because we say they have to have it ... they are going to get it because we build ways in which they will get it.

"The important points I want to share with you are these: First, you don't teach someone to use a calculator by telling him how to do it ... you teach by getting him to do it. In the process, he will learn a lot more than the calculation you are using as an example.

"Secondly, I believe that the calculator offers us a new way to get at basic ideas.

"Thirdly, I really believe that the calculator is going to substantially affect the mathematics curriculum. It is going to change the teaching parameters. Some things are going to become more important, some things less important. The calculator opens a complete new set of objectives in the problem-solving dimension.

"Here's an example. I happen to be involved in an experimental project in problem-solving. We are using the calculator because we can get at problem-solving in a way we never did before. Besides calculating more quickly, we can get more 'experiences' in a given period of time.

"During this project, I once asked fifth-grade kids to determine how many pieces of paper they could tear from a sheet in one minute. The teacher I was working with thought I was a bit crazy, but everyone in the class tore little pieces of paper. Then, we asked ourselves: 'How long would it take the class to tear a piece of paper for every citizen of Iowa?' And: 'How much longer would it take to tear a piece of paper for everyone in the U.S.?' These are hard problems if you are working with pencil and paper, but not if you have a calculator."

(Dr. Bell suggested that the audience be prepared to ask Prof. Immerzeel practical questions about calculator usage during the panel discussion to come later. Dr. Bell noted that Prof. Immerzeel's long-term classroom experience with calculators - first mechanical and later electronic - was "the best kind of experience to have."

The next panelist to speak was Dr. Jack Kavanagh. Dr. Bell prefaced his introduction by explaining that advice on the future use of electronic calculators in classrooms might well be garnered by studying "the lessons that have been learned about the use of mechanical calculators in the past."

He cautioned the audience, though, that "There is a sense in which this is an analogous experience, and a sense in which this is an irrelevant experience. Mechanical calculators have certain flaws and difficulties that make the classroom use of electronic calculators a new and different proposition.")

DR. KAVANAGH:

"Basically, I have compiled a technical paper. But, instead of reading it to you (and having half of the audience go to sleep) I want to summarize what we did and what our objectives were.

"Essentially, we questioned if mechanical calculators could be used to improve the achievement and attitude of low achievers in mathematics. Work done as early as 1937 showed that the achievement of normal students could be improved by the use of calculators.

"We randomly chose 125 junior high students who were at or below a 'C' average for two years. We distributed them into five groups: a pilot group; two experimental groups; and two control groups. We then ran a pilot first quarter, an experiment and control the second quarter, and an experiment and and control the third quarter.

"The pilot was used to determine methodology. We took 25 basic concepts (communitivity, associativity, and the like) and split them into 10 teaching units. We gave students a unit at a time, and asked each student to complete each one to predetermined criteria. At the end of each unit, we gave the students a test devised to measure mastery of the work unit.

"We were interested in several things. First, we looked at the achievement level at the end of each unit. Next, we wanted to know how many students in each group completed all 10 units. Finally, we were interested in attitudes, and we used a standardized mathematics attitudinal inventory to measure this.

"Our results were interesting. Although we could not measure an increase in attitudinal growth of the experimental group compared to the control group, the second quarter experimental group showed significantly greater achievement than the control group.

"We later found, though, that the situation was reversed in the third quarter ... the control group achieved better results than the experimental group. We interpret this reversal to the fact that our mechanical calculators began jamming frequently in the third quarter. Students frequently wound up with incorrect answers, or else didn't have a machine to use.

"There was one other positive result we observed. In both experimental groups, a significantly larger number of students completed all 10 units compared to the control groups. This means that even though students often got wrong answers, they were better motivated to complete each exercise. I think that this in itself says something about the use of calculators in the classroom.

"I am certain - based on my own intuitive feelings - that a new experiment using jam-free electronic calculators would show positive results regarding academic growth."

(The next speaker was Dr. Joy Rogers. Dr. Bell introduced her by noting that "Joy Rogers, like the rest of us, has been through numerous revolutions which turned out not to be. She has some advice for us about the use of calculators as teaching aids, and some ground rules for determining whether or not a new device - such as the calculator - is likely to be truly useful in a classroom."

DR. ROGERS:

"The people I really want to talk to today are the ones who influence the spending of money. I think that is probably all of you. If you walk through a school and find the dustier corners of that school, you will find a place with a sign, slightly mildewed, that says 'Language Laboratory'. Only 15 years ago, language laboratories were really 'in'. If you wanted to get a Federal grant, write the words 'Language Laboratory' anywhere in the application, and your grant would be funded. About five years ago, I noticed the mildew had started to stick to the signs, and you simply could not find any new references to language laboratories in professional or non-professional journals. The super-sensational trend lasted 10 years at the very most.

"I think we can learn something from the Language Laboratories. You are sitting there now with a delightful little gadget in your hands. Something you have enjoyed using for an hour-and-a-half or so, and something that you can see has important instructional applications. However, I think there are some cautions to keep in mind when we talk about using calculators in schools, and, more importantly, spending school money on them. I don't think the tale of the Language Laboratory need necessarily be a forecast for every instructional innovation that comes along. This one, for example, I think could be successful. I'd like to identify some things that one ought to look for in a teaching aid ... things that make it a decent and enduring teaching aid.

"Briefly, I think there are three points to keep in mind. First, I think a decent, enduring, respectable, lasting teaching aid should be inexpensive and it should be durable enough to be used by a learner. Second, I think it has to be controlled by the learner. Third, I think it has to solve problems or do things that the learner wants done.

"When I say I mean that the teaching aid has to be inexpensive or durable enough to be used by a child, I don't mean inexpensive in a relative sense. I think I mean that in an absolute sense.

"Take, for example, the cassette recorder, which is, in one sense, inexpensive. It is inexpensive enough that parents buy cassette recorder for the children as gifts. But, in the classroom the children are not allowed to use cassette recorders as if they were inexpensive. Teachers tremble if the cassette recorder is about to fall off the table or if the child does something the least bit unusual with it. Often, they are locked in vaults and are not accessible to children except under specific instructional circumstances. Children do learn things from cassette recorders ... the ones their parents buy them. The ones in the schools don't aid learning. I feel that if something isn't inexpensive enough to let relatively unsupervised children use it freely, then it's too expensive to be a valuable teaching aid.

"An obvious essential feature of any teaching aid is that it does actually teach. That is, it responds to the learner's behavior in some consistent way that is satisfying or interesting. Notice that when you picked up those calculators after the first experience of turning them on, you had an opportunity to discover a device that responded to you. Given enough opportunity to play with it, you just might discover for yourself the kind of principles (mathematical) that a calculus professor would love to have the time to teach you.

"But, if a classroom is stocked with a lot of electronic calculators that are locked safely in a closet - except during arithmetic class, at which time they are cautiously placed on the tables to prevent any danger of them being dropped - I think it likely that the calculator will be as disappointing as the cassette recorder. To be a genuinely effective instructional tool, calculators have to be inexpensive enough to be taken to the gym, the playground, the lunch room, the band practice room, or any place where mathematical questions are of interest to learners.

"The second feature of a lasting teaching aid, I think, is a logical outgrowth of the first ... that it can be controlled by the learner. An excellent example of an existing teaching aid like this is the book. The design of the book has long given it the potential to be a useful teaching aid. Before Gutenberg, books were chained to the walls. Only monks, who would treat them very gently, were allowed to read them. But, after printing presses made them reasonably inexpensive, books became readily available to real learners. And the reader controls many of the aspects of the operation of that book.

"If those of you who are thumbing through your calculator exercises right now would look at what you are doing, you will realize how many aspects of the book you actually control. You can read it at any rate you want, you can refer to any part of it you want, it can be read in a variety of different places, or different circumstances, or even different body positions. If you get interested in the book, you can continue with the book; if you find something that I am saying interesting, you can direct your attention away from the book to me, and when I get a little boring, you can go back to your book.

"In contrast, consider something clumsy like the 16 mm motion picture projector. Most frequently the projector has to be operated either by the teacher or by a specially trained student assistant. It can't be conveniently stopped for an individual learner because it's usually shown to a large group at one time. It's difficult to assess only a particular portion of a film for reference. It is almost impossible for a student to view a film if he was absent, or whatever, anywhere other than the classroom in which it is presented at the time at which it is presented. In consequence, it's easy to find schools that are relatively well supplied with books but have only a few relatively rarely used motion picture projectors. The point here is that an effective teaching tool needs to be at hand whenever there is a problem to be solved or any object of curiosity to be investigated.

"The calculator has the potential versatility comparable to a book. A learner can move it, or use it, in about the same range of situations that he would use a book. It is kind of amusing to speculate what the world would be like if we all had the calculator habit to the extent that so many have the book habit. You can imagine an ordinary individual thinking through some new mathematical concept with the aid of his pocket calculator ... in his car while he is waiting for a light to change ... on the computer train in the morning ... at breakfast, in lieu of his paper ... or even in the tub while he is soaking. And the size and shape of existing calculators make these uses very possible.

"The only barrier is that most people, who are now adults, have grown up without having mathematical freedom. Simply, the calculations involved in solving practical problems are likely to be so forbidding that the adult would never have thought about playing with mathematics as a pastime. If, however, it is now possible to produce a generation of children who can calculate any item of curiosity, their own baseball averages, or imaginary problems, such as how much gas would it take if we drove the car around the world - and without the computation for this sort of thing, it would be a totally burdensome chore - then maybe they will grow up to be adults who enjoy mathematical thinking.

"A tiny, inexpensive calculator ought not to present a problem for either educators or manufacturers. It is true that educators will have to begin to think of mathematics as something more than computation. The fact that you are here, and the trends that I have seen in modern math, would suggest that mathematicians are interested in teaching something more than computations.

"This leads to the third feature of an enduring teaching aid. It should solve problems or do things that the learner wants done. A good example of that kind of tool is the slide rule. But, remember, a slide rule requires user sophistication that a pocket calculator does not. They are complex, and you have to know at which end of the scale to read the answers, and where the decimal point should go. These limitations prohibit use by a nine-year-old who wants to know how many sunflower seed packages he could buy if he had a million dollars. But, an electronic calculator can give this child speculative capacity.

"I think that we will pay a price if we fail to keep these things in mind. If you want to understand fully the dimensions of that price, go back to your school Monday morning, pick up the storeroom key, and go into the storeroom. I think you will find the whole language laboratory library stored in that storeroom somewhere. You are probably going to discover several broken projectors that no one has either the expertise or the interest to fix. There are probably a couple of yellowing teaching machine programs and several crumpled audio tapes with a stack of empty take-up reels that no one wants to use. There will be the laminating machine that no one ever did figure out how to use and probably a box of dried-out overhead projector crayons that would crack if you attempted to use them. And you might even see the acoustic coupling for the computer terminal that was taken out two years ago when the budget dried up. I don't think schools need any more expensive dust catchers, and I think storerooms are quite full enough.

"So in buying electronic calculators, let's buy them and put them in the KIDS' hands and NOT in the storerooms ... that's my point."

(Dr. Bell expanded on Dr. Rogers' comments by noting that "the classroom practicality of calculators" is an important factor in determining the ultimate utility of calculators as teaching aids.)

DR. BELL:

"I became convinced a couple of years ago, when I first bought a primitive calculator, that they were fun to use, and couldn't really help but turn kids on. But, could we safely put calculators in the kids' hands? Was it a practical thing to do?

"When calculator prices came down to the level where a few hundred dollars would buy a reasonably big collection of machines - in general, 'practicality' is a function of price - the University of Chicago bought me 25 calculators. I scattered them around the Hyde Park area in the hands of teachers I trusted.

"Some teachers I gave one or two calculators; other six or eight. My instructions were to take a calculator home, see what you think it is good for, then (if you want to) bring it into your classroom.

"My first results weren't promising. I asked a teacher: 'How do you like the calculator?' The teacher answered: 'I love it! It's a beautiful little thing ... I put it in the vault.'

"'Why is it in the vault?' I asked. 'Well,' she replied, 'I was afraid that I would catch hell if I lost the thing.' I told the teacher that the point of the experiment was to see if the machines were tough enough for kids to use ... to find out if the kids will 'rip them off' at an excessive rate ... to see if they will break down in service. I told her that the University will stand the cost of lost or busted machines. She finally did take the machines into her classroom and I received enthusiastic reports dedescribing the ways her kids used them."

(In addition to the three university-affiliated panelists, Dr. Bell invited three Chicago city school teachers to comment on actual teaching experiences with calculators for the use of these teachers.

Mrs. Sharye Garmeny gave a detailed presentation recounting her experiences with two groups of students. Dr. Bell introduced Ms. Garmeny as "a gifted and perceptive teacher.")

MS. GARMENY:

"My first experience involved a summer school program for remedial students. They were young people who had not been placed in any particular grouping and were between the ages of seven and 10. They had been identified by their home school teachers as in need of remedial instruction in reading. It was necessary to divide up a three-and-one-half hour session into areas other than pure reading, and so mathematics was included.

"My initial reason for bringing the calculators to the students was to see what would happen in terms of their motivation to find out things about numbers.

"As it turned out, I had six calculators and an average of 12 young people each day. We set up an alternating schedule. No child had a calculator every day. My main interest in the beginning was just to give them out to the young people and let them see what they could do with them. One of the first and most interesting things was the fact that the young people perceived from the keys that were indicated that they could use the machines to add, subtract, and multiply. They were not too sure about that other symbol with the two dots and the slash line in between.

"They began by working out problems that they knew answers to. Well, I know that 2 + 2 = 4, and when I feed this into the calculator, does the calculator say that it is 4? It did. They began to trust the calculator. The calculator had proven itself. They knew some things and the calculator knew the same things they knew.

"Soon, they began to feel that the calculator could give them answers that they wanted. A play 'store' was set up using empty containers that the students brought from home. Here was an activity-based learning situation that allowed the children the opportunity to work with numbers in different ways. Of course, we had a cashier, and people would go to the store and shop. The cashier would use a calculator to determine costs for the people as they shopped.

"These kinds of experiences were very interesting to the young children. It was always a question of, 'When will it be my turn to do this?' and 'How accurately can I work with the calculator? and 'What kinds of things can I find out?' They checked not only the calculator, but they began to check themselves and each other.

"It wasn't necessary for everyone to have a calculator in order for the calculators to be effective. And I think this is an important fact, because if you are thinking of instituting a program using calculators, it might not be realistic to think initially of a calculator for every child.

"Multiplication became a big issue because my students wanted to find out what happened when they used big numbers. And, of course, when they would see these big answers coming out on the calculator, they would want to know, 'Well, how does the calculator come up with a number like this ...', because somehow or other their understanding of multiplication had not reached the same level as that of the calculator. This gave me an opportunity to work with them on the distributive property. Now, we all know about the distributive property and what that does to children, but in this particular instance they were prompted to be curious about it because of something that the calculator had told them. So then when we did get around to the kind of rote instruction that consists of a teacher lecturing and students listening, they were very interested in what I was explaining to them because it was going to help them to understand something that the calculator knew how to do that they did not yet know how to do.

"The presence of the calculator seemed to make the idea of mathematics instruction more appealing to the children. The word got around and kids came from other classes to see the calculators. The students would share the knowledge and the information that they had about what the calculator could do, and they became more assured about themselves, not simply as students working in math, but as people and in their relationships with other children in the school.

"The program proved to be a very positive kind of experience for the young people, and the end-of-term evaluation suggested that they had in fact become more proficient in terms of their understanding of basic principles of elementary school mathematics.

"The second group that I am going to talk about are eighth-graders in my home school who are in a remedial class. I assume that some of you know that some of the young people we find in remedial classes are there, not purely because of their level of ability, but because of their acceptance or disapproval or dislike of patterns as they exist. I have found this to be true of some of the young people in this class. They really have good skills, but for whatever reasons, they have not chosen to indicate their skills and abilities. Instead they choose to exhibit behavior that labels them as unable to perform.

"The class is composed of twenty young people, 13 and 14 years of age, and we have a total of eight calculators. Students are grouped in clusters so that their use of the calculators again is not on an individual basis all of the time. In their initial introduction to the calculators, the questions were, 'What is this?', 'What does it do?', 'Who do they belong to?' So, initially, here again the activity with the calculator was just a kind of exploration.

"In terms of activities, their primary concern initially was in checking for accuracy. Many of the young people in the class can perform, but their rate of performance is slower than that of an average student. Their use of the calculator as kind of a follow-up experience to an exercise was really to check themselves out. If a student takes the calculator and checks himself instead of waiting for the teacher to come over, then there is more immediate response which is very important to these kids. They really cannot wait too long to find out that they are doing okay. Waiting for the teacher can delay this to the point that they lose interest and desire to participate.

"Secondly, when word problems are involved, the calculator becomes an important tool for solving problems. The question still is 'what operations do I need to perform in order to do this problem?' And this is the question I am interested in having them answer for themselves. Because, to my way of thinking, this is what I am concerned about.

"In terms of the benefits of using the calculators, I think that one very obvious advantage is that they provide an opportunity for students to check up on what they are doing. That is very fundamental; but, calculators also motivate a kind of curiosity that I as a teacher am not always able to come up with. I have seen an actual change in the performance level of these students. I have seen an enthusiasm and a desire to really become involved in mathematics and what it means; so that as far as a teaching aid, it is a teaching aid that is very important to me now, and I think that the students can see the advantages of it."

(Ms. Garmeny's comments completed the formal presentations. Dr. Bell continued the program with a few of his own observations.)

DR. BELL:

"The question is not: 'Will calculators come to be a factor in modern life?' - they will! As price continues to drop, they are certain to be a factor in the life of every man, every citizen, because people will buy them and use them. The real question is: 'Will calculators become a significant factor in the classroom?' I believe that the answer will depend on whether or not we can find ways to exploit them for pedagogical reasons, and thus use them to advance the knowledge of mathematics in our classrooms.

"We have always had slide rules. They are very handy, but are of limited applicability. You cannot add on a slide rule, they don't place decimal points, and they require a fair amount of instruction in use. Consequently, although they are useful teaching aids, they are limited in applicability.

"At the very least, calculators give us a tool that lets kids experiment with numbers ... they provide opportunities for simple experimentation and play and fiddling, some things we don't let kids do enough.

"Early reports of their use suggest that they are highly motivating. But, I caution you that almost anything new is highly motivating for a while. Prof. Immerzeel has been using calculators long enough to be able to answer the question: 'Does the motivation wear off?' - George?

PROF. IMMERZEEL:

"The novelty of calculators is fairly short-lived. Students keep finding new ways to develop ideas with them. You will find for yourself that you need a source of things to do with the calculators in your classroom. Calculators represent a tremendous capability, but the play period is fairly short. Both you and your students need input ... you need things to do with your calculators. So, in the long run, it depends upon your imagination as a teacher as to whether or not the motivation disappears."

DR. BELL:

"Okay, so motivation strictly from a standpoint of novelty wears off quickly. But, motivation based on the opening up of new possibilities of calculator use keeps going. For example, I have seen kids use a calculator for instant verification of the things that they suspect and to check out the things that they are doing.

"For many years, I have preached that the fundamental problem with school mathematics is that there are too few links between the arithmetic pushed at the kids and the real world. Here is where the calculator can do some lovely things.

"With widespread use of calculators in the classroom, the whole artificial barrier to understanding what the world is really like ought to drop. Real-world problems - with real-world numbers - can be handled as easily as the nicely-rigged textbook problems of today.

"The clever use of algorithms, the efficient ways to do certain calculations, the generation of data and patterns, the verification of hunches, the checking out of things which the student suspects ... all of these provide possibilities for exploiting calculators in a pedagogically fruitful way.

"Let me say at once that I am not optimistic that we will do this sensibly. Some years of being in education has indicated to me that it is never safe to be optimistic about such things. I only say that there is substantial potential here which, if we behave in a reasonable way, can be exploited. Though I am not optimistic that it will be done, I say that it can be done.

"The emphasis will inevitably change if calculators become commonplace in the lives of citizens. Fractions will have very little use anymore except in order to set up ratios. Fractions will be practically useless as indeed they are already. Decimals will come to the fore very early. The business of how to place decimal points by making sensible estimates will have to come up and get attention. The artificiality in school books will no longer need to be there.

"One question that seems to come up again and again is: 'Will they get stolen by the kids?' None of the calculators I distributed were stolen. Will you comment, Mr. Nelson."

(Mr. Charles Nelson was another of the Chicago city school teachers who participated in Dr. Bell's experiment and was invited to speak at the symposium; Ms. Mary Page was the third.)

MR. NELSON:

"I have 167 students, and I believe that they feel that the calculators belong to the group as a whole. The calculators have not 'walked off', in spite of the fact that I don't lock them up. I leave them in my drawer, and everyone knows that they are in my drawer. Perhaps students inclined to take a calculator are afraid to do so because they are afraid that the group will report on them. For whatever the reason, the calculators have not 'walked off'."

PROF. IMMERZEEL:

"We lost two calculators for a period of time, but they were recovered. The teacher who had them left them - in full view - on a shelf in an office, and so two kids had an opportunity to take them home. I don't believe there is much danger of loss in a classroom situation, but I think there may be some danger if we are not careful about visible storage."

(Dr. Bell announced that the panelists and guest teachers would now answer questions from the audience.)

QUESTION:

"What is the failure rate of calculators in the hands of students?"

DR. BELL:

"We have not had any calculators break because kids use them. However, there is a definite failure rate ... usually in the first hours of use. Reputable manufacturers will exchange calculators that fail electronically, and so astute administrators will order a few extra machines to allow for this phenomenon. If you can get over the first few uses of the machines, then they seem to work well. Also, be sure to buy your calculators from a source that offers repair facilities, because any machine will eventually need some kind of repair."

QUESTION:

"Are calculators 'crutches'?"

DR. BELL:

"There is an objection to the use of calculators that is not easily overcome ... a moral objection. I hear over and over again that it is wrong that something (arithmetic) that has always been so hard should now suddenly be so easy. There is a deep streak of puritanism in us - a kind of hair shirt philosophy - that I don't think will be easy to overcome. I recognize this in many of the arguments I hear against the use of calculators in the classroom. I think that this may be our most difficult problem after cost."

MS. GARMENY:

"If a calculator were off-limits in dealing with mathematics in life, then I would say we have an important issue here. But, then, I don't see any conflict. My experience with calculators shows that the young people still have to know how to perform the operations in order to be sure that their calculators are going about their business properly. I don't see a conflict of interest in using calculators in instruction."

PROF. IMMERZEEL:

"I would like to respond by saying that what are acceptable crutches for adults are also acceptable crutches for children. That is, if you expect adults to use calculators, then they shouldn't be forbidden to children, at least in most cases."

MR. NELSON:

"If you have a student who reaches the seventh or eight grade and who doesn't know his multiplication facts, it is pretty hard to teach them to him. What do you do with the higher mathematics that makes use of these facts? Do you stop right there with this child? Or, do you give him a multiplication table and let him do the rest of the work he has to do?

"I think of calculators like that. I would give a child a calculator to to do multiplication if it meant he could continue with other work he is required to do."

QUESTION:

"I want to modify the original question. What devices are tolerated as crutches in school?"

DR. BELL:

"What do schools tolerate as crutches? I think they tolerate (panelists, tell me if I am wrong) whatever an individual teacher will tolerate."

PROF. IMMERZEEL:

"I am bothered by the moral issues that are being raised because they are unreal. We aren't looking at the world in terms of what it really is. The calculator is not going to go away. And, in addition, the calculator is merely a tool.

"We have on a given day in my classroom something that we want to accomplish. If the calculator helps us accomplish it more efficiently, then we should use the calculator. The students are not worse off if we use calculators to accomplish a particular instructional objective. We should ask if the calculator is the best tool for accomplishing an objective, rather than whether its use is moral or immoral. I don't think that the calculator has any morality at all."

DR. BELL:

"There is a question raised here that is more than a moral question. What do you say to a school board or parent who is concerned about a student's ability to continue without the aid of a calculator in the future?

"One hopeful answer would be that using a calculator won't lead to the rotting of the student's mind, and won't lead to the loss of ability to do ordinary calculations. I suspect we will have to document this answer by more research."

QUESTION:

"Are we doing kids a disservice by making them memorize computational facts? Is memory training an essential part of education?"

PROF. IMMERZEEL:

"It's my opinion that without basic computation reflexes you are a cripple. Why? Because I care a lot about estimation and being unable to develop a quick 'more-or-less' answer to a specific problem. To get this kind of answer you have to be able to manipulate the flow of numbers quickly and easily."

DR. BELL:

"I don't think you can do mathematics without being able to compute. I think computing is part of thinking."

QUESTION:

"What is the effect on the class if the school doesn't provide calculators for students, but some students use their own machines?"

DR. BELL:

"This problem has already come up in universities. Initially, it was answered in different ways, but now, if a student needs a calculator, he is expected to get one. Many colleges negotiate arrangements for the students to get them cheaply, or they rent them to students (often through the university bookstore)."

CALCULATING MACHINES IN SCHOOLS

Scottish Central Committee on Mathematics

Dundee, Scotland

This article, which appeared in The Times Educational Supplement (Scotland) and in the Bulletin of the Scottish Centre for Mathematics, Science and Technical Education (June, 1977), has been reprinted with the permission of Editor D.C. Fraser and Director John A.R. Hughes of the Scottish Centre.

The continuing rapid development of technology has brought about widespread changes in the everyday lives of all of us. Since the end of the second world war, our lives have been transformed by jet aircraft, the spread of television, transistor radios, the computer and many other less obvious technological advances. All of these, especially the computer, have influenced curricular thinking in some measure, but the sudden appearance on the market of cheap pocket-sized electronic calculators has had a powerful direct impact on education, an impact so sudden that a majority of senior school pupils in Scotland (and not a few younger children) may well be in possession of a calculating machine by the time this paper appears. The purpose of this paper is to review the priorities in arithmetical and mathematical education in the light of the new situation, to make recommendations regarding the extent to which the use of calculating machines should be encouraged at various stages in education, and to consider the types of machines that are most appropriate for school use.

1. It is probably fair to say that the instinctive initial reaction of many teachers of mathematics is to be somewhat alarmed by the widespread availability of calculating machines. People have been saying for many years (probably since the adoption of the Arabic numeral system in Europe some five centuries ago) that arithmetical skills are on the decline, and whether they are right or wrong, there is good reason to suppose that skill in arithmetical computation will indeed be eroded by the habitual use of machines. On the other hand, skill in computation is not an end in itself, and if it can be shown that the skill is now irrelevant, then there is little point in bewailing its disappearance. What we must do is to make as rational an assessment as possible of the new priorities in arithmetical education, accepting that these priorities will be materially affected by the availability of calculating machines.

2. Computational aids, in the shape of logarithmic tables and slide rules, have of course been used in schools for many years. Both are open to the objection that they erode the more elementary computational skills, and it is doubtless the case that some slide-rule-using Sixth Year pupils would be more than a little shaky on long division. That this has never been a cause for serious concern is probably attributable to the fact that logarithms and

slide rules are used only by relatively senior school pupils who have already acquired a fair measure of arithmetical competence and understanding. This delay in use arises not because of any arbitrary edict from the mathematical/educational establishment, but because both these computational aids require a considerable measure of mathematical sophistication before they can be used at all. The child who attempts to work out 7 x 9 by logarithms needs to be able to look up tables twice, to add 0.845 and 0.954, to look up a table yet again, and finally to use the characteristic of the logarithm to sort out that the answer is 63 rather than 6.3 or 630. It is fairly easy to persuade such a child that it is easier simply to remember that 7 x 9 = 63. In many ways the slide rule is still more treacherous to the unsophisticated, since it makes no distinction between numbers whose ratio is a power of 10 and so requires a considerable feeling for numbers on the part of its user. The essentially new educational issue introduced by the advent of the pocket electronic calculator arises from the fact that a calculator is usable by quite young children. It may indeed be hard to persuade the child in primary school who presses 7 $\boxed{x}$ 9 $\boxed{=}$ ___ and instantaneously reads off 63 that there is some point in committing to memory the 36 non-trivial multiplication facts that constitute the traditional multiplication tables and the comparable addition and subtraction facts that are the basis of the standard arithmetical algorithms. Should we care? We believe that we should, and offer justification for our belief in the next few paragraphs.

3. First, there is the sheer matter of time. Properly used, calculators can save huge amounts of time, but if one happens to *remember* that 7 + 9 = 16 or that 7 x 9 = 63, then one has the answer before the instructions can even be fed into a calculator. It is quite simply more efficient to remember some arithmetical facts of this type. There is, no doubt, room for difference of opinion regarding which facts should be remembered and how many, but surely they should include the previously mentioned multiplication tables up to 10 together with addition and subtraction facts to an extent indicated by the examples 8 + 9 = 17 and 15 - 7 = 8.

4. Secondly, though it is interesting and useful that 7 x 5 = 35, it is also of interest that 35 = 7 x 5, and a calculator is not nearly so useful in establishing a factorization statement of this sort. The arithmetical competence that is the basis of numeracy consists of knowing facts like this *both ways round*, and it may help to persuade primary school children of the importance of basic arithmetical knowledge if more emphasis is placed on partitions and factorizations.

5. Thirdly, it is unrealistic to suppose that one will always have a calculator within reach when an arithmetical problem arises, any more than one always has a dictionary within reach when one is in doubt about a meaning or spelling. If pupils are to be properly equipped for life, they must be able to do some simple arithmetic without a calculator.

6. We are arguing that the importance of 'tables' is unaffected by calculators and that competence in this area should continue to be a major aim of primary school arithmetic. We are not, however, arguing that calculators create no change at all in teaching priorities. Let us examine some of the traditional arithmetical skills and attempt to reassess their usefulness in the

new situation. The algorithms for addition and subtraction of large numbers continue to be useful, and many people in suitable cases still find it quicker and more reliable to do such calculations without a machine. On the other hand, when the numbers get larger or the column of figures gets longer, there comes a point where even the most dedicated arithmetician is glad to have a calculator to relieve him of the drudgery of computation. It seems reasonable to suppose that the adults of the future will reach this 'cut-off' point sooner than those of the present time. This surely does not matter a great deal. The conclusion is that, while the algorithms of addition and subtraction should continue to be taught and practiced, there need not be the same emphasis as formerly on the development of speed and accuracy in the extended written application of these algorithms. Speed and accuracy in mental arithmetic remain important, as we shall see later.

7. The same conclusion applies to the more difficult algorithms for 'long' multiplication and division. The fact that algorithms exist is a point of some mathematical importance, and we believe that children should continue to be able to apply them in simple cases. However, these skills are nowadays very little applied in real adult life, and we must expect that they will tend to fade away.

8. The question of when to use a calculator is an important one, on which pupils will require guidance. The 'cut-off' point referred to above even now among adults varies widely from individual to individual, and one must expect that variation will continue in the future. If one takes a graded series of arithmetical tasks such as

$$5 + 7,\ 8 \times 6,\ 17 + 15,\ 23 + 48,\ 17 \times 15,\ 23 \times 48,$$

$$668 \div 23,\ 27.48 \times 133.72,\ 2876.52 \div 346.9,$$

people will vary in their judgment as to when a calculator is necessary. We would not regard it as important if a generation were to grow up with only a general idea of how to tackle the last two of these without a calculator, but we would regard it as quite disastrous if the first two could not be answered from memory, and extremely disappointing if the third and fourth could not be done mentally.

9. The matter of mental arithmetic is a very important one. It is a feature common to all calculating aids that errors, when they occur, tend to be of a gross nature, leading to an answer that is not even approximately correct. It is an essential aspect of numeracy to be able to know whether an answer is of the right order of magnitude, and the unwillingness of pupils to perform even the most rudimentary checks on their calculations is a constant source of disappointment. Anyone with a calculating machine handy will use it to multiply 19.35 by 21.77; the numerate person knows in advance that the answer will not differ very much from 400. Mental arithmetic, unlike paper and pencil arithmetic, remains an indispensable skill, but possibly the emphasis ought to change. We would suggest increased emphasis on oral questions in class such as "About how much is 53.1 x 32.7?," to which there is no one answer, but to which the first answer might be "a bit more than 50 x 30, that is, a bit more than 1500," while a more refined attempt might

be "about the same as 50 x 35, that is, about the same as 1750." (The exact answer is 1736.37) There is a vast scope for activity of this sort, and one could even make a game of comparing approximate methods offered by different class members, using a calculator as an adjudicator. Other examples:

> About what percentage of 178 is 57? (About the same as 60/180, that is, about 1/3, that is, about 33%)
>
> What approximately is 17/43 as a decimal? (About the same as 16/40, that is, about 2/5, that is, about 0.4)
>
> Would you believe that 1/13 is approximately equal to 0.187? (No, because $\frac{1}{13} < \frac{1}{10} = 0.1$)

The ability to answer questions like these is the quintessence of numeracy and is totally unaffected by calculators. The skills involved are complex and take time to acquire. Perhaps one may hope that calculators will release more time for the development of these skills.

10. It is probably fair to assert that the unease felt by many mathematics teachers regarding the use of calculators is in inverse proportion to the age of the pupils involved. Their worry is that children will begin to use calculators at too early an age and will fail to develop the kind of numeracy we have described. By contrast, pupils in Higher or Sixth Year Mathematics can concentrate all the better on genuinely mathematical ideas if they are released from the drudgery of repetitive arithmetic. In any event such pupils have all for many years used logarithmic tables or slide rules as aids to calculation, and it would be excessively puritanical to object to this new computational aid solely on the grounds that it is better and easier to use. Calculators also enable mathematics to be applied (whether in statistics, science, or geography) to real rather than artificial data, for the arithmetical drudgery formerly involved in handling real data is reduced to manageable proportions. This, too, is surely a change for the better. We would in fact see it as an important part of mathematical education in secondary schools that pupils should learn how to make intelligent use of calculators. This learning should not consist merely of finding out which buttons to press. It should emphasize the wisdom of checking a calculation either (as previously described) by making an approximate mental calculation to verify that the order of magnitude is correct or by performing the calculation twice. It should also emphasize the necessity of giving answers with an appropriate level of accuracy. For example, if the sides of a rectangular piece of paper are given as 10.1 cm and 8.3 cm respectively, a calculator readily gives that the diagonal has length 13.07287268 cm, but it is essential in any real-life situation to realize that the sensible answer is 13.1 cm, that is, that the answer should not be given to a higher level of accuracy than the original data justify. This, of course, is a general point but is especially important in the present context since calculators give so many decimal places.

11. As regards primary schools, we would not go so far as to suggest that calculators should never be seen in the classroom. We did suggest above that a calculator could be used by a teacher as an 'adjudicator' in an approximation game. Additionally, the children might use a classroom machine

themselves as a check on sums done by paper-and-pencil methods. It might be also that children could be stimulated into greater interest in arithmetic by handling and using what is in fact a very remarkable and highly entertaining toy. Nonetheless, we see a very limited place for calculators in the primary school and reiterate that they must not be allowed to prevent the development of the kind of numeracy we have described. We would emphasize also that a certain level of mathematical sophistication is required before a calculator can be used with any confidence. Many of the simpler and less expensive calculators assert, for example, that $(35 \div 9) \times 9 = 34.99999999$, and some arithmetical understanding is required if one is not to be disturbed by this.

12. The first years of the secondary school are a time of consolidation and extension of arithmetical understanding. We see danger in too much reliance on calculators at this stage, though with growing understanding the danger would diminish, and we certainly would not advocate a situation whereby pupils might be using calculators in every classroom except the mathematics one.

13. It is, of course, a corollary to the use of calculating machines that there is now no point in teaching logarithms in the arithmetic syllabus as an aid to computation. This does not, of course, mean that logarithms have no place in the curriculum. They remain an immensely important idea in mathematics and their position in the S5 syllabus is quite unaffected by the irrelevance of common logarithms as a computational aid.

14. It is time now to consider the social issue of whether the pupil who owns a calculator has an unfair advantage over the pupil who does not. It is, of course, possible to exaggerate the importance of this point. Success in examinations, whether in mathematics or in other numerate disciplines, depends far more on understanding than on computation - depends, that is, far more on knowing which sum to do than on being able to do the sum. Nonetheless, it must be conceded that a time advantage does lie with the pupil who has a calculator. He also has an advantage stemming from the greater confidence he can have in the results of computations, for he has time to check them. This may be an argument for setting out to provide a calculator for each pupil in S3 or above. It is not an argument for banning calculators altogether.

15. Is it possible to provide calculators out of public funds? The simplest calculators (which are certainly capable of doing everything that logarithms previously did) seem to cost between £6 and £10. This is in fact not a great deal of money: schools currently buy books for their pupils, and at the present rate of progress books will soon cost more than calculators. The total sum that would be required to provide every pupil in S3 to S6 with a calculator sounds quite large, but viewed as a fraction of the total cost of education it does not appear so significant. At this time of restraint on public expenditure it is probably not feasible to enter into a specific new commitment to purchase calculators in the large numbers that would be necessary, but it is important to measure the financial problem against an appropriate scale and to realize that in the long term such a policy is by no means out of the question.

16. The low cost of calculators is also a relevant consideration if one considers how many parents are likely to be able and willing to buy a machine for their children. There must be comparatively few families in Scotland who would be seriously inconvenienced by the outlay of £6, and the evidence is that huge numbers of parents have already demonstrated their willingness in this respect. Indeed, many have spent considerably greater sums by purchasing more expensive and sophisticated machines for their children.

17. The most likely outcome, unless we specifically make plans for something different to happen, is that schools will provide an inexpensive basic calculator to the minority of children whose parents are unable or unwilling to buy one. This will certainly keep public expenditure to a minimum and will minimize the disadvantage to children not in possession of their own machines. For the advantages at school level to be gained from having a more expensive and sophisticated machine are slight. It does sometimes save a little time to be able to read off trigonometric functions from a machine, or to be able to change a number instantaneously into scientific notation ($a \times 10^n$, $1 \leq a < 10$), but at school level the main benefit of a calculator is in performing the basic arithmetical operations, and even the simplest machine does that. Indeed there is probably an actual disadvantage in having too sophisticated a machine, for in the hands of an unsophisticated user the very complication of the machine presents perils. One of the disturbing aspects of the present situation is that parents are in many instances buying over-sophisticated machines for their children, in the mistaken belief that the advantage to their child is in direct proportion to the cost of the machine.

18. Reference has been made to basic 'four-function' machines, and the opinion has been expressed that such machines are adequate for school purposes. It is time now to consider the specifications of a machine that would be ideal (as opposed to merely adequate) for school purposes. Here we shall limit ourselves to the years S3 to S5. For Sixth Year Studies work a full 'scientific' calculator could be useful. It should be emphasized that at the present time there seems to be no calculator on the market specifically designed for school use and that the specification given below does not correspond exactly to any machine currently available. It comes closest to the 'semi-scientific' machines which presently retail at about £20.

   a. The machine should have a clear, bright display. Its keyboard should be easy to read and use, with firm, positive action switches.

   b. It should have a fully rechargeable battery. Experience in schools indicates that machines are frequently left switched on.

   c. It should employ algebraic rather than reverse Polish logic. As an example of the difference, on a machine employing algebraic logic the routine for determining 5 - 3 is

   $$5 \boxed{-} 3 \boxed{=} \_\_,$$

   While on a machine using reverse Polish logic the routine might be something like

   $$5 \boxed{+} 3 \boxed{-} \_\_$$

We feel that this latter system can confuse a beginner.

d. The machine should have a square root key.

e. The machine should have at least one memory.

f. Sums of squares should be easy to handle. This is often achieved by having an $\boxed{x^2}$ key, a single memory, and a facility for adding to the number in the memory.

g. It should include the trigonometric functions and their inverses.

h. It should have a 'clear last entry' key.

i. It need not have a percentage key.

j. It should not include an automatic constant facility. Such a facility enables (for example) 8 x 5, 8 x 6, 8 x 7 to be computed by a routing such as:

$$8 \ \boxed{\times}\ 5\ \boxed{=}\ (40)$$
$$6\ \boxed{=}\ (48)$$
$$7\ \boxed{=}\ (56)$$

The constant multiplier 8 is 'held' in the machine and need not be re-entered each time. This device can be useful for repeated calculations, but can certainly be confusing to a beginner, and for school use the disadvantages probably outweigh the advantages.

19. If 'semi-scientific' or 'scientific' machines come into widespread use in in the senior classes in secondary schools, then trigonometric tables will virtually disappear. While we see some advantage in showing pupils a page of tables so that the general behavior of (say) the sine function in the range 0° - 90° can be seen, we do not believe that it would matter greatly if the use of tables ceased to be taught in schools. It is a skill that has little virtue in its own right and can very easily be acquired if necessary at a more advanced stage in education.

20. Summary of conclusions:

a. Calculators are valuable to senior school pupils, and their use should be encouraged.

b. They should be used by pupils only when a substantial degree of arithmetical understanding and competence has been achieved.

COMPUTATIONAL SKILL IS PASSÉ

Editorial Panel
The Mathematics Teacher

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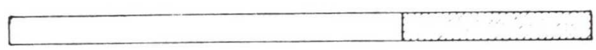
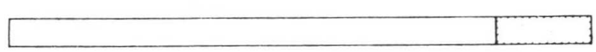
Soon nearly everyone who faces an arithmetic problem will be able to call on a low-cost electronic calculator as an aid. For more elaborate calculation, remote communication with a computer will be almost as easy. This development has led many teachers and students to question the high instructional and testing priority currently assigned to speed and accuracy in arithmetic computation. Their doubts have been expressed in a variety of controversial propositions and proposals for curricular change.

The Mathematics Teacher Editorial Panel posed the following seven issues to a sample of teachers, mathematicians, and laymen. Their responses are given here in percentage form, along with some of their positions and justifications, which help to identify the consequences of emerging technology and alternative instructional policies.

1. Facility with arithmetic computation is the major goal of elementary and junior high school mathematics teaching today.

68% Agree  32% Disagree

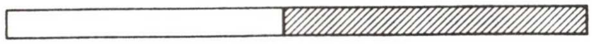
2. Speed and accuracy in arithmetic computation are still essential for a large segment of business and industrial workers and intelligent consumers.

84% Agree  16% Disagree

- One always needs to check mechanical contrivances.
- Ability to make accurate mental estimates quickly is now most important.
- One doesn't carry his pocket calculator around at all times.
- Calculators will not soon be as readily available as pencil and paper; more time will be wasted getting to a machine than saved using it.
- Unit pricing decreases the importance of calculation for consumers.
- Inexpensive pocket and desk calculators provided at company or personal expense have all but eliminated the need for computational skill.

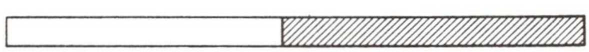
- When is the last time you saw a salesperson do mental or paper and pencil computation?

3. Impending adoption of metric measurement implies that computation with rational numbers should be largely confined to decimal fractions.

48% Agree  52% Disagree

- The housewife will still want fractions of a recipe, and these fractions and ratios are not always best expressed as decimals.
- This is true for the low achievers; others should spend more time with p/q.
- Working with algebraic fractions requires considerable knowledge of all kinds of fractions.
- As long as we measure time to the quarter of an hour, sell shares of stock at 35 7/8, measure material by eighths of yards, or even concern ourselves with degree and radian measure, we must continue to work with all kinds of fractions.
- Probability requires common fractions.
- Halves, thirds, and quarters are too commonly needed.
- Eventually the average citizen may have no use for fractions of the type p/q.
- We need more experience with the metric system before we can express an opinion.
- We should examine the European experience.
- The decimal notation has always made more sense than common fractions; it makes use of the basic whole-number computational algorithms and avoids far-out common denominator problems, and so on.

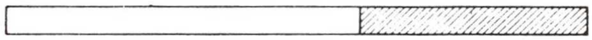
4. In the face of declining arithmetic computation test scores, the energies of mathematics instruction should be concentrated on these skills until achievement reaches mastery levels.

48% Agree  52% Disagree

- Mastery of computation is not essential to do the kind of arithmetic thinking that is important in programming, estimating, and checking answers from machine computation.
- Yes, particularly for those teachers who have misunderstood the call for emphasis on concepts and excluded drill.
- The business person or consumer who is arithmetically illiterate and dependent on machines would be lost when the machines are not available or in good working order.
- Kids compute as well now as they ever have.
- Not everyone can be expected to achieve mastery of computation skills.
- Continued practice with straight computation is tedious and boring to teacher and student.
- Introducing geometry into lower grades is as important as computational skills.
- Spend the money on making calculators available so we can get on to more important mathematics, like learning *when* to perform arithmetic operations.

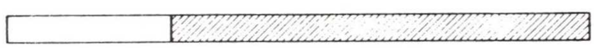
. Mastery level might be a long way off, but we should certainly strive for significant improvement in skills.
. What we need is to establish individual mastery levels for each child, reflecting his or her ability, expectations, and so on.
. The energies of mathematics instruction should be spent on problem-solving skills and applications of mathematics.

5. Weakness in computational skill acts as a significant barrier to learning of mathematical theory and applications.

61% Agree  39% Disagree

. Yes, to the extent that errors in calculation prohibit building on that result.
. There is correlation, but doubtful causal relation.
. It is through arithmetical examples that one gets the feel of what theory and applications are about.
. How can one expect to learn algebra as generalized arithmetic without knowing arithmetic?
. If a person is not sure of an algorithm, he cannot concentrate on the theoretical issue or application.
. I know a brilliant mathematician who cannot do basic computation.
. Students today show *increased* knowledge of theory and applications.
. This is true when it comes to applying theory to applications; those who make contributions to theory but have weak computational skills are exceptional.
. What electronic calculator helps factor trinomials?
. Many instructional texts use simple skills in illustrating mathematical theory and applications.
. Many slower students lacking computational skill have, when given access to a calculator, easily mastered some difficult theories and applications.
. The more important understanding of concepts and quantitative relationships can easily be supported by computation equipment if student skills are weak.

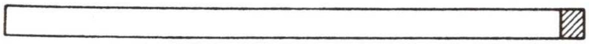
6. Every Grade VII mathematics student should be provided with an electronic calculator for his personal use throughout secondary school.

28% Agree  72% Disagree

. Cost would be prohibitive for the initial supply and the replacement due to theft, damage, and so on.
. With costs plummeting downward, a school system can no longer consider the cost prohibitive.
. Students would become too dependent on the calculator as a crutch.
. This should wait until Grade VIII or IX or when students demonstrate computational proficiency without the calculator.
. Maybe even at an earlier age!
. In China, students are taught to use the abacus early.

- Access at least should be provided.
- Students' minds would get lazy and operate less efficiently if the machines were available.

7. Availability of calculators will permit treatment of more realistic applications of mathematics, thus increasing student motivation.

96% Agree  4% Disagree

- We won't have to avoid the messy real-life situations or reach for a set of tables; we can deal with the approximations encountered in real measures.
- In my experience, student interest and success increase when calculators are available for use.
- The motivation is often short-lived and artificial.
- The standard text problems rigged with "nice" answers deceive students.
- Calculators will support the efforts of less able students.

HOW DO YOU FEEL?

Have the above questions identified the key issues in determining the impact of calculating equipment on mathematics education?

Do the opinions of our preliminary sample reflect the beliefs of most NCTM members?

Have you had exceptionally good or bad experiences with calculator use that can be shared profitably with other readers of *The Mathematics Teacher?*

The Panel welcomes your reaction.

THE INFLUENCE OF CALCULATORS ON MATHEMATICS CURRICULUM

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The combination of technological advancement and price reduction has made the electronic calculator available to virtually anyone in our society who wants one. Pocket-size calculators and desk models alike today have capabilities beyond the wildest dreams of 15 or 20 years ago. And the price of these machines has gone down while prices of most items have gone up. The curriculum has already been affected, and the changes have only begun.

The curriculum has already been changed by the availability of these machines. An ever-widening segment of the student population has access to machine computation. Students will use machines to save labor whenever they can. Teachers can and should prevent calculator use at those times when calculator use would interfere with specific learning objectives. But at other times teachers should not, and many times teachers can not prevent student use of calculators.

Changes in the curriculum can result from sheer pressure, or they can come out of thoughtful work by educators. Pressure has resolved the issue of whether students should be allowed to use calculators. Careful consideration by educators is needed to determine what curriculum changes will be positive and therefore should be encouraged, and how to minimize the negative influences. Educators in many fields are facing the need for such thoughtful planning, but those in mathematics education have a special interest and responsibility in guiding the influence of electronic calculators in positive directions.

The calculator is a powerful thinking tool in its own right, and students must be taught to use calculators effectively. We need new experiences in the curriculum which are designed for student mastery of the calculator. Other changes may be more difficult, but perhaps even more important. The calculator must be considered in the design of student exercises on other topics. Most exercise material for present mathematics curricula is designed to be done by a student without the aid of a calculator. Many of those exercises are well designed to help a student master a particular educational objective or set of objectives. Students who use calculators to do those exercises probably will not master the objectives for which the exercises were designed, because the calculator will prevent the student from gaining the experience the exercises were designed to provide. Student experiences must be designed to enable students to master the same objectives in the presence of a calculator.

That part of the curriculum related to the teaching of computation may be influenced most. The activities designed to teach computation to a student without a calculator may be inadequate to teach computation to a student with a calculator. But the availability of calculators is no reason to neglect the teaching of computational skills. The availability of the machines will require educators to modify the student activities used to teach such skills. Some work in this direction has already begun.

Some of the programmable calculators have been programmed to provide drill and practice routines to help students learn computation. Research on the use of these and other immediate feedback devices seems to indicate a positive effect. Creative teachers are also designing various activities for using calculators to help teach computation. Among these activities, games are becoming popular. One game is called Target and requires one calculator to be used by two players. The name comes from a target interval that is established before the game starts. The first player enters any number as one factor into the machine. The second player enters a number as a second factor and obtains the product. If the product is in the target interval, the second player wins. If not, the product stays in the machine as one factor; the first player enters a number as another factor and obtains a new product. The game continues until a player wins by obtaining a product inside the target interval.

This particular game provides practice in computational estimation in multiplication (or perhaps division). The example is presented here not to show teachers how to play the game but as an example of how a calculator which is or is not programmable can be used to teach computational estimation. The key to the game lies in the fact that students who can estimate computations well are more likely to win. The desire to win may provide motivation to learn to make better estimates. The teacher can control the difficulty level for individual students by judicious pairing of students and corresponding selection of the target interval. Difficulty increases as the length of the interval decreases. Practice in computational estimation can thus be provided in a game setting. Other games and activities can be used to teach other computational skills, but games will not be the only innovations.

Fractions may actually be easier to teach with a calculator than without one. The idea that the fraction 3/8 means 3 ÷ 8 becomes immediately useful to the learner since the calculator is restricted to decimal fractions. Learning decimal equivalents can be a shortcut to calculating them each time that they are needed. Fractions may be introduced at a much earlier age for most students of the future. Decimal fractions will certainly become of greater relative importance while the value of common fractions may diminish somewhat.

Students may need to be convinced that learning to compute has value now that the calculator can compute for them. Perhaps they have even heard adults take the absurd position that students no longer need to learn to compute. Not only are calculators often unavailable when needed, but many calculations are easier to do without a calculator. Some examples of exercises that are easier without a calculator are 300 x 60, 48 - 2, 10% of $367.50, and 3/8 x 24. Of course, one must have some computational skills before these can be done more easily without a calculator. People who fail to learn to compute are handicapped, and the calculator may provide some help, but it cannot overcome that handicap. Whether "Modern Mathematics" deserves the criticism it is receiving for a de-emphasis in the teaching of computational skills is another issue, but similar criticism would certainly be appropriate for a program that would attempt to substitute the teaching of machine skill for the teaching of computational skills.

The point is that the activities used to teach computational skills in the presence of calculators probably will differ from those activities used to teach the skills without calculators, but the skills can still be taught and must still be taught. As the computational skills are taught, the calculator can become a powerful tool in the teaching of other mathematics. Electronic calculators may help teachers put computation into its proper perspective with respect to the rest of mathematics.

Computation has always been and will continue to be a necessary component of the elementary and secondary school mathematics curriculum, but it has never been a sufficient component. Perhaps the advent of the inexpensive calculator will help to point out that fact. Indeed, students have a legitimate complaint if everything they learn to do in mathematics can be done faster and more accurately by an inexpensive machine. As teachers of mathematics we need and have always needed to teach computation, but we need and have always needed to teach more than that. The calculator may very well serve to make both teachers and laypeople more aware of the needs in mathematics above and beyond computational skills, and such awareness may prove to be the greatest contribution of the calculator.

Many mathematical ideas are not computational ideas in themselves but are based on computations. Students who follow the computations may comprehend the idea better, but teachers often find students unwilling or unable to do enough computation to understand the ideas. Even students who have mastered computational skills are reluctant to do excessive computation as a part of other learning. The calculator places a whole new perspective on what computation is excessive.

An old activity for teaching π involves having students measure the circumference and diameter of a variety of circular objects using a variety of measuring units. For each object the student is asked to compute the quotient: circumference divided by diameter. The activity involves much computation and, due to computational error, teachers often find it difficult to even convince students that the quotient is a constant, let alone a particular constant. The objective itself, "the understanding of π," is often lost due to the focus on the computation. With electronic calculators, students can concentrate on the objective, check and re-check measurements, and the activity has a much higher probability of success.

Similar examples abound in the studies of sequences and series. Who wants to sum $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} \ldots$ by hand? With a calculator it can be rather enjoyable, and the experience can be powerful in helping a student understand convergence.

Electronic calculators with programming capabilities lend themselves to a whole variety of mathematical learning situations. Programming itself involves logical thought and mathematical reasoning. Functions can be programmed and studied. Students can literally put in X and get out f (X).

Iterative processes are worth studying and yet the computation can be overwhelming without machine help. One example of iterative thinking is a process for computing square root usually attributed to Newton. For the initial step in the computation of the square root of 17 one might guess 5 as a first approximation. (Any guess will work, but close guesses are more efficient.) Divide 17 by 5 and the quotient is 3.4. Average 5 and 3.4 to get 4.2 as the second approximation, which is better than the first. To repeat the process $17 \div 4.2 = 4.0476$. The average of 4.2 and 4.0476 is 4.1238 which is the third approximation, and better than the second. The process can be continued indefinitely and requires a decision to terminate when closer approximations are no longer desirable. (When a calculator is used, limits are imposed on the process by the capability of the calculator used and the ingenuity of the person operating it.) Any process is said to be iterative if it involves the repetition of an algorithm where the output of one cycle is used for the input of the next cycle in order to obtain successively better approximations. Newton's method applies not only to square root, but can be applied to a root of any polynomial. Iterative thinking is very productive in a variety of applications, some of which involve complex algorithms and computation that would be overwhelming. But with a programmable calculator, all steps of the algorithm can be programmed once, and then repeated easily and accurately. Thus iterative thinking becomes accessible to study at the high school level.

Compound interest is a topic that has been hard to teach because of the volume of computation involved. Typically, we assign a few exercises to show a student that interest is computed by multiplying principal by rate, and that the sum of the principal and the interest becomes the principal for the next period of time, repeating the process for the total time of the investment. Then we show the student that the value of "one plus the rate" can be used as the base of an exponential function which simplifies the whole process. Too often the exponential simplification is given to the student before the student has experienced enough computation to know just what is being simplified. With the calculator more computations can be done prior to the simplification, and perhaps the whole experience will be more meaningful.

Many textbook examples of mathematics applications involve carefully selected numbers to keep the students from being overwhelmed by the calculations. Students often recognize such problems as contrived, and approach them with little enthusiasm. With calculators available, the exercises can be more realistic. Perhaps we will see more problems in which students collect their own data from the world around them. Making applications more realistic could have a positive effect on students' desire to study mathematics in the first place.

In summarizing, then, educators will not be allowed to decide the issue of whether students will or will not use calculators. Students will use them. If the students use calculators to complete student activities which were designed for students without calculators, the effect on student learning will probably be negative. But if curricular materials are designed to take full advantage of the power of the calculator as an educational tool, then perhaps student use of calculators will lead to increased mathematical achievement. Students may even find mathematics more interesting and more useful.

CALCULATORS — A REVIEW

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INTRODUCTION

For at least the past two decades, educators have been investigating the potential of using calculating machines as instructional aids in elementary school mathematics. More recently, the availability and increasingly lower cost of the hand-held calculator prompted most teachers and mathematics educators to ask if it should be used in the classroom. The past few years have seen dramatic progress. Mini-calculators are widely available now for less than 10 dollars. Many adults are buying them for everyday use. Children are using them at home and bringing them to school. Today, the question no longer seems to be whether or not calculators should be used in school but, rather, how they can be used most effectively.

Hopkins cites an historical analogy to the present situation. In one of Plato's dialogues, *Phaedrus*, concern was expressed that written materials would cause decay of the oral tradition of memorization and recitation. Hopkins concedes such a loss, but points to the greatly increased amount of literature which can be appreciated through reading. In the same vein, he says that widespread use of calculators in mathematics education, like any technological advance, will not be easily accepted. He refutes the argument that calculators should not be used because they do not carry understanding of basic operations by asserting that algorithms can also be done mechanically without meaning. He says that using a calculator does not mean that understanding is not a goal. It is merely a faster and more accurate replacement for pencil and paper algorithms. Gains will be greater than losses because more time can be spent doing more complicated word problems and studying the theory of mathematics.

Hawthorne says that the anticipation of inexpensive calculators influenced the lack of drill in modern arithmetic programs. Few jobs require lengthy calculations with paper and pencil, but basic operations must still be understood.

Etlinger distinguishes between using the calculator as a functional device for tedious computation and using it as a manipulative device to facilitate learning. Gawronski says that the type of use could be good or bad, depending on the task and age of the student: both views can be rationalized in the curriculum. If used as a functional tool for all computation, current objectives would be changed from using algorithms to using a calculator.

Rogers warns that the calculator, like other teaching aids in the past, could become obsolete. She presents features of enduring teaching aids, for which the calculator has potential. It must be inexpensive and durable enough to be used anywhere that mathematics questions might arise. It must be controllable by the learner in terms of starting, stopping, and rate of working;

and be adaptable to a variety of physical positions. It must satisfy individual needs and be self-contained to prevent planned obsolescence.

Sullivan reports on classroom trials carried out in two Grade VI classes in New York during the 1973-74 school year. The major goal was to try to find if and how the calculator could enrich, supplement, support, and motivate the regular mathematics program. The calculator proved to be a successful motivator. It led to more sophisticated calculations, was useful in checking answers and intermediate steps of algorithms, facilitated verbal problem-solving, encouraged exploration of topics requiring complex computation, and supported regular topics. Sullivan feels that the outstanding impact of the calculator may have been its power to motivate increased attention to decimal fractions, and interest in their relation to common fractions.

Bell reports that the initial motivation of using calculators is high and persists for a long time, providing that interesting activities are available. In fact, he found that children 'demand' such activities. Even kindergarten children can benefit by using calculators for number readiness activities, as reported by Scandura. For this age level, desk model calculators seemed more suitable than hand-held models because the keys and display are larger, making them easier to use and read. While specific instructions on calculator operation appeared preferable to open-ended exploration, all the kindergarten children observed were motivated to learn mathematics, and their attention spans were greatly increased.

The National Council of Teachers of Mathematics Instructional Affairs Committee identified the following nine justifications for using the hand-held calculator in the schools:

1. to encourage students to be inquisitive and creative as they experiment with mathematical ideas;
2. to assist the individual to become a wise consumer;
3. to reinforce the learning of basic number facts and properties in addition, subtraction, multiplication, and division;
4. to develop the understanding of computational algorithms by repeated operations;
5. to serve as a flexible "answer key" to verify the results of computation;
6. as a resource tool that promotes student independence in problem-solving;
7. to solve problems that previously have been too time-consuming or impractical using pencil and paper;
8. to formulate generalizations from patterns of numbers that are displayed; and
9. to decrease the time needed to solve difficult computations.

We have at our disposal a small, inexpensive calculator that computes quickly and accurately. Its inevitable impact on the elementary school mathematics curriculum has concerned educators warning against its wholesale use in the classroom until the large problem of how it should be used has been rigorously examined. Many pros and cons have been raised concerning this issue. While formal research is really just beginning, many informal studies and observations have been and will continue to be made.

COMPUTATION

One of the biggest concerns regarding the use of calculators in school is that children will become dependent on them and lose, or not develop, mental computational skills. These concerns are legitimate, but both formal and informal studies are showing that calculator use does not undermine meaning. It can, in fact, facilitate understanding.

Many educators including Bruni, Ockenga, Gibb, and Immerzeel have pointed out the necessity for sharpening estimation skills when using the calculator. Bell reports that while children do tend to accept the results shown on the calculator, they also accept the results of paper and pencil algorithms. In fact, poor judgment of significant figures is revealed by calculator use. Calculator errors tend to be large, and estimation skills, which are important in any case, must be learned. Bell also found that children seem to quickly gain good judgment in deciding when to use their heads or the calculator.

Bruni has found the calculator to be useful in developing standard algorithms and in discovering alternative ones. Ockenga advocates use of sequenced calculator exercises which can be effective with hard-to-teach ideas. An example of this is the placement of the decimal point in the division algorithm.

Fehr, in 1955, conducted a two-week controlled experiment which tested the effect of hand-operated computing machines as an aid in learning both the meaning and pencil and paper skills of multiplying by two-digit multipliers. Though he found no significant difference in favor of either the control or experimental group, there were factors which indicated that prolonged use could be advantageous. Both students and teachers enjoyed using the machines, but the teachers had to simultaneously learn how to use them and teach with them. Though the experimental groups had to learn pencil and paper as well as machine methods at the same time, they still made normal gains in pencil and paper achievement.

Fehr followed up with a half-year experiment involving Grade V children. He tested the hypothesis that pupils who use computing machines to learn arithmetic will gain significantly in both pencil and paper computations, and in arithmetic reasoning compared to a control group not using machines. The experimental group gained 4.4 more months in reasoning and three more months in computational ability than the control group. There was, however, no statistically significant difference in the final achievement standing between the two groups. Again, the experimental group had learned both methods in the same time, and they enjoyed the experience.

Rudnick reports preliminary findings from a study in progress. It was designed to measure the effect of the availability and use of a mini-calculator on students' total math achievement and their ability to perform pencil and paper skills. It is a full-year study involving 600 seventh graders in two schools. Classes were randomly assigned to experimental and control groups. The curriculum was not changed for either group. Calculators were not used on pre- or post-tests, but the experimental groups used them on the second post-test. At the time of writing, no statistical differences in achievement were found between the two groups, but slight differences favored the experimental group. A survey of parental attitudes revealed that parents were evenly divided on whether or not calculators should be used in school, but most parents felt that the students should learn how to use calculators.

Schnur reports on a controlled experiment conducted with a summer compensatory education program where most students were bicultural. Both groups received the same instruction on the four basic operations. The experimental group used calculators to verify or do some problems, but neither group used them on pre- and post-tests. Analysis of these tests revealed significant differences in the computational ability of the two groups, favoring the experimental group. There was no significant interaction between ethnic background and calculator usage. Neither was there significant interaction between sex and calculator usage, but there was a slight trend to favor females.

Schafer, Bell, and Crown reported on a study conducted with fifth-grade children at the University of Chicago Laboratory School. About 120 children were divided into an experimental and a control group. Following a computational pre-test, on which the two groups showed no significant mean differences, the control group was given calculators to experiment with and solve problems. A parallel form of the pre-test administered one week later again revealed no overall significant mean differences between the two groups. The calculator group scored significantly better on examples which could be worked by simple calculator manipulation, but did not score as well on examples requiring additional information or more than one operation. Concern was raised that the control group might have become dependent on the calculator, using it inappropriately. The authors suggest that more experience may lead to better judgment in this respect. Also, as the children involved, overall, tended to score above national norms on standardized computational tests, the authors question whether lower achievers may produce different results. General impressions from the study were that the calculator necessitates sharper estimation skills, and has great potential for motivating children to learn mathematics and discover new concepts.

CONCEPTS

Concept learning is proving to be an area where the calculator has great potential. Immerzeel describes the calculator as a "portable, hand-held math lab" providing a source of experience with numbers in a fast, efficient manner. Topics not pursued previously because of the computation they require can be introduced earlier and developed further.

The calculator itself seems to promote discovery. Bell reports that, in learning to use the calculator, children tend to ignore unfamiliar keys, but their presence eventually sparks curiosity and interest in exploring new ideas. Gibb says that the decimal solutions of calculators will cause the study of rational numbers in decimal form to appear earlier in the curriculum - a change already anticipated by the shift to the metric system. Similar changes have been predicted by Elder and Scandura, who have found that children are eager to use large numbers because the calculator handles them so easily, and they discover negative numbers by experimenting with the subtraction function.

Van Atta suggests using the calculator in an intuitive approach to learning laws of exponents, Pythagoras' theorem, square roots and logarithms. Such an approach is only marginally possible without a calculator. The large amount of paper and pencil work required is prone to error and mistaken conclusions. Patterns in mathematics have long intrigued children, but interest quickly wanes when the computation becomes tedious and boring, and undetected errors cause frustration. The immediate feedback of the calculator lends itself well to trial and error procedures involved in the discovery of number patterns. Number properties and relationships can also be formulated faster and inductively with the use of a calculator.

PROBLEM-SOLVING

The calculator's greatest potential may well be in the realm of problem-solving. Many educators, in advocating use of the calculator as a problem-solving tool, have quickly pointed out that the nature of the operations must still be understood.

Gawronski says the calculator should not be used for problems which can and should be done mentally. If handled properly, the calculator will save time and relieve computational drudgery. More time can be spent on problem-solving skills, and the curriculum can be expanded in a problem-solving direction.

Another advantage recognized by Immerzeel, Gibb, and Schumway is that problems can be more realistic. Numbers do not have to be chosen to have integer or 'smooth' answers. Children can generate their own problems from their own experiences. Immerzeel says that the calculator enables students to tackle more complex problems, and solve verbal ones much faster.

CONCLUSIONS

The issue of calculator use in schools continues to be debated and the questions arising are being investigated by educational researchers. Lewis expresses the thoughts of those who support calculator usage by predicting that the calculator "may revolutionize mathematics teaching." But curriculum changes will not appear overnight; they will only occur if and when teachers are convinced of the calculator's potential.

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PART TWO
Specifications

SO YOU WANT TO BUY A CALCULATOR

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Mathematics educators throughout the country are faced with a new crisis in the classroom, the hand calculator. Prices have decreased dramatically in the face of widespread inflation, and continued lower cost coupled with increased sophistication is predicted. It is now possible to find a simple calculator at the same or lower cost than that of a mathematics textbook.

If teachers intend to begin this work with students, the choice of machinery becomes a fundamental problem. Selection today is varied, and prices fluctuate almost daily. Thus far, teacher input has not been significant with respect to the machine features most desirable in the classroom, so we must take what the manufacturers produce.

During the summer of 1975, 11 junior high teachers from Montana were involved in writing and developing materials for a junior high school curriculum. A segment of this program includes materials and activities utilizing the hand calculator. As the program was to purchase 100 calculators for pilot schools, several firms made calculators available to the team for analysis and testing. Wallace Judd, who has published a number of curricular materials for hand calculators, worked for three days as a consultant with the writers. He was extremely helpful in directing careful analysis of each calculator. As members of the team worked with a variety of available machines, it was obvious that features varied widely, even where keys were identical.

After careful analysis, the teachers involved concurred that anyone considering purchase of calculators for schools should spend considerable time in studying each machine and its functions. The following features were found to be most desirable for use with junior high students.

1. ALGEBRAIC LOGIC. Most calculators today are being produced in this mode as opposed to arithmetic or adding machine mode. Algebraic logic allows the problem to be entered as it is written. For example, $5 \times 2 + 6 = 16$ can be solved by pressing the keys in exactly that order, similarly $14 - 3 = 11$ or $24 \div 6 \times 2 \div 5 = 1.6$. Laws of operation must be considered in problems such as $6 + (2 \times 5) =$ __. It must be entered as 5 (x) 2 (+) 6 (=) __.

A machine with arithmetic logic is immediately distinguished by the $\pm$ $\mp$ keys. An addition problem such as $5 + 3 + 7 =$ __ must be entered as 5 $\pm$ 3 $\pm$

7 (±) __. A subtraction problem such as 17 - 5 = __ is entered as 17 (±) 5 (=) __; however, it can be entered as 5 (=) 17 (±) __. The latter may possibly lead to the suggestion that subtraction is commutative.

The algebraic logic is most consistent with the natural way that a student thinks and does mathematics. Thus, it would seem more appropriate for increasing understanding, rather than introducing a new mode of operation along with the calculating tool.

2. FULL FLOATING DECIMAL. Most of the newer calculators are being constructed with this capability. One can check this feature quickly by entering 17 ÷ 3 = __. If the answer is displayed as 5.6666666 on an eight-digit display, then the machine has a full floating decimal. If, however, 5.6666667 is displayed, the machine is rounding off in the last place.

Calculators which contain a selector switch with F, 0, 2, 4, allow both floating and fixed output. One should check these outputs with the selector at each position to find out if the calculator is rounding off the answers.

The capability of selecting the output mode can be very desirable in the junior high school classroom. In many situations it may be desirable to use an integer output along with a decimal input. For business or monetary problems, the round-off at two decimal places can also be useful. Analysis of repeating patterns in conversions of fractions to decimals necessitates the full floating decimal.

A caution is necessary here concerning the calculators that claim 12- or 16-digit display with the use of special key (↑). On one such machine, any problem such as 17 ÷ 3 = __ results in a terminating decimal. The answer displayed is 5.33333 (lower register), 330000 (upper register). The final six digits are displayed by pressing the (↑) key. Thus, *all* rational numbers result in terminating decimal in the eighth place.

3. A SEPARATE CLEAR ENTRY AND CLEAR KEY. We found that some machines have only a (C) key and if a mistake is made with a single digit entry, the entire problem must be cleared and one must start over. The clear entry key (CE) allows one to clear only that which is on the display, without destroying other data which has

been entered into the machine. This can be of great value to students that have not yet mastered the keyboard.

Some calculators with eight-digit display will indicate an error if the number entered, or the result of an operation, exceeds eight digits. Others will display the answer with the decimal point in the correct place, and indicate that the number must be multiplied by 10^8 or some other power of ten. The latter machine is certainly more appropriate. In both cases, most machines which were investigated are then frozen; that is, no further calculation can be done until the machine is cleared with the C key, thus, clearing the entire problem.

On one machine, however, it was possible to press the CE key in order to cause the overflow indicator to vanish; then one could continue with the operations. It must be remembered, however, that the displayed number is still a number multiplied by 10^8. One might think that this feature has little or no value, but several instances were investigated where it became a definite advantage.

4. CONSTANT FACTORS. Be careful of this one. Some older models have a K key for this purpose. Thus, the student must remember to set it. He may also find that the constant is stored in the memory and this will affect his use of both the memory and a constant. Again, one must be careful as machines were found where the first factor became a constant for one operation and the second for another.

This is easily checked by doing the following:

Press	6	X	5	=	30	5, the second factor, is constant for multiplication
Press			7	=	35	
Press			1	=	5	
Press	15	÷	5	=	3	5 is constant for division
Press			20	=	4	
Press			30	=	6	

Any machine used in the classroom should be consistent using the same factor as a constant for all operations. It seemed most appropriate to us that the second factor should be the constant. This alleviates possible confusion with respect to the commutative property for subtraction and division.

5. CHANGE OF SIGN KEY +/- . This is most desirable if one wishes to use calculators with integers. Care should be taken to note the location of the minus

sign on the display. Some models place it to the right; others have a small light indicator. Our consensus was that it is most desirable for students to have it placed to the left of the display just as it would be written on paper.

One calculator with upper and lower registers indicated the minus sign to the left, sometimes. However, the result of multiplying a large number (five digits or more) by a negative number was displayed as a positive number. The minus sign was displayed only if the ↑ key was depressed, and then it was indicated on the upper register to the right. A student using this machine might easily get the idea that the product of a positive integer and a negative integer is positive, especially if you use very large positive or very small negative numbers.

6. <u>MEMORY</u>. While this feature is not an absolute necessity for junior high or elementary school students, it can certainly save a great deal of time in more complicated problems. Some machines have a simple store STO and recall RM key, whereas others have an accumulating memory M± M=. The latter will add or subtract the displayed number to the contents of the memory with the M± key or M= key.

Once again, be careful in analyzing calculators with the memory feature. We found one machine with a functioning memory, M± and M=, that added the answer obtained, when using the equal key, automatically to the memory. If the M± key was then used, the result was added a second time. Thus, if one pressed 3 x 2 = 6, six was entered into memory. If M± key was then pressed, 12 would be the result in memory.

This particular method of memory usage could be most confusing to young students. They would certainly wish to see the results of each step in a problem and then enter it into the memory. It can also cause trouble if one forgets, and presses the equal key before the M± key.

If a memory machine is to be purchased, be sure that it contains a separate clear memory CM key. Some calculators we analyzed had one key to clear the machine and the memory C/CM. One press cleared the machine and a second cleared the memory. We found this to be a definite disadvantage. If a problem involved several calculations which were to be placed in memory, one had to be sure that the memory was clear before starting. At various times, we found ourselves clearing the problem and the memory because of lack of foresight. Students would certainly have this same problem.

Another feature important on a memory machine is the memory indicator. Students may forget from one problem to the next that data has been stored. Thus, a light or display indicator of some kind is most beneficial.

Each of the teachers involved in this writing project had previous experience with calculators. Some had machines in their classrooms, while others had investigated various models for purchase in their district. Since then, each of us who looks at a new model calculator will carefully scrutinize each feature claimed by the manufacturer. We will also make certain that those capabilities are consistent with the mathematics being taught and the level of understanding of the students who will use the machines. Teachers and administrators desiring to purchase calculators will serve themselves and their students well to take ample time in the same type of analysis.

SPECIFICATIONS FOR ELECTRONIC CALCULATORS

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The day of the £10 calculator has arrived. Maybe even this remark will seem to express unwonted surprise if the downward spiral in prices continues. But, whatever the future holds, there is no doubt that many teachers in schools of all types are beginning to consider buying one or more of these machines. With this in mind a group of interested teachers at the 1974 ATM Conference set out to provide some guidelines which would specify what to look for in a basic machine. This is a machine which could be used almost universally in the classroom, with age groups of at least five to 16.

Perhaps a word of explanation is necessary here because this article is concerned with machines without memories and without scientific function keys like LOG, SIN, COS, and so on. It is not that these machines are unimportant, but that with limited funds available a basic machine which meets nearly all the calculation requirements of children up to 16 would appear to have an overwhelming priority. The basic machine's price already matches that of the cheap transistor radio, and there are numbers of cases on record where these have been bought for children by fond relatives. There are many apparently similar machines on the market and it was felt that some guidelines would be useful to get best value combined with maximum applicability from the money spent.

A basic machine should have the following, in order of preference:

1. natural order arithmetic;
2. floating point;
3. underflow;
4. constant key to operate on all four operations;
5. eight-digit display;
6. not too small, fingertip sized keys;
7. rechargeable batteries, with alternative mains operation; and
8. CLEAR ENTRY key.

These items can be checked by using the following tests:

1. *Does it use natural order arithmetic?*

(a) Key in: 7 - 2 = ___ (5)

Look for a separate "=" key. Beware of machines that have "+=" or "-=" keys since these often demand an 'unnatural' arithmetic in the sense that calculations on the machine do not follow the written order even with simple binary operations.

(b) Key in: 8 - 5 x 13 ÷ 7 = __ (5.5714285)

Each operator is a binary operator, working sequentially from the left. This calculation should be evaluated as though bracketed from the left; that is, (((8 - 5) x 13) ÷ 7).

2. *Does it have full floating point arithmetic?*

(a) Key in: 123 ÷ 456 x 789 = __ (212.82236)

This will indicate whether the intermediate calculation result (0.2697368) is carrying maximum decimal places. This should be so, and even if a fixed decimal place setting is provided, the intermediate calculation should still be carried out to the limit of the machine's capability. A severe fault in the decimal handling is evident if this chain calculation is not carried out satisfactorily.

Some machines demand that each part of a chain calculation be completed with an "=" key, and this may lose significant figures in the final result, because the displayed digits are only part of those used in a calculation. Depression of the "=" key may destroy the non-displayed digits.

(b) Key in: 1 ÷ 81 = (0.0123456)
and 1 ÷ 81 x 100 = (1.2345679)

If the machine has the capacity for doing arithmetic to more than eight digits, the last operation will give the maximum set of significant digits that can be displayed on the eight-digit calculator. Clearly, if the machine has this facility, it will be more accurate, in the sense of avoiding rounding errors more successfully.

3. *Does it indicate when figures have been dropped?*

Key in: 88888888 x 2 = __

The full answer is 177777776, one digit too many for the eight-digit calculator. Observe how the machine handles this.

Some machines give 1.7777777, with the decimal point indicating one place dropped, associated sometimes with a flashing display. Other machines indicate this condition by the absence of a decimal point. This is less satisfactory because it appears to give an authentic result, whereas it is only a partially appropriate result. Normally all calculation should be termintated when overflow appears.

4. *How does the constant key operate?*

Almost all of the latest calculators have a constant facility, most usually indicated by a 'K' key, which allows entry of a given value to a store. The basic machine should allow this constant to operate on all four operations.

(a) Key in: CLEAR 2.3917 = K
Then 5.56 = x
should produce 13.560939.

(b) Key in: CLEAR 2.3918 = K
Then 5.67 = +
should produce 8.0617(000).

(c) Key in: CLEAR 2.3917 = K
Then 5.67 = -
should produce 3.2783(000).

(d) Key in: CLEAR 2.3917 = K
Then 5.67 = ÷
should produce 2.3706986.

The clear key on many machines clears not only the display but also the contents of the 'constant' store. This store is best considered as holding a constant and an operator. On the same machines the operator needs to precede the constant. This is best checked using the multiplication example, 4(a).

5. *How many digits does the calculator display?*

There are a number of machines which can give more than the eight figures we feel sufficient. The important point to note is that eight figures is the maximum likely to be of use, so that more figures either cost more or alternatively are displayed in two parts. There seems no advantage in this. However, the apparent capacity of the machine may be limited to its displayed digits, in which case rounding errors may occur. The test at 2(b) is useful for deciding this.

6. *Is it large enough?*

If too small, it can easily be knocked on the floor, or slipped into a pocket or briefcase. Keys need to be tested to see that fingers don't touch more than one key (inadvertently) at a time. The display should be clearly visible from a wide angle of view, clearly readable in daylight conditions.

7. *Can it be easily used in a classroom?*

This implies rechargeable batteries and a mains unit for recharging. The time between charges is also important. In general, machines that keep all eight digits' positions lighted will drain the batteries more quickly. Light emitting diodes (leds) are more efficient than other forms of display. If a mains unit is bought for recharging, check that it is rated at 240v AC. A number of those inspected are rated at 220v AC, and will have a diminished life on the mains supply in U.K.

8. *Does it have a 'clear entry' (CE) key?*

As already noted, the CLEAR key will clear both the display and the constant store, which may be undesirable in the middle of a calculation if a mistake has been made in entering a figure. A CLEAR ENTRY key allows the re-entering of a corrected number if the mistake is noted before the next operation key is pressed. This can be very frustrating in a sequential calculation if no CE key is available.

These eight questions and checks can be put in the form of a matrix, and individual machines assessed comparatively. There is an order of priority here, although there may be rather less agreement over the order of items 6 to 8.

During this study, it emerged that a number of machines already on the market came near to meeting the 'basic' specification. This is either because we (luckily) agree with some manufacturers, or that we have not been far-sighted enough. It does, however, lead one to ask whether we should leave it to chance in the future. If the educational market is one of the large growth areas, then when are manufacturers going to take seriously the need to produce an adequate basic calculator at the right price? We have heard a figure of £5 quoted. Who will take up the challenge?

PART THREE
Research

SURVEY OF THE USE OF HAND-HELD CALCULATORS IN MATHEMATICS CLASSES IN THE SECONDARY SCHOOLS OF BRITISH COLUMBIA

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This article is based on a research project funded by The Educational Research Institute of British Columbia.

INTRODUCTION

A questionnaire concerning different aspects of the use of hand-held calculators was mailed with the 1975/76 school year's last issue of *Vector*, the journal of the British Columbia Association of Mathematics Teachers (BCAMT).

The survey was initiated due to the rapidly increasing interest in the use of hand-held calculators. Since the price of hand-held calculators fell below $20 per unit, many schools have been buying laboratory and/or class sets of hand-held calculators. The use of hand-held calculators by public school students has spread so fast that the National Council of Teachers of Mathematics (NCTM), the world's largest association of mathematics teachers, has created a special subcommittee to look into the matter. Also the NCTM has dedicated the November, 1976 issue of *The Arithmetic Teacher* to articles concerning the use of hand-held calculators. Before the special issue on calculators, the NCTM had already published 15 articles in the last five years concerning hand-held calculators. The NCTM recently published a brochure, *Minicalculator Information Resources*, which lists over 50 publications and articles that have been produced since 1972 concerning hand-held calculators.

Given such increased interest concerning hand-held calculators it was felt that some baseline data were needed for British Columbia.

METHOD

The questionnaire was created and piloted. Based on the results of the pilot, the questionnaire was revised and shortened. The final form of the questionnaire was then turned over to the BCAMT. The questionnaire was included in the next mailing of *Vector*. The returned questionnaires (N = 75 for secondary mathematics teachers) were turned over to the keypunching service at UBC.

The data cards were used in a computer program to gain a frequency count for each item on the questionnaire. Since this is an exploratory study to gather initial data, no tests of significant differences were used to analyze the data. A weighted score was computed for each item by assigning a value of 2 to Strongly Agree, 1 to Agree, 0 to Undecided, -1 to Disagree, and -2 to Strongly Disagree. The values were then multiplied by the frequency in each choice category, and the five products were added; to obtain the weighted score one then divided by 75, the number of respondents.

RESULTS

The survey is not an attempt to gather information from every teacher in the province. The survey report will not be judging the data, only presenting and interpreting the data. The survey gathered data from about one out of every five members of the BCAMT. The sample is biased in the sense that the characteristics of teachers that belong to the BCAMT may differ from those of mathematics teachers who do not belong to the BCAMT, and teachers who returned the questionnaire may have different characteristics than the teachers who did not return the questionnaire.

The teachers who did return the questionnaire took the task very seriously. Of the 3,264 possible responses only 33 were left blank. Of the 62 percent of the teachers who marked that they have used hand-held calculators in their classes, 100 percent took the time to write comments concerning the ways they use the hand-held calculators in their classes.

Even though the questionnaire had been pilot-tested, the data from one item (item 16) was lost due to a typographical error.

Since the survey instrument was distributed by the BCAMT, teachers from Grade I through university returned the questionnaire. The data from all 136 questionnaires are presented in this section, but for the purposes of this report only the data for the secondary mathematics teachers are analyzed.

AVAILABILITY OF HAND-HELD CALCULATORS

Items 1, 2, 9, 24 and 25 were concerned with the availability of hand-held calculators. The data for these items are presented in Tables 1 to 5. The title for each table presenting raw data is the statement of the item as it appeared on the questionnaire.

Table 1

Item 1: More Than Half of the Students I Teach Have a Calculator at Home

	SA[1]	A	U	D	SD
Primary	0	1	0	2	8
Intermediate	1	1	5	8	5
Secondary	3	13	7	40	9
College/University	2	3	0	2	0
Total[2,*]	7	21	19	56	26

[1]SA - Strongly Agree, A - Agree, U - Undecided, D - Disagree, SD - Strongly Disagree

[2]The numbers appearing as the total may not be the sum of the four categories since 19 subjects did not mark their teaching level.

*5% of the subjects left this item blank.

Table 2

Item 2: I Personally Have a Calculator and Use It Often

	SA	A	U	D	SD
Primary	2	3	0	3	3
Intermediate	1	10	1	4	5
Secondary	27	32	2	9	5
College/University	4	3	0	0	0
Total*	40	57	4	18	0

*3% of the subjects left this item blank.

Table 3

Item 9: The School Should Buy the Calculators for Each Class, Just as They Do Books

	SA	A	U	D	SD
Primary	0	2	2	6	1
Intermediate	4	5	2	4	9
Secondary	8	18	13	21	13
College/University	2	2	0	0	3
Total*	16	31	19	36	32

*1% of the subjects left this item blank.

Table 4

Item 24: The Sooner We Get the Calculator into the Classroom the Better

	SA	A	U	D	SD
Primary	0	3	6	1	1
Intermediate	4	3	10	4	1
Secondary	15	26	17	12	4
College/University	3	3	1	0	0
Total*	27	40	36	20	9

*3% of the subjects left this item blank.

Table 5

Item 25: I Have Used Calculators in My Classes

	Yes	No
Primary	3	8
Intermediate	7	17
Secondary	57	17
College/University	7	0
Total*	84	51

*1% of the subjects left this item blank.

As mentioned before, in presenting an interpretation of data in an exploratory study no tests of significance will be given. A weighted score will be given to indicate trends in the data. The formula for the weighted scores (see Methods section) does not allow the group marking "Undecided" or leaving the item blank to have any effect in the numerator, but does allow them to have an effect in the denominator. Those two groups will have a "dampening" effect upon the results.

Table 6 contains the weighted scores for the secondary mathematics teachers for the data presented in Tables 1 to 4. The weighted scores reflect the trends in the data concerning the availability of hand-held calculators.

Table 6

Weighted Scores for Items 1, 2, 9, 24

Number of Table in Which the Data Appear	Weighted Score
1	-0.52
2	0.77
3	-0.17
4	0.48

While the teachers tend to own a calculator, they do not feel their students have a calculator at home. The data also show that the secondary mathematics teachers (responding to the survey) feel that hand-held calculators should be placed in the classrooms as soon as possible, but the teachers are undecided as to whether the school should buy the calculators for the classes (though the tendency is that the school should not buy the class calculators).

WHO SHOULD USE HAND-HELD CALCULATORS?

Items 3 to 7 dealt with the type of student who should be allowed to use hand-held calculators in the classroom. The data for items 3 to 7 are presented in Tables 7 to 11.

Table 7

Item 3: Calculators Have No Place in the Elementary Classroom (K - 7)

	SA	A	U	D	SD
Primary	0	2	2	2	5
Intermediate	4	1	5	5	9
Secondary	14	13	15	21	12
College/University	0	0	0	3	4
Total	22	18	24	36	36

Table 8

Item 4: Calculators Have No Place in the Junior Secondary Classroom (8 - 10)

	SA	A	U	D	SD
Primary	0	0	1	5	5
Intermediate	0	1	3	10	10
Secondary	3	6	3	38	25
College/University	0	0	0	2	5
Total	4	11	7	61	53

Table 9

Item 5: Calculators Have No Place in the Senior Secondary School (11 - 12)

	SA	A	U	D	SD
Primary	0	0	0	3	8
Intermediate	0	0	1	9	14
Secondary	0	2	1	23	49
College/University	0	0	0	2	5
Total	0	3	3	42	88

Table 10

Item 6: Calculators Should Only Be Used by "Good" Students

	SA	A	U	D	SD
Primary	0	1	0	3	7
Intermediate	0	2	3	7	12
Secondary	0	1	2	40	32
College/Universtiy	0	0	0	3	4
Total	1	4	5	59	67

Table 11

Item 7: Calculators Should Be Used by Students Who Cannot Remember Their Basic Facts or Skills

	SA	A	U	D	SD
Primary	2	2	1	1	4
Intermediate	3	7	5	3	5
Secondary	1	14	12	23	25
College/University	1	3	0	2	1
Total*	9	31	20	32	42

*1% of the subjects left this item blank.

The weighted scores for the data presented in Tables 7 to 11 are in Table 12. This group of data is concerned with what grade level and type of student should use the hand-held calculator in mathematics classes.

Table 12
Weighted Scores for Items 3 - 7

Number of Table in Which the Data Appear	Weighted Score
7	-0.05
8	-1.01
9	-1.59
10	-1.37
11	-0.76

The trend of the data is obviously that calculators should be used in the secondary mathematics classroom, but the secondary mathematics teachers were undecided about the use of hand-held calculators in the elementary classroom.

The subjects felt very strongly that students other than the "good" students should use hand-held calculators. The subjects also felt that students who cannot remember their basic facts or skills should not use calculators. Later it will be shown that this result does not mean that the "slower" students should not use hand-held calculators to drill on basic facts or skills.

HOW SHOULD HAND-HELD CALCULATORS BE USED?

Items 10, 11, 13, 14, 15, 22 and 23 were concerned with "how" the hand-held calculators should be used. The data for the items in this section are presented in Tables 13 to 19.

Table 13

Item 10: Special Courses Should Be Designed That Would Use Calculators for Most of the Computation

	SA	A	U	D	SD
Primary	0	2	6	3	0
Intermediate	2	10	4	4	3
Secondary	7	21	17	22	8
College/University	0	3	0	2	2
Total*	12	42	30	34	17

*1% of the subjects left this item blank.

Table 14

Item 11: Most Parents I Know are Against the Use of Calculators in the Schools

	SA	A	U	D	SD
Primary	0	0	9	2	0
Intermediate	0	1	16	5	2
Secondary	0	10	44	18	2
College/University	0	0	1	5	1
Total*	0	13	83	34	5

*1% of the subjects left this item blank.

Table 15

Item 13: Students Should Be Allowed to Use the Calculator on Tests Designed to Evaluate Problem-Solving Ability

	SA	A	U	D	SD
Primary	3	6	1	1	0
Intermediate	4	13	4	1	1
Secondary	13	43	12	3	3
College/University	2	5	0	0	0
Total*	27	74	19	7	6

*2% of the subjects left this item blank.

Table 16

Item 14: Calculators Should Be Used to Extend a Student's Problem-Solving Ability

	SA	A	U	D	SD
Primary	4	6	0	1	0
Intermediate	6	17	1	0	0
Secondary	21	46	5	3	0
College/University	4	3	0	0	0
Total	44	78	9	5	0

Table 17

Item 15: Students Should Be Allowed to Use Calculators Only to Check Their Work

	SA	A	U	D	SD
Primary	0	2	2	7	0
Intermediate	1	1	6	14	0
Secondary	1	4	8	53	9
College/University	0	0	0	5	2
Total	2	7	17	93	17

Table 18

Item 22: I Am Convinced That "How To" Use the Calculator Should Be Taught to Every Student Before Leaving Secondary School

	SA	A	U	D	SD
Primary	2	4	3	1	0
Intermediate	8	9	5	2	0
Secondary	19	35	13	8	0
College/University	3	4	0	0	0
Total*	41	55	22	17	0

*1% of the subjects left this item blank.

Table 19

Item 23: The Calculator Will Be a Big Help to Other Classes Besides Mathematics

	SA	A	U	D	SD
Primary	0	9	2	0	0
Intermediate	6	10	5	3	0
Secondary	22	45	6	2	0
College/University	3	4	0	0	0
Total	39	76	13	8	0

The weighted scores for the data presented in Tables 13 to 19 appear in Table 20. This group of data is concerned with ways the hand-held calculator should be used in secondary school mathematics classrooms.

Table 20

Weighted Scores for Items 10, 11, 13, 14, 15, 22, 23

Number of Table in Which the Data Appear	Weighted Score
13	-0.04
14	-0.16
15	0.80
16	1.13
17	-0.87
18	0.87
19	1.16

The subjects felt rather strongly that the hand-held calculators would be beneficial in non-mathematics classes and that hand-held calculators should be used to extend a student's problem-solving ability. The students should be allowed to use a hand-held calculator on tests, if the test is designed to evaluate problem-solving ability. The subjects also felt that before leaving secondary school every student should have been taught how to use a hand-held calculator and the use of hand-held calculators should go beyond simply checking one's work.

The subjects were undecided as to whether special courses should be designed which would use hand-held calculators for most of the computation. The subjects were also split as to whether parents are for or against the use of hand-held calculators in the schools.

THE HYPOTHESIZED EFFECT OF USING HAND-HELD CALCULATORS

Items 8, 17, 20 and 23 are concerned with the possible effects of using hand-held calculators. The data for item 23 are presented in Table 19. The data for items 8, 17, and 20 are presented in Tables 21 to 23.

Table 21

Item 8: Calculators Will Keep Students from Learning Their Skills If Used Before Junior Secondary School

	SA	A	U	D	SD
Primary	0	3	2	4	2
Intermediate	2	2	2	8	9
Secondary	7	17	15	26	8
College/University	0	1	1	3	2
Total*	12	28	25	43	25

*2% of the subjects left this item blank.

Table 22

Item 17: Calculators Could Lead to a Complete Breakdown in Students Learning Basic Skills

	SA	A	U	D	SD
Primary	0	2	1	7	1
Intermediate	1	3	4	9	7
Secondary	3	15	13	32	11
College/University	1	2	1	2	1
Total*	9	24	20	58	24

*1% of the subjects left this item blank.

Table 23

Item 20: Calculators Will Inspire Students to Continue in Mathematics Since They Take Away the Drudgery of Computation

	SA	A	U	D	SD
Primary	1	3	4	2	0
Intermediate	3	9	7	3	1
Secondary	11	31	17	15	1
College/University	2	4	1	0	0
Total*	18	56	34	23	3

*1% of the subjects left this item blank.

The weighted scores for the data presented in Tables 21 to 23 and Table 19 are presented in Table 24. This group of data is concerned with the hypothesized effect of the use of hand-held calculators.

Table 24

Weighted Scores for Items 8, 17, 20, 23

Number of Table in Which the Data Appear	Weighted Score
21	-0.15
22	-0.44
23	0.48
19	1.16

As mentioned before, the respondents felt very strongly that the hand-held calculators would benefit other classes besides mathematics classes.

The data in Tables 7 and 21 may be closely related. Just as the subjects were undecided (weighted score of -0.05) as to whether hand-held calculators should be used in the elementary grades, they were undecided as to whether using hand-held calculators in the elementary grades would keep the students from learning their skills. The data, however, do show that the subjects tend not to agree that calculators could lead to a complete breakdown in students learning basic skills.

The respondents also tended to feel that using hand-held calculators would inspire students to continue in mathematics since the hand-held calculators take away the drudgery of computation.

PREPARATION FOR USING HAND-HELD CALCULATORS

Items 9, 12, 18 and 19 are concerned with the preparations that are needed to make use of hand-held calculators in secondary school mathematics classrooms. As with item 23 in the last section, item 9 in this section is being repeated. The data for item 9 appear in Table 3. The data for items 12, 18, and 19 appear in Tables 25 to 27 below.

Table 25

Item 12: Special Materials Should Be Written to Be Used with the Hand-Held Calculators

	SA	A	U	D	SD
Primary	2	2	7	0	0
Intermediate	7	11	2	4	0
Secondary	14	38	5	15	3
College/University	1	3	0	1	1
Total*	29	62	16	23	5

*1% of the subjects left this item blank.

Table 26

Item 18: Textbooks Should Have Special Sections Devoted to the Use of Hand-Held Calculators

	SA	A	U	D	SD
Primary	1	3	5	2	0
Intermediate	6	10	3	5	0
Secondary	7	39	15	12	2
College/University	2	2	2	0	1
Total	20	65	27	20	4

Table 27

Item 19: Special Inservice Courses Should Be Given to Teachers Who Wish to Use Hand-Held Calculators in the Classroom

	SA	A	U	D	SD
Primary	2	7	2	0	0
Intermediate	7	12	2	2	1
Secondary	14	42	8	9	2
College/University	2	4	1	0	0
Total	30	75	16	12	3

The weighted scores for the data presented in Tables 25 to 27 are in Table 28. Also in Table 28 is the weighted score for item 9 (see Table 3). This group of data is concerned with preparations that could be made for the use of hand-held calculators in secondary school mathematics classes.

Table 28

Weighted Scores for Items 9, 12, 18, 19

Number of Table in Which the Data Appear	Weighted Score
3	-0.17
25	0.60
26	0.49
27	0.76

The trends shown by the data are that materials and training are desired. Special materials should be written to be used with the calculator, and textbooks should have special sections devoted to the use of the calculator. Besides the materials, there should be inservice courses for teachers who wish to use the calculator in the classroom.

COMMENTS

As mentioned earlier, every subject who marked "Yes" in item 25 took the time to make comments concerning "how" they made use of hand-held calculators in their classes. It is always difficult to organize written comments in such a manner that one can see if there are any trends indicated by the comments. Many of the subjects used hand-held calculators in several different ways. In Table 29 are listed the categories and the number and percent of the subjects stating that they use hand-held calculators in the specified manner.

Table 29

The Uses of the Hand-Held Calculators

Category	Number of Subjects	Percent of Subjects
To reduce manual computation	47	82
To check work	24	42
To do computation when the computation is of minor concern for the concept being taught	20	35
To motivate the learning of basic skills	14	25
To teach a unit on calculators	11	19
To drill students	8	14
To demonstrate a concept	2	4

By far the main use of the hand-held calculator in secondary school mathematics classes in British Columbia is to reduce computational time and effort. The reasons for wanting to reduce computational time and effort seem to be to spend more time and attention on the primary concept of concern, to motivate students to work on their skills, and to be able to cover more examples of a concept or more concepts.

SUMMARY

With the rapidly increasing interest in and use of hand-held calculators, it became necessary to gain some indication of the use of hand-held calculators in the secondary school mathematics classes of British Columbia.

The survey instrument was distributed by the BCAMT via its journal, *Vector*. Of the 136 replies, 75 were from secondary school mathematics teachers and are the object of concern of this report.

The strongest responses from the subjects were that hand-held calculators have uses throughout the secondary school grades in mathematics and non-mathematics classes by all ability groups. Equally strong was the response that hand-held calculators should be used to extend a student's problem-solving ability.

Other fairly strong responses were that while the subjects have calculators and use them often, the subjects feel that the students probably do not have calculators available to them at home. Special materials and special inservice courses concerning the use of hand-held calculators probably should be developed. Everyone should learn how to use hand-held calculators before leaving secondary school. A student should be able to use a hand-held calculator for more than just checking his work; in fact, a student should be able to use a hand-held calculator on a test, if the test is designed to evaluate the student's problem-solving ability.

Given the size of the sample and the fact that this was an exploratory study to gather initial data in an area of increasing concern to secondary mathematics teachers, it is preferred to classify the remainder of the results as too equivocal to include in a summary section.

APPENDIX — The Survey Instrument

In order to determine how some mathematics teachers feel about using the hand-held calculator and/or use of the hand-held calculator in mathematics classes, the members of the British Columbia Association of Mathematics Teachers are requested to please fill in this questionnaire.

This form has been designed to take a minimum of time to complete. The information you provide will be kept confidential and only summary data will be made public.

Please take the time to complete the form and return it to Jim Sherrill, Faculty of Education, U.B.C., Vancouver, B.C. V6T 1W5.

Please circle the grade(s) in which you teach a mathematics class:

K 1 2 3 4 5 6 7 8 9 10 11 12 College University

For all but the final two items you are asked to mark how you feel about each statement using the categories Strongly Agree (SA), Agree (A), Undecided (U), Disagree (D), or Strongly Disagree (SD).

1. More than half of the students I teach have a calculator at home.	SA	A	U	D	SD
2. I personally have a calculator and use it often.	SA	A	U	D	SD
3. Calculators have no place in the elementary classroom (K - 7).	SA	A	U	D	SD
4. Calculators have no place in the junior secondary classroom (8 - 10).	SA	A	U	D	SD
5. Calculators have no place in the senior secondary classroom (11 - 12).	SA	A	U	D	SD
6. Calculators should only be used by "good" students.	SA	A	U	D	SD
7. Calculators should be used by students who cannot remember their basic facts or skills.	SA	A	U	D	SD
8. Calculators will keep students from learning their skills if used before junior secondary school.	SA	A	U	D	SD

9. The school should buy the calculators for each class, just as they do books.	SA	A	U	D	SD
10. Special courses should be designed that would use calculators for most of the computation.	SA	A	U	D	SD
11. Most parents I know are against the use of calculators in the schools.	SA	A	U	D	SD
12. Special materials should be written to be used with the calculator.	SA	A	U	D	SD
13. Students should be allowed to use the calculator on tests designed to evaluate problem-solving ability.	SA	A	U	D	SD
14. Calculators should be used to extend a student's problem-solving ability.	SA	A	U	D	SD
15. Students should be allowed to use calculators only to check their work.	SA	A	U	D	SD
16. When my students finish their formal education mathematics will (sic) computation only.	SA	A	U	D	SD
17. Calculators could lead to a complete breakdown in students learning basic skills.	SA	A	U	D	SD
18. Textbooks should have special sections devoted to the use of the calculator.	SA	A	U	D	SD
19. Special in-service courses should be given to teachers who wish to use the calculator in the classroom.	SA	A	U	D	SD
20. Calculators will inspire students to continue in mathematics since they take away the drudgery of computation.	SA	A	U	D	SD
21. Most people in the future will not have to do very much computation because machines will do it for them.	SA	A	U	D	SD
22. I am convinced that "how to" use the calculator should be taught to every student before leaving secondary school.	SA	A	U	D	SD

23. The calculator will be a big help to

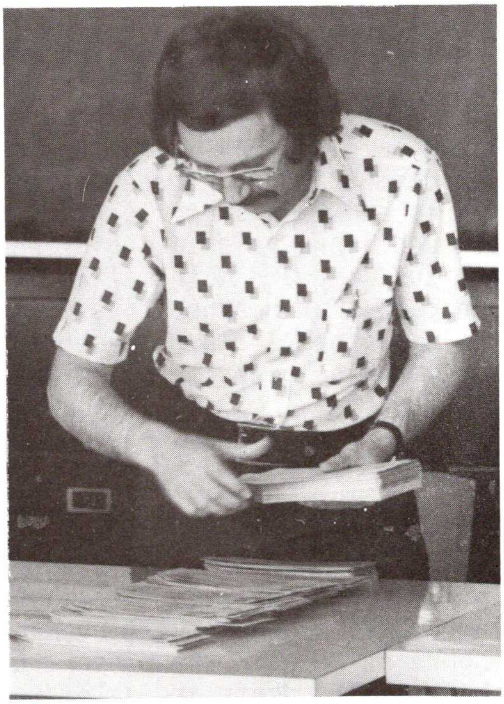
other classes besides mathematics. SA A U D SD

24. The sooner we can get the calculator into the classroom the better. SA A U D SD

25. I have used calculators in my classes. YES NO

26. If you marked "YES" for number 25, would you please describe how you make use of the hand-held calculator in your class(es).

Thank you very much for taking the time to complete the form! We hope that you will return your completed form to the address on the other side of this form.



Tabulating the data.

THE EFFECT OF THE USE OF DESK CALCULATORS ON ACHIEVEMENT AND ATTITUDE OF CHILDREN WITH LEARNING AND BEHAVIOR PROBLEMS

Kan Advani

Frontenac County Board of Education
Kingston, Ontario

This article, which appeared in The Ontario Mathematics Gazette, September, 1973, has been reprinted with the permission of Editor Arn W. Harris.

The report describes an experimental study conducted from January to June 1972 in a special class of 18 children (ages 12 to 15 years) with learning and behavior problems for the purpose of determining the feasibility of using desk calculators in conjunction with classroom instruction in mathematics and their effect on the achievement, attitude and behavior of these children. Mathematics achievement was based on the results of the pre-test and post-test scores of the Stanford Achievement Test. Four calculators were placed in a corner of the classroom, and students were encouraged to check their answers of math problems related to classroom work. The teacher also used the machines as a tool for the enrichment and reinforcement of a new unit. Comparison of the pre- and post-test scores showed a significant difference (0.01 level of confidence), and an analysis of questionnaire data indicated marked increases in student interest and positive attitudes toward mathematics. The author concludes that the use of calculators can facilitate mathematics instruction in a special class, provide variety in the classroom, help release frustrations children have due to inaccessible numbers, and help teachers in individualizing the math instruction.

INTRODUCTION

"Probably the greatest revolution in the teaching of arithmetic is the introduction of calculating machines." (6) Technology is gradually playing an important role in the present-day education. We do find that some of these mechanical gadgets are able "to execute rather elaborate series of instructions (programs) and in many cases adequately serve the instructional needs of the students." (14)

STATEMENT OF THE PROBLEM

To determine whether or not desk calculators can be used in conjunction with classroom instruction in mathematics with children with learning and behavior problems.

To determine whether or not the use of these machines will have any effect on the achievement, attitude and behavior of the students.

HYPOTHESES

In an attempt to answer these problems, the following hypotheses were formulated:

a. the use of the desk calculators in a class of children with learning and behavior problems will result in their greater achievement on Standardized Tests, and

b. the use of the calculators will have a positive effect on the students' self-confidence and attitude toward learning, thus improving their classroom behavior.

PROCEDURE

The criteria used to determine mathematical achievement of the 18 students were the results of pre- and post-tests on Stanford Achievement Test (Intermediate I - Partial Battery). The change, or difference, in the scores of each student was determined and the t-test of the differences between these mean differences was used, with the 5 percent level of confidence as the criterion for rejecting the null hypothesis.

EDUCATIONAL TREATMENT

Because these students were working at their individual levels in mathematics, they were encouraged to complete their unit of work and then go to the calculators to check the answers.

When the teacher introduced a new unit with one student or a group of students which lent itself to reinforcement and enrichment by supplementary use of the calculators, he brought that student or group to the Mathematics Centre set in the far corner of the classroom and encouraged them to solve the problems of similar nature from the teacher-prepared laboratory worksheets.

Quite often students brought the receipts of the grocery and other purchases made by their families and checked the totals on the machines.

SOURCES OF DATA

The study was conducted with 18 boys and girls attending a special class at Victoria Public School, Kingston, Ontario. These youngsters, who were 12 to 15 years in age, were educable mentally retarded and neurologically impaired. They had I.Q. scores of about 68 to 116.

EARLIER EXPERIMENTS

After reviewing past issues of several leading journals, a few articles regarding the use of calculators in regular classrooms were found, but none referred to their use with children with learning and behavior problems. Schlauch reported that the machines were a strong motivating force with no sacrifice of analytic skills. Marsh (9) and Keough et al (7) found that the use of calculators resulted in better achievement in mathematics.

Shoemaker has developed materials employing the desk calculators in the high school classroom, claiming:

> The calculator is a very important segment of the mathematics laboratory. During the past few years, hundreds of instructors have found very favorable results in using the calculator as a means of reaching the slow achievers.

Broussard, Fields and Reusswig (3) stated that a program for low achievers from disadvantaged areas which emphasized real world applications and use of flow charts, calculators, and other materials resulted in significant achievement gain.

Beck (2); Stenzel; Groenendyk (5); Longstaff, Stevens, and King (8) found that motivation of the students was improved by the introduction of desk calculators into the classroom thus resulting in better attitude toward the subject. Reynolds, and Traverse and Knaupp have urged the early introduction of computational aids in the elementary school curriculum. On the other hand, Cech (4) did not find improvement in the achievement or attitude of the ninth-graders. His study, however, supported the hypothesis that the students could compete better with calculators than without them.

FINDINGS

Table 1
Comparison of the Pre- and Post-Test Scores (N=18)

	Mean Score	t
Pre-Test	40.5	3.09
Post-Test	43.0	
$p < 0.01$		

Table 1 shows that the post-test scores were higher than the pre-test scores and the difference was significant at the 0.01 level of confidence.

STUDENT INTEREST AND ATTITUDES

An analysis of questionnaire data indicated marked increases in student interest and positive attitude toward mathematics. When asked whether they would like to work on the calculators on a permanent basis, 17 out of 18 students answered affirmatively. One girl did not like the noise of the calculators while she was trying to concentrate on her work.

Students offered the types of responses categorized in Table II to an open-end item asking what was of most value in doing math with the aid of the calculators.

Table II
Strength of Calculator-Assisted Instruction
(N=18)

Responses	Frequency
It helped me understand better	6
It gives you the correct answer when you are wrong	5
It is fun	4
It is faster	3

Success breeds success; failure does not provide motivation for learning, nor does it release energy for learning. The self-image of a child with learning and behavior problems, as with any other child, is enhanced by experiencing success and diminished by frequent failure. Many of these children lack both hope of achievement and fear of not achieving. Machines helped in removing the fear of failure and provided structure through which the child understands the computation process more clearly. Interest in math picked up and the children who were formerly disinterested started finding problem-solving great fun.

In the beginning, students appeared rather overwhelmed with the complexities of the machines, but after this initial demonstration by the teacher, the enthusiasm and desire to master the "mechanical genius" rose. It didn't take them long to handle the 'tool' successfully.

Through these calculators it was easy to explain the "whys" of multiplication and division to some students. Most of the students gained an understanding of negative integers by using the calculators.

The presence of the calculating machines in the classroom brought a remarkable change in the behavior of the students. The calculators brought variety to the class. These children, who have comparatively short attention spans, find it difficult to sit at one place for 40 to 45 minutes and concentrate on a task. A chance to go to the calculator did provide a change or a kind of rest and eased the tensions. Disruptive students became pleasant and the classroom atmosphere calmer and more controllable.

CONCLUSIONS

Based upon the data gathered from testing procedures and questionnaires, the following conclusions were drawn:

1. The student showed significant gains in Stanford Achievement Test scores from pre-test to post-test.
2. The attitudes of the students after working with the calculators for six months improved.
3. The calculators proved a help to the teacher in individualizing his instruction.
4. Calculators provided variety in the classroom.
5. Calculators helped to release some of the frustrations children had due to inaccessible numbers which resulted in better behavior.

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THE USE OF THE MINI-CALCULATOR IN THE CLASSROOM

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INTRODUCTION

A few years ago, people thought of calculators as costly, bulky machines that added, subtracted, divided, and multiplied, and were used only in business offices. With the advent of the mini-calculator in 1971 that whole image changed. Prices dropped to under $100 a unit, with some now selling for as little as $10, and even cheaper calculators may be on the market soon. These pocket-size electronic calculators are sprouting everywhere, even in the pockets of school children.

Many of these calculators are very powerful machines if used in the proper way. Apart from performing the basic functions of addition, subtraction, multiplication, and division, they have such features as memory banks, floating decimal (8 to 10 places), automatic constant, automatic squaring, square root, reciprocal, percentage, negative sign, and trigonometric functions. These calculators can handle problems involving computations of any type and give instantaneous answers.

It is now quite common for students to bring calculators to class and request permission to use them. Many teachers and administrators are undecided about whether to accept or reject these calculators in the classroom. Traditionally, educators have been slow to adopt new ideas. We usually lag behind and are forced by business and social pressures to catch up with the world around us. It is about time that we began to make our own decisions.

Electronic technology has made the calculator possible but, as educators, we should realize that we are in control of this technology. If our students are going to use these machines, it is imperative that we, as teachers, be aware of their uses, applications, implications, and limitations. We have to examine ways of getting the most benefit from these machines without allowing our students to become passive button-pushers. The time saved in using the mini-calculator must be used in such a way as to improve the quality of our education.

SHOULD WE ACCEPT THE MINI-CALCULATOR INTO OUR CLASSROOMS?

How should educators decide on the acceptance or rejection of the mini-calculator, or do we really have a choice? We are not going to be able to keep calculators out of the hands of children although we could outlaw their use in our schools. However, if we examine the facts carefully, it can easily be seen that the calculator does have an important role to play in our classrooms.

Is the ability to compute long and tedious addition, subtraction, multiplication, and division problems in longhand one of our main objectives in our mathematics, physics, and chemistry classes? Maybe the answer is yes in Grades I to VI, but there comes a time when we have to assume our students know the basics and we must proceed to bigger and better things. This is not to say there are no uses for the calculator in Grades I to VI (these uses will be discussed in another section), but to subject students to long and tedious calculations after they know the basics seems to be a great waste of valuable school time. We spend much time on computation and yet our students don't seem to get any better at it. Many of our students fail to comprehend important concepts because they get bogged down in the computation involved.

It could very well be that we have overdone computation. Our students have become bored with so much tedious work. We should remember that computing is not an exciting process. The mini-calculator may be just what we need to get our students remotivated.

The main point to remember is that we, as educators, decide who uses the calculator, when it is used, and in what subjects it is used. None of these things are predetermined; we are in control of this marvelous device.

SURVEY OF LOCAL TEACHERS

We thought it would prove interesting to see what teachers at the local level feel about the electronic calculator. We decided to conduct a very limited survey to determine the local use of the electronic calculator in the classroom and also teacher attitude toward these machines. Realizing that teachers are bombarded with surveys every year and are becoming turned off by them, we decided that the survey must be simple and to the point, and that there must be personal contact.

Consequently, the questionnaire consists of nine yes-or-no questions, a question on grade level preference, and spaces to list advantages, disadvantages and further comments. Contact was made with one person whom we knew personally in each school and each was asked to be responsible for the distribution and collection of questionnaires in his school.

A total of 19 schools took part in the survey. In St. John's there were 6 high schools, 3 junior high schools, and 2 elementary schools. In Conception Bay we had 2 high schools, 1 junior high, and 5 elementary schools. The response was excellent with a total of 97 mathematics, physics and chemistry teachers completing the questionnaire. The results of the first 10 questions are summarized in Tables 1 and 2. It should be noted that the top number in each block represents the number of responses and the bottom number represents the percentage.

Table 1

Question	Grade Intervals 4 - 6 yes	4 - 6 no	7 - 8 yes	7 - 8 no	9 - 11 yes	9 - 11 no	4 - 11 yes	4 - 11 no
1. Have any of your students requested permission to use the mini-calculator in class?		18 100%	11 55%	9 45%	29 49%	30 51%	40 42%	57 58%
2. If the answer to question number 1 is Yes, was permission granted?			3 30%	7 70%	18 64%	10 36%	21 55%	17 45%
3. Do you believe that most of the time pupils spend on calculations is a waste?		15 100%	6 32%	13 68%	21 37%	35 63%	27 36%	48 64%
4. Do you think that schools should provide calculators (in limited number) for their students to use?	10 55%	8 45%	14 74%	5 26%	35 61%	22 39%	59 63%	35 37%
5. If your school board purchased these machines, would you encourage students to use them in class?	6 33%	12 67%	14 74%	5 26%	40 70%	17 30%	60 64%	34 36%
7. Do you think that the extensive use of calculators would result in the loss of basic computational skills?	14 74%	5 26%	14 70%	6 30%	39 67%	19 33%	65 68%	30 32%
8. Do you have access to an electronic calculator in your school?		18 100%	10 50%	10 50%	31 53%	27 47%	41 43%	55 57%
9. Do you (or would you) use the electronic calculator for computing grade scores, and so on?	12 67%	6 33%	18 90%	2 10%	52 88%	7 12%	82 85%	15 15%
10. Do you think that Grade XI students should be permitted to use calculators in Public Exams?	9 50%	9 50%	17 89%	2 11%	41 68%	19 32%	67 69%	30 31%

The results of item 6. on the questionnaire could not be included in Table 1 since it required the teacher to indicate a grade level at which students should be permitted to use calculators. The results of that question are given in Table 2.

Table 2

Grade level at which students should be permitted to use the mini-calculator	Grades taught by teachers answering this question 4-6	7-8	9-11	4-11
4 - 6	3 19%	2 12%	2 4%	7 9%
7 - 8	4 25%	5 29%	5 11%	14 17%
9 - 11	9 56%	10 59%	40 85%	59 74%

Although it would not be fair to draw any conclusive generalizations from such a limited survey, the results do suggest a few interesting points. Some of these are:

1. Calculators are not being used in the elementary schools surveyed.

2. In junior high schools, 70 percent of the students who requested permission to use the calculator were refused, whereas in high school 64 percent were given permission.

3. Computation is still considered as a major objective in mathematics at all levels. Sixty-four percent of all teachers questioned did not believe that most of the time spent on computation is a waste.

4. Most teachers (68 percent) believe that the extensive use of calculators could result in the loss of computational skills.

5. Less than half of the teachers surveyed (43 percent) have access to an electronic calculator.

6. The vast majority of teachers (74 percent) believe that calculators should only be used at the high school level.

The eleventh item on the questionnaire asks: What, in your estimation, are some possible advantages of having students use calculators?

The advantages most often mentioned by teachers were:

1. They save time and allow students to work on more important topics.

2. They provide a means whereby students can check their answers.

3. They allow students to concentrate on the material rather than get bogged down with the calculations involved.

4. They bring more complex problems within reach of the student.

5. They may reduce the difficulty that a person has with a subject because of less computational difficulty.

The final item on the questionnaire asks teachers to list some disadvantages of having students use electronic calculators. The disadvantages most often mentioned by teachers were:

1. If used too early (elementary grades), they may replace computational skills.

2. Students may become too dependent on the machine.

3. They may make students lazy.

4. If used too early, students would lose motivation to learn basic skills.

5. Not all students can afford calculators.

The comment section of the questionnaire provides some good insights into what the local teachers really feel about this issue. Some of the comments are:

"Really, I'm not against them as an aid, but I am against them as a substitute, an easy way out."

"If blanket approval for the use of calculators is given in lower grades, how much of the resultant math will be a result of understanding mathematical principles and how much will be the result of knowing which button to press."

"If calculators were used in elementary school, then finding a way to motivate children to learn the basic skills could prove a problem."

"We have found that unless you have quite a knowledge of basic Math principles, the use of a calculator is limited."

"This is the age of the computer and anything we can do to help the student fit into the world of technology after high school, we should do."

"It makes them aware of the technological era."

"It should not be automatically assumed that the machine is an ogre in the teaching of computational skills. The machine does not necessarily deny the acquisition of various inductive and deductive processes and capabilities. It seems to be a matter of approach. If mechanistic to the exclusion of cognitive, the loss of computational skills would result. But this need not be so!"

This last comment has the crux of the matter well in focus. It says that it seems to be a matter of approach; it depends upon how the calculator is used, upon how we as teachers approach the mini-calculator in the classroom.

In going through the questionnaires one cannot but notice an air of caution on the part of local teachers with respect to the introduction of electronic calculators into the classroom. They insist that pupils must learn the basic computational skills first and the majority think that if used at all, it should be at the high school level. This caution is good in one sense, and that is, we should first determine how to make the best possible use of them.

It is obvious that most of us are looking upon the electronic calculator as a tool for getting quick answers to our computational problems, and, after all, this is how they are being presented to us as the consumer. As educators, however, we now have to accept the electronic calculators as a fact of life and capitalize on whatever value they may have in the *learning* situation.

SPECIFIC USES OF MINI-CALCULATORS IN THE CLASSROOM

While producing no grand changes in programs, hand-held calculators offer important advantages to elementary school teachers. Obviously they make it possible for a child to check the accuracy of his answer, thus providing immediate verification, which is an important motivational factor.

One definite possibility that comes to my mind is asking a young child if he can teach the calculator to count. We cannot begin to imagine the insights and appreciation of number that would take place if he eventually discovers that by adding one each time the machine will count for him.

Imagine the possibilities for the calculator in trying to teach the child that multiplication is really repeated addition. That is, the fact that 5 x 7 is the same as 7 + 7 + 7 + 7 + 7 may seem obvious to us, but is it to a child? On the other hand, are things that we *know* completely without possibilities for new insights? Is it so obvious that 7000 + 400 + 50 + 8 = 7458? Can a young person learn anything when you ask him to find the value of 25 x 384 and then you ask him to find the value of (20 x 384) + (5 x 384)? Granted, this can be done without a calculator, but the insight might get lost in the paper and pencil computation.

If we examine the concept of division more closely, we can see other excellent examples of the use of the calculator. How many students realize that division is really repeated subtraction. In other words, if you divide 2 into 10 we are really saying how many twos can be subtracted from ten. We could help elementary students grasp this concept by allowing them to check their division problems using only the subtraction key on the calculator. How many of our students realize that when we divide 71.265 by 29.6, we are really asking what number multiplied by 29.6 will have a product of 71.265? We could have elementary students appreciate this fact by allowing them to check their division using only the multiplication key.

The possible uses for the calculator in the elementary school are limitless. Students at this age could be given the calculator and asked to find something the machine can not do. Many students would soon discover that you cannot divide by zero or get the square root of a negative number.

The important point to emphasize is that at the elementary grade level, or at any grade level, students would not have a calculator all day, every day. Every addition, subtraction, multiplication, and division problem would not be done on the calculator. The teacher would decide when the calculator is to be used.

The junior high school mathematics program offers many areas of application for the calculator. Van Atta suggests two problems that are marginally possible without a calculator, but that would be considerably easier with one.

In the first problem a teacher simply asks a student to prepare a table of values for the powers of some numbers (for example, 3) and then answer such problems as $3^2 \times 3^5 = \_\_$. The student will probably, at first, *translate* 3^2 into 9 and 3^5 into 243, multiply and give the answer 2,187. In fact, if he reaches the conclusion that $3^2 \times 3^5 = 3^7$ and attempts to generalize for this one example, we would be a bit lax if we did not ask such questions as: Does this always work? Does it work with other numbers than 3 as a base? Does the same pattern work for division? In order to answer such questions the student must do an incredible amount of computation. He runs the risk of error, and so also risks mistaken conclusions. At this point it seems reasonable to ask whether you are trying to set up a situation in which the laws of exponents become obvious, or whether you are supplying practice in multiplication and division. If it is the former, you might choose to use calculators; if it is the latter, it is time to buy another ream of paper.

The second problem, that of discovering the Pythagorean theorem relationships, would involve having students construct numerous right angle triangles, measure each side, square each measure, and examine the relationship that exists between the squares of the measures. Hopefully, after several examples, the students would discover the relationship. Without a calculator this could be a very time-consuming process. In fact, many teachers would avoid wasting time by telling their students the relationship and deprive them of the excitement of discovering it for themselves.

The high school algebra, geometry, trigonometry, and general mathematics courses provide numerous opportunities for the use of the calculator. In our general mathematics course we do simple interest, compound interest, discount buying, installment buying, bank statements, and income tax forms. In all of these topics the computation obscures the concepts and the students get bogged down with the computation and miss the concepts. The mini-calculator would in this case allow students to concentrate on the concepts and principles involved.

One excellent example of the use of the calculator is in the Grade XI algebra course. It deals with helping students understand the definition of logarithms. We say to a student that a logarithm is the power to which you would raise a given base to obtain the number desired, but how many of our students do you think understand it? Have any of them ever used 10 to the power 1.5, let alone 10 to the power 3441? We assume that since logarithms are simple to use, they are simple to understand, but in reality we do so little work with fractional exponents that very few students understand the meaning of logarithms. Why do we do so little work with fractional exponents? They are too difficult for a student to do with a paper and pencil. The mini-calculator could solve that problem.

The sine and cosine laws in Grade XI trigonometry involve much tedious computation. For example, students would have to solve an equation such as $a^2 = 26^2 + 35^2 - 2(36)(35)(6734)$. The solution would require two squares, four products, an addition, a subtraction and a square root. The student should not be denied the use of calculators here.

The physics and chemistry courses in high school are loaded with computational problems. All science teachers agree that most of their students would have less trouble with the concepts if they didn't concentrate so heavily on the computational skills. The real question is whether the main objective of teaching science is to understand the principles of science or to reinforce computational skills. We believe it is mainly the former and so in many cases the calculator should be used.

The uses for the mini-calculator are numerous. The creative teacher could easily turn this tool into an important teaching device. As was said previously, the calculator is more than a tool which gives the correct answer quickly; it is also a very important instructional device that can be used in teaching many concepts.

CONCLUSION

The mini-calculator is a device that is eventually going to become an essential piece of equipment in our classrooms. As educators we have to examine the potential of the mini-calculator from two viewpoints. First, as an aid in computation. This does not mean that we substitute the calculator for learning to compute with the pencil, but we can allow our students to use the calculator in long and tedious calculations which would otherwise occupy much valuable time. Secondly, as an instructional device that can help students get a better grasp of many mathematical concepts. This aspect is the one that is probably the most important and exciting feature of these machines. It may very well serve to be the motivational factor that we have been groping for in our mathematics classes for years. Quite often instead of really giving our students some way to grasp the concept of division, multiplication, addition and subtraction more fully, we gave them more and more practice. We know this doesn't work.

As with any teaching aid, the calculator could easily be misused. It could lead to the destruction of computational skills if used improperly in the classroom. As teachers we should know these misuses and always guard against them.

Finally, as educators we should always remember that we are in control of these fantastic little *black boxes* called calculators. We can use them to our advantage and to the advantage of our students, or we could let them turn our students into mathematical illiterates. Let's hope that we, as educators, have enough foresight to assure that the former happens.

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A COMPARISON OF ACHIEVEMENT AND ATTITUDES OF STUDENTS USING CONVENTIONAL OR CALCULATOR-BASED ALGORITHMS FOR OPERATIONS ON POSITIVE RATIONAL NUMBERS IN NINTH-GRADE GENERAL MATHEMATICS

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A growing number of mathematics educators consider the electronic calculator to be a useful and practical aid in the teaching/learning process. Its value as a motivating device has been suggested by teachers and researchers (Mastbaum, 1969). Its utility in reducing computational drudgery is apparent; its applicability in many instructional settings with children of divergent skills and abilities attests to its utility in the mathematics laboratory (Johnson, 1970). Most children are easily able to master operation of calculators.

Since many variations of strategies and methodologies within the framework of the conventional algorithmic processes for performing operations on positive rational numbers have been proposed and tried with apparently little success in Grade IX general mathematics, the efficacy of the conventional algorithms for teaching low-ability or low-achieving students is questionable. Accordingly, an algorithm set that is calculator-dependent suggests a reasonable alternative to the conventional algorithm set and provided the basis for the investigation.

The purpose of this study was to assess the differential effects on achievement and attitude of conventional or calculator-dependent algorithms for performing the four fundamental operations on positive rational numbers in ninth-grade general mathematics. No research has been reported that relates directly to the problem under investigation. However, the alternative algorithmic set is dependent on the use of the calculator. Therefore, selected research relative to machine use in the classroom is examined. The literature was also surveyed to study effects on student achievement of variations within the conventional algorithm set.

Fehr, McMeen, and Sobel (1956), Schott (1955), and Findley (1966) report studies suggesting that student use of calculators improves student achievement in arithmetic fundamentals when machines are used over extended periods of time. These findings differed from those observed in a later study by Johnson (1970), who found that comparison of a group using activity-oriented lessons without the calculator to a group using activity-oriented lessons with the calculator in a rational numbers unit resulted in the former group scoring significantly higher. However, Johnson also reported that low- and middle-ability students who had used machines regularly displayed more positive attitudes toward mathematics than did similar groups that did not use the calculator.

Mastbaum (1969) reported an extensive calculator-oriented study which showed that use of machines as a teaching aid with slow learners in seventh- and eighth-grade mathematics did not significantly improve attitude, increase mathematical achievement, noncalculator computational skill, mastery of mathematical concepts, or ability to solve mathematical problems. Mastbaum also found that seventh- and eighth-grade students could learn to use the calculator in solving one-step problems, could solve problems faster, and could work more accurately than students not using machines. It was also reported that ability to solve problems with machines did not transfer to noncalculator situations.

Research on algorithms for performing rational number operations has generally been limited to examining the effects on achievement of making variations within the conventional algorithm sets. The findings of the following studies suggested the strategies to be used for performing operations on positive rational numbers according to the conventional algorithm set.

Brownell (1933), Anderson (1965), and Capps (1962) reported studies showing that alternative strategies for presenting the conventional algorithms of addition and multiplication of fractions do not significantly alter achievement. Brownell also found that students who used the least-common-denominator method of adding fractions achieved as well as students who used the process whereby the denominators of the fractions were multiplied to obtain "a" common denominator. Anderson reported no significant difference in achievement resulting from adding fractions by finding the least common denominator by (1) setting up rows of equivalent fractions or (2) finding different prime factors of each denominator and using their product as the least common denominator.

Brueckner (1928) found that major difficulty with all four operations resulted from three sources: lack of comprehension of the processes involved, difficulty in reducing fractions to lowest terms, and difficulty in changing improper fractions to mixed numbers. Scott (1962) found that children make more errors in subtracting common fractions where regrouping is necessary than in subtracting whole numbers involving regrouping. Capps (1962) found no significant difference in the achievement of students dividing fractions by the common-denominator method as compared to the inversion method. Dutton and Stephens (1960) obtained similar results with respect to retention.

METHOD

The study was conducted during fall, 1971 in three ninth-grade general mathematics classes at each of two schools - Marshall-University High School

(School A) and Coon Rapids Junior High School (School B). The former institution, a secondary school containing Grades VII through XII, is a Minneapolis public school and the laboratory school for the University of Minnesota. The latter school is part of the suburban Anoka-Hennepin Independent School District #11 and contains Grades VII through IX.

The investigator, assisted by a student teacher, taught all three classes at School A. A volunteering teacher, assisted by a paid teacher aide, instructed all three classes at School B. Fifty-three students at School A and 48 students at School B were randomly assigned to classes during spring, 1971. Table 1 summarizes the numbers of students involved in the treatment groups (classes) at the two schools. The duration of the study, including all pretesting, remediation, and retention period, was 10 weeks.

TABLE 1
NUMBERS OF STUDENTS IN TREATMENT GROUPS AT SCHOOL A AND SCHOOL B

School	Treatment 1	Treatment 2	Treatment 3
A (Marshall-University)	18	16	19
B (Coon Rapids)	20	16	12

Three different treatments (developed in a pilot study during the fall of 1970) for performing operations on positive rational numbers were used in the experiment. Treatment differences resulted from the use of the electronic calculator or from the use of two different algorithm sets. The two algorithm sets may be briefly described as follows:

CONVENTIONAL ALGORITHM SET (CAS) - Operations on positive rational numbers are performed according to the "usual" text approaches.

ALTERNATIVE ALGORITHM SET (AAS) - Each fractional operand is converted to a decimal on the calculator (truncated to thousandths). The indicated operation is then performed on the decimals using the calculator. The three treatments for performing operations on positive rational numbers are defined as follows:

T1 - CAS without calculators
T2 - CAS with calculators
T3 - AAS with calculators

Students in the T2 and T3 groups were allowed to use machines throughout the unit on rational numbers and on the post-test and retention test. Each student had his own calculator.

The units of instruction used by the T1 and T2 groups were adapted by the investigator from several current junior high mathematics texts, including recent publications of Houghton-Mifflin Co.; Allyn and Bacon; Holt, Rinehart, and Winston; and SRA. The T1 and T2 units were constructed to adhere strictly to the conventional algorithmic techniques for performing operations on positive rational numbers and were identical but for the following exception: Instructions for use of the electronic calculator in performing the four fundamental operations on positive rational numbers were incorporated into the T2 instructional materials. The T1 group was not exposed to machines. The instructional materials used by the T3 group were constructed in their entirety by the investigator and presented rational numbers according to the AAS.

All students participating in the study were pre-tested for (1) possession of certain skills prerequisite to the study of operations on positive rational numbers, (2) ability to perform operations on positive rational numbers, (3) reading level (Metropolitan Reading Test), and (4) IQ level (Lorge-Thorndike Verbal Battery). Remediation was provided to students showing deficiencies in prerequisite skills. Students scoring above the 80-percent level on the pre-test on operations with positive rational numbers were excluded from the study.

All treatments were conducted within a mastery-learning model as advocated by Bloom (1969). The criterion tests of the mastery of operations on positive rational numbers consisted of a set of five test worksheets, one for each of the five units of instruction. Eight parallel forms of each of the five test worksheets were constructed to conform to the specific objectives of the units of instruction. Parallel forms made it possible for two students in a given treatment group never to work on the same form of a test worksheet at the same time. All completed test worksheets were kept on file by the teacher. Mastery criterion was set at 80 percent (12 of the 15 items on the test worksheets).

The five units of instruction were (1) adding fractions, (2) subtracting fractions, (3) multiplying fractions, (4) dividing fractions, and (5) operating on fractions. The fifth unit, operating on fractions, was a combination of the previous four units, and the first test worksheet a student attempted in this unit served as the post-test for the unit's objectives. If the student did not score at the 80 percent level on any test worksheet, he completed additional forms until mastery was achieved.

The T2 and T3 groups were given instruction in calculator operation. In addition, the T3 group was instructed in changing fractions to decimals on the electronic calculator prior to commencing the unit, adding fractions. Table 2 summarizes the experimental sequences for the three treatment groups.

TABLE 2

UNITS COMPRISING THE EXPERIMENTAL SEQUENCES FOR THE THREE TREATMENTS

	Treatment		
Unit	1	2	3
Operating the Calculator		X	X
Changing Fractions to Decimals			X
Adding Fractions	X	X	X
Subtracting Fractions	X	X	X
Multiplying Fractions	X	X	X
Dividing Fractions	X	X	X
Operating on Fractions	X	X	X

NOTE: The X's denote the units studied by a particular treatment group. The order in which the units are listed in the table corresponds to the order in which they were presented to the students.

The calculators used in the study, provided by the Minneapolis office of the Singer Company, Friden Division, were Friden 1115, 1116, and 1117. The three models were operated identically with respect to performing the four fundamental operations.

The criterion measures used in the investigation were either constructed by the investigator or computer generated via the Honeywell Arithmetic Test Generator (ATG) using a wide range of common rational number skill forms selected by the investigator. The ATG system is an operational example of what measurement theorists term domain-referenced testing. It is made available through remote teletype terminals and is capable of generating large numbers of parallel forms of arithmetic tests. In the present investigation, all test worksheets, the fractions pre-test, and the fractions retention test were generated through the ATG system. A study by Hively, Patterson, and Page (1968) established that reliability estimates for tests generated through the ATG system ranged from .80 to .90 when a variety of skill forms and multiple items within skill forms were used.

The transfer-oriented post-test was constructed by the investigator to measure the performance of the three treatment groups with respect to selected tasks not taught as part of the students' present mathematics course. The test consisted of six subtests concentrating on the following areas: (1) general rational number concepts (not operations), (2) estimation of fractional values, (3) ordering rational numbers, (4) combining rational numbers involving more than two operands or more than one operation, (5) solving open sentences involving rational numbers, and (6) selecting the appropriate operation to use in solving verbally stated problems involving rational numbers. Reliability estimates for the transfer-oriented post-test ranged from .70 to .92.

Students' attitude toward mathematics was measured through a semantic differential (SD) instrument containing 17 sets of bipolar adjectives arranged on seven-point scales. The SD was administered prior to the onset of differentiated instruction and immediately following the students' completion of the unit on operations on positive rational numbers. The SD pre-test carried the stimulus "Mathematics and Me," and the post-test carried the stimulus "Studying Fractions."

Students completed all units of instruction and all post-treatment tests on an individualized basis. When an individual student achieved mastery of the unit operating on fractions, he immediately completed the SD post-test followed by the transfer-oriented post-test.

Upon completion of the post-treatment tests, the student began a two-week retention period during which he completed units related to topology, symmetry, area, spatial representation, and volume. No rational number operations were used during the retention period. At the end of the retention period, the student completed the fractions retention test, a parallel form of the fractions pre-test.

The investigation was concerned with testing the following general null hypotheses:

H1: There is no difference with respect to the achievement of computational skill with rational numbers among the three treatment groups.

H2: There is no difference with respect to the mean performance on selected transfer-oriented, rational number related tasks among the three treatment groups.

H3: There is no difference with respect to the retention of skills for performing operations on positive rational numbers among the three treatment groups.

H4: There is no difference with respect to the attitude toward studying operations on positive rational numbers among the three treatment groups.

H5: There is no difference with respect to the rate of the mastery of operations on positive rational numbers among the three treatment groups.

H6: There is no interaction effect when the treatment groups are blocked on the basis of high or low reading ability with respect to the six subtests of the transfer-oriented post-test, the fractions retention test, and the SD attitude instrument.

To test the null hypotheses of the investigation, the following statistical techniques were employed: (1) one-way ANOVA (unequal frequencies), (2) two-way ANOVA (unweighted means analysis), and (3) two-way ANCOVA (unweighted means analysis). Significant F-ratios ($p < .05$) from the one-way ANOVA prompted further examination of all possible ordered pairwise contrasts on the treatment group means using the Newman-Keuls procedure (unequal frequencies). Figure 1 shows the variables being examined in a given contrast. In discussing ordered contrasts on treatment group means, the treatment group having the higher mean is listed first. For example, T3 versus T2 indicates that the T3 group had the higher mean in the comparison of the T3 group mean with the T2 group mean.

Each school was considered a separate experiment and results from each school are reported separately for each criterion measure.

Treatment Description	T1 CAS	T2 CAS Calculators	T3 AAS Calculators
T1 CAS	—	compares effect of calculator under CAS	compares effect of calculator and AAS to CAS without calculator
T2 CAS Calculators	compares effect of calculator under CAS	—	compares effect of AAS v. CAS under calculator
T3 AAS Calculators	compares effect of calculator and AAS to CAS without calculator	compares effect of AAS v. CAS under calculator	—

Fig. 1. Interpretation of Ordered Pairwise Contrasts

RESULTS

Tables 3 and 4 summarize treatment group means and standard deviations for all criterion measures of the investigation.

TABLE 3
TREATMENT GROUP MEANS AND STANDARD DEVIATIONS FOR THE CRITERION MEASURES AT SCHOOL A

Criterion Measure	Possible Score	T1 M	T1 SD	T2 M	T2 SD	T3 M	T3 SD
Operating on Fractions	15	12.30	1.70	12.10	1.60	13.80	1.10
Transfer-Oriented Posttest							
Pt. 1 (general)	5	2.88	1.28	3.19	1.56	2.95	1.27
Pt. 2 (estimation)	5	1.89	1.08	2.69	1.20	2.63	1.01
Pt. 3 (order)	7	2.78	1.70	3.62	1.54	4.79	1.40
Pt. 4 (combining)	5	2.22	1.11	1.94	1.53	3.32	1.45
Pt. 5 (open sent.)	4	2.50	1.10	2.31	1.35	2.32	1.29
Pt. 6 (select. oper.)	5	2.39	0.98	2.62	1.31	2.53	1.43
Fractions Retention Test	20	11.11	4.56	11.75	5.21	17.89	1.97
SD—Studying Fractions	119	75.70	20.40	79.40	23.30	84.20	19.80

TABLE 4
TREATMENT GROUP MEANS AND STANDARD DEVIATIONS FOR THE CRITERION MEASURES AT SCHOOL B

Criterion Measure	Possible Score	T1 M	T1 SD	T2 M	T2 SD	T3 M	T3 SD
Operating on Fractions	15	12.90	2.00	11.70	2.10	13.10	3.1
Transfer-Oriented Posttest							
Pt. 1 (general)	5	3.45	1.23	2.94	1.12	2.75	1.16
Pt. 2 (estimation)	5	2.40	1.05	2.13	1.22	2.42	1.11
Pt. 3 (order)	7	3.70	1.13	3.38	1.54	3.67	2.01
Pt. 4 (combining)	5	2.80	1.70	2.63	1.71	2.75	1.54
Pt. 5 (open sent.)	4	2.65	0.59	3.13	0.81	1.75	1.05
Pt. 6 (select. oper.)	5	2.95	1.10	2.88	0.62	2.33	1.15
Fractions Retention Test	20	14.15	3.96	13.50	5.14	17.25	1.82
SD—Studying Fractions	119	63.50	28.40	77.60	14.70	78.30	25.90

Tables 5 and 6 summarize the results of the statistical analyses of the experiments conducted at the two schools. The tables include F-ratios, the p-value associated with each F-ratio, the significant contrasts associated with a significant F-ratio ($p < .05$), the p-value associated with a given contrast, and the p-value associated with the test for interaction effects.

Findings Pertaining to H1.

Mean treatment group performance with respect to the individual student's first-attempted form of the test-worksheet for the unit, operating on fractions, was analyzed to provide evidence of achievement of computational skill with positive rational numbers. Results obtained at School A supported rejection ($p < .01$) of the null hypothesis of no difference in treatment group means. The results favored the T3 group. Consequently, the contrasts T3 v. T2 and T3 v. T1 were significant at the .01 level.

TABLE 5

SUMMARY OF ACHIEVEMENT AND ATTITUDE TEST RESULTS AT SCHOOL A

	Treatment Effects				Interaction	
Criterion Measure	F	p	Sig. Contr.	p	F	p
Operating on Fractions	7.08	$p < .01$	T3 v. T2	$p < .01$		
				$p < .01$		
Transfer-Oriented Posttest						
Pt. 1 (general)	0.70	$p > .25$	—	—	0.64	$p > .25$
Pt. 2 (estimation)	3.27	$p < .05$	T2 v. T1	$p < .05$	0.26	$p > .25$
Pt. 3 (order)	7.57	$p < .01$	T3 v. T1	$p < .01$	0.49	$p > .25$
			T3 v. T2	$p < .05$		
Pt. 4 (combining)	5.06	$p < .05$	T3 v. T2	$p < .05$	0.28	$p > .25$
			T3 v. T1	$p < .05$		
Pt. 5 (open sent.)	0.82	$p > .25$	—	—	1.12	$p > .25$
Pt. 6 (select oper.)	0.15	$p > .25$	—	—	0.95	$p > .25$
Fractions Retention Test	15.52	$p < .01$	T3 v. T1	$p < .01$	0.21	$p > .25$
			T3 v. T2	$p < .01$		
SD—Studying Fractions	0.23	$p > .25$	—	—	0.65	$p > .25$

TABLE 6

SUMMARY OF ACHIEVEMENT AND ATTITUDE TEST RESULTS AT SCHOOL B

	Treatment Effects				Interaction	
Criterion Measure	F	p	Sig. Contr.	p	F	p
Operating on Fractions	1.59	$p < .25$	—	—	N/A	
Transfer-Oriented Posttest						
Pt. 1 (general)	1.52	$p < .25$	—	—	2.35	$p < .25$
Pt. 2 (estimation)	0.56	$p > .25$	—	—	0.39	$p > .25$
Pt. 3 (order)	0.23	$p > .25$	—	—	0.08	$p > .25$
Pt. 4 (combining)	0.05	$p > .25$	—	—	0.03	$p > .25$
Pt. 5 (open sent.)	10.34	$p < .01$	T2 v. T3	$p < .01$	0.59	$p > .25$
			T1 v. T3	$p < .01$		
Pt. 6 (select oper.)	1.61	$p < .25$	—	—	0.24	$p > .25$
Fractions Retention Test	3.30	$p < .05$	T3 v. T2	$p < .05$	0.09	$p > .25$
			T3 v. T1	$p < .05$		
SD—Studying Fractions	0.88	$p < .25$	—	—	1.70	$p < .25$

Results obtained at School B with respect to the individual student's first-attempted form of the test/worksheet for the unit, operating on fractions, did not support rejection of the null hypothesis of no difference in treatment group means.

Findings Pertaining to H2 and H6.

Results obtained at School A on the transfer-oriented post-test supported rejection of the null hypothesis of no difference in treatment group means with respect to Part 2 (estimation of fractional values), Part 3 (ordering rational numbers), and Part 4 (combining rational numbers involving more than two operands or more than one operation). In all instances, the results favored the T3 group. The only significant ($p < .05$) contrast on Part 2 was T3 v. T1. The contrasts T3 v. T1 and T3 v. T2 were significant at the .01 level on Part 3. Part 4 yielded treatment-group means that were significantly different at the .05 level for the contrasts T3 v. T2 and T3 v. T1. The tests for reading ability level by treatment interaction effects showed no significant effects on any of the six parts of the transfer-oriented post-test at School A.

Results obtained at School B on the transfer-oriented post-test supported rejection of the null hypothesis of no difference in treatment group means only for Part 5 (solving open sentences involving rational numbers). The T2 group was favored at the .01 level of significance. Pairwise comparisons of the treatment group means showed the contrasts T2 v. T3, T1 v. T3, and T2 v. T1 all significant at the .01 level. The tests for reading ability level by treatment interaction effects indicated no significant effects at School B.

Findings Pertaining to H3 and H6.

The results obtained at School A on the fractions retention test supported rejection ($p < .01$) of the null hypothesis of no difference in treatment group means. The results favored the T3 group. The contrasts T3 v. T2 and T3 v. T1 were significant at the .01 level. Interaction effects were not significant. An additional result of the fractions retention test was the proportion of students in each treatment group at School A who retained the mastery level (80 percent correct) performance following the two-week retention period. The proportions were 5/18 for the T1 group, 5/16 for the T2 group, and 16/19 for the T3 group. These results were subjected to no further statistical analysis, but clearly favor the T3 group.

Results obtained at School B with respect to the fractions retention test supported rejection of the null hypothesis ($p < .05$) of no difference in treatment group means. The differences favored the T3 group. The contrasts T3 v. T2 and T3 v. T1 were significant at the .05 level. The data did not support rejection of the null hypothesis of no reading ability level by treatment interaction effects. An additional result of the fractions retention test was the proportion of students in each treatment group at School B who had retained mastery level performance (80 percent correct) following the two-week retention period. The proportions were 9/20 in the T1 group, 7/16 in the T2 group, and 11/12 in the T3 group. These results were subjected to no further statistical analysis.

Findings Pertaining to H4 and H6.

Results on the semantic differential attitude instrument at School A did not support rejection of the null hypothesis of no difference in treatment group mean scores. However, according to test results all three treatment groups exhibited a positive attitude toward mathematics. The data at School A also did not support rejection of the null hypothesis of no reading ability level by treatment interaction effect with respect to the SD instrument.

Results obtained from attitude testing at School B indicated no significant difference in treatment group mean scores of attitude. In addition, the data did not support rejection of the null hypothesis of no reading ability level by treatment interaction effects.

Findings Pertaining to H5.

As one means of determining the relative efficiency of the three treatments, the rate of mastery of operations on positive rational numbers by each of the three treatment groups was examined descriptively for School A. The results showed that the mean time required by the T1 group to complete the experimental treatment was 6.2 days; the mean time required by the T2 group to complete the experimental treatment was 7.7 days; and the mean time required by the T3 group to complete the experimental treatment was 6.3 days. These mean numbers of days include instructional time for calculator operation in the T2 and T3 groups, and a unit on changing fractions and mixed numbers to decimals in the T3 group. Calculation of the mean number of days minus calculator-related instruction revealed that the T1 group required 6.3 days, the T2 group required 6.3 days, and the T3 group required 4.8 days.

No data on time to complete the experimental treatments was available from School B.

LIMITATIONS OF THE INVESTIGATION

Certain limitations more or less specific to this investigation should be recognized:

1. Fifteen calculators were used at each school. This number was probably appreciably larger than the number available at many schools.

2. The six subtests of the transfer-oriented post-test contained relatively small numbers of items (from four to seven), with no reliability estimates determined for the separate subtests.

3. Because the Newman-Keuls procedure keeps a constant level of significance for all ordered pairs of contrasts, the power of the collection of all tests conducted is less than for a single F-test, thus increasing the likelihood of committing a Type II error.

CONCLUSIONS

The experimental results at the two schools involved in the investigation suggest the following conclusions subject to the limitations just identified:

1. When development of computational skill with positive rational numbers is a goal of instruction, the alternative algorithm set with the calculator appears to be a viable alternative to the conventional algorithm set with or without the calculator for low-ability or low-achieving students in the ninth-grade general mathematics.

2. When development of computational skill with positive rational numbers via the conventional algorithm set is a goal of instruction, use of a calculator does not significantly and consistently affect performance.

3. The alternative algorithm set with the electronic calculator can produce success for students in dealing with semi-novel problem situations such as estimation of fractional values, ordering rational numbers, and simplifying rational numbers involving more than two operands or more than one operation.

4. When a goal of instruction is retention of skill for performing operations on positive rational numbers in ninth-grade general mathematics, use of the alternative algorithm set with the electronic calculator can promote superior performance when compared to use of the conventional algorithm set either with or without the electronic calculator.

5. When development of a positive attitude toward mathematics is an instructional goal, no one of these is more effective than the other: use of the conventional algorithm set without the electronic calculator, use of the conventional algorithm set with the electronic calculator, or use of the alternative algorithm set with the electronic calculator.

6. When deployment of the conventional algorithm set with or without the electronic calculator or the alternative algorithm set with the electronic calculator depends on the relative efficiency of the two algorithm sets in promoting computational skill and retention of skill, the alternative algorithm set with the calculator is apparently more efficient in terms of the rate of mastery, the performance on the individual student's first-attempted form of the test/worksheets for each unit, and the proportion of students retaining mastery-level performance two weeks after termination of instruction.

DISCUSSION

In today's technological society, where sophisticated electronic processing media are becoming available to the "average" American citizen, teachers with low-ability or low-achieving children should consider a machine-based curriculum for students unable to master the skills necessary for learning practical mathematical concepts by conventional procedures. The opinion is held by some mathematics educators that the advent of a machine-based curriculum would lead to the creation of machine-dependent learners. However, the students to whom this investigation was directed were, typically, youngsters who had repeatedly demonstrated their inability to attain and retain the skills that the experimental treatments were attempting to develop.

Therefore, one implication of the study is clearly indicated: the alternative algorithm set tested in this investigation provides a means by which low-ability and low-achieving children can compute with rational numbers. Students are able to apply their learned skills in semi-novel situations and they are able to retain the skill to a significantly greater degree than students of like ability using conventional algorithms.

The results of the investigation indicated that the groups using the alternative algorithm set with the electronic calculator exhibited transfer of skill for operating on positive rational numbers to such areas as (1) ordering rational numbers and (2) combining rational numbers involving more than two operands or more than one operation to a significantly greater extent than did the groups using the conventional algorithm set. This suggests a step in the direction of making the study of rational number operations applicable to further study of mathematics appropriate for slow-learning children. If further research shows the alternative algorithm set to indeed have applicability to a wide range of areas of mathematics appropriate to low-ability or low achieving children, then the likelihood of its acceptance by mathematics educators will be increased.

It must be remembered that the alternative algorithmic approach with the electronic calculator was directed only toward low-ability or low-achieving children. Mathematics teachers must consider the future educational plans of their students before adopting such a machine-oriented approach to performing operations on positive rational numbers. If a particular group of students is capable of studying high school algebra to a relatively high degree of sophistication, then the alternative algorithmic approach should not be stressed (or possibly not used at all) in light of its impracticality in solving equations and in combining or simplifying algebraic fractions. But, if the study of computation-related or computation-dependent topics represent the future mathematics of the children, the alternative algorithm set and the electronic calculator are worthy course inclusions.

Hopefully, additional uses of the electronic calculator in the general mathematics classroom will be developed. It is logical to propose that machine use could allow some aspects of many topics to be included in the curriculum that currently are omitted from the general mathematics course because of arduous computation. Selected material from topics such as estimation, area, volume, maxima-minima, ratio-proportion, probability and statistics, conversion, trigonometry (numerical), linear interpolation, sequences and series, and evaluation of polynominal expressions might be included in courses for students classified as low-ability or low-achieving.

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THE EFFECT OF THE USE OF DESK CALCULATORS ON ATTITUDE AND ACHIEVEMENT WITH LOW-ACHIEVING NINTH GRADERS

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There appear to be two important reasons for the use of calculators with low-achieving students. One is to provide motivation, and the other is to improve achievement. The study discussed here addressed itself to the effect of the calculator in these two areas. In particular, this experimental study tested the following three hypotheses:

1. The use of calculators in the instructional program with ninth-grade, low-achieving mathematics students improves their attitude toward the study of mathematics.

2. The use of calculators in the instructional program with ninth-grade, low-achieving mathematics students improves their computational skills.

3. Ninth-grade, low-achieving mathematics students can compute better with calculators than without calculators.

SOURCE OF DATA

This study was conducted at the Maine Township (East) High School in Park Ridge, Illinois. The school is in a suburban area with students from the middle and upper-middle economic brackets and is highly regarded in the state as having excellent facilities and an outstanding staff and curriculum. The school used calculators in about half the General Mathematics classes during the 1968-69 school year. For the 1969-70 school year the General Mathematics students (about a hundred of them) were scheduled into five classes of about 20 each in a random manner. This scheduling of mathematics classes was done by a computer before other classes were scheduled.

The students were placed in General Mathematics on the basis of their IQ scores and standardized achievement-test scores. These students had IQ scores of about 75 to 95 and had scored two years or more below grade level on achievement tests.

EXPERIMENTAL DESIGN AND PROCEDURE

Two experienced teachers taught the four classes used in the experiment. Each teacher had one control group and one experimental group; so the teacher variable was held relatively constant. The choice of which class was the experimental group and which was the control group was made by a flip of a coin. Lesson plans were developed during the summer prior to the fall semester by a teacher who had previously taught a class using the calculators. These lesson plans were used by both teachers for both groups in order to control the instructional activities, assignments, and time for both groups. One variable in instruction was that each experimental group, which used calculators, received four days of additional instruction time dealing with the operation of the calculator. Both the experimental and the control groups had seven weeks of instruction (45 minutes a day) dealing with addition, subtraction, multiplication, and division of whole numbers. The experimental groups differed from the control groups in that they used the calculators to verify paper and pencil computation; the students in an experimental group were told to check their work by using the calculators, while those in a control group were told merely to check their work.

The teachers encouraged students in the experimental groups to check each problem as soon as they completed it but did not force them to do so. This was done in order to reduce the time lapse between response and reinforcement. The teachers indicated that the students did follow this suggestion willingly.

The lesson plans utilized worksheet materials, which were written by teachers in the district several years ago, and the textbook *Trouble Shooting Mathematics Skills* (Bernstein and Wells 1963). The lesson plans also included specific instructions on the administration of the pre-test and the post-test to ensure that all students received the same instructions and were given the same time to complete the tests.

Every Monday morning before school started, the investigator met with the two teachers in the experiment to discuss the lesson plans for that week. This was done to ensure that both teachers followed the experimental design.

All groups were given pre-tests and post-tests that measured attitude toward mathematics and computational skills with whole numbers. The attitude test that was used as the pre-test and post-test was the PY 011 Pro-Math Composite Test, developed by the SMSG for its longitudinal study (Wilson, Cahen, and Begle 1968, p.183). The pre-test and post-test used to measure computational skills with whole numbers was the Stanford Diagnostic Arithmetic Test, test 2, parts A, B, and C (Beatty, Madden, and Gardner 1966, pp.4-5). This test has two forms, W and X. Form W was used as the pre-test and post-test with both the experimental and the control groups to deal with the hypothesis that the use of calculators in the instructional program increases computational skills. In addition, Form X was used as a post-test with the experimental groups to deal with the hypothesis that these students can compute better with calculators than without calculators.

The attitude pre-test was used to verify the comparability of both kinds of groups. The attitude post-test was used to determine the attitude of both after the seven-week experimental period. The t test for the difference between the mean scores of both was used, with the five percent level of confidence as the criterion for rejecting the null hypothesis.

The computation pre-test and post-test were used to determine the change in computational skill for each student in both kinds of groups. The change, or difference, in the scores for each student was determined, as was the mean and variance for these differences for the experimental group and the control group. The t test for the differences between these mean differences was used, with the five percent level of confidence as the criterion for rejecting the null hypothesis.

To deal with the hypothesis that these students can compute better with calculators than without them, the comparable parts of Form X of the Stanford Diagnostic Arithmetic Test were administered to the experimental groups the day after Form W was completed. Whereas the students were not allowed to use the calculators for form W, they were told they could use the calculators to get results for the test items when taking Form X. The mean and variance were computed for the distribution of the Form-X scores. The t-test for the difference between the means of the Form-W and the Form-X distributions was used, with the five percent level of confidence as the criterion for rejecting the null hypothesis.

FINDINGS AND CONCLUSIONS

Analysis of the attitude-pre-test scores, as shown in Table 1, does not support the view that the experimental and control goups were different. Analysis of the attitude-post-test scores, as shown in Table 2, does not support the hypothesis that the use of the calculators in the instructional program with ninth-grade, low-achieving mathematics students improves their attitude toward mathematics.

TABLE 1
PRETEST SCORES ON ATTITUDE TEST

Mean of Control Group	31.18
Variance of Control Group	33.50
Mean of Experimental Group	31.76
Variance of Experimental Group	23.50
t Score	.49
Significance Level	31%

TABLE 2
POSTTEST SCORES ON ATTITUDE TEST

Mean of Control Group	32.85
Variance of Control Group	30.10
Mean of Experimental Group	32.07
Variance of Experimental Group	53.62
t Score	.53
Significance Level	30%

Analysis of the test scores on the two forms of the computational-skills test, as shown in Table 3, does not support the hypothesis that the use of calculators in the instructional program with ninth-grade, low-achieving mathematics students improves their computational skills.

Analysis of the scores on the two forms of the post-test on computational skills, as shown in Table 4, does support the hypothesis that ninth-grade, low-achieving mathematics students can compute better with calculators than without them.

GENERAL COMMENTS

Low-achieving, ninth-grade students are subjected to a host of social, academic, and physiological pressures. These shape their attitudes toward the study of mathematics and are monumental when compared to the salutary effects the use of the calculator for a short time may have in improving their attitudes. In this study a seven-week experience did not have a significant effect. However, given enough time, the use of the calculator may improve attitudes toward mathematics.

TABLE 3
DIFFERENCES BETWEEN PRETEST AND POSTTEST SCORES ON COMPUTATION TEST

Control Group	Experimental Group
−2	15
−12	−1
3	10
−3	7
2	4
12	15
3	12
21	8
7	6
17	11
2	6
9	13
−1	7
11	8
−2	5
8	1
3	7
13	5
13	11
14	10
9	9
5	2
9	16
10	17
16	13
8	21
8	18
17	11
13	3
2	6
3	4
8	3
21	−2
19	7
20	6
19	0
4	6
6	1
8	13
17	2
	8

Mean of Control Group	8.50
Variance of Control Group	55.10
Mean of Experimental Group	7.90
Variance of Experimental Group	28.72
t Score	.42
Significance Level	34%

TABLE 4
POSTTEST SCORES ON COMPUTATION TEST BY EXPERIMENTAL GROUP

Student	Form W	Form X
1	43	52
2	25	48
3	37	48
4	49	52
5	37	53
6	40	55
7	36	48
8	39	49
9	43	52
10	49	39
11	49	55
12	49	55
13	43	48
14	38	53
15	35	52
16	30	38
17	42	53
18	51	50
19	47	56
20	53	32
21	40	49
22	35	37
23	50	53
24	42	35
25	40	44
26	40	44
27	46	54
28	48	53
29	34	40
30	20	37
31	49	54
32	41	47
33	27	38
34	48	54
35	55	56
36	45	50
37	47	56
38	32	56
39	32	43
40	22	48
41	45	55

Mean (Form W)	40.80
Variance (Form W)	68.60
Mean (Form X)	48.56
Variance (Form X)	44.30
t Score	5.92
Significance Level Below	1%.

The calculator was used as an aid for computation in the instructional program of most of the studies reviewed by this investigator and was so used in this experiment. It was not used to build insights into the understanding of mathematical principles. This experiment, as well as those experiments found in the literature, indicates that the use of the calculator as a means to improve computational skills through reinforcement is not effective when so used for less than a year. What effects its use may have when used over a number of years is unknown. However, schools cannot expect the use of calculators to have results significantly better than the results of conventional means to improve computational skills when calculators are used only in the General Mathematics class in the ninth grade. It is the investigator's opinion that the use of calculators to improve computational skills is strategically unsound, for its impact, if it has any, is too light for the time it has to operate. Though the calculator may be of no value in improving computational skills, this does not preclude its desirability when other objectives are being sought. An objective such as the ability to solve meaningful problems *possibly,* in some cases, can be achieved only with the help of a calculating device such as a calculator or computer; for the complexity of computation may make some problems beyond the capabilities of the student.

Perhaps the one objective that the calculator *may* be useful in achieving is the objective that students should understand mathematics. The calculator *might* be used effectively to illustrate some mathematical principles. In such circumstances the calculator is a teaching device, just as models and graphic representations of mathematical ideas are. However, its effectiveness in achieving this objective has not been tested, nor are there easily available instructional materials or software designed to achieve this objective through the use of calculators.

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A POCKET CALCULATOR EXPERIMENT WITH FIFTH- AND SIXTH-GRADERS

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INTRODUCTION

The effectiveness of pocket calculators as a teaching tool is in question. Hard data is needed to determine to what degree these devices are useful. Opinions are strong on each side of the question with subjective appraisals ranging from "the panacea" to "the ruination of science." To help clarify the situation, an experiment was undertaken to determine the effectiveness of calculators with below average arithmetic achievers in the fifth and sixth grades. With a small experiment it was determined that the pocket calculator used in a specific manner can be effective.

THE EXPERIMENT

Twelve students from Lincoln Middle School of Pullman, Washington, were selected for treatment with pocket calculators, and another 12 were matched with the treatment group to be used as control. The selection was made on the basis of scores on the Iowa Test of Basic Skills administered during October, 1974. Each subject was judged at least a year behind grade level, with one exception each in the experimental group and in the control group. Twenty 40-minute classes were devoted to the instruction of the experimental group. Each student in the experimental group was supplied with a Minuteman 3-M calculator. Since funds were not available to employ the recognized master teacher, the authors conducted the 20 periods of instruction for the experimental group. The control group received their regular classroom instruction.

The types of activities in which the pocket calculators were utilized were:

1. Calisthenics - drill on the basic multiplication, division and subtraction facts using the calculator for immediate correction and/or reinforcement.

2. Repairing misconcepts on algorithms - borrowing across zeros, estimating divisors, and placement of partial products.

3. Equivalent fractions - fractions were defined to be equivalent if numerator divided by denominator agreed to the eight places on the calculator.

4. Multiplication and addition concepts - calculate the number of tiles on the floor of the school building; calculate the number of holes in the acoustical tiles on the ceiling of the room, and the number of mailboxes in the array.

5. Consumer usage - a visit to a grocery store where the students calculated the price of all the potato chips, cans of peaches, and bags of flour.

6. Area and perimeter - find the perimeter and area of the playfield. Find the number of square metres of glass in this building; find the number of metres of aluminum trim necessary to trim the windows.

7. Self-correction of work - does the calculator agree with your answer? If it doesn't, you have likely made a mistake. Check your work and find your mistake. (This alleviates the problem when students give random answers just to finish an assignment.)

8. Averages - what is the average height and weight of the people in this class?

9. Reading blueprints - some measurements have been left out of the design. Fill in the missing numbers. How far is it from point A to point C?

In February the 24 students were administered the Iowa Test of Basic Skills as a post-test. During October, 1975 they took the regularly scheduled Iowa Test of Basic Skills with their class as an additional time-lag post-test. The data from these tests was analyzed. No additional in-school treatment was given the students by the authors. Two students in the experimental group failed to return to school in September, 1975 so data concerning their status in October, 1975 is not available.

Table 1: Raw Scores and Means

	Experimental			Control		
	Oct. 1974	Feb. 1975	Oct. 1975	Oct. 1974	Feb. 1975	Oct. 1975
			5th Grade			
1	3.6	4.7	-	3.6	5.9	6.2
2	3.9	5.4	-	3.8	4.4	4.8
3	4.0	4.8	5.4	4.1	4.8	5.6
4	4.1	6.6	6.4	3.9	6.1	6.0
5	4.2	5.4	5.7	4.2	4.2	5.2
6	4.4	6.4	7.6	4.4	6.4	6.2
			6th Grade			
1	3.8	5.2	4.2	4.0	5.4	4.7
2	3.9	4.8	6.0	4.0	5.2	6.0
3	4.4	6.0	6.9	4.5	4.9	6.3
4	4.6	5.6	6.8	4.6	4.9	5.8
5	4.8	6.8	7.5	4.8	7.2	7.4
6	5.0	6.0	5.6	5.0	6.7	6.4

				$\underline{N}$			
$\bar{X}_1$	4.22	5.64	-	12	4.24	5.51	5.88
$\bar{X}_2$	4.31	5.76	6.21	10	4.35	5.58	5.96

Table 2: Grade Level Gain

	Experimental				Control		
	0-74 F-75	F-75 0-75	0-74 0-75		0-74 F-75	F-75 0-75	0-74 F-75
				<u>5th Grade</u>			
1	1.1	-	-		2.3	0.3	2.6
2	1.5	-	-		0.6	0.4	1.0
3	0.8	0.6	1.4		0.7	0.8	1.5
4	2.5	-0.2	2.3		2.2	-0.1	2.1
5	1.2	0.3	1.5		0.0	1.0	1.0
6	2.0	1.2	3.2		2.0	-0.2	1.8
				<u>6th Grade</u>			
1	1.4	-1.0	.4		1.4	-0.7	.7
2	0.9	1.2	2.1		1.2	0.8	2.0
3	1.6	0.9	2.5		0.4	1.4	1.8
4	1.0	1.2	2.2		0.3	0.9	1.2
5	2.0	0.7	2.7		2.4	0.2	2.6
6	1.0	-0.4	.6		1.7	-0.3	1.4
				$\underline{N}$			
$\bar{X}_1$	1.42	-	-	12	1.27	.38	1.64
$\bar{X}_2$	1.44	.45	1.89	10	1.23	.38	1.61

Using the paired observation technique to analyze data for the interval from October, 1974 to February, 1975 the grade level gain yields a t value of 0.678 for the 12 students and 0.971 for the 10 students. The first t score is significant at the 0.275 level and the second t score is significant at the 0.20 level.

From February, 1975 to October, 1975 the grade level gain yields a t value of 0.398 significant at the 0.36 level. In the combined grade level gain from October, 1974 to October, 1975 the paired observations yield a t value of 1.2522 significant at the 0.13 level. The complete data from the October, 1974, and October, 1975 ITBS shows that fifth grade students at this school averaged 0.9 years of gain and the sixth-graders averaged 1.5 years of gain. Since data is available on only four fifth-graders and six sixth-graders, the weighted mean of 1.26 years of growth was used between the two test periods. The corresponding t score is 2.2141 which is significant at the 0.03 level.

Each pairing was sex-consistent. In the October to February test interval, the four girls were exactly even with their control on grade level gain so the margin of gain was entirely attributable to the eight boys. The paired observation on the eight boys gave a t value of 1.043, significant at the 0.20 level. The girls in the control group outperformed the girls in the experimental group from February, 1975 to October, 1975, and the boys accounted for more than the slight edge held by the experimental group. In the combined total the boys produced most of the advantage and had a t value of 1.2026 which is significant at the 0.15 level. Compared to the class average grade level gain the seven boys produced a t value of 2.41815 which is significant at the 0.03 level.

COMMENTS

Although the results are quite impressive, several points of caution should be noted:

1. The sample size was small and consisted of students from one school in one small community.

2. The control group scored considerably higher than class average on growth (significant at the 0.05 level). This may have had something to do with removal of half of the slower students from a class giving the teacher more time to concentrate on the others who were having some difficulties.

3. Other programs for slow learners were in progress and may have been having some effect on the observations.

4. The instruction was conducted by two persons with more mathematical training than the usual fifth grade teacher, but with less experience at this grade level.

CONCLUSIONS

It appears that using pocket calculators in the manner of this experiment doesn't seriously erode the basic mathematical skills of the children. The calculators may be more effective with boys than with girls. Possibly the boys derive more excitement out of this mechanical toy than do girls. Fifth-graders outperformed sixth-graders. There was less regression between February and October than expected. It seems as though once started, the students continued to grow without further treatment.

PART FOUR

Activities - Junior and Senior High

USING ELECTRONIC CALCULATORS

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This article, which appeared in Mathematics Teaching No. 76 (September 1976), the journal of the Association of Teachers of Mathematics, has been reprinted with the permission of Editor David S. Fielker.

In the summer of 1975 Laurie Buxton, Staff Inspector for Mathematics for the Inner London Education Authority, obtained some development money to purchase 200 simple electronic calculators to use in a year's experiment in schools. The machine chosen was the *Citizen 120R*, then the current recommendation to London schools as a four-function machine.

One hundred machines were given to a secondary school to be shared by the mathematics, science, commerce, geography and technology departments. Twenty were given to each of four primary schools.

The aim of the experiment was to see what schools could do with a number of machines that they would not normally buy for themselves, and we were fairly open-minded about the results, being guided mainly by the idea that since calculators were around, they could not be ignored, and we wanted to see what benefit teachers and children could obtain from them, with a reasonable amount of assistance from the inspectorate, the local mathematics center, and the local college of education.

The informal report which follows was published locally in December 1975, as a summary of the first term's developments, and, it was hoped, as a stimulus to the schools involved.

GETTING STARTED

For various reasons, things took some time to get off the ground (or out of the boxes). The secondary school was, rightly, security-conscious about their large number of machines; each had to have a reference number painted on, and wooden boxes were made, each holding just 10 or 15 machines, so that they could be transported easily and quickly checked back. Initial charging up was even more complicated than subsequent recharging. The physics department wired five chargers into each plug. One school's 20 were plugged in at the Mathematics Centre over a weekend. At another I helped the head unpack machines and plug in two in each classroom.

Distribution was easier to plan. Most of the primary schools decided to use machines with their older children, but one thought it would be interesting to have one or two in all classes, including top infants.

After that, it was a case of getting the machines into the hands of children. As with much other apparatus, teachers sometimes felt that they had to be experts themselves before they could teach the children. In fact, when they were persuaded just to give them out without instructions, they found that in about half an hour most 10-year-olds had discovered how to perform all four rules and perhaps other operations as well. (Perhaps it takes longer with older children; a sixth form commerce group who had received careful instructions from their teacher were very unsure about what to do!) The *Citizen* machines, wisely chosen by the Electronic Calculator Sub-Committee of the I.L.E.A. Maths Advisory Panel, lent themselves ideally to discovery since the order in which one pressed the buttons corresponds exactly to what one would write.

INITIAL ACTIVITIES

Ten-year-olds, once they had found out how to operate, soon found things to use the machine for. A pair of boys tested each other on questions like 7 x 5 and 8 + 23. Two girls were checking some subtractions that they had done in their books; strangely, one was doing it by calculating one column at a time, so that for 65 - 23 she would use the machine for 5 - 3 and 6 - 2 separately, and if decomposition was necessary, she did that on paper first! "What year were you born?" asked Victor, and before I had remembered to add on 10 years, he had subtracted it from 1975 and told me my age; then he told me his teacher's age!

However, there were some interesting problems about comprehension, which perhaps had not come to light before in the context of paper and pencil calculations:

- A girl wanted 9 - 7, but pressed buttons for 7 - 9, and complained that the answer was wrong. This prompted a discussion with the teacher.

- Nicola and Louis had calculated 700 ÷ 400 and were worried about the answer. I asked them to do it on paper. It went in once, they decided after some time, and after more discussion they thought that there would be 300 left over, but they were not sure how to write it down. They wrote "700 ÷ 400 1300." We returned to the answer on the machine, 1.75, which they read as "one hundred and seventy-five." I indicated the decimal point. "Oh," they said, "one *point* seventy-five." They didn't know what that meant. I asked them to calculate 800 ÷ 400. "Two point," read Nicola.

- Julia and Andrea were trying to find the highest number. They had entered "111111" and kept pressing the "=" button, which previously had kept adding 111111 but now didn't work. I asked what the highest number was. "I used to know but I've forgotten," said Julia.

- "What's the smallest number?" I asked. "A half," said Andrea. "That's hard," said Julia; "... a quarter ... half of a quarter ... a quarter of a quarter."

- "What's half of 16006?" asked Paul. "8003," I replied. "Thank you," he said! I asked him how he found a half of something. "I don't know," he said; "find half of it and take it away." I asked him how he had halved 16006. "Half of 16 is 8 and half of 6 is 3."

- "What are seven eights?" I asked. The boy used the machine to produce 56. "What are eight sevens?" "56," he said straight away. "Why?" "Because it's the same." "What's 56 ÷ 7?" "Can I do it on the machine?" "If you wish." He used the machine for 56 ÷ 7 = 8, and then for 56 ÷ 8 = 7.

It seemed that all these examples revealed some misunderstanding about arithmetic, partly because the children chose the questions, but mainly because the important part of the work was not in *getting the answer*, as is usual with paper and pencil calculations, but in *framing the question* and *interpreting* the answer. Obviously an earlier understanding of decimals is going to be desirable, but notice Nicola and Louise's incomplete concept of division, which they always referred to as 'sharing', and how this relates to Paul's efforts at halving.

Other activities raised some different problems.

- Simon had entered 78 - 456 and the machine registered '-378'. He thought the answer should be "nought." I asked him to try 1 - 2 and 1 - 3, which produced respectively '-1' and '-2'. Victor thought it might mean "difference." I asked what he would expect from 1 - 4, and he said "a difference of 3."

- "Give me a sum," said Stephen. "Three take away seven," I said. The machine answered '-4'. "Four," said Stephen. "What's this in front of it?" I asked. Stephen didn't know. "What's seven take away three?" "Four." "What's three take away seven?" "You can't do it." "But you *did* it," I said! Stephen was puzzled.

Other children also came across negative numbers in this way, usually on their own because no teacher normally asks 'impossible' questions! Maybe we can teach negative numbers earlier, and perhaps they are easier to teach than fractions. The connection between fractions and division and decimal notation, however, rears its ugly head all too easily.

"4 ÷ 4 = 1," said Victor. "What's 4 ÷ 8?" I asked. Victor was mystified by the answer '0.5'.

Compare the activity of three bright 10-year-olds. They found that pressing 'x' and then '=' squared the number entered, so '3 x=' produced 9, '5 x=' produced 25 and '8.5 x=' produced 72.25. "What must you square to get 10?" I asked. They knew it would be between 3 and 4, and because it would be nearer 3, they tried 3.3, which gave 10.89. 3.2 gave 10.24; 3.1 gave 9.61. So it must be between 3.1 and 3.2 They tried 3.15! This gave 9.9225. 3.16 gave 9.9856 and 3.17 gave 10.0489, so the next thing to try was 3.165. They continued in this way, watching their answers get closer to 10, and one was able to say, for instance, that 9.998244 was closer than 10.004569. Note the exceptional understanding of decimal notation as well as the perfect systematic approach.

By contrast some 13-year-olds were having far more trouble. Some of the class were finding the square root of 63, and no one seemed to have discovered the squaring facility on the machine so they were having to enter a number twice, for example '7.9 x 7.9' rather than '7.9 x='. One knew it should be "seven point nine something," but multiplied 7.9 successively by 6.3, 9.4, 7.9, 9.8 and 8.7. Those who were squaring numbers seemed to do so at random. (One sequence was 7.98, 7.96, 7.95, 7.98, 7.943, 7.912, 7.901, 7.911, 7.900.) "Have you got to start with seven point?" asked a girl; "You ain't gotta," was the reply, "but that's the way to do it."

The less able in the class had been asked to calculate things like 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 and 6 x 8; they were surprised that these gave the same answer! I entered '8' on one machine and was told it was 8 units; '85' was 8 tens and 5 units; '855' was 8 hundreds 5 tens and 5 units; '855.9' was 8 thousand ... Other simple activities, primary and secondary, have included building up tables by repeated addition, and multiplying numbers by 10 or 100.

Teachers have commented on the stimulation given by the machines, the increase in confidence achieved by the less able, some reluctance to use them by the more able, and the implicit faith in them some children seem to have, even when the answers are wrong!

OUTSIDE MATHEMATICS

The sixth form girls taking commerce seemed to be getting least benefit. As was mentioned above, they were virtually taught by rote, and, perhaps in consequence, were far less confident about their use than primary children. Although the machines stimulated some interest, the girls preferred to work on paper "where you can see what's going on," and some of them expressed some worry about their availability when they started work!

Other subjects seemed very satisfied that the difficulties and wasted time caused by calculation had been removed, so that ideas and principles could more easily be discussed. In engineering technology, third year boys could discuss gear ratios and levers without being held up doing the arithmetic, and since this subject was working toward a mode iii C.S.E., the use of machines could be allowed in the examination. In geography, second year children calculated average rainfall and temperature; the geography teacher used to bring in his own calculator because, except with the most able children, calculations on paper took much too long and killed off all interest in the geography. Fourth and fifth years doing urban geography were able to calculate percentage increases in house prices in Reading, Chippenham, and Bristol to investigate the effect of opening the M4 Motorway, a task virtually impossible without machines.

In science there was enormous benefit, particularly in physics. Previously a whole lesson on density could be taken up calculating the density of one material; now the densities of many different materials could be calculated on the machines. There was increased confidence, and much more time to spend on the science. However, there were still worries about other mathematical ideas, the concepts of multiplication and division, area and volume, and particularly ratio. In other words, it is still important to understand, and the relief from the tedium of calculation seems to uncover the misunderstandings and throw them into sharper relief. One wonders in particular whether ideas like area and volume are more appropriately dealt with in mathematics or in science, and I discussed with the head of the faculty the difference between the ways ratio is dealt with in the two subjects. It is worth noting, however, that the removal of arithmetical difficulties has enabled these discussions to take place.

WHAT NEXT?

In mathematics, once the initial enthusiasm has been used up, the children know how to use their machines, everyone has tackled the square root problem, and both pupils and teachers have run out of ideas, there seems to be a sudden vacuum. Teachers are then reluctant to interfere with their planned schedules by taking in the machines, and, in any case, they have asked, should there be some worksheets to go with them? (One secondary teacher was using a hastily prepared worksheet to test a third year class's use of the machines, based on old-fashioned primary arithmetic, which raised some interesting problems about how many weeks there were in a month!)

There are several types of activity which could be undertaken.

1. If possible, machines should always be available so that whenever calculation is necessary in some other work, they can be used to do or to check it. This raises problems about exercises in mechanical arithmetic, and it may be interesting to see whether children are willing to try these on paper first and check them afterwards. We have not yet explored enough how much arithmetic children actually learn from using the machines, and teachers are naturally worried about any possible deterioration in arithmetical abilities. We also have to investigate what help machines can be toward understanding whatever is left when the actual burden of calculation is removed, as opposed to the help they can be, as illustrated earlier, in revealing what understanding is missing!

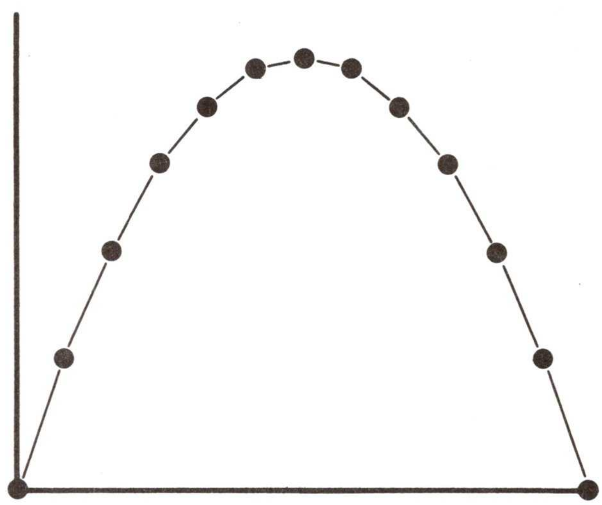
2. The practical ideas promoted by Edith Biggs 10 or more years ago do not seem so prevalent these days. One possible reason is that many of the activities in 'weighing and measuring' or statistics often produced numbers that were too unwieldy to deal with, and this either required drastic approximation, or the teacher eventually resorted to artificial examples with easy numbers that 'worked out'.

   Now, however, a combination of metrication and electronic calculators makes it possible to go back to practical activities and deal with *any* numbers that crop up. One can concentrate then on (i) the strategies of measurement, (ii) processing the information, (iii) identifying the problems that occur, (iv) deciding on which arithmetical processes are appropriate (with machines teachers can say again, as they used to years ago, that children can add, subtract, multiply and divide but they don't know *which* to do *when*) and (v) interpreting the results. Come back, trundle wheel, all is forgiven!

3. When teaching any mathematical principles, it is customary to keep the numbers simple so that the principles are not obscured by the difficulties of

calculation. Machines make it possible to include the awkward cases too. The following are some examples.

(a) When calculating AREAS OF RECTANGLES, we usually confine children to whole numbers by drawing rectangles on squared paper or making them on nail-boards. Now we can measure any rectangle to the nearest millimetre, and perhaps think of something more sensible to do with the results. A recent piece of work for C.S.E. involved a graph of areas of rectangles with a perimeter of 24 units (graphed against base) where only integral points were plotted, and invariably these were joined up with straight lines.



Machines would enable some intermediate points to be plotted very quickly. A graph of squares raises similar problems.

(b) Normally, calculating PERIMETERS is a fairly futile occupation, but at least we can now deal with anything we can measure. There is an inverse problem to the rectangle one above; fix the area and draw a graph of perimeters; this is more interesting numerically. Perimeters of polyominoes are all whole numbers, but what perimeters can you get from shapes made from tangrams?

(c) When investigating areas or circumferences of CIRCLES, we usually start with whole number diameters. Now we can measure all the circles we have around as accurately as possible.

(d) Operations with MATRICES OR VECTORS invariably involve simple whole numbers. Some applications can usefully involve large numbers or decimals.

(e) CHANGING FRACTIONS TO DECIMALS is usually a limited operation. We will avoid a discussion here of what is involved in dividing numerator by denominator, but having established the principle we feel obliged to limit the denominator to a maximum of 10 or perhaps 12, so that the only really interesting recurring decimals are the sevenths. Now we can investigate thirteenths, fourteenths, and so on. (And see 4d below.) Incidentally, this technique rather spoils the exercises on comparing two fractions by rewriting them, with a common denominator; it is more sensible to express them as decimals!

(f) Any activities involving RATIO are usually limited to easy whole numbers, and perhaps this is why pupils have such incomplete ideas about it. The science department may be happier if we spend some time on the following ideas; volume/surface area ratios for different size cubes, spheres, people, and so forth; mass/volume ratios for different materials; weight/height ratios for people; waist/height ditto; ratio of weight/height to waist/height (are you heavy or fat?); pressure on feet, that is, weight/area of feet; and of course price/weight or price/volume ratios for the same commodities in different shops or in different weeks.

These activities involve no great change, if any at all, in present syllabuses. They just require a little extra thought when dealing with any topic to make sure that some extra calculation, suitable for machine work, is encouraged. Occasionally this will make possible some new ideas.

4. Some mathematical activities can arise as a result of the particular characteristics of the machine.

(a) The finding of SQUARE ROOTS is a typical example. This is an exercise that becomes accessible to younger children because it relies on an understanding of decimal notation rather than on an ability to multiply decimals. A typical introduction to square roots with 12-year-olds used to be exactly the same method, for example, guess the square root of 2, say 1.5; square 1.5 ; 2.25 is too big; try 1.4; 1.96 is too small; try something in between 1.4 and 1.5, say 1.45. A class discussion would clarify ideas about the limiting process, the best strategy, and so on. The trouble was that using paper and pencil it took one lesson to find the square root of 2 to about 4 decimal places. Now, however, the calculation is so much quicker, and many roots can be found, making the principle much clearer, and opening up possibilities for plotting a graph, calculating intermediate points, and discussing alternative methods. Usually one progressed to the Newton-Raphson method, where the first guess is divided into 2 to produce a larger estimate if the first estimate is too small (or vice versa), and so a better estimate would be halfway between the two; this is then divided into 2, and the process is repeated. Strangely, this is a more tedious method on the machine, and perhaps the 10-year-olds' method is more efficient now.

(b) How do you find CUBE ROOTS?

(c) NUMBER PATTERNS abound in the literature, but the numbers are usually simple. Ray Hemmings, in *Mathematics Teaching* No. 48 (p.56), suggested an activity for the old-fashioned mechanical calculator which is easily adapted for the electronic one. Enter, say, 123456, and keep adding it. (The first time you press '+=' and after that only '='.) Record successive entries. Ignore digits after the first six (but you can put these on the display if you wish). Investigate the numbers produced in any way you like. How long before the original number returns, and why? Start with other numbers. The situation is quite different from that of the mechanical calculator, but just as interesting. Try starting with 142857!

The simple number sequences can still be built up. Add successively 1, 3, 5, 7, 9, ... or 1, 2, 3, 4, 5, ..., recording answers. Younger children can add successive 1s, or 2s or ..., and we are back to building up multiplication tables. Try successive multiplication by 2, and see how quickly the numbers grow. Explore successive multiplication by 10. Explore successive division by 2 or 10.

(d) When exploring RECURRING DECIMALS, the sevenths become immediately interesting, and then possibly the thirteenths. One then looks for a recurring cycle of more than 6 digits. The seventeenths are the next longest, with a cycle of 16 digits, but only 11 of these can appear on the display. Problem: How do you obtain the others? (Hint: When you have calculated $1 \div 17$, what happens if you multiply the answer by 17?) When you have solved this, you are ready for longer cycles. For older pupils some are particularly interesting, for example, 1/49 begins 0.0204081632 ...; why? After this 1/24 = 0.0416666 ... is exceptionally curious, if you consider first that it may be equal to 0.04 + 0.0016 + 0.000064 + ..., and secondly that it is the sum of two fractions, respectively equal to 0.041 and to 0.0006666 ...

Incidentally, is there a way of using the calculator to convert a decimal, recurring or not, back to a fraction? Which reminds me that the third year pupils were asked to "reduce 1980/4620 to its lowest terms." What is the best way of tackling that on the machine?

(e) There are some problems which involve the tabulation of figures, especially those that require a MAXIMUM OR MINIMUM. The maximum area for a rectangle of fixed perimeter becomes fairly obvious from the symmetry of the graph (see 3a above), but the inverse problem gives a graph which is not symmetrical. David Hale quotes a similar problem in *Mathematics Teaching* No. 67 (p.20), the one about making an open box from a square by cutting off square corners.

The graph of volume against size of corner is also asymmetrical, and some calculations are necessary around the peak of the graph if the corner size is to be found to any accuracy. Obviously we are getting close to ideas of calculus here, and it is interesting that W.W. Sawyer suggested a numerical approach to differential calculus in *Mathematician's Delight* (Penguin), which is most suitable for calculators.

(f) Many of these examples, like the square root problem, involve the idea of a LIMIT. Limits are not often dealt with numerically, but it is possible to calculate certain numbers from their sequences, series or even continued fractions. For example, a sequence that approaches the square root of 2 is

$$\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \ldots$$

in which m/n is followed by $m + 2n/m + n$. (This can be generalized for other square roots.) There are well-known series which can be summed by successively adding more terms, and the same procedure can be used for continued fractions.

(g) Finding the square root of 2 can be viewed as solving the EQUATION $x^2 = 2$. One can use the same process for solving, say,

$$x^2 + x = 17,$$

that is, guess x and calculate $x^2 + x$ to see how near 17 you are.

Younger children have apparently used machines to solve simpler equations like $x + 3 = 10$ or $2x = 16$ in the same sort of way (perhaps using a 'box' notation for the 'missing number'). They are exploring number facts in a different way from usual and they are also learning the 'guess and improve' technique.

Obviously we have other techniques for solving simple or quadratic equations, which should also be used, but the calculator method has much to commend it intrinsically, and also it will in fact solve any polynominal equation, say

$$x^2 - x^2 + x = 20,$$

though there may be some snags about roots which are near each other.

Another interesting point concerns decreasing functions. If in solving $5x - x^2 = 1$ we guess $x = 4$, then $5x - x^2$ gives 4 which is too much. We may then try $x = 3$, giving 6, which is even larger! A graph will help sort out what is happening if the numbers themselves do not.

(h) Some PROGRAMMING will be necessary. $x^2 + x$ is easy to calculate, because one can square x and then add on x. $x^2 + 2x$ is not so easy, because it looks as if one must square x, record it and clear the machine, double x, then add x^2 from the record (or vice versa). But in fact

$$x^2 + 2x = (x + 2)x,$$

and this can be calculated successively on the machine. Note that this can but $x(x + 2)$ cannot. Similarly the area of a trapezium is $\frac{1}{2}(a + b)h$; when using the machine, $a + b$ must be calculated first. This all gives some point to the distributive, associative, and commutative laws. Other examples are more interesting. How do you calculate

$$\frac{1}{u} + \frac{1}{v} \quad ?$$

Although letters have been used here, most pupils when calculating will be dealing with numerical examples. A complexity of operations does not often occur naturally, but we should make the most of it when it does, exploring different ways of performing the calculation, and deciding which is best, either for pencil and paper, or for the machine. Graph plotting is a useful exercise, since whenever we have to evaluate a function for different numerical values we want the most efficient way of doing it.

(i) A useful C.S.E. project could be to produce a small BOOK OF TABLES. This could involve interpolation, some graphical work, and perhaps methods of differences in a simple way, but it would be based mainly on the use of calculators.

All these activities are only suggestions. While some are based on well-tried classroom situations where often a calculator would have been useful, many remain to be explored. I should apologize that they are not in any order of difficulty; primary teachers will have to sort out what is appropriate for them.

We should be pleased to hear from any other schools who are making use of machines.

PROGRAMMABLE CALCULATORS AND MINI-COMPUTERS IN HIGH SCHOOL MATHEMATICS:

A SURVEY OF POSSIBILITIES

Evert Karman

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This article, which appeared in the Colorado Mathematics Teacher, an official publication of the Colorado Council of Teachers of Mathematics, April 1976, has been reprinted with the permission of Editor Marc Swadener.

The value of a large-scale computer, complete with time-sharing terminals, in teaching high school mathematics in a large metropolitan school district has been proven. A computer has a twofold educational value: (1) Computer programming requires rigorous logical and mathematical thinking, and provides an arena in which mathematical abstractions become very concrete indeed; and (2) properly channeled, the motivation and enthusiasm generated by a successful response from a computer can greatly accelerate a student's mathematical growth.

Smaller school districts, however, may find it difficult to justify the acquisition of a full-sized computer system, even if they pool their resources with neighboring districts. On the one hand, the costs involved include not only the substantial initial outlay for the computer itself, but the additional expenses of telephone lines, CRT or teletype terminals, *service contracts* for all this equipment, and the ongoing costs of paper, ribbons and other supplies. On the other hand, for one or more small school districts to invest in a large-scale multiprocessing system capable of doing 10 times more than these districts could ever demand of it would be a wasteful misuse of the computer's capacity.

Technological developments of the past few years have created a wide range of possibilities involving computers and programmable calculators for small- to medium-sized school districts. Generally speaking, there are now three levels of calculating equipment available for educational uses: at the lowest level, there are pocket-sized programmable and non-programmable calculators capable of enhancing instruction in every area of mathematics; in the intermediate range, there are desk-sized programmable calculators with formatted printed output and substantial program and data memory; and, at the top level, there are mini-computer systems, programmable in BASIC or other widely-used computer languages, which can be expanded to meet a school district's growing needs. At Denver's George Washington High School, we have had the opportunity to experiment with the full range of equipment, and to examine the educational potential of each type.

A mathematics department with a restricted budget should invest in "scientific" or "slide-rule" calculators, such as Hewlett-Packard's HP-45 or HP-21, Texas Instruments' SR-50 or SR-51, or, if the money is available, the programmables: the HP-65 or HP-25, or TI's new programmable calculator, the SR-52. If a full class can have access to such calculators, they can be an indispensible aid at every level of high school mathematics. For example, perhaps the hardest concept encountered in intermediate algebra is that of logarithms. The presence in the classroom of a calculator with logarithmic functions can be an enormous help in getting that concept across. (The reservation should be made, however, that the presence of the calculator cannot justify omitting instruction in such matters as how to interpolate from logarithm tables; whether calculators are available or not, the assumption in teaching should be that they won't *always* be there.)

A slide-rule calculator can be particularly helpful in such a course as analytic geometry. If enough calculators are available so that no student is discriminated against by not having one, they can be used to speed up the necessary calculations, leaving teacher and students free to concentrate their attention on the concepts. Some hand-held calculators even come equipped with keys for instantaneous polar-to-rectangular (and vice versa) coordinate conversion.

The subject of statistics is presently struggling to find a place in the high school curriculum; one reason for the struggle is the tedium of calculation involved with most statistical analyses. The present generation of hand-held calculators has many statistical functions hard-wired. The availability of calculating machines makes statistics more teachable, and opens up the possibility of an alternative track for students who are interested in higher mathematics, but who don't feel they are able to handle calculus.

The specific keystroke functions available on a hand-held calculator becomes an issue of less importance if the calculator is programmable. The user is then in a position to program in his own functions. Hand-held calculators currently available provide enough programming capacity for any student to learn the rudiments of algorithmic thinking, so that he will have a head start when he is exposed to computer language programming later in his education. On any of the current models, it is easily possible to generate the algorithms to solve such problems as quadratic equations, systems of linear equations, or the production of prime numbers. The more expensive hand-held models even offer magnetic card program storage.

Hewlett-Packard's programmable model HP-25 has proven to be extremely popular in our school. Programming the calculator is easy (once the logic is understood), and it provides a congenial tool for teaching the conversion of mathematical algorithms into computer instructions. One measure of the usefulness of a hand-held programmable calculator like the HP-25 is the fact that many advanced-placement math and chemistry students have bought their own. Any high school could find ample justification for having a few of them around.

Of course, if a school is investing in small, pocket-sized programmable calculators, it must be conscious of the danger of theft. The availability of a security cradle for the calculator minimizes that danger; with security cradles, we have found that we can leave our hand-held calculators out in the open and accessible to all students with complete confidence. Hewlett-Packard offers security cradles for all its programmable and non-programmable calculators at a cost of about 30 dollars; Texas Instruments provides a security cradle for the new SR-52 only in conjunction with that machine's printer. The printer costs about 300 dollars.

If a school is more ambitious in its desire for programmable equipment, then the next step up is the desk-sized programmable calculator. Such calculators are offered by Wang Laboratories, Monroe, Hewlett-Packard, Canon, Tektronix and others. These calculators offer several advantages by comparison with the hand-held models.

With a desk-sized calculator it becomes fully possible to teach programming in its own right. Such a calculator typically has a sizeable amount of program and data memory, making possible larger and more sophisticated programs than on the hand-held models. While the calculator languages are not as easy to learn (or teach) as conversational BASIC, they are not hopelessly difficult; and once a student has learned to program the calculator, he can write programs of the same level of complexity as the BASIC programs featured in many math curricula which rely on a full-scale computer system. At the advanced-math level, programs performing such tasks as numerical integration or approximations to the derivative are not difficult to write for a programmable calculator; on a calculator with 64 data registers (which is practically the minimum available) it is possible to program a Gaussian reduction of a system of seven equations with seven unknowns. It is easy to use desk-size calculators in such introductory programming applications as the generation of "perfect numbers," Pythagorean triplets, or the solution of systems of linear equations.

The desk-size calculators have the added advantage of providing hard-copy printed output; the format of this output is usually, within the limitations of the machine's printer, formatible. And permanent storage of programs and data is possible, in the case of Monroe on magnetic cards, and on tape cassettes with Wang and Hewlett-Packard calculators.

If you decide that the desk-size calculator is the route that your school should travel, there are some cost factors you should consider in choosing the particular calculator you buy: for example, there is the questions of supplies - *thermal* printers are *very* much more expensive than conventional printers (although they are faster), so you would have to buy more paper; the cost of magnetic cards or cassettes varies from calculator to calculator, as does the cost of service contracts, and so on. The best choice of a calculator depends on the degree of frugality necessary for your school system, and on the specific uses you envision for the calculator. Before deciding, it is highly desirable to test the calculator out at work in your own school, and see if it suits your needs.

Certainly, in choosing a programmable calculator, a key factor is the system's capacity for expansion - that is, the availability of peripheral. Most calculators currently on the market are capable of powering such devices as typewriters and plotters. Some, in addition, can operate paper tape readers, porta-punch and mark-sense card readers, external tape drives and even disk drives. In terms of student motivation, the choice of peripherals is frequently as important as the calculator itself; and peripheral input devices such as mark-sense card readers can greatly increase the number of students having access to the calculator.

A most important peripheral for any calculating system is a mechanical plotter. A plotter is interesting to watch, and it can provide a graphic demonstration of such fundamental concepts in algebra as linear equations and conic sections; complex functions, which could take a student hours to visualize or a teacher hours to portray, can be presented by a programmable plotter in minutes. A plotter is one of the most effective teaching tools available in mathematics at the high school level; and a plotter is usually *not* available in conjunction with a large-scale system.

In ascending order of expense, Monroe, Wang Laboratories and Hewlett-Packard all offer analog flatbed plotters. In addition, Wang offers a large (42" by 31") digital plotter; this can be run by a moderately priced programmable calculator (the model 600).

The most ambitious level of entry into the world of programmable equipment is the acquisition of a mini-computer system, programmable in BASIC or a similar compiler language. Such systems are currently offered by Wang Laboratories, Hewlett-Packard and IBM; and the educational market may soon be entered on a large scale by the makers of "micro-computers," such as Altair.

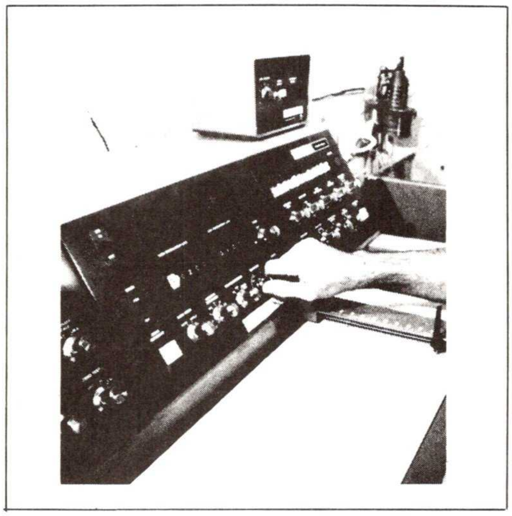
We have had the opportunity to experiment with a typical mini-computer system, the Wang model 2200. The 2200's capabilities, coupled with its moderate cost, make it, in our opinion, an ideal educational computer system.

The basic configuration for a mini-computer system consists of a central processing unit (CPU), a CRT terminal and keyboard for input and output, and a tape cassette drive for storage. CPUs start at 4,000 bytes of core storage, and are expandable in increments of 4,000 bytes; an ideal size is 16,000 bytes. Such a configuration as this is available for under $10,000.

Programming in BASIC makes this system considerably more accessible to a wide range of students than a programmable calculator. We find it possible to use this system, for example, in the consumer math curriculum, with programs which simulate the processes of payroll deductions, the credit arrangements for buying a car, amortization, and so forth. The Wang 2200 and the corresponding Hewlett-Packard system also make available the programs created by the Huntington Project which cover a large number of areas of mathematics, physics, chemistry and earth sciences.

The chief advantage of a system such as the Wang 2200, however, is its capacity for expansion as the needs of the user grow. As funds become available, a school district might want to acquire an output typewriter to get printed copies of program output; another peripheral which could be added is a plotter, either of a small analog type or a large digital type. Programming can be made easier for the student by the addition of an editing package which greatly speeds up the correction of mistakes. Eventually, the system could be expanded to include program and data storage on disks; at this point, the mini-computer system would be fully usable for administrative as well as instructional purposes, and would rival in its efficiency the large multi-processing systems used by metropolitan school districts. In short, it is now possible for a school district, with a relatively small initial investment, to acquire a computer system which can be expanded virtually indefinitely to meet future needs.

Knowledge of computer programming in and of itself might well be an expendable educational goal; but its usefulness as a tool to increase the student's understanding of all levels of mathematics and the sciences has been demonstrated beyond dispute. At the present level of the art of computer manufacture, no school district can use the excuse of inadequate funds for depriving its students of some programming experience.



But we have learned one thing more in our past few years' experience with various types of equipment. If you don't feel you can afford to invest in a computer system, or even a small programmable calculator, now, wait a minute! The diversity of equipment available is increasing at a furious pace, and the cost at all levels continues to go down.

SOME USES OF PROGRAMMABLE CALCULATORS IN MATHEMATICS TEACHING

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The purposes of this article are to suggest uses of programmable calculators in mathematics teaching, and to disseminate certain programs I have developed that may be useful in mathematics teaching. It has been written because of a conviction that programmable calculators are now, or soon will be, available to many classrooms, and, for many purposes, are viable alternatives to computers (LaBar).

KEYBOARD

Most people like to "fiddle" with calculator keyboards. A little fiddling by an inquisitive student, or a little directed fiddling by one that isn't so inquisitive, will often teach in minutes what lecture and blackboard example may not teach in hours. Flashing displays and/or printed tapes are a powerful reinforcement.

Many mathematical operations are directly available on programmable machines. The Texas Instruments SR-56, one of the smallest programmable calculators, has natural and base 10 logs, powers and roots, trigonometric functions and inverses, polar to rectangular conversion and the reverse, mean, variance and standard deviation directly accessible, as well as other operations such as degrees to radians and the reverse and factorials, available through programming. Most of these operations, of course, are not unique to programmable calculators (NCTM).

PROGRAMMING

Programming itself is a valuable learning experience. Having a student develop a flow chart will probably teach him or her a great deal about logical thinking and about mathematics. As examples, developing Program A (programs are indicated by letters throughout, which are briefly outlined in the final section of this paper) taught me some things about primes, and developing Program K forced me to apply the quadratic equation for the first time ever outside of a mathematics class.

Assigning students to program on a calculator is also training for computer programming. The principles are the same, without the necessity of learning a language.

ITERATION, LIMITS AND PROBABILITY

Besides their use in teaching programming, PCs may be used to repeat a calculation over and over. This is useful for at least three reasons.

The first of these is that the calculator will do the same calculation, or a similar one, over and over, automatically. Determining prime numbers is an example of this. If you wish to determine whether 1 000 001 is prime, a simple and exact method would be to divide 1 000 001 consecutively by 1, 2, 3, 4, ... 1 000 000, determining, in each case, whether there was a fractional remainder following division. If at any division there is no such remainder, then 1 000 001 is not prime. This operation is the basis of Program A.

Secondly, the iterative process is also useful in determining limits (Johnsonbough). Thus for example, $\sum_{k=1}^{\infty} 1/2k = 1/2 + 1/4 + 1/8 + 1/16 \ldots$ is an expression for Zeno's paradox of Achilles and the tortoise. Adding this sum for K of 1 through 10 versus K of 1 through 100 should satisfy students that this is an expression for a finite sum. (Warning - all calculators are subject to rounding errors. Also, they will print only a finite number of digits. In dealing with very small numbers, the effects of rounding errors will be reflected in answers. Significant rounding errors may occur in limit problems.) Programs H, J, M find approximate limits by iterative processes.

A third usefulness of iterative processes is for Monte Carlo simulations. That is, simulations wherein some quantity is found by random means. An example is Program L. I have called this the random archer program because of the following analogy: Assume an archer shooting randomly at a square 2 units on a side, within which a circle of radius 1 is inscribed. The ratio of the times he hits within the circle to the total number of shots (assuming he always hits somewhere on the square) should be $\pi/4$. A development of this method, using another analogy for the generation of random hits, is given by Vervoort. Another random approach to determine π is Program K (Schroeder).

Random simulations may involve game theory. A highly modified application of game theory is Program G. A non-random simulation is Program F (Gardner).

PROGRAMS

Each of the programs in this section is available in the form of an annotated listing of steps for the program.[1] To program a Monroe 1880 calculator, it is only necessary to repeat these steps in sequence. For other machines, including computers, the annotation attempts to be specific enough that necessary changes can be made. The annotation includes mathematical explanations, references, and examples, where necessary. Programs requiring indirect memory addressing are

[1]Program lists and annotations are available from the author and will be sent on receipt of 40¢ per program to cover handling and postage.

marked with an asterisk. Where the number of memories required is more than 10, or the number of steps more than 250, this is indicated.

A. PRIME NUMBER GENERATOR

Generates primes, starting with 1, or may be made to generate primes starting from any desired number. Useful as an algorithm example.

B. FACTORS

Factors any number, printing out all factors. (If number is prime, prints zero.) Useful as an algorithm example.

C. ITERATIVE SQUARE ROOT

Determines square root of any positive number to the ninth place by approaching it as a limit, starting with the number itself and 1 as outer bounds. Can be modified to determine any integral root, or to determine more or less places. Useful as an algorithm example.

D. RATIONAL NUMBER GENERATOR

Generates rational numbers which are ratios of integers between 0 and 100, randomly. Will print the integers or not, as desired.

*E. NUMERICAL SORTER

Accepts input numbers (depends on number of memories available) and ranks them from algebraic lowest to algebraic highest. Can be simply modified to determine quartiles, deciles, and so on, and to give mean and standard deviation. Illustrates principles used in automatic alphabetizing.

*F. LIFE

Simulation, or game, with simple rules for play which are used by one "generation" of "cells" to replicate the next. Has been a great hit with "terminal freaks." Size of field depends on number of memories. At least 25 are suggested. About 450 steps.

*G. FOOTBALL

A game is played, between the machine and the operator, complete with downs and touchdowns. The game is based on a matrix of rewards and losses which can be input by the instructor, or randomly generated by the machine. Operator chooses a play, with or without knowing this matrix. Machine responds randomly, but is programmed to change its responses based on the matrix and the play of the operator (that is, it "learns"). About 350 steps. Useful as an illustration of game theory, and as an amusement.

H. DETERMINATION OF $y = x^2 - 5$

Approximates limit of the area under the curve between any specified xs. Useful in illustration of methodology of calculus, as you can use Δxs of different sizes, and thus approximate the limit as closely as desired.

I. STRAIGHT LINE

Operator punches in coordinates of any two points. Program determines slope and y intercept, and will then output y for any x.

J. *Pi* BY ARCHIMEDES' METHOD

Pi is approximated, based on simple geometric considerations, between an upper and lower bound. Method based on perimeters of inscribed and circumscribed regular polygons of greater and greater number of sizes, as circumference of circle is approached more and more closely. Annotation uses knowledge of simple geometry, gives references.

K. BUFFON'S NEEDLE

Monte Carlo determination of *Pi* by simulating random dropping of a needle onto a background of parallel lines. Two versions, one using polar coordinates, the other the quadratic equation. Annotation gives references.

L. RANDOM ARCHER

Monte Carlo determination of *Pi* by simulating random shooting of an arrow into a circle inscribed in a square. Annotation uses the formula for a circle, cartesian coordinates.

M. LIMIT APPROXIMATION OF *Pi*

Approaches *Pi* by establishing a square matrix of paired coordinates and determining whether or not each pair is in a circle. Approximation may be made better or worse, depending on size of Δx and Δy. Annotation uses the formula for a circle.

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THE POCKET CALCULATOR AS A TEACHING AID

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Since its first appearance in the market about five years ago, the pocket calculator has quickly become everyone's home tool, as common as the transistor radio or the cassette tape recorder. The increasing variety of calculators, combined with their generally declining prices, has made them available to more and more people. As a result, even people who previously had no interest in mathematics have begun to express an interest in the properties of numbers, the operations with them, and the relations that exist among them.

The stimulation of exploring with these calculators apparently has a psychological effect on people's attitude toward numbers and arithmetic. The boring, time-consuming, and ever-dreaded use of tables - whether they are the ordinary multiplication tables of elementary school days or the logarithmic and trigonometric tables of high school - suddenly disappears. Instead, one plays with a small, pocket-sized, attractive instrument, operated by the touch of a few keys, and generates answers that appear electronically in the display.

Undoubtedly, the introduction of the pocket calculator will bring about a drastic change in the field of elementary computations, that is, computations that are too trivial to be processed by a computer, yet too time-consuming for manual calculation. This trend will have an obvious bearing on the teaching of mathematics in schools. Not only will the use of most numerical tables become unnecessary, but the teaching of many topics in elementary as well as college mathematics will be given new insight, interest, and even fun by the use of these instruments. It is the purpose of this article to offer several examples and suggestions on how this might be accomplished.

ARITHMETIC AND ALGEBRA

The proofs of all the basic formulas and identities of elementary algebra can, and should, be supplemented by testing their validity for various numerical values of the symbols appearing in them. This is where the pocket calculator is of great help. Not only will it increase students' faith in the formula, but it will also encourage them to test and search for new relations and patterns.

As a first step, however, some simple experiments should be performed in class in order to test the calculator's logic in processing sequences of operations. Calculators may be classified into two categories according to their algebraic hierarchy, that is, according to the method used to process sums of products. In the one category, an expression such as $ab + cd$ is computed exactly as we would understand it, for example, 1 x 2 + 3 x 4 = 2 + 12 = 14. In the other category, the calculator will interpret the same expression as $(ab + c) \times d$, that is, (1 x 2 + 3) x 4 = 20. Generally, the more sophisticated models, such as Texas Instruments' SR-50, belong to the first category, and the simpler types, such as Texas Instruments' SR-10, belong to the second.

A similar situation exists with respect to sums of fractions, sums of squares, sums of roots, and so on. It is very important to be aware of these differences before a class goes on to more complex calculations. The interpretations of several expressions for two categories of calculators can best be summarized in Table 1. The last column in Table 1 indicates the ways the expressions in column one should be interpreted for calculators in the second category.

TABLE 1
TWO INTERPRETATIONS FOR SOME SPECIFIC ALGEBRAIC EXPRESSIONS

Expression	Interpretation: Category 1	Interpretation: Category 2	Correct Input Notation for Category 2
$ab + cd$	$ab + cd$	$(ab + c)\,d$	$\left(\frac{ab}{d} + c\right)d$
$\frac{a}{b} + \frac{c}{d}$	$\frac{a}{b} + \frac{c}{d}$	$\frac{\frac{a}{b} + c}{d}$	$\left(\frac{ad}{b} + c\right)/d$
$a + b^2$	$a + b^2$	$(a + b)^2$	$b^2 + a$
ab^2	ab^2	$(ab)^2$	b^2a

A discussion can now be initiated on these differences, and the students can practice with their own calculators to discover their various features.

The explorations with some of the more sophisticated operations of the calculator might turn into exciting discoveries for the beginner. Let us take the reciprocal key as an example. A simple formula like $\frac{1}{1/x} = x$, with which many students are familiar only as a formal means of handling composite fractions, is suddenly given a very realistic meaning. Choose any number (except zero, of course), press the "$1/x$" key twice, and the very same number appears in the display! The discovery can be made even more exciting by asking the students to watch the results as they press the "$1/x$" key first an even number of times and then an odd number of times. Almost without noticing it, we have introduced the students to the transformation equations $T^{2n} = I$ and $T^{2n+1} = T^{-1}$, which characterize all involutory transformations such as $y = 1/x$.

Similar experiments can be performed with the square and square-root keys. On some instruments, such as the SR-50, any successive application of two inverse functions will restore the exact original number; whereas in other types, such as the SR-10, the result will be only approximately equal to the original number. In this latter case, it will be interesting to observe what happens when a sequence of x^2 and $\sqrt{x}$ operations is performed on the same number x. Does the error remain bounded, or does it accumulate?

It is also fun to test the calculator's reaction to "forbidden" operations, such as zero division. Some types will protest with a violent flash of the display; other, shyer types will merely indicate with an error sign that the operation is invalid. An interesting situation is exhibited on the SR-50: if you take the common logarithm of zero, you get, as expected, the largest negative number the display can show, -9.999999999 times 10^{99}, which is the instrument's interpretation of minus infinity. If, however, you take the *natural* logarithm of zero, you get the unexpected flashing result -4.342944819 times 10^{99}! Each type of mini-calculator has its own idiosyncracies.

Finally, many interesting and quite surprising numerical relations can be discovered with the calculator. Who would guess, for example, that e^{π} and π^{e} have almost the same value? ($e^{\pi} = 23.14069 \ldots$; $\pi^{e} = 22.45915 \ldots$). Or that $2 - \sqrt{3}$ is exactly the reciprocal of $2 + \sqrt{3}$? Of course, after discovering these relations on the calculator, the class can go on to analyze them and find out their origin. Thus, $e^{\pi} \sim \pi^{e}$ because the value of π is very nearly equal to that of $e \ln \pi$, so that $\pi^{e} \equiv e^{e \ln \pi} \sim e^{\pi}$. In the second example we have, of course, $(2 - \sqrt{3})(2 + \sqrt{3}) = 4 - 3 = 1$; incidentally, $2 - \sqrt{3}$ is the tangent of 15° and $2 + \sqrt{3}$ is the tangent of 75°, and our relation becomes the trigonometric equation tan 15° x tan 75° = 1. There is almost no limit to the ingenuity of numerical relations and properties that can be discovered with the calculator, and it can turn many a boring mathematics lesson into an exciting and unforgettable experience.

TRIGONOMETRY

There are obvious advantages in using the calculator over the conventional trigonometric tables: greater speed and accuracy and the lack of the need to calculate proportional parts. Most important, it is not necessary to reduce any angle to the range $0° \leq \alpha \leq 90°$, for the calculator will do this automatically. Anyone who has ever worked with angles whose measures are greater than 90° knows how time-consuming these reductions can be. These advantages, however, are somewhat counterbalanced by the fact that all angles must be entered either in radians or in decimal form, not in degrees/minutes/seconds. Only in some very advanced models, such as Texas Instruments' SR-51, is there direct conversion from decimal parts of a degree to minutes and seconds.

The pocket calculator can be of great help in demonstrating the validity of trigonometric identities. It is one thing to prove an identity but quite another to grasp the nature of the formula under consideration. Has anyone once bothered to test in the classroom the validity of, say, the formula $\sin^2\alpha + \cos^2\alpha = 1$ for various values of α, not just for 30° or 45°? Most probably, the answer is no. With the calculator, such a test becomes real fun (Table 2). Now it depends on the specific type of calculator being used whether the result, after "=" key has been pressed, is exactly 1 or 0.999999 ... The latter result is due to the calculator's truncating property, a point that should be explained to the class.

TABLE 2
VERIFYING $\sin^2\alpha + \cos^2\alpha = 1$

Choose any number for α	
Keyboard	Display
α	α
sin	$\sin\alpha$
x^2	$\sin^2\alpha$
+	$\sin^2\alpha$
α	α
cos	$\cos^\alpha$
x^2	$\cos^2\alpha$
=	

Another class of identities that can be demonstrated on the calculator are the trigonometric-inverse trigonometric relations. Identities such as sin(arcsin x) = x, arcsin (sin x) = x, or arcsin x = arcos $\sqrt{1 - x^2}$ appear quite abstract and meaningless to many students. A simple numerical demonstration will at once make these relations meaningful (Table 3).

TABLE 3
VERIFYING ARCSIN ($\sin\alpha$) = α

Choose any number for α	
Keyboard	Display
α	α
sin	$\sin\alpha$
arcsin	α

NOTE: The procedure of the operation "arcsin" differs from one instrument to another. Often it is required to press two keys, "arc" and "sin" or "inv" and "sin."

Finally, a very common error in the first lessons of a trigonometry course is to confuse, for example, sin $2x$ with 2 sin x. One or two demonstrations on the calculator will remove any notion that the two expressions are the same. Indeed, the correct relation, sin $2x$ = 2 sin x cos x, can easily be demonstrated in the same way.

FUNCTIONS AND LIMITS

In the calculation of numerical values of functions, the pocket calculator is almost indispensable. Even simple functions, such as $y = ax^2 + bx + c$ or y = sin ($ax + b$), require quite involved computations, unless the constants a, b, c have integral values and only as long as we confine ourselves to integral values of x as well. But sometimes assigning only integral values to x will make the values of y too sparse for a graph to be meaningful. This is particularly true near a singular point of the function. For example, the function

$$y = \frac{2x + 1}{2x - 1}$$

yields ever-increasing values as x approaches the value 0.5. It is imperative to calculate the values of y for xs very near to this point. This is a task that fits exactly the capabilities of even the simplest calculator (Table 4).

TABLE 4
EVALUATING $y = \frac{2x+1}{2x-1}$ FOR $x = 0.7$

	Keyboard	Display	Keyboard	Display	
denominator	0.7	0.7	0.7	0.7	numerator
	×	0.7	×	0.7	
	2	2	2	2	
	−	1.4	+	1.4	
	1	1	1	1	
	=	0.4	=	2.4	
	STO	0.4	÷	2.4	
			RCL	0.4	
			=	6.	

NOTE: If the calculator has no memory (STO key), one would have to write down the intermediate result, 0.4, manually and then use it again in the final division. In more advanced models, where there is an interchange function ("$x \rightleftarrows y$" key), one may calculate first the numerator, store it, then calculate the denominator, divide it into the result of the numerator, and then press the interchange key (before pressing the "=" key); or, alternatively, press the "=" key and then the reciprocal "$1/x$" key.

The same calculations should then be repeated for $x = 0.6, 0.55, 0.525$, and so on; then for $x = 0.3, 0.4, 0.45, 0.475$, and so on.

One may use similar procedures to show how the quotient $\frac{\Delta y}{\Delta x}$ of a continuous function $y = f(x)$ approaches the value of the derivative $y' = \frac{dy}{dx}$ as Δx becomes smaller and smaller near the point x_0 where the value of the derivative is sought (if the derivative exists at that point).

Perhaps the most elegant use of the calculator as an educational tool is in the demonstration of limiting processes, such as the convergence of infinite sequences, series, and products. The calculator will not only calculate the values of the terms themselves but will automatically record the value of the partial sum (or the product, as the case may be) at each step. Moreover, one can test the *rate* of convergence of the sequence or series, a crucial question that, unfortunately, is often neglected in teaching the subject in school. Let us mention just a few simple examples.

1. The sequence $\sqrt[n]{a}$, a any positive number. It is well known (Courant 1956, p.31) that

$$\lim_{n\to\infty} \sqrt[n]{a} = 1.$$

Choose any positive number, then press the square-root key as often as you wish. It is really exciting to observe how the numbers in the display successively approach one. (We must be aware of the fact, though, that in this way we obtain the sequence

$$\sqrt[2^n]{a} = \quad \sqrt[2]{a}, \quad \sqrt[4]{a}, \quad \sqrt[8]{a}, \ \ldots,$$

which is a partial sequence of the original one.) Moreover, when the difference between two successive terms becomes smaller than the last digit the display can hold, the final limiting value will automatically appear (in our example, the number 1). This is a vivid demonstration of the fact that, in practice, we can expect a numerical result to be of any significance only as long as the result is larger than the worst inaccuracy of the measuring device used: the final result, in our example, is displayed as 1 and not as 1.000000.

2. The geometric series 1 + 1/2 + 1/4 + 1/8 + ... = 2.

Notice (Table 5) that each pressing of the "+" key automatically gives the value of the partial sum that has already been accumulated (the underlined results). Thus we can literally observe the convergence of the partial sums to their limiting value 2. The sequence of operations can best be represented by a flow chart (Fig. 1).

TABLE 5

EVALUATING $\sum_{n=0}^{k} \frac{1}{2^n}$

Keyboard	Display
1	1
+	1.
2	2
$1/x$	0.5
+	1.5
4	4
$1/x$	0.25
+	1.75
8	8
$1/x$	0.125
+	1.875
⋮	⋮

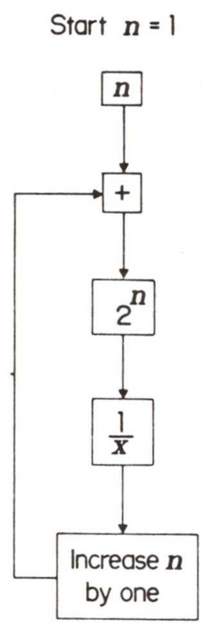

Fig. 1

It should again be noticed that the sequence of operations in Figure 1 will be valid only on instruments that perform directly the addition of products; otherwise, the operations 1 + 2 $1/x$ will be interpreted as (1 + 2)$1/x$; that is, we shall get for the first two terms 1/3 = 0.3333 instead of 1 + 1/2 = 1.5. In such a case, we must calculate each term separately and add it to the previous terms.

We can now contrast this example with an example of a *divergent* series, such as the harmonic series 1 + 1/2 + 1/3 + 1/4 + ..., and observe how the partial sums gradually increase. This would be a vivid demonstration of the fact that the condition for a series to converge, namely, that $|a_{n+1}| < |a_n|$, is necessary but not sufficient.

Other series representing important constants can likewise be tested and compared. Thus, one could compare the well-known limit for *e*,

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n,$$

with the Taylor series for *e*, $e = 1 + 1/1! + 1/2! + 1/3! + \ldots$, and observe how much more quickly the second expression converges than the first. Likewise, it would be interesting to compare some of the many series for π or π^2 that result from the Fourier expansion of various square and triangular functions (Courant 1956, pp. 440, 443, 446). An infinite *product* - which is much more rarely encountered than an infinite series - can also be tested; a well-known example is Wallis's product, (Courant, pp.224,445).

$$\frac{\pi}{2} = \frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdots$$

As a final assignment, the class can be given the task of calculating the numerical values of some of the transcendental functions, such as sin x or ln$x$, from their power series - in the case of ln$x$, one must use the series for ln(1 - x) or ln(1 + x). The teacher can, at this point, tell the class how laborious and time-consuming the very same calculations must have been for the mathematicians of the previous centuries, when logarithmic and trigonometric tables were first introduced. Much historical information on this subject exists, such as the works of Klein (1953) and Smith (1953). We are actually introducing the class, through the pocket calculator, not only to numerical analysis but also to some of the more exciting chapters in the history of mathematics!

I should like to thank Professor Henry Mullish, of the Courant Institute of Mathematical Sciences, who inspired in me the interest and enthusiasm for using the pocket calculator as an educational tool and without whose ideas and help this article would never have been written.

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THE HAND CALCULATOR IN SECONDARY MATHEMATICS

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Calculation is an integral part of elementary school mathematics. In the junior and senior high school it has, until somewhat recently, been downgraded relative to the ideas, theorems, and formulas of secondary school mathematics. True, numbers were involved in factoring quadratics, in computing areas, and so on, but there was a tendency to make numerical situations artificially simple so as not to distract from the central ideas. More recently, there has been a trend to bring computation into the mainstream of mathematics development with a recognition that calculation can promote the growth of ideas as well as play its former, more pedestrian role. Some examples are found in computer-oriented mathematics texts which have appeared on the market over the past decade. Related to all of this has been the use of computer-related techniques of flow-charting and setting down precise algorithms for performing classes of calculations. The computer enforces a precision in the writing of algorithms which goes substantially beyond that required in more informal communication. The computer also focusses attention on the ideas related to computing while it takes over the more mechanical aspects of the computation itself.

Although it is generally recognized that the computer can make the kind of contribution referred to above, many secondary school mathematics students have limited, if any, access to the computer. The hand-held calculator makes possible an intermediate stage between conventional text book presentations and the more numerical approach of a computer-oriented course. With the increased popularity of these calculators, their use in mathematics courses becomes increasingly feasible. Teachers, and others, are attempting to determine how best to employ these fascinating electronic devices. Currently, we appear to be at a stage where a sharing of ideas is needed. Following are some areas which the writers believe warrant further exploration:

1. the development of mathematical concepts contained in the regular curriculum through an approach based on calculation;

2. the exploration of enrichment problems which have a strong numerical component with a view to generating mathematical ideas;

3. the use of the calculator to increase the accuracy and to reduce the tedium of arithmetic calculations, particularly repetitive ones, where calculation is supplementary to the ideas being developed; and

4. the development of computer-related techniques such as flow-charting and the writing of algorithms motivated by the availability and use of the hand calculator.

The selection of problems below illustrates the above applications. An emphasis is on the first application in problems 1, 2 and 3 (provided that they are used in the introductory phase), 4, 6, 18; on the second in problems 5, 7, 8; on the third in problems 9a, 10, 11, 12, 13, 14; and on the fourth in problems 9b, 15, 16, 17. Clearly most of the problems involve more than just one of the four areas listed above.

The lowest grade level at which the average student will likely be equipped with the ideas for solving the problem is indicated beside the problem. Where the problem appears to be accessible to any interested and competent secondary school mathematics student, it has been labelled with a "G," for general interest. In many instances the teacher may be able to make modifications that will enable students to attack the problem at a grade level below that indicated.

The solution to the problem depends not only on the teacher and student but upon the particular calculator available as well. In giving sample solutions, the writers do not claim that these are the best and, further, they have made assumptions about the calculator available to the student. Most of the problems can be done readily with a calculator which provides a memory along with the four arithmetic operations. For a few, additional functions such as the square root are useful.

1. (a) Given that measurements are accurate to the nearest cm, find the greatest and the least possible value of the area of a rectangle whose sides are measured as 56 cm and 83 cm. How many significant digits should be given in the product? How is the product best shown? (Grade VIII)

   (b) During a science experiment, students were required to complete the following calculation based on measurements they had made:

   $$K = \frac{2.4 \times 3.6}{1.8 - 1.6}$$

   All measurements were made to the nearest 0.1. What is the largest and smallest possible value of K? (Grade VIII)

2. Newton discovered a method for calculating the square root of a number to any desired accuracy by using successive approximations. The following example shows how you would use his method to find $\sqrt{151}$

STEPS	CALCULATIONS (On a calculator with 10 display digits)
(i) Guess $\sqrt{151}$	12
(ii) Divide $\sqrt{151}$ by the guess	$12.58\overline{3}$
(iii) Thus $12 < \sqrt{151} < 12.58\overline{3}$	

Use the average of these two numbers as the next guess.	$12.291\overline{6}$
(iv) Repeat steps (ii) and (iii) until the divisor and quotient are equal and retain this value for $\sqrt{151}$.*	(ii) 12.28474576 (iii) 12.28820621 (ii) 12.28820524 (iii) 12.28820572 (ii) 12.28820573 (iii) 12.28820572

*As this example indicates, the divisor and quotient may never be equal. In this case the last digit oscillates between 2 and 3. In cases such as this you could retain either number or round the answer back one digit.

(a) Use this mehod to find $\sqrt{17}$, $\sqrt{7.3}$, and $\sqrt{1295}$. (Grade VIII)

(b) Write a flow chart to demonstrate this method of finding square roots (Grade VIII).

3. (a) Given the function $f(x) = 2x^2 - 5x + 6$ and the point P (2, 4) on its graph, calculate the slope of the secant line $\overleftrightarrow{PQ}$ where Q has the following x-coordinate:

   (i) 3

   (ii) 2.5

   (iii) 2.1

   (iv) 2.01

   (v) 2.001

   (b) Make a conjecture about the slope of the tangent line to the graph at P. (Grade XI)

4. (a) Do each of the following and compare the result with what it should be theoretically.

   (i) $(457 \div 3) \times 3 =$

   (ii) $(458 \div 3) \times 3 =$

   (iii) $(459 \div 3) \times 3 =$

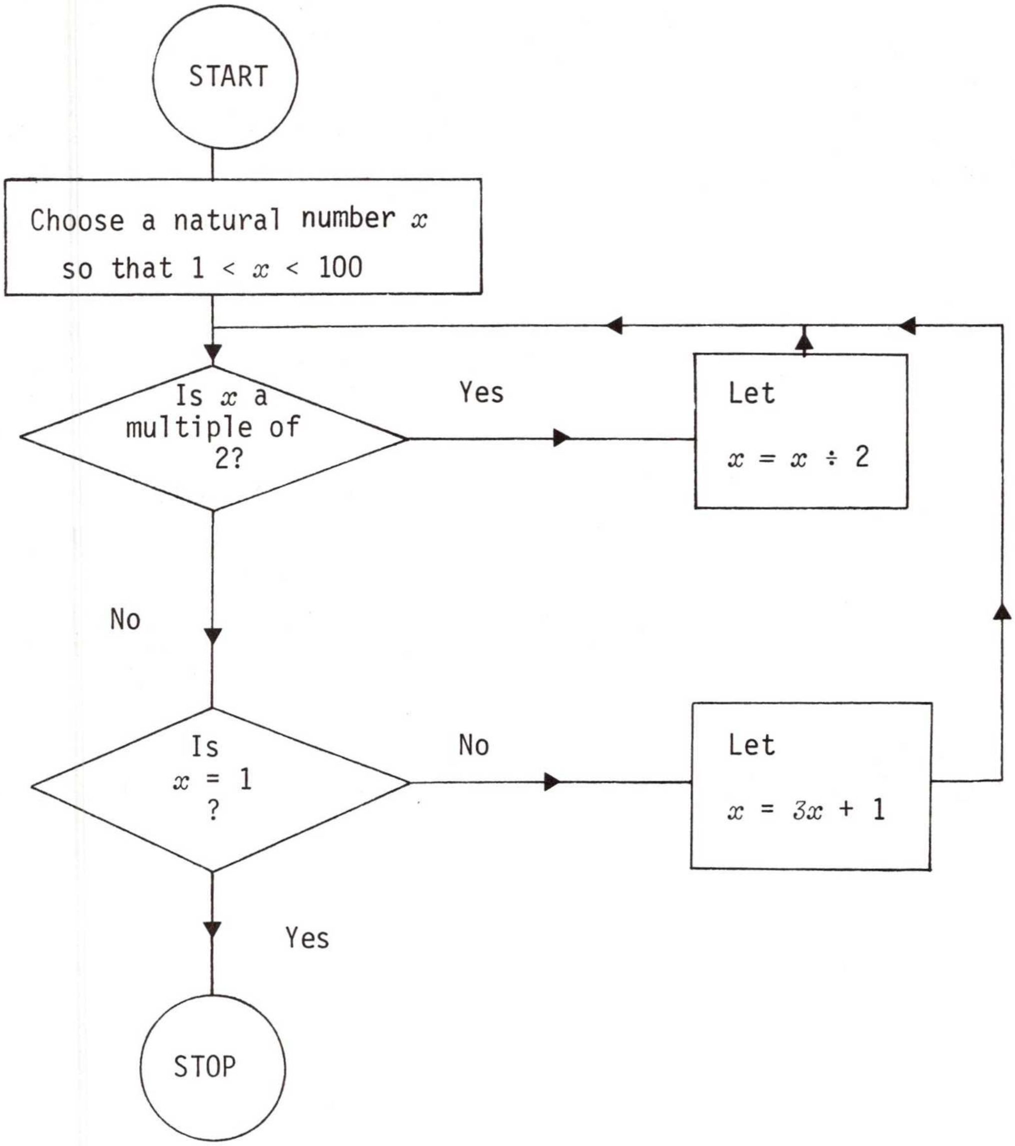
   (iv) $(15 \div 497) \times 497 =$

   (v) $(7 \div 1234) \times 1234 =$

   (b) Decide whether your calculator rounds or simply drops digits when there are more digits in the quotient than can be shown in the display. (G)

5. In some problems about natural numbers you are shown that a certain result occurs for several specific natural numbers. You may then be asked if this result holds for every natural number. A problem of this type is given in

flow chart form below. Work through the calculations for a few natural numbers, starting with small ones.



Pool your results with other class members. Cross-check any doubtful cases. Can you make a conjecture? Would checking 1,000 cases, all of which confirmed your conjecture, convince you that your conjecture is true? Would checking 1,000 cases prove your conjecture? What kind of a result would convince you that your conjecture is wrong? Discuss these questions with other class members. (G)

6. Solve

$x + y = 5$ (1)
$xy = 2$ (2)

by the following method of approximation, giving the solution for x and for y to 6 decimal place accuracy. First get a crude approximation to the answer, say, $x = 4$ and $y = 1$ (which satisfies the first equation, but not the second). Now, using the second equation to find y when $x = 4$ we get $y = \frac{2}{4} = 0.5$. The second equation is now satisfied but the first one no longer is. Now using this y-value in (1) we refine the approximation for x, getting $x = 5 - y = 4.5$. Continue in this manner (alternating between the two equations to refine successively the y-value and then the x-value) until you are satisfied that you have the solution to 6-decimal accuracy. (Grade XI)

7. In the Fibonacci sequence the first two terms are "1" and each successive term is the sum of the two previous ones. Thus the third term is 1 + 1 or 2, the fourth is 2 + 1 = 3, and so on. The first few terms are 1, 1, 2, 3, 5, 8, 13, . . .

   (a) Use the hand calculator to extend the sequence to the 20th term.

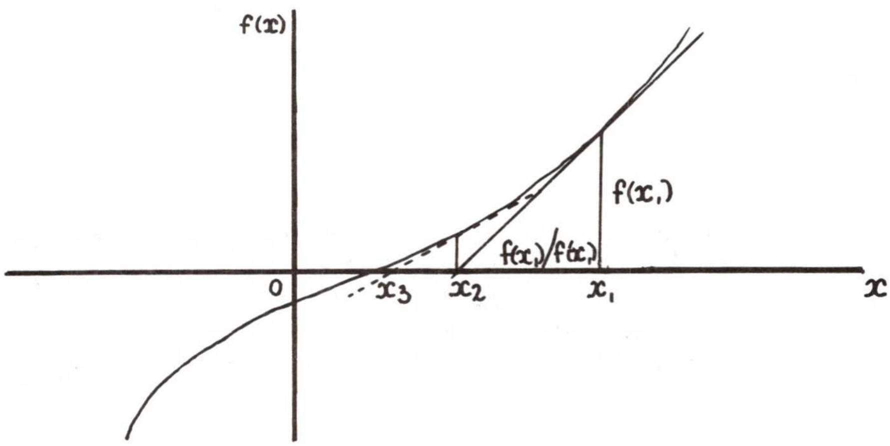
   (b) Find, and write down, the ratio of each term to the following one for the first 19 terms of the sequence. What do you notice?

   (c) The Greek "golden ratio" is expressed by $(\sqrt{5} - 1) \div 2$. Calculate this to 6 decimal place accuracy and compare with results obtained in (b). (Grade VIII)

8. The Newton-Raphson rule for successively refining approximations to a root of an equation $f(x) = 0$ is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

The geometric picture looks like this:



With $x^3 + 3x^2 + 1 = 0$ and $x_1 = -3$, apply the Newton-Raphson rule twice to refine the approximation. (Math 31)

9. (a) Throughout history, many people have developed different approximations for π. Test each of the following to determine the number of significant digits in its decimal form.
(Recall $\pi = 3.141592653589...$) (G)

	Year	Developed by	Value for π
(i)	1800 - 1600 B.C.	Babylonians	$3\frac{1}{8}$
(ii)	1580 B.C.	Ahmes Papyrus (Egypt)	$(\frac{16}{9})^2$
(iii)	200 B.C.	Archimedes (Greece)	$3\frac{1}{7}$
(iv)	150 A.D.	Ptolemy (Greece)	$\frac{377}{120}$
(v)	470	Tsu Chung-chih (China)	$\frac{355}{113}$
(vi)	510	Aryabhata (India)	$\frac{62832}{20000}$
(vii)	1220	Fibonacci (Italy)	$\frac{864}{275}$
(viii)	1580	Tycho Brahe (Europe)	$88/\sqrt{785}$
(ix)	1583	Simon Duchesne (Europe)	$3\frac{69}{484}$
(x)	1685	Adamas Kochansky (Poland)	$\sqrt{\frac{40}{3} - \sqrt{12}}$
(xi)	1769	Arima Shūki Sampō (Japan)	$\frac{428\ 224\ 593\ 349\ 304}{136\ 308\ 151\ 570\ 117}$ *

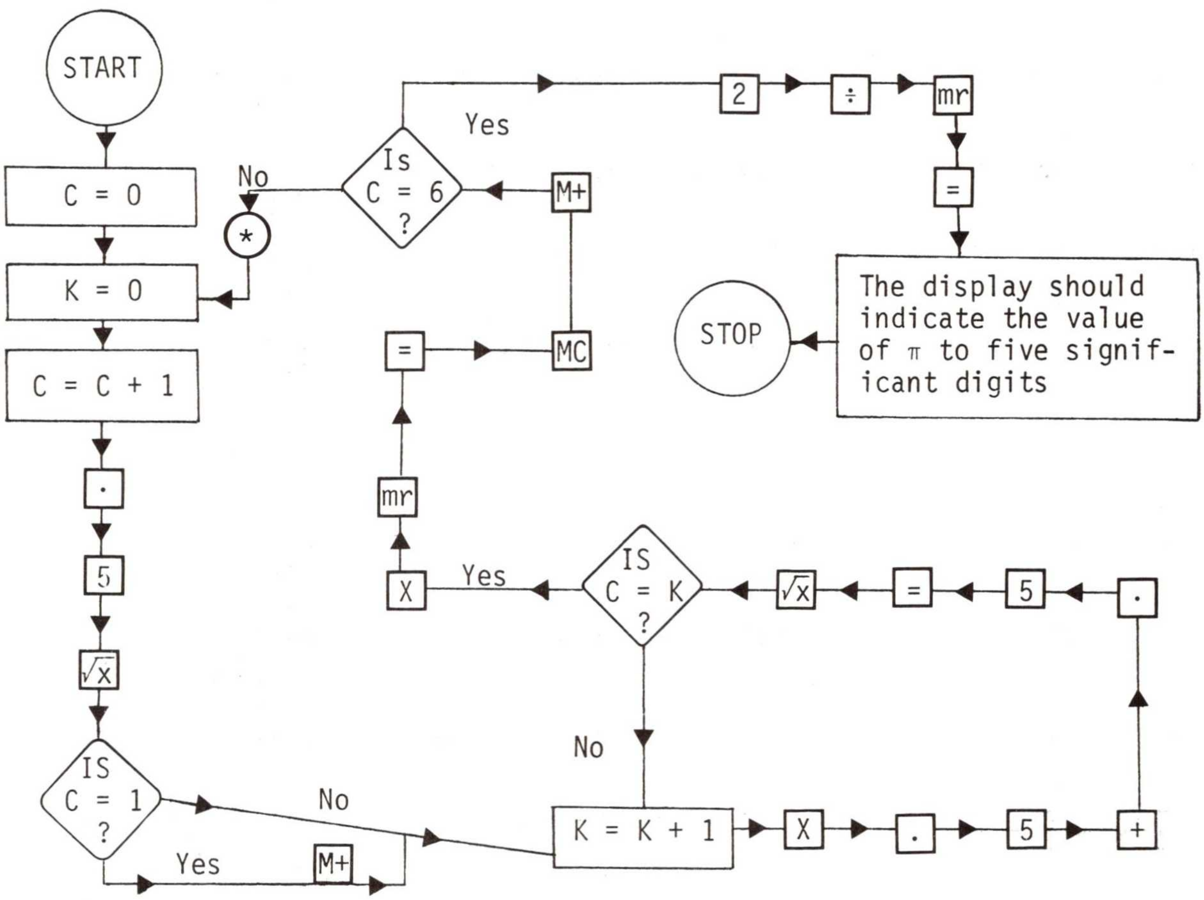
*Since this value likely exceeds the display digits in your calculator, you may have to retain only as many digits as your calculator will allow. The value will still be quite accurate.

(b) Quite a number of methods were developed for calculating π as accurately as one pleased. Unfortunately these methods involve excessive calculations; thus they were of little practical use for determining the value of π until the invention of the modern computer.

Francisco Vieta (1593) used the following continued product:

$$\frac{2}{\pi} = \sqrt{\tfrac{1}{2}} \cdot \sqrt{\tfrac{1}{2} + \tfrac{1}{2}\sqrt{\tfrac{1}{2}}} \cdot \sqrt{\tfrac{1}{2} + \tfrac{1}{2}\sqrt{\tfrac{1}{2} + \tfrac{1}{2}\sqrt{\tfrac{1}{2}}}} \cdots$$

Use Vieta's method to calculate π to show 5 correct digits. The following flow chart indicates the sequence of buttons you would press. (A square root function and memory are required.) (Grade X)



*Check point: you can check your readout with the answer key.

10. Following is a list of prime numbers between 1 and 100:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

(a) Using the above information and a hand calculator, show how you could systematically check any whole number less than 10,000 to determine if it is prime and, if not prime, to determine its factors.

(b) Find the prime factors of each of the following numbers using the scheme you have devised in (a) above. If the number is not factorable, state that it is prime:

(i) 456

(ii) 4356

(iii) 9991

(iv) 9027

(v) 1151

(Grade VII)

11. A student takes a student loan of $2,500 at the beginning of each of the years 1973, 1974, 1975, 1976. Interest on the loan is at 9% per annum, calculated monthly. He intends to repay the loan on a monthly basis commencing on January 1, 1977. Using a hand calculator, find the amount of his loan on that date. (Grade XII)

12. Find the product of the following two matrices:

$$\begin{pmatrix} 1.57 & 3.42 \\ -1.96 & 5.08 \end{pmatrix} \begin{pmatrix} 2.66 & -1.73 \\ -4.17 & 3.56 \end{pmatrix}$$

(Math 31)

13. six $x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$

Find sin 31° 17' to 4 decimal places and compare your result with what you obtain from a set of tables. (Grade XII)

14. The binomial theorem yields the following series for $\sqrt{1 + x}$, which converges for $|x| < 1$,

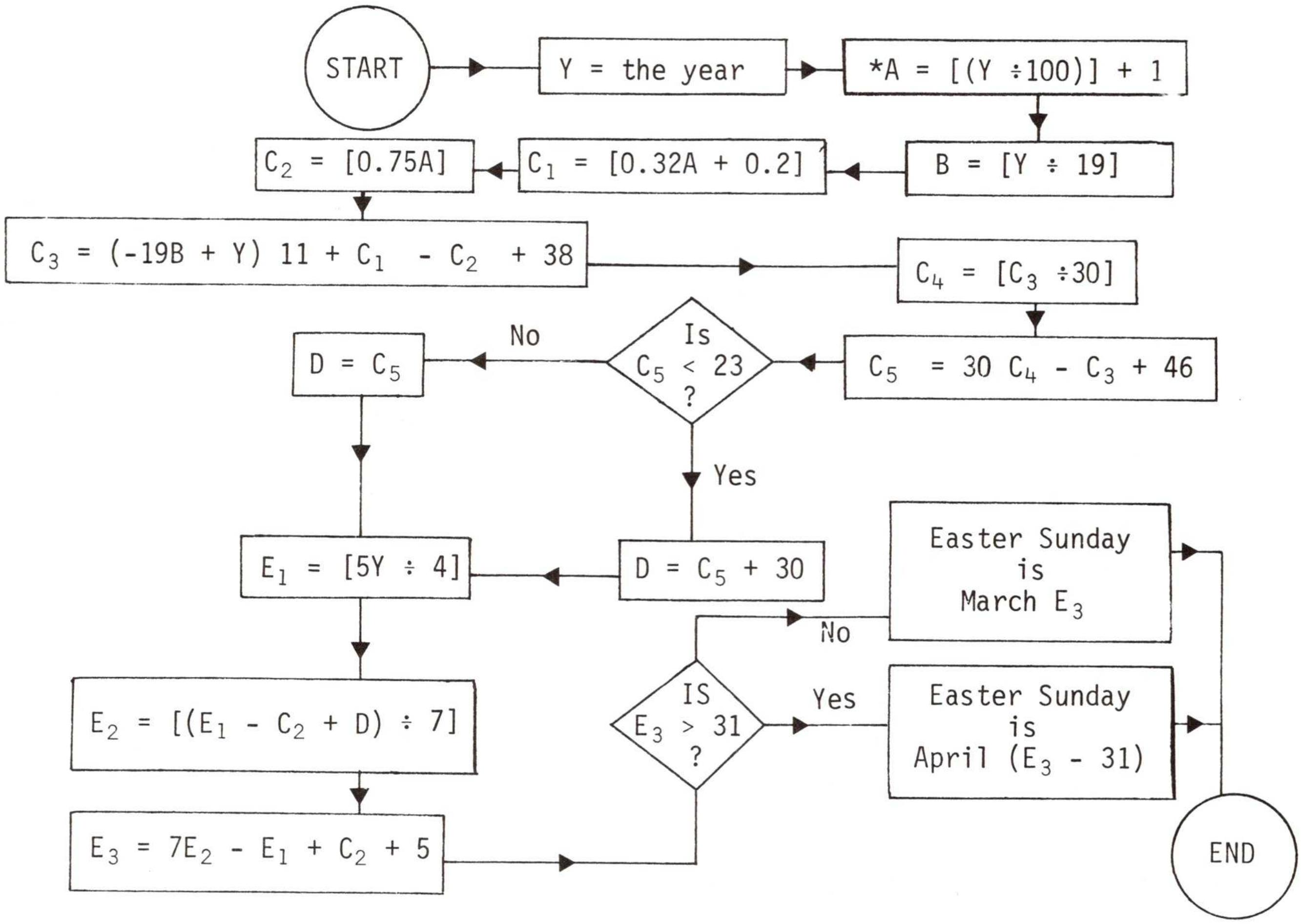
$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \cdots$$

The first few terms give a good approximation to $\sqrt{1 + x}$ if x is small.

(a) Using the first 4 terms, find $\sqrt{1.12}$ to 4 decimal places.

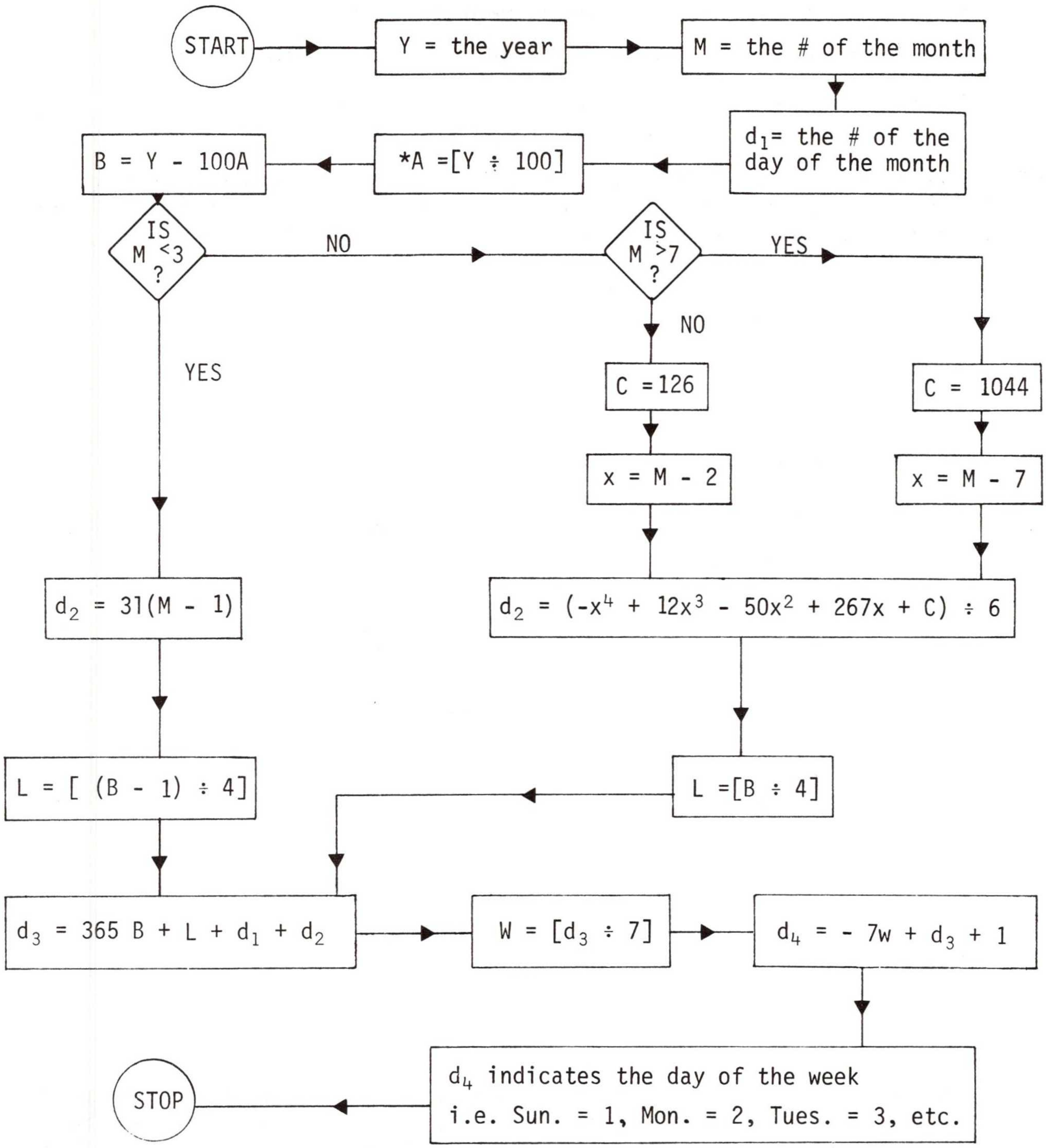
(b) Find $\sqrt{26}$ using the above series. (Hint: note that $26 = 25 \times 1.04$ so that $\sqrt{26} = 5\sqrt{1.04}$.) Square your result for $\sqrt{26}$ and compare with 26. (Grade XII)

15. The following flow chart can be used to determine the date of Easter Sunday for any year after 1582. Use this procedure to find the date of the next Easter Sunday. [G]



*The symbol [x] refers to the greatest interger not greater than x.

16. The following algorithm will give the day of the week for any date between the years 1900 and 2000. (a) Calculate the day of the week that you were born. (b) How would you alter the algorithm to provide for future centuries? (Grade IX)



This algorithm also gives:

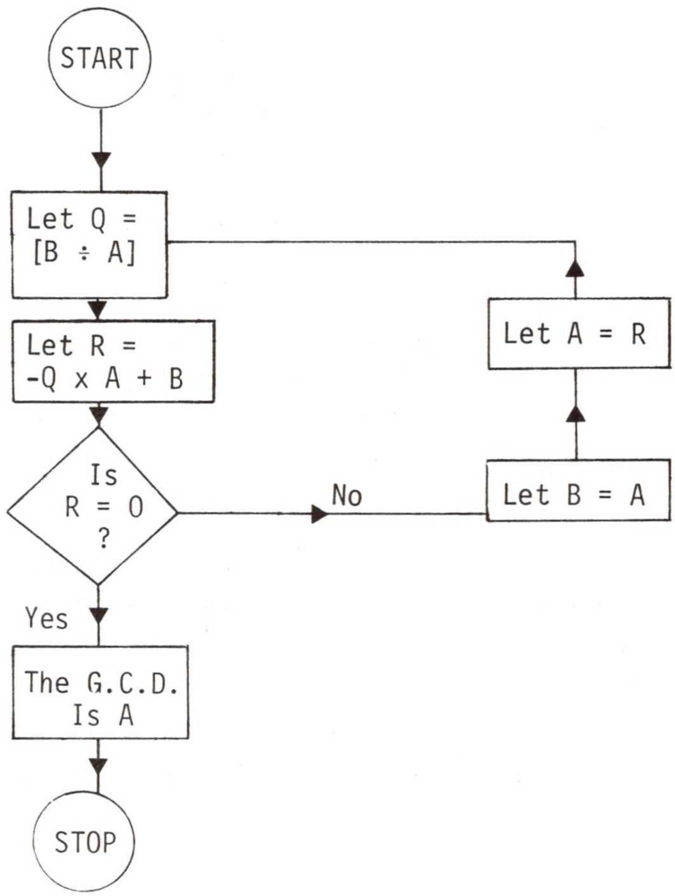
(i) The number of days into the year ($d_1 + d_2$). Add 1 for dates after February 29 in a leap year.

(ii) The number of days into the twentieth century (d_3).

(iii) The number of complete weeks into the twentieth century (w).

*[x] refers to the greatest integer not greater than x.

17. Following is the Euclidean Algorithm, in flow chart form, for finding the G.C.D. of the whole numbers A and B, (A < B). Use the algorithm and a hand calculator to determine the G.C.D. of 4469 and 4687. (Grade VII)



18. A student group did an experiment to get a rough measurement of the length of a molecule of oleic acid. They first obtained a mixture of oleic acid and alcohol in which the oleic acid was 1 part in 1,000 by volume. They dusted the surface of the water in a large shallow pan with lycopodium powder and then put one drop of the oleic acid mixture on the surface of the water near the center of the pan. The drop spread out over the surface in approximately the shape of a disc. (The powder made it easy to see the boundary of the film.) As alcohol dissolves readily in water, the students could assume that the film was entirely oleic acid. The students read in a book that the oleic acid molecules "stand on end" against the surface of the water in a layer one molecule thick.

The student group found that it took 48 drops of oleic acid to increase the volume in a measuring device by 1 cm^3. The diameter of the film was measured at approximately 9 cm.

Using the students' data, calculate the length of the oleic acid molecule, expressing the answer with one significant digit. (Grade XI)

SOLUTIONS AND COMMENTS

---

1. (a) If measurements are to the nearest cm, then the least possible value of the area of the rectangle would occur if the true measures were 55.5 cm and 82.5 cm. Thus, the smallest possible area is 4,578.75 cm^2. Similarly, the largest dimensions that would be recorded as 56 cm and 83 cm would be 56.5 cm and 83.5 cm, respectively. Thus, the largest possible area is 4717.75 cm^2. It is clear that only two significant digits should be shown, and that the second digit is in some doubt. Although the product of 56 and 83 yields an area in cm^2 of 4,648, quoting all four digits implies an accuracy that is misleading. The product is best shown in scientific notation as 4.6×10^3 cm^2.

   (b) The largest possible K-value occurs with the maximum value of the numerator and the least value of the denominator.

   The largest possible K-value is $\dfrac{2.45 \times 3.65}{1.75 - 1.65} = 89.425$

   The least possible K-value is $\dfrac{2.35 \times 3.55}{1.85 - 1.55} = 27.808$

   Thus, the data do not yield even one significant digit.

2. (a) $\sqrt{17}$

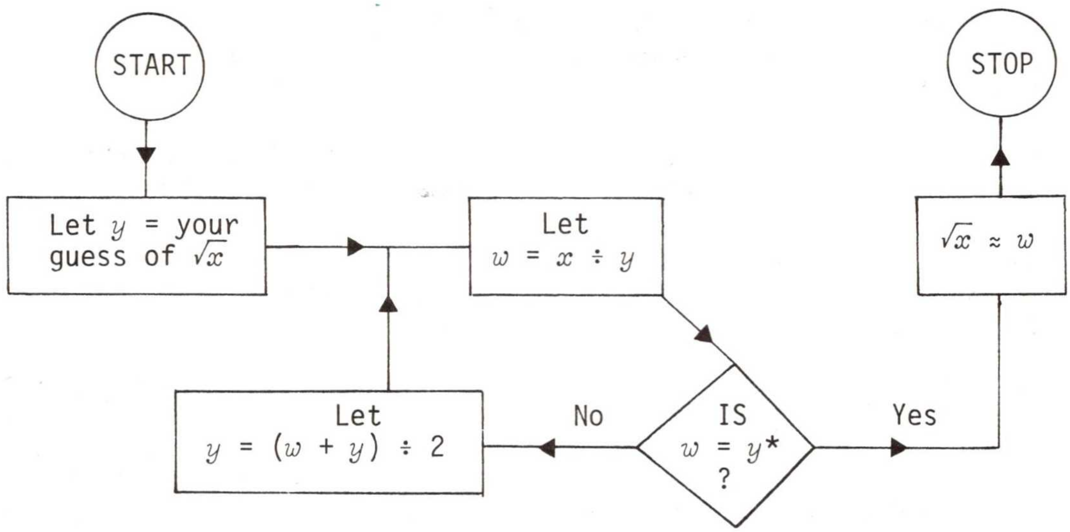
(i)	guess	4 (intermediate results will vary with different guesses)	
(ii)	$17 \div 4 =$	4.25	
(iii)	Average =	4.125	
(iv)	Repetitions:	(ii)	$4.\overline{12}$
		(iii)	$4.123\overline{106}$
		(ii)	4.123105191
		(iii)	4.123105625
		(ii)	4.123105626
		(iii)	4.123105625

   $\sqrt{17} \approx 4.123105625$ or 4.123105626

   $\sqrt{7.3} \approx 2.701851217$

   $\sqrt{1295} \approx 35.98610843$

   (b) To find $\sqrt{x}$



*Note the possibility that w may never actually equal y as illustrated in the example.

3. (a) (i) 5
(ii) 4
(iii) 3.2
(iv) 3.02
(v) 3.002

(b) It appears reasonable to conclude that the tangent line would have a slope of 3, since the slope of $\overleftrightarrow{PQ}$ gets closer to 3 as the point Q gets closer to P.

4. (a) Results depend on type of calculator used.

(b) If the results to part (a) are never too large, your calculator drops the extra digits. If they are sometimes too large and sometimes too small, the calculator is rounding off the last digit.

5. A group of students could check all the odd numbers between 1 and 100. In some cases, such as 27, for example, some persistence is needed; the numbers climb to over 3,000, then in a few more cycles tumble down to 1. A group of students could explore ways of sharing the work effectively. For example, if a student is checking 51, he will rapidly turn up 29 in his sequence. If that number has been checked, or if another student has been assigned the task of checking 29, then the first student goes on to the next number of his list. Students may notice that every second odd number, namely, each of 5, 9, 13 . . ., yields a smaller number at the end of one cycle. It is, therefore, unnecessary to check numbers of the form 4n + 1. A reasonable conjecture, based on the results, is that the procedure described will always result in 1 being attained. Checking 1,000 cases gives one some confidence in the conjecture, but this does not prove the conjecture, of course. The conjecture would be disproved if we could exhibit a

single example of a number for which the procedure did not lead to 1. If we could find any number which trapped us in a loop (that is, a cycle of the same numbers over and over again) then the conjecture would be disproved. To the best of the knowledge of the writers this problem is still open, that is, the conjecture is still a conjecture which no one has yet been able to prove or disprove. It will be of interest to some students that such an apparently innocent looking problem turns out to be so difficult.

6.

$x = 5 - y$	$y = 2 \div x$
	1
4	0.5
4.5	0.44444444
4.5555556	0.4390243
4.5609757	0.4385026
4.5614974	0.4384525
4.5615475	0.4384476
4.5615524	0.4384472
4.5615528	0.4384471
4.5615529	0.4384471

7. (a) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, . . .

(b) 1, 0.5, 0.6666666, 0.6, 0.625, 0.6153846, 0.6190476, 0.617647, 0.6181818, 0.6179775, 0.6180555, 0.6180257, 0.6180371, 0.6180327, 0.6180344, 0.6180338, 0.618034, 0.6180339, 0.6180339.

The ratios alternately get larger and smaller but seem to be settling down toward a fixed value.

(c) $(\sqrt{5} - 1) \div 2 = 0.6180339$.

8. $f(x) = x^3 + 3x^2 + 1$, $f'(x) = 3x^2 + 6x$

$x_1 = -3$. $f(x_1) = 1$, $f'(x_1) = 9$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -3 - \frac{1}{9} = -3.1111111$$

$f(x_2) = -0.075446$, $f'(x_2) = 10.37037$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -3.1111111 - \frac{-0.075446}{10.37037} = -3.1038$$

9. (a) The number of significant digits is:

(i)	2	(vii)	4
(ii)	2	(viii)	4
(iii)	3	(ix)	3
(iv)	4	(x)	5
(v)	6	(xi)	29
(vi)	5		

If your calculator has 10 display digits, you would get 7 significant digits.

(b) The following records the values obtained on a calculator with 10 display digits.

C	K	* Check Point Read-Out
0, 1	0, 1	0.653281481
2	0, 1, 2	0.64072886
3	0, 1, 2, 3	0.637643574
4	0, 1, 2, 3, 4	0.636875504
5	0, 1, 2, 3, 4, 5	0.636683688
6	0, 1, 2, 3, 4, 5, 6	0.636635746

At this stage $\pi \approx 3.141513828$

If you continue this procedure, you will eventually get as many correct digits as you have on your calculator's display.

10. (a) For any number $N < 10{,}000$ check to see if 2 is a factor (and, if so, how many factors of 2 there are); then move up the list of given primes (3, 5, . . .) checking, as above, as far as is necessary. If, after removing a certain number of factors, the remaining factor is any number on our list of primes, then our factorization is complete. If, in attempting to factor the number (or a factor of that number) by repeated trial - by trying 2, 3, 5, . . . in order - we reach a point where the quotient is less than the divisor, then the number in question is a prime. The display on the calculator readily reveals whether one number is divisible by another.

(b) (i) $2 \times 2 \times 2 \times 3 \times 19$
(ii) $2 \times 2 \times 3 \times 3 \times 11 \times 11$
(iii) 97×103
(iv) $3 \times 3 \times 17 \times 59$
(v) prime

11.

1973	1974	1975	1976	1977
\$2500	\$2500	\$2500	\$2500	

$$A = 2500\,[(1.0075)^{48} + (1.0075)^{36} + (1.0075)^{24} + (1.0075)^{12}]$$

$$A = \frac{2500(1.0075)^{12}\ (1.0075^{48} - 1)}{(1.0075)^{12} - 1}$$

Enter 1.0075

Press $\boxed{X}$ $\boxed{=}$ $\boxed{X}$ $\boxed{=}$ $\boxed{M+}$ $\boxed{X}$ $\boxed{=}$ $\boxed{X}$ $\boxed{MR}$ $\boxed{=}$

Record display minus 1, .0938062, as denominator of A

Press $\boxed{CM}$ $\boxed{M+}$ $\boxed{X}$ $\boxed{=}$ $\boxed{X}$ $\boxed{=}$ $\boxed{-}$

Enter 1

Press $\boxed{X}$ $\boxed{MR}$ $\boxed{X}$

Enter 2500

Press $\boxed{\div}$

Enter .0938062

Press $\boxed{=}$

Record display 12,575.65 as answer in dollars.

$\boxed{M+}$ - add to memory

$\boxed{MR}$ - recall memory

$\boxed{CM}$ - clear memory

12. $$\begin{pmatrix} -10.0852 & 9.4591 \\ -26.3972 & 21.4756 \end{pmatrix}$$

13. First convert 31° 17' to 31.283 3333 radians by multiplying by π and dividing by 180 and then do the necessary calculation. Check against a table of values.

14. (a) With $x = 0.12$ the terms of the series given yield 1.058308 which checks to 4 decimal places with tables.

    (b) By substituting 0.04 into the terms of the series given we get 1.019804 which when multiplied by 5 gives 5.099020 which checks with tables to 6 decimal places.

15. $Y = 1977$
    $A = 20$
    $B = 104$
    $C_1 = 6$

$C_2 = 15$
$C_3 = 40$
$C_4 = 1$
$C_5 = 36$
? No
$D = 36$
$E_1 = 2471$
$E_2 = 356$
$E_3 = 41$
? Yes

Easter Sunday is April 10

16. Example: Suppose you were born April 7, 1960:

$Y = 1960$	$C = 126$
$M = 4$	$X = 2$
$d_1 = 7$	$d_2 = 90$
$A = 19$	$L = 15$
$B = 60$	$d_3 = 22012$
? No	$W = 3144$
? No	$d_4 = 5$ i.e. Thurs.

You were born on a Thursday

17.

A	B	R
4469	4687	218
218	4469	109
109	218	0

The G.C.D. is 109.

So $\frac{4489}{4687} = \frac{41}{43}$.

18. Volume of oleic acid in drop $= \frac{1}{1000} \times \frac{1}{48}$ cm^3

Area of oleic acid film $= \pi \times 4.5^2$ cm^2

$$\text{Thickness of film} = \frac{\frac{1}{1000} \times \frac{1}{48} \text{ cm}^3}{\pi \times 4.5^2 \text{ cm}^2} = 3 \times 10^{-7} \text{ cm}$$

Length of molecule $= 3 \times 10^{-7}$ cm, approximately.

PART FIVE

Activities - Elementary and Junior High

EXCITING EXCURSIONS IN NUMBER THEORY WITH AN ELECTRONIC CALCULATOR

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The activities reprinted here have been selected from a booklet which is available from the author.

PALINDROMES

DEFINITION

Palindromes are positive integers such as 4735374 or 461164 that read the same forward as backward.

PROJECT 1

Take a positive integer N.	N = 139
Add N to its reverse.	139 + 931 = 1070
If sum is a palindrome, STOP.	
If not, continue process.	1070 + 0701 = 1771
	1771 is a palindrome in two cycles.

PROJECTS FOR YOU TO DO

1. Investigate whether or not N = 5, 6, 7, ... 99 all produce palindromes and, if so, how long are the cycles.
2. If you wish, extend your investigation to more than two digits.
3. Can you determine several sets of starting values that will always produce palindromes in one cycle? Two cycles?
4. What else can you discover about starting values that produce palindromes?

HAPPY NUMBERS

Take a number, say 13. 13

Square each digit and add. $1^2 + 3^2 = 1 + 9 = 10$

Repeat the above with 10. $1^2 + 0^2 = 1$

A number for which this pattern yields finally a 1 is a HAPPY NUMBER. The number 13 is therefore a happy number.

Take the number 2. 2

Square it and add. $2^2 = 4$

$$4^2 = 16$$

$$1^2 + 6^2 = 1 + 36 = 37$$

$$3^2 + 7^2 = 9 + 49 = 58$$

$$5^2 + 8^2 = 25 + 64 = 89$$

. =

. =

. =

You are back to the number you started with. . = 4

This pattern will cycle forever.

The numbers 2 and 4 are not happy numbers.

EXERCISE

Find all happy numbers less than 100.

FIBONACCI SERIES

QUESTION

How many pairs of rabbits can be produced from a single pair in a year if each pair begets a new pair every month, each new pair produces from the second month on, and no rabbit dies?

ANSWER

In an effort to answer the above question, the following series of numbers evolved:

1, 1, 2, 3, 5, 8, 13, 21, ...

This series is called the Fibonacci Series, after the man who discovered it.

QUESTIONS FOR YOU TO ANSWER

1. Can you write the next five numbers in the series?
2. Study the question about the rabbits and see if you could generate the series.
3. Using your calculator, write the first 50 terms of the Fibonacci series.
4. Calculate the sum of the n terms of a Fibonacci series.
5. Calculate the sum of the squares of the terms.
6. Study your findings for possible patterns. Can you make any generalizations?

ALGORITHM to help you generate a series of Fibonacci numbers. General algorithm:
a = 1st term, b = 2nd term

a $\boxed{+}$ b M $\boxed{+}$ $\boxed{=}$ M $\boxed{+}$ $\boxed{+}$ M

EXTRAS FOR EXPERTS

FOR YOU TO INVESTIGATE

Let N_0 be any 4-digit positive integer.

Let L_k = the largest integer obtainable by rearranging the digits of N_k.

Let S_k = the smallest integer obtainable by rearranging the digits of N_k.

THEN, $N_{k+1} = L_k - S_k$

EXAMPLE

N_0 = 7162

N_1 = 7621 - 1267 = 6354

N_2 = 6543 - 3456 = 3087

$N_3 = 8730 - 0378 = 8352$

$N_4 = 8532 - 2358 = 6174$

$N_5 = 7641 - 1467 = 6174$ which clearly repeats forever.

Your problem is to investigate this recurrence.

The results may surprise you!

NOTE

It is not necessary to investigate all 9,000 possible 4-digit numbers to completely solve this problem. Can you tell why?

How many possible values for N_1 are there?

EXPERIENCES WITH THE HAND-HELD CALCULATOR IN TEACHING COMPUTATION, PROBLEM-SOLVING, AND FRACTIONS

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The activities reprinted here have been selected from booklets which are available from the author.

ROOTS

Play a guess-and-test game. Solve these root problems in less than 10 tries.

GAME 1	$\square^2$	=	65536
GAME 2	$\square^3$	=	117, 649
GAME 3	$\square^4$	=	456,976
GAME 4	$\square^2$	=	53.29
GAME 5	$\square^3$	=	91.125

GAME — GUESS THE FRACTION

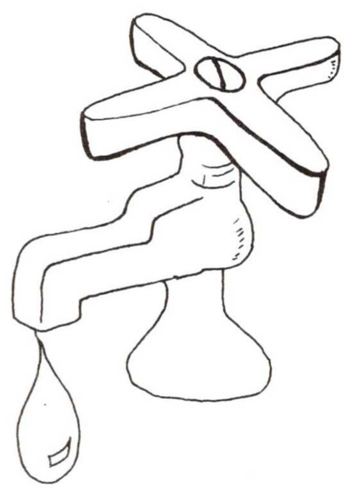
1. Each player writes down a "hidden fraction" in $\frac{a}{b}$ form, (b less than 10) and computes the decimal name.
2. Players take turns reading the decimal, while the other players guess the original fraction.
3. Correct guesses score 1 point. The first player with 5 points is the winner.

Practice guessing for these decimals:

Calculator display	Guess	Calculator display	Guess
0.5000000	______	0.4000000	______
0.1111111	______	0.2222222	______
0.3333333	______	1.1250000	______
0.3750000	______	1.6666666	______
0.1666666	______	.28571428	______

MULTIPLE STEP PROBLEMS

A. If a water faucet drips 18 drips in 10 seconds, how much water will be lost in 24 hours? Which of these guesses would you choose?



1 gallon	10 gallons
100 gallons	1000 gallons

B. A dollar bill weighs about 1 gram and \$10 bills, \$20 bills, and \$50 bills weigh the same. Of course, you would rather have 100 grams of \$10 bills than 100 grams of one dollar bills, but which of these amounts would you rather have?

1. 20 grams of \$10 bills, 40 grams of one dollar bills, and 40 grams of \$20 bills.

2. 10 grams of \$50 bills, 70 grams of one dollar bills, and 20 grams of \$10 bills.

3. 100 grams made up of half one dollar bills, one-fourth \$10 bills, one-fifth \$20 bills and the rest \$50 bills.

TARGET

Partners ____________

____

Play with a partner.

1. First player punches any numeral on the keyboard and punches "add" key.
2. Second player chooses any digit and punches the "add" key.
3. Play continues until one player chooses to punch the "equal" key.
4. The player scores the difference between the target number and the display.
5. The player with the smallest total score wins.

Game 1	Game 2	Player Scores
Target 40	Target 40	1. _____ _____
Game 3	Game 4	2. _____ _____
Target 50	Target 50	3. _____ _____
Game 5	Game 6	4. _____ _____
Target 65	Target 75	5. _____ _____
		6. _____ _____

What strategy will win the game?

Can you make up a game for multiplication?

GAMES WITH THE POCKET CALCULATOR

Sivasailam Thiagarajan

Harold D. Stolovitch

Dymax
Menlo Park, California

The activities reprinted here have been selected from a book which is available from Dymax, P.O. Box 310, Menlo Park, California 94025.

NUMBER PLEASE

This is another fast-paced party game which is easy to learn. It involves the addition of two telephone numbers. Players predict which digit will be repeated most frequently in the resulting sum. You win by being fast and accurate or just plain lucky.

NUMBER OF PLAYERS: Two or more. Since each player has to choose a different digit, more than 10 cannot participate.

APPROXIMATE TIME REQUIREMENT: Each round of the game lasts for about three to five minutes.

SKILLS INVOLVED: Addition of big numbers and a quick draw.

CHANCE FACTOR: There is an element of chance depending upon the skill levels of the players.

SPECIAL REQUIREMENTS: With a small number of players, you may permit players to shout out different digits to indicate their selection. To avoid more than one player claiming that he was the first to select a particular digit, it is a good idea to use number cards or tiles with a large group. There are 10 of these cards, each with a different digit - 1, 2, ... 9, 0 - placed in the middle of the play area within reach of all players.

PLAY OF THE GAME:

1. The first player announces a telephone number slowly and clearly. The second player does the same, using a different telephone number from another exchange so that the first three digits are not the same.

   Kathy announces her number, 631-4214. Vince checks in his little black book and comes up with 976-2285 (and, smiling lecherously, writes down 631-4214 in his book).

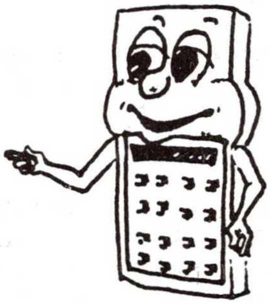
2. Each player (including the two who supplied the telephone numbers) grabs a number card to indicate what he/she thinks will be the most repeated digit in the sum.

Lucy attempts to add the two numbers in her head and gets a headache. She gives up and grabs a random card which has the number 7. Charlotte begins to add and gets two nines as the last two digits of the sum. She decides that is good enough and reaches for the card with the 9. However, Vince makes a faster grab and Charlotte picks up a zero in disgust. Lillian uses her feminine intuition and selects a card with a 3. Kathy has been doing some careful adding and sees 6 as a strong contender.

3. One player now uses the calculator to add the telephone numbers. Each player gets a score which equals the number of times her/his selected digit appears in the display.

The sum in the display is 16076499. Here's how the scoring goes:

Player	*Selected Digit*	*Score*
Lucy	*7*	*1*
Charlotte	*0*	*1*
Vince	*9*	*2*
Lillian	*3*	*0*
Kathy	*6*	*2*



Thus at the end of the first round, Vince and Kathy are tied with a score of 2 points each.

4. The game continues with the next two players announcing the telephone numbers. The first player to accumulate a total score of 11 points wins the game.

During the next round, Lucy announces the phone number of her hair dresser, 439-2615. Charlotte gives her office number, 331-6161. The players perform their respective mental gymnastics and again snatch digits from the pile of tiles. With a characteristic flourish, Vince adds the two phone numbers on the calculator and gets a total of 7708776. Here is the summary table:

Player	*Selected Digit*	*Score for this Round*	*Total Score*
Lucy	*8*	*1*	*2*
Charlotte	*0*	*1*	*2*
Vince	*7*	*4*	*6*
Lillian	*5*	*0*	*0*
Kathy	*6*	*1*	*3*

So, at the end of the second game, Vince jumps ahead with a total of 6. However, in the later rounds, Lillian comes from behind to reach the total of 11 points and wins the series.

TOWARDS A MILLION

This is a game for people who think big. Players listen to a storyteller as he provides a running commentary on what he is doing with a bunch of big numbers in the calculator. All players know what numbers and what operations are being used, but they are left to guess the outcomes. Whenever a player thinks that the resulting number is close to a million, he may stop the storyteller and peek at the display. Each player's score is the difference between the number he peeked at and 1,000,000. The player with the *smallest* score wins the game.

NUMBER OF PLAYERS: Three or more. For a two-person game, see variations.

APPROXIMATE TIME REQUIREMENT: Depending upon the storyteller's line and the number of players, the game may last from three to six minutes.

SKILLS INVOLVED: Addition and multiplication of big numbers. Estimating results.

CHANCE FACTOR: A little. Sometimes if you get lost, you may score by being lucky. Also depends on the whims of the storyteller.

PLAY OF THE GAME:

1. One player is selected to be the storyteller. He performs a series of additions and multiplications using any numbers he wants. He does not permit the other players to see the display in the calculator, but reports to them each new number and each operation he is using.

   Raja, the first storyteller, spins this yarn: "I multiply 198 by 40 ... I multiply the result by 125 ... I add 9800 ... I add 200 ... I multiply by ten ... "

2. The other players attempt to keep track of the shifting number. Whenever a player feels that the number has become a million or nearly so, he stops the storyteller, takes a peek at the display and secretly writes down what he sees. More than one player may stop the storyteller at the same time. Play continues until all players have their peeks at the display.

   Here's a summary of what really happens during Raja's story and how each player interprets it: When Raja multiplies 198 by 40 he gets 7920. Harold and Charlotte mentally round off the 198 to 200 and estimate the product as 2000. Thiagi and Lucy, the other two players, attempt the actual multiplication in their heads. Thiagi gets 7620 which is incorrect and Lucy gets 7920 which is correct.

   When Raja multiplies by 125, he gets 990,000. Harold multiplies his estimate of 8000 by 125 and gets a million. Excitedly, he asks Raja for the display. Harold writes down the 990,000 on a piece of paper. Charlotte gets the same estimate, but she remembers that she rounded off upwards earlier and decides to wait one more round. Thiagi now decides to round off his earlier incorrect product of 7620 to 7500, multiplies that by 125, loses a zero at the end of the result, and gets a way-off estimate of 93,750. Lucy gives up trying and begins to daydream.

When Raja adds a 9800, he gets 999,800. Charlotte decides to stop him. She writes down the 999,800 from the display and feels happy about being close to a million. Thiagi does not understand why people are rushing to stop the story; his estimate is still around 100,000.

When Raja adds the 200 next, Lucy wakes up and decides to halt the storyteller. She is pleasantly surprised to find that she has hit the million on the nose. Thiagi still thinks the number is close to a hundred thousand.

When Raja multiplies the number by ten, he gets 10,000,000. Thiagi feels that his patience is rewarded. He halts the storyteller and expects to find the million on display. He discovers the ten million figure with a shock.

3. Game ends after all players have a peek at the display. Each player now finds the difference between the number he has written down and 1,000,000, using the calculator if necessary. The player with the smallest difference wins the game.

   Here are the various differences:

Harold:	1,000,000 - 990,000 = 10,000	
Charlotte:	1,000,000 - 999,800 = 200	
Lucy:	1,000,000 - 1,000,000 = 0	
Thiagi:	1,000,000 - 100,000 = 900,000	(sic)

VARIATIONS

1. Instead of the million, any other number may be selected to be the goal number. The play of the game remains the same. This is especially desirable if your calculator has only a six-digit display!

2. The storyteller may begin with a million and use subtraction and division to gradually reduce the number to zero. Players try to stop him as close to zero as possible. When some players peek, the number may have already become negative.

3. With just two players, two rounds of the game have to be played to see who stops the other closest to the million. An additional rule may be added to require that both players get within a thousand of the million mark at least once during their story.

THE HAND-HELD CALCULATOR

Iowa Council of Teachers of Mathematics
Monograph 1976
Ann Robinson, Editor

The activities reprinted here have been selected from a monograph which is available from Ann Robinson, 2712 Cedar Heights Drive, Cedar Falls, Iowa 50613.

ESTIMATION

Select the largest number in each row by estimating an approximate answer for each problem. Then use the calculator to check your accuracy.

	a	b	c
1.	27 x 42	88 x 18	47 x 54
2.	17 x 28	23 x 27	500
3.	227 ÷ 46	462 ÷ 71	89 ÷ 14
4.	32 x 53	142 x 15	668 x 7
5.	613 + 486	37 x 32	44 x 24
6.	18 ÷ 8	25 ÷ 16	37 ÷ 25
7.	92 x 39	23 x 65	118 x 9
8.	1000 - 598	793 - 425	715 - 203
9.	13 x 9 x 6	7 x 4 x 8	15 x 2 x 8
10.	490 ÷ 45	378 ÷ 16	98 ÷ 5

DECODE THE MESSAGE!

You should have a quotation by a famous mathematician.

G = 400 - 273	E = 59 + 83
N = 7 (13)	T = 7 · 8 · 6
C = 42 (9)	F = 19 + 18 + 17
S = 14 + 18	D = 18 (7)

(W) = 39 + 39 + 39

(B) = 9^2

(M) = 18 + 12 + 13

(U) = 48 + 82 + 17

(S) = 28 + 42

(N) = 78 + 32

(O) = 5^3

(L) = 13^2 - 18

(Y) = 26 + 49 + 18

(H) = 48 + 32

(I) = 65 + 35

(T) = 2^3

(A) = 7^3

(R) = 625 ÷ 25

(I) = 2 · 3 · 4

(K) = 25 + 18 + 19

"43 343 8 80 142 43 343 8 100 378 32   100 70   343

81 125 126 93   125 54   62 110 125 117 151 142 126 127 142 ,

81 147 336   24 8   378 125 91 8 343 100 110 32   91 125

8 25 147 8 80 70 ."

by Morris Kline

CALCULATOR TALK

1. Enter 0.7734 on the calculator. Turn the calculator so that you see the display upside down. What does it say?

   Enter these numbers: 34, 0.5, 7714. What does each say?

2. A man was trying to decide whether to keep his car or trade it. While fiddling with his calculator, it told him what to do. To see what it said, find the sum of 1234 and 6501.

3. If you spend money recklessly, where do you wind up? Do this subtraction to find out.

   $13,815 - $10,000 - $100 - $10 - $1 = ?

Make your calculator spell out these words. Use addition, subtraction, multiplication and division.

4. She

5. Sole

6. Hose

7. Bell

8. Bill

9. HO HO HO

Try these:

10. What did Amelia Earhart's father say the first time he saw her fly the airplane? Calculate the following: [.023 (3) + 10141] (5) =

11. What did the cannibals say when they saw their dinner guest getting angry? Calculate: 228440 ÷ 4 - 1.67 + .01 =

12. What did Farmer Smith throw at Peter Rabbit to chase him out of the garden? Compute: ([27 (109) + 4 - .027] (2)) (9) =

CALCULATOR TALES

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1. JAWS - The movie "Jaws" has been very popular. Recall that Jaws had a very large appetite. In one day Jaws might have had 100 snacks. Multiply this number by 240,000 minnows in *one* snack. Next add the 268,869 baby squid he ate for breakfast and divide by three (the number of times Jaws burped!). Then subtract the 14,010 fish that swam out of his mouth when he yawned. Finally, add the five people he ate in the movie. Now turn the calculator around and you will know that most people think Jaws is a ______________________.

2. THE T-MART SPECIAL - The T-Mart department stores have a cafeteria that serves their customers. After a shopper slips by the blue light special, he can stop for a snack. The cafeteria cooks know how to make 667 different soups. The number of customers they serve a week is 3,000. Add the number of soups to the number of customers. All of the varieties of soup are homemade. Multiply by the two cooks who make the soups. Turn the calculator around and you will find that one of the soups has a ______________ as an ingredient. Some of the ingredients are unusual, but the soups are very tasty. Clear the calculator and enter 2 (remember this is the number of cooks!). There is one customer who loves the soup so much that he has eaten 189,403 bowls of soup (multiply this). Other customers in the store walk by and they often say, "Look at the man ____________ his soup."

SHORT STUFF - Jenny was a very tiny girl. Sometimes she wished she was a little taller. One day she purchased a calculator. A very strange thing occurred as she was working this problem: Enter 49040, subtract 411, divide by 15, multiply by 3, add 200, multiply by 5, add 7704. To her amazement the calculator which was also very tiny began spelling out words. Jenny began asking the calculator questions. When she asked the calculator what she could do about being so short, the calculator printed, "Buy ________________."

4. NERVOUS PIERRE - The French chef was very excited about his new TV show. Tonight, March 13 (enter 313), was his first broadcast. He was introducing a dish named Noodle Surprise. To the 313 already entered, multiply 43 which is the number of noodles Pierre is going to use. Before the show began Pierre had some preliminary preparations to perform. First he peppered 47 noodles (add this). But alas! He had added too much and had to subtract 6,398 grains of pepper. Pierre's next step was to take the noodles and ______________.

5. SELOH?? - There is an unusual animal called a Seloh which lives in the freshman lockers at Northville High School. Marvin was a curious freshman who wanted to see a Seloh. It is not easy to see a Seloh since they are very quick. Marvin began his search by first looking in locker 224. He did not find a Seloh, but add 26,020 which is the number of peanut shells he found. Next he looked in locker 99 (add this) but instead of a Seloh he found 200 orange peels (add this). Multiply your total by 2. Add 618, the locker he ran to next. He frantically opened the locker, but it was in vain, for all he found was a bunch of _________.

PROBLEM-SOLVING PRACTICE VIA STATISTICAL DATA

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The activities reprinted here have been selected from a booklet which is available from the author and editor of this Monograph.

ETHNIC POPULATION

(Addition and Subtraction of Whole Numbers)

1. The table below gives the numbers of people of differing ancestry (distribution of population by ethnic origin) for Canada in 1971.

British	9 624 120	Polish	316 425
French	6 180 120	Native Indian and Eskimo	312 765
German	1 317 200	Jewish	296 940
Italian	730 820	Asiatic	285 540
Ukrainian	580 655	Russian	64 475
Netherlands	425 945	Others	1 046 510
Scandinavian	384 795		

   Find the total population of Canada in 1971.

2. Choose the three groups from the following four which together have a total population closest to that of the German group: Ukrainian, Italian, Netherlands, and Scandinavian.

3. Choose the two groups from the Polish, Native Indian and Eskimo, Jewish and Asiatic groups which together have a total population closest to that of the Ukrainian group.

4. How many more are there of British and French ancestry than of all others combined?
   Hint: Look for a shortcut using the total population.

5. Twelve groups are listed by name in the table. What is the difference between their total and the total of those listed as "Others"?

6. Except for "Others," the groups are listed in decreasing order. Name the two consecutive groups whose difference from one another is the least.

7. Excluding the first three groups and the "Others," name the two consecutive groups whose difference is the greatest.

8. Try to find the distribution of population by ethnic origin for your province. Arrange the population in decreasing order of numbers. Is the order different from that given in the table?

TRAFFIC INJURIES AND DEATHS

(Division of Whole Numbers)

1. The chart gives the population and number of traffic injuries in one month for four Alberta cities and towns. The figures are for 1968.

City	Population	Traffic Injuries
Edmonton	376 925	184
Calgary	330 575	223
Lethbridge	37 186	16
Red Deer	26 171	7

We can show how safe or dangerous each city or town is by saying that one person was injured out of each set of n inhabitants

For Red Deer $26\ 171 \div 7 = 3738.7$

1 person was injured out of every 3 739 inhabitants.
We could call this number a monthly personal safety index.
Compute this index for the other cities and towns.
Which was the safest? Which was the most dangerous?

2. Obtain the data on traffic injuries and deaths for each month in cities and towns near you.
Compute the monthly personal safety index for each city or town.
Compute an annual personal safety index also.
Is any month especially dangerous?

3. Insurance companies use indexes like these to set automobile insurance rates. This chart shows the populations, the number of traffic deaths and drivers in some states of the U.S.A. and provinces of Canada.

Compute a personal safety index and a driver safety index for each.
Where do you think insurance rates might be highest? Lowest?

State or Province	Population	Number of Traffic Deaths	Number of Automobiles
British Columbia	2 067 143	542	811 590
California	19 953 134	5 080	11 646 000
Hawaii	769 913	133	501 179
Missouri	4 677 399	1 528	2 568 633
Newfoundland	514 394	96	102 295
New York	18 190 740	3 164	8 055 785
Ontario	7 684 658	1 667	2 864 979
Texas	11 196 730	3 551	6 380 057
Utah	1 059 273	308	615 014
Wisconsin	4 417 933	1 142	2 459 539

4. Find more recent data for your own province or state. How does it compare with the data above?

POPULATION CHANGE

(Addition and Subtraction of Decimals)

Complete the chart on population change for Canada from 1861 to 1971. All figures are in millions. In some cases you will be adding and subtracting positive and negative numbers.

	1861-1871	1871-1881	1881-1891	1891-1901	1901-1911	1911-1921	1921-1931	1931-1941	1941-1951	1951-1961	1961-1971
(P) Population at Beginning of Period	3.23	3.69									
(A) Births	1.37	1.48	1.54	1.55	1.93	2.34	2.42	2.29	3.19	4.47	4.11
(B) Immigrants	0.19	0.35	0.90	0.33	1.76	1.61	0.20	0.15	0.55	1.54	1.43
Total "Arrivals" A + B	1.56										
(C) Deaths	0.72	0.75	0.82	0.83	0.81	0.99	1.06	1.07	1.21	1.32	1.50
(D) Emigrants	0.38	0.44	1.12	0.51	1.04	1.38	0.97	0.24	0.38	0.46	0.53
Total "Departures" C + D	1.10										
Net Increase (A+B) - (C+D) = N	0.46										
Population at End of Period P + N	3.69										
Natural Increase A - C	0.65										
Immigration - Emigration Increase B - D	-0.19										

PRECIPITATION AND SUNSHINE

(Division of Decimals)

1. Copy and complete this chart for precipitation and sunshine for Inuvik. The first line has been completed. Check it carefully.

	Monthly Average		Daily Average	
Month	Precip. (mm)	Sunshine (h)	Precip. (mm)	Sunshine (h)
January	20.3	9.3	0.7	0.3
February	10.4	67.6		
March	16.5	173.1		
April	14.0	254.3		
May	17.5	288.9		
June	13.0	365.7		
July	34.3	314.0		
August	46.2	208.4		
September	21.1	110.1		
October	33.8	53.4		
November	14.7	19.1		
December	18.5	0		
Annual				
Average				

2. On the average, Sept Iles, Quebec, has a total annual snowfall of 423.2 cm. This is the highest average for any city or town in Canada. The lowest average is at Estevan Point, British Columbia, with 34.3 cm. How many times greater is Sept Iles' average than Estevan Point's average?

3. Estevan Point has an average total rainfall of 2993.6 mm per year. The lowest average is at Alert in the Northwest Territories, with 11.4 mm. How many times greater is Estevan Point's average than Alert's average?

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