Reading in Mathematics

by

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An examination of curriculum outlines leads one to the conclusion that reading in the content fields is one of the most neglected aspects of public school education. This can probably be attributed to the fact that traditionally it was assumed that if a student was taught to read (that is, narrative material), there would be more or less automatic carry-over from this general reading instruction to reading in the content fields. The assumption resulted in the teaching of reading during one specific period and content field material during other specific periods. Thus, little or no consideration was given to the specialized reading skills required within each content field. Gradually it was realized that too much carry-over was assumed, and that general reading skills were not automatically applied successfully to reading content area materials.

Despite the recognition that reading programs, of necessity, must teach the students to read in the content areas, instruction is still to a great extent concentrated on narrative (basal) type of reading material. This, according to Jackson (1968), can be largely attributed to the fact that the reading program is limited by time, and as such, teaching for the transfer of training to reading other types of material remains as the basis of instruction in reading.

In the content area of mathematics, the current trend at all grade levels places emphasis on teaching basic mathematical ideas that structure the discipline in addition to routine skills. This requires increased intellectual rigor and as a result places greater demands upon the reading ability of the students.

The Nature of Reading in Mathematics

Reading in mathematics is very different from reading narrative material. While it would be agreed that general reading ability is operative in reading mathematics, there are also specific, unique skills necessary for successful reading of mathematics.

General Reading Skills

Although the research is inconclusive, there appears to be general agreement between mathematics educators and reading educators on the skills of reading mathematics that are common to many content areas. These common skills are listed below, with numerals referring to the references cited:

1. Noting details (4,10,28,32,33,34,35)
2. Following directions (4,10,26,32,33,35)
3. Organizing and relating facts (4,10,29,32,33,34,35,43)
4. Judging the relevancy of information (4,10,32,35,43)
5. Recalling important facts (26,28,33)
6. Locating information (26,29,33,34)
7. Forming visual impressions (4,43)
8. Reading graphic materials (26,29,32,38)

Specific Reading Skills

Research seems to indicate rather clearly that specific reading abilities must be developed in content areas where they are to be used. In the literature reviewed there is a general consensus of opinion between mathematics educators and reading educators regarding what the specific reading skills unique to mathematics entail. These skills are briefly discussed below, under five major headings with numerals indicating references.

ADJUSTMENT TO VOCABULARY

1. Words specific to mathematics
   (1,3,4,5,8,10,11,18,19,20,24,26,28,32,34,35,43)

Before a student can read mathematics, he must learn the language of mathematics. A very careful definition of words is necessary because it is often individual words which give meaning to what is being read. Thus, understanding every word becomes critical when a single word may influence the reading of an entire problem or example. Mathematics has a technical vocabulary characterized by a high degree of precision in meaning. Such words as perimeter, quotient, numerator, subtrahend, exponent, diameter, denominator, and decimal point serve as examples of the many words that are rarely met, with the same meanings, outside of mathematics. In many cases, these words are abstract and backgrounds of experience in the real world have to be carefully laid so the student does not merely verbalize, but understands as well.

2. Words with special meanings in mathematics
   (1,3,4,5,8,10,18,19,20,24,26,28,32,34,35,43)

Also part of the highly technical vocabulary of mathematics are common words and phrases that have special meanings in mathematics which are different from the common meanings the student knows. Meanings such as mixed in improper fraction; words such as root, table, and, and between are all examples of such specialized interpretations.

VOCABULARY OF SYMBOLISM
(1,3,4,5,9,10,23,26,28,32,33,35,40,43)

In order to interpret the shorthand of mathematics, the symbols used must be recognized and must have meaning. That is, the student must be able to read a new symbolic system, different from the symoblic system of words. There are symbols for numbers: 1,35,240; for geometric figures: <AB, $\overleftrightarrow{CD}$; for operations: +, -, x, ÷; for relations: =, <, >; and highly specialized uses of common symbols such as parentheses and brackets. To add to the difficulty, the symbols and their meanings are abstract, that is, are considered apart from any application to a particular object.

Not only does the student need to know the symbols, he also needs to know the pattern which governs their arrangement. For example, to read a numeral, each digit must be observed and read according to its positional value. This value is derived from the plan of the decimal system based on grouping by tens and assigning a value (place value) to each position. This is basically a simple scheme which applied repeatedly provides numerals for any number considered. However, there is not always a pattern to follow. The names of the numbers the student first learns may be confusing in that there is no pattern to follow for awhile. For example, ten suggests no cue relationship to either nine or eleven; twelve gives no clue as to the nature of the next name, thirteen. The three and five in thirteen and fifteen have strange names. The decades - twenty, thirty, forty, et cetera - do not follow the names originally assigned to two, three, or four. It is not until sixty that the numerals are named by the names originally assigned.

Another form of mathematical shorthand is the use of abbreviations, particularly with measures. Again, there appears no pattern to follow, as witness lb. for pound, min. for minute, and ft. for foot. Yet, these symbols must be learned in order to read mathematics.

Because of the hierarchical structure of mathematics, it is not surprising that knowledge of symbols is rapidly extended to knowledge of combinations of symbols. Those combinations which use only symbols to convey complete ideas are commonly called mathematical sentences, and represent a new form of sentence structure for the student. These generally begin as equations: $4 + 5 = 9$; then a variable is introduced: $4 + n = 9$; and a new use for a common symbol, a letter, must be learned. Another combination of symbols; one step further on the hierarchy is the algorithm: the form used in writing the symbols for computation purposes. Reading these computational procedures is a very specialized type of reading in which the student not only must know the symbols but must know the basis for each step in the computation.

ADJUSTED RATE OF READING
(1,3,4,5,26,28,32,34,35,39)

Mathematics must be read slowly, deliberately, carefully, and with intense concentration. The number of pages of mathematics that a student reads each day is small when compared to the number of pages he reads in other content areas, or for pleasure. This is due to the nature of the mathematical material. It is concise, contains more ideas per line and page than most other writing, and is

written at a highly abstract level. That is, it is concerned with ideas and symbols rather than with actual objects. Rereading and vertical reading are often required, and particular attention must be paid to individual words. Thus, the student must make a definite adjustment in his rate and style of reading.

READING CHARTS, GRAPHS, AND TABLES (1,4,9,26,32,34)

Although suggested by many writers as a special skill in reading mathematics, this might be interpreted to mean that it is a reading skill to be taught in mathematics classes because of the quantitative nature of the data involved. Certainly the ability to get information from these sources is widely used in other content areas. In mathematics, the skill of reading charts, graphs, and tables could be extended to include their construction as well.

READING IN VERBAL PROBLEM SOLVING (3,4,5,10,26,28,31,32,34,35,43)

Verbal word problems are often considered *the* reading material of mathematics. The above sections should indicate that this is not the case. However, it is in the problem solving phase of mathematics that reading skills have their major application, because this is considered the central, crucial area of mathematics - "the very essence of mathematical behavior." The skills of reading most often related to problem solving are the literal, interpretive, and critical comprehension skills. Because the position of solving problems is uppermost in the hierarchy of mathematical skills, all the specialized reading skills discussed in the above sections also apply to problem solving.

CHARACTERISTICS OF MATHEMATICS READING MATERIALS (3,4,5,14,16,21,26,28,30,34,35,36,43)

There is available at all school levels an ever-increasing array of mathematics materials which require reading. This is particularly evident with the advent of individualized programs and published enrichment materials. However, the "traditional" textbook is still considered by many teachers as *the* course of study. This makes the selection of mathematics textbooks very cruicial for the development of an effective mathematics program. Yet, textbook selection committees often completely ignore readability of textbooks, and make the assumption that a mathematics textbook written for a specific grade level will be able to be read by all pupils in that grade. The research on textbook readability does not confirm this assumption.

The mathematics materials requiring reading are unique in that they involve content which is markedly different from any other materials the student must read. The vocabulary, both words and symbols, has been discussed previously. The typical textbook material has been described as lacking continuity, being very terse and concise, having little contextual relationship, and mixing technical vocabulary and vernacular vocabulary with the symbols of mathematics. Thus, in reading such materials, many of the habits or techniques the student has learned that are suitable for reading narrative materials must be extensively revised.

Research on Reading in Mathematics

Research on the role of reading abilities in mathematics may be classified into five categories, which are overlapping to different degrees: (1) mathematics and general reading ability, (2) mathematics and vocabulary, (3) mathematics and specific reading skills, (4) verbal problem solving and reading ability, and (5) readability of mathematics textbooks. Several studies in each of these categories are summarized in the sections which follow.

Mathematics and General Reading Ability

Studies may be found dating as far back as the 1910s. Monroe (1918) found that the same problem could be stated verbally 28 different ways in arithmetic textbooks; obviously reading was involved in these different statements. Stevens (1932) concluded that ability in the fundamental operations of mathematics was more closely correlated with ability in problem solving than with general reading ability. Coffing (1941) found no relationship between silent reading ability and ability in mathematics. Mortion (1953) reported skill in problem solving correlated highly with skill in the fundamental operations and with intelligence, but showed a low, though positive, correlation with general reading speed. Balow (1964) used sixth grade students in an effort to determine, among other things, whether or not general reading ability was significantly associated with problem solving ability. He concluded that general reading ability does have an effect on ability to solve problems, noting that this differs from the findings of most of the previous studies in this area. He suggested the reason for this was that rather than using two groups, good and poor readers, he used pupils with a total range of reading ability. He concluded that his findings "point out the importance of considering children's reading ability... when teaching problem-solving skills." (Balow, 1964)

All of these results necessarily depend upon the reading and mathematics tests used. Studies in this area seem to indicate some positive correlation between general reading ability and mathematics. However, the correlations are not particularly high; this is to be expected because general reading scores very often include tests of paragraph reading, reading literary materials for main ideas, and general vocabulary items.

Mathematics and Vocabulary

Foran (1933) found that technical terms and other unfamiliar words interfered greatly with performance in problem solving, at different age and grade levels. Eagle (1948), Johnson (1944) and Johnston (1949) all reported that mathematics vocabulary was closely related to achievement in mathematics and could even be considered a main factor. Johnson (1952) concluded that a program of word enrichment was necessary for the understanding of mathematics textbooks in Grade 5. As a result of an experiment to determine the effects of a systematic, direct study of mathematical vocabulary on fifth graders' achievement in problem solving, Vanderlinde (1964) recommended incorporating vocabulary direct-study techniques into the mathematics curriculum. Lyda and Duncan (1967) identified and

studied directly, with second grade pupils, 178 terms considered to have quantitative meaning. They concluded that a significant growth in problem-solving ability resulted from this direct study of quantitative vocabulary.

In contrast to the studies on general reading ability, research in the area of vocabulary indicates a consistently positive and strong connection between mathematics and vocabulary, especially mathematics vocabulary.

Mathematics and Specific Reading Skills

Lessenger (1925) found that specific instruction in reading the signs of operation had favorable effects on mathematics computation scores. In a study involving general language ability, vocabulary, and specific reading skills, Hansen (1944) found significant differences between good and poor problem solvers. Two studies which appear to report conflicting results are those of Treacy and Fay. Treacy (1944) showed clear evidence in favor of specific instruction in reading skills for mathematics; Fay (1950) found that arithmetic achievement was not specifically related to any group of reading abilities. The apparent conflict is explained by Russell (1960) as stemming from the fact that the reading abilities which Treacy and Fay tested were not the same. Koenker (1941) found no differences in reading comprehension ability between sixth grade students classed as good or poor according to their ability to handle the long division algorithm. Coulter (1965) concluded that pupils receiving special skills instruction relating to vocabulary, literal interpretation of problems, and selection of the proper solution process, appeared to gain in both reading and mathematics performance. Gilmary (1967) reported that students receiving help in both arithmetic and reading achieved higher than those receiving help in arithmetic only. The reading skills stressed were reported as having significant transfer value for the arithmetic classes.

The research in this area seems to indicate a need for direct teaching of certain skills and abilities applicable to reading in mathematics. Generally, the reading skills stressed in the studies were reading symbols, gaining meanings from symbols, transforming and applying symbols, and those reading skills most closely related to quantitative thinking.

Verbal Problem Solving and Reading

Many of the above studies overlap this area and will not be referred to in this section. Problem solving is included as a spearate section because the solving of verbal or word problems is a most important part of elementary school mathematics and is the most obvious area of relationship between reading and mathematics. In the field of mathematics education, there is no area which has received greater attention than problem solving: the very essence of mathematical behavior is the solving of problems. For over fifty years researchers have been inquiring into the improvement of instruction in this area, yet we still know very little about this complex mental process, problem solving.

Many lists and summaries of research on problem solving have been compiled, one of which is a selected list of research sources done by Riedesel (1969) for

The Arithmetic Teacher. The summaries indicate that there is some agreement that problem solving involves "a group of skilled and interrelated activities marked by relational thinking in a variety of patterns." (Spencer, 1960) There also seems to be general agreement that problem-solving ability is not related to any one specific reading skill, but rather is related to reading abilities such as word recognition and word meaning skills for general and mathematical vocabularies, the ability to grasp quantitative relationships and to draw inferences, and the ability to integrate ideas.

Readability of Mathematics Textbooks

Heddens and Smith (1964) examined the readability of five commercially-available series of mathematics textbooks for Grades 1-6. They concluded that the readability of the selected texts was generally above the assigned grade level, although there was considerable variation within each textbook and among the texts of each series.

Stauffer (1966) compared vocabularies in primary grade basal readers and textbooks of three content areas, including mathematics. The results indicated very little overlap of vocabularies between the basal reader and the content texts, and a lack of uniform vocabulary usage in the content texts. The recommendation was made for a program of word attack skills emphasizing meaning in each of the content areas.

Kerfoot (1961) examined the vocabulary in six arithmetic textbook series for Grades 1 and 2. He compiled a list of 49 basic words for Grade 1 and 370 basic words for Grade 2. Of the Grade 2 words, 62 did not appear on either the Gates List of Vocabulary for Primary Grades or the Dale List of 769 Easy Words. This would indicate that the 62 basic mathematics words should be taught.

Reed (1968) analyzed the vocabulary of California state-adopted mathematics textbooks for Grades 1 through 3. She found little agreement between the vocabularies of the state-adopted texts in mathematics and reading.

Hill (1967) cites research indicating the density of concepts, the difficult vocabulary, and the generalized impersonal style of presentation as criticisms of content area textbooks, including mathematics textbooks. Evidence reported tends to confirm that content textbooks may be a hinderance to the pupil. The suggestion is made that successful mastery of content textbooks depends on a systematic program of instruction in comprehension study skills.

Summary and Conclusions

Reading in mathematics is highly specialized. Improvement in reading of mathematics takes place when instruction and practice are provided to strengthen the unique skills needed. These skills center around the highly technical word and symbol vocabulary, the rate and style of reading, the reading of word problems, and reading data presented in charts and graphs.

Teachers of mathematics must accept a large part of the responsibility for teaching pupils the special reading skills necessary for understanding the basic

processes and quantitative relationships of mathematics. This is not to imply the old cliché that "every teacher must be a reading teacher." Rather, all teachers should become more proficient teachers in their own areas of specialization, through teaching *of* their subject area material rather than teaching reading *in* their subject field. The person best equipped to teach reading of mathematics is the mathematics teacher, who should see the teaching of mathematical reading as an integral part of the learning of mathematics. To do this, however, mathematics teachers should seek assistance from reading specialists.

The fact that steps must be taken to make mathematics teachers more aware of the importance of teaching reading of mathematics is apparent from a review of the related literature. The vast majority of the writing dealing with reading of mathematics appears in the field of reading education. There is comparatively little attention given to this problem in mathematics education literature. It would appear that at least some members of the mathematics education community should turn their attention to this problem. Two areas of concern are evident: making mathematics teachers aware of their responsibility for teaching pupils the special skills necessary, and making specific suggestions as to how these skills can most effectively be taught.

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