Reading in Mathematics

by

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Introduction

ARITHMETIC is where numbers fly like pigeons in and out of your head.

ARITHMETIC is numbers you squeeze from your head to your hand to your pencil to your paper till you get the answer.
(from "Arithmetic" by Carl Sandburg)

READING is externally guided thinking.
(from *Cognitive Psychology* by Ulric Neisser, 1967)

READING is reasoning.
(reprinted in *Reading Research Quarterly* by E.L. Thorndike, 1917)

READING is thinking with someone else's head instead of one's own.

Mathematics (or arithmetic), poetry, and reading involve complex cognitive processes. To understand them is an awesome task. We achieve it only in degrees.

Mathematics and reading, as suggested by the definitions, are in fact closely related. Pitts (1952) found a highly significant relationship (.53) between functional competence in mathematics and reading grade level. In fact the relationship was higher than between math and IQ (.46).

Krantz (1957) in trying to predict success in the content areas in senior secondary schools (from measures taken in the seventh grade) found that reading *vocabulary* was more closely related to all content areas on the ninth grade level than any other measured ability. Reading *comprehension* was the best predictor for the eleventh grade. *Study skills* was the third most persistent predictor.

These are some of the reasons, then, that I have selected this particular approach (that is, to consider vocabulary, comprehension, and study skills) to reading in the field of mathematics. The following outline (moving from Testing to teaching) will provide a preview of my presentation.

testing

Vocabulary	*Comprehension*	*Study Skills*
Vocab-Concept	Readability	Magic number
General-Specific	Concept load	7 ± 2
Analysis of basic list	Concept development	Flexibility
		Model Approach
Invariant word parts	Advance organizer	Book parts
Invariant prefixes	Reasoning guides	Making Assignments

teaching

Diagnosing

To identify functional reading problems and where within the reading process the problem occurs, a number of formal and informal evaluative procedures may be used.

First, standardized reading test results from the student's permanent record should be collected. This will quickly reveal the range of reading competence within a class - generally a very wide range. A study was done using seventh and eighth graders of two schools in Winnipeg. The study examined the ranges of difficulties encountered in the students' reading competence. It was found that the seventh graders ranged from below a third grade reading ability to college level with a mean of 7.9. The eighth graders ranged approximately the same but the mean was slightly higher at 8.2.

Second, one may glean some indications of problem areas from reading test data. If rate scores are available, two sub-problems will be identifiable: (1) readers exhibiting high rate with low comprehension, and (2) readers exhibiting low rate with low comprehension.

Third, vocabulary scores below grade placement will probably interfere with the reading of expository materials and with verbal problems.

Thomas and Robinson (1972) suggest, however, that standardized tests do not answer the following questions: What are the special reading competencies for mathematics? How strong are mathematics vocabularies? These authors suggest the following technique:

1. Select a passage or section which could be handled by the average student.

2. Instruct students to read the passage as if they were being tested on it. Inform them that questions will follow and to use paper and pencil to make notes if they wish.

3. Give a specified time period within which to work, perhaps 25 minutes. Indicate the point to which the student has worked.

4. Analyze these papers for ability to handle technical terms, key concepts, and diagrams. Does the student use a scratch pad actively? Does he make sketches? Does he fill in the "inner steps" of explanations that the author assumes?

   An analysis chart may be readily compiled to pinpoint areas of weaknesses.

Another informal measure suggested by Muelder (1966) is to assign a 100 percent reading assignment in the mathematics class. This is followed by a quiz covering the following items which are discussed when giving the assignment:

1. Note the author's style (as compared to other subjects), the density of ideas, and the high level of abstraction - for example, square root.

2. Read through the assignment at moderate speed, without stops, to get the general idea.

3. Reread with pencil in hand. Check the author's figuring.

4. Notice positive and negative examples. Supplement with own examples.

5. Is the description clear enough to allow drawing a sketch?

6. Question (mentally) the author.

7. Why does it make sense? How could I explain it to someone else?

8. Maintain a glossary (read, say, write).

9. Be prepared to use the same words in different ways (specialized sense).

10. Feel free to read out loud parts causing difficulty. We all do this when interference reaches a certain level of intolerance.

A further prognostic device has been suggested by Aukerman (1972). Four columns of mathematical terms are presented to the student which he is asked to read orally from left to right. Any hesitation, repetitions, regressions, uncertainty, gross mispronunciation, or incorrect stress is to be recorded. A performance of less than one word per two seconds indicates general vocabulary problems.

Example (Form A, p.216):

horizontal	conclusions	numerical	equivalent
vertical	polygon	measurement	perpendicular

The teacher might well prepare a similar test by randomly selecting vocabulary from the index of the particular textbook in use.

Vocabulary

Having considered some procedures for diagnosing mathematics reading difficulties, let us consider the roots of the difficulties.

Earp (1967) has suggested that in mathematics we are frequently dealing with two or three sets of symbolic meaning within the same line of text. That is, traditional English orthography is interspersed with Hindu-Arabic numerals and mathematical symbols with specialized meanings.

Example: How is this expression read?

$\{ g \in W \mid g \text{ is between 5 and 15} \}$

It has also been pointed out by Aukerman (1972) that mathematical language has little redundancy, fewer contextual clues, and more symbolization and abbreviations than many other forms of writing. He further identified three types of mathematics vocabulary: (1) words not unique to math but difficult, (2) words in general usage but with specialized meaning in math, and (3) technical terms peculiar to math.

Since a one-to-one relationship does not necessarily exist between a word (printed or spoken) and a concept, principle, or other referent, several mediation processes are necessary in order to understand the meaning. Schnepf and Meyer (1971) have diagrammed the above relationship like this:

{ Stimuli → Percept → Spoken word → Printed word }

↘ Concept ↗

In fact considerable evidence exists, according to DeCecco (1968), that the learning of certain names or labels (as verbal mediators) facilitates the student's learning of a concept. Generally the class labels (as opposed to the specific object, for example, tree - fir) have been more facilitative. DeCecco's nine steps to *concept* teaching can be most valuable:

1. Describe the level of performance expected of the student after he has learned the concept.

2. Reduce the number of attributes to be learned in complex concepts and make important ones dominant.

3. Provide the student with useful verbal mediators.

4. Provide positive and negative examples of the concept.

5. Present the examples in close succession (continuity).

6. Present a new positive example of the concept and ask the student to identify it.

7. Verify the student's learning of the concept.

8. Require the student to define the concept (note, however, that concept learn- is prelingual or alingual).

9. Provide occasions for student responses and the reinforcement of these responses.

A facet of vocabulary already alluded to is the specialized nature of certain mathematical terms. Not only is it necessary for the student to have the proper conceptual background but he must also recognize whether it is a cognate (having the same technical and general meaning) or not.

In order to assist the student in handling a heavy vocabulary load, the teacher may wish to capitalize on the analogies with grammatical terms suggested by Aukerman (1972):

Grammar	*Mathematics*
noun function	numeral, variable, point
verb function	equality (=), size ( ), congruence (=)
punctuation	parentheses, braces
abbreviations	+

The difficulty of dealing with symbolism cannot be overstated. The example already stated underlines the point that several symbol systems are required to read such a statement.

indicates any element an element of whole numbers such that "sets"

Read: The set of all whole numbers such that g is between 5 and 15.

In order to give maximum assistance to students in learning vocabulary the teacher may wish to utilize the Vocabulary Analysis Chart adapted from Herber as illustrated on the following page.

The vocabulary words from a given selection are written in a list in the left-hand column. To the right of these words are various subsections into which vocabulary can be categorized such as whether or not the words have been defined and/or explained, if the word has been illustrated, the key-word parts involved, the most efficient word meaning skill, the key concept involved, and the desired level of understanding. The teacher can fill out this chart and from here determine the difficulty of the selection and the degree of assistance the children may need.

Leading directly to difficulties in comprehension of mathematical reading is the shorthand of mathematics. George Boole, a nineteenth century English mathematician (*Modern Mathematics 9*, p.54) applied algebraic symbols to logic - a procedure now commonly used.

Example: "men minus Asiatics, which are both white" becomes z (x-y) where z means white, x means men, and y means Asiatics. This may also be written z x - z y.

Vocabulary Analysis Chart ______________________________

Selection: ______________________________

Part of Selection: ____________________ Grade: ________ Teacher: ____________

Analysis Page: ________

	1-A	1-B	1-C		1-D	2			3	
Vocabulary	Defined and/or Explained	Illustrated	Key Word Parts Involved	Most Efficient Word Meaning Skill (i.e., c/s/d)	Key Concept	Relative Value	Students' Competence	Reinforcement through Context	Desired Level of Understanding	Students' Level of Understanding

Comprehension

The readability of a textbook or passage refers to the ease or difficulty with which it may be read. A whole host of factors affecting readability have been identified by Smith and Dechant (1961):

word length
percent of different words
personal reference
number of syllables
number of pronouns
number of affixes
prepositional phrases
number of difficult words
abstractness of words
sentence length
simple or complex sentences
density of facts
number of illustrations
interest and purpose
organization and format
interrelationship of ideas
clauses

Readability can easily be varied to cater to the needs of students. Much progress has been made in this regard, the SMSG materials being an example. These materials, *Secondary School Mathematics* published by Byrne Publishing of New York in 1972, have three versions - a Blue Version for the above-average reader (top 25 percent), a Green Version for the average and low-average, and a Gold Version for the low achievers.

A practical example is found in *Modern Mathematics 9* (Thomas Nelson, 1970) on page 21, the last page of the first chapter. While this passage is probably meant for independent reading, its average sentence length is 18 words and it contains 32 three-syllable and longer words. To translate these figures into a more meaningful grade equivalent score, one may use one of the many readability formulas - the FOG Index (Technique of Clear Writing, 1968).

1. Take approximately a hundred-word selection from the material you are working with.
2. Count the number of three or more syllable words found in the approximate hundred-word selection omitting any words made into three-syllable words by adding *ing* or *ed* or by being compound words (for example, bookkeeper).
3. Count the number of sentences in the hundred-word selection.
4. Determine the average sentence length by dividing the total number of words in the selection by the total number of sentences in the selection.
5. Then add the total number of three or more syllable words in the approximate hundred-word selection to the average sentence length which you calculated in number four. This result is then multiplied by .4 and the end product is the readability level of the material.

In this selection there are 180 words, 30 three or more syllable words (according to the rules), sentences averaging 18 words in length, and a readability of 19.2 - college level.

Another aspect of mathematics reading comprehension is the interpretation factor. Earp (1967) has estimated that approximately two-thirds of the differences in solving verbal problems were a result of different interpretation, not vocabulary. While this problem has not been carefully documented, it may be illustrated easily.

If I read to you the statement, "Find the square root of the sums of the squares of the differences from the mean and divide it by the number of students in the sample," you will not likely be able to process it unless you happen to be well acquainted with statistical calculations. Note how the symbolization clarifies the meaning to some extent:

$$SD = \sqrt{\frac{(X - M)^2}{N}}$$

Another example used by Earp (1967) is particularly interesting since it illustrates the value of "easy number" substitution: "The square of the sum of two numbers is equal to the square of the first number added to twice the product of the first and second numbers added to the square of the second number."

Example: $(2 + 3)^2 = 2^2 + 2\ (2X3) + 3^2$

or $(a + b)^2 = a^2 + a(aXb) + b^2$

Comprehension becomes a special problem when word or verbal problems are encountered. Generally the difficulty is due either to (1) low reading ability, (2) inadequate knowledge of mathematics shorthand or punctuation, (3) inadequate knowledge of the process, or (4) poorly worded problems.

Several procedures for overcoming these difficulties may be utilized.

Aukerman (1972) suggests a method of finding the "core" of verbal problems by (1) crossing out unnecessary words, without distorting the meaning, (2) requiring the student to paraphrase the core statement, and (3) classifying words according to their functional role; for example:

Math Individuals	*Operators*
length	twice
width	is equal to
perimeter	plus

Thomas and Robinson (1972) recommend the use of a "translation device," that is, writing out the statement in one long line on the board, then writing below it the equivalent mathematical symbolization.

Two more comprehension techniques follow. Advance organizers come from the theoretical work of Ausubel (DeCecco, 1968, p.448) and are "concepts or principles, the subsumers, introduced before the presentation of the main body of instructional material. They are chosen for their usefulness in explaining and organizing that material." The organizers provide a framework within which facts may be referenced.

Reasoning guides as proposed by Herber (1970, p.106) are the last comprehension strategy described here. The basic idea behind reasoning guides is that they will aid the student "in focusing on information that contributes to the organizational patterns which formulate the intrinsic and extrinsic concepts." The guides are "stimulators," not tests, and are discussed with the student.

Levels of difficulty according to some acceptable taxonomy are generally assigned to these exercises. Herber prefers to use the terms literal (*), interpretive (**), and applied (***) to classify the difficulty of exercises.

Study Skills

The last major area of mathematical reading is the area designated as study skills, although this classification is by no means discrete from that of vocabulary and comprehension; it serves as an umbrella term.

Perhaps one of the governing constructs of all learning - short-term memory - is even more crucial to concepts packed with mathematics reading. George Miller, a psychologist and communications authority, has discovered the "magic number 7 ± 2" in a variety of learning tasks. Apparently our short-term memory (which allows information storage for up to one-fifth of a second without relearning) is able to process 7 ± 2 discrete bits of information simultaneously. When the limit of nine bits is reached no more information may be handled, except through recoding (concept formation). This may easily be illustrated by speaking a sentence slowly with one-second gaps between words. Those who can recall the sentence invariably will have consciously or subconsciously repeated the sentence adding one word at a time.

One study skill involved in reading mathematics is working with parallel structures, that is, the reader following both verbal context and worked-out example or he relates context to tabular material (Earp, 1967). The short-term memory is severely taxed. This is probably why several readings are often required before full comprehension occurs. Realizing this source of difficulty should aid the teacher in devising strategies to help students follow the parallel processing. Some recent textbooks provide colored symbols to aid the reader in moving from text to example.

Another study skill is reading flexibility, made by the reader to adapt his speed to the textual demand. Several reading rates have already been mentioned in the section on diagnosis. Generally speaking, mathematics materials are read at a rate slower than narrative materials. Oral reading generally has the side-effect of slowing down reading to less than 200 wpm as well as reinforcing aurally (through the listening channel) the visual stimulus. Earp (1967) suggests that much more varied eye movements are exhibited and that more right-to-left and regressive movements occur in oral reading.

A third study skill, the "model approach," has been suggested by Thomas and Robinson (1972) and combines a number of sound practices. The teacher leads students through these steps:

1. Pre-read and discuss major ideas (use subheads or read summary first)
2. Orally read key parts

3. Interrupt at crucial points to -
   draw diagrams
   do figuring
   examine tables or graphs
4. ask questions (use visual aids or colored words)
5. reread at a moderate rate
6. close books and take quiz

A number of well-known study techniques such as PQRST, SQ4R, and others may be adapted to certain types of mathematics reading as well.

A fourth study skill relates to the reading of graphs, charts, and tables. Graphs present relatively concrete data which are easily visualized and may become the basis of developing more abstract equations. Often, however, the specialized skill of reading graphs is assumed.

Aukerman (1972) suggested the following four steps in reading graphs, charts, and tables:

1. Read title. Know *what* is being compared to *what*.
2. Read labels and figures on graph. Read titles on axis.
3. Study graphs to make comparisons among different items on it. Which items give the most meaningful data?
4. Interpret significance of graph. Draw conclusions.

A further study skill is the efficient use of book parts, a skill so basic that it is commonly overlooked. To help students become proficient, functional exercises are recommended. The use of several of the following parts may be planned into each lesson (or may form the basis of the first class of the year):

1. table of contents and index
2. list of mathematical symbols
3. headings
4. italics, colored words, boxes
5. aids for pronunciation
6. chapter summaries
7. self-check tests
8. tables of squares, square roots
9. list of axioms, theorems, etc.
10. glossary

The final study aid discussed here is giving assignments. It is perhaps one of the most poorly-used techniques. Who has not heard, "Read pp.48-50 for next day"? The foremost fault in this command is that the student has no real purpose for reading - no conceptual framework in which to store information. Furthermore, if no purpose is set it becomes a *testing* rather than a *teaching* exercise. Therefore, it is necessary to *relate* the reading assignment to previous work, give guide questions or specific purpose, do follow-up to check on comprehension.

Lastly, idea books such as the following are helpful in sparking interest in methods:

Barnard, D. *A Book of Mathematical and Reasoning Problems: Fifty Brain Twisters.* Princeton, N.J.: Van Nostrand, 1963.

Earle, R.A. *Teaching Reading & Mathematics,* pp.79-84. IRA, 1976.

Gardner, M. *Mathematical Puzzles and Diversions,* Books 1 and 2. New York: Crowell, 1961.

Assignments can frequently be made challenging in the form of puzzles, problems, games, building models or manipulating materials. Perhaps, as Carl Sandburg wrote:

- Arithmetic is where numbers fly like pigeons in and out of your head.
- Arithmetic tells you how many you lose or win if you know how many you had before you lost or won.
- Arithmetic is seven eleven all good children go to heaven - or five six bundle of sticks.
- Arithmetic is numbers you squeeze from your head to your hand to your pencil to your paper till you get the answer.
- Arithmetic is where the answer is right and everything is nice and you can look out of the window and see the blue sky - or the answer is wrong and you have to start all over and try again and see how it comes out this time.
- If you take a number and double it and double it again and then double it a few more times, the number gets bigger and bigger and goes higher and higher and only arithmetic can tell you what the number is when you decide to quit doubling.
- Arithmetic is where you have to multiply - and you carry the multiplication table in your head and hope you won't lose it.
- If you have two animal crackers, one good and one bad, and you eat one, a striped zebra with streaks all over him eats the other, how many animal crackers will you have if somebody offers you five six seven and you say No No No and you say Nay Nay Nay and you say Nix Nix Nix?
- If you ask your mother for one fried egg for breakfast and she gives you two fried eggs and you eat both of them, who is better in arithmetic, you or your mother?

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Thomas, E.L., and H.A. Robinson. *Improving Reading in Every Class.* Boston: Allyn and Bacon, 1972.