Reading in Mathematics and Cognitive Development

by

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Introduction

There are many aspects to reading, including the following:

- word recognition,
- vocalizing the word (oral reading),
- understanding individual words,
- understanding groups of words,
- understanding sentences and groups of sentences,
- remembering.

This paper focuses on understanding. A student may "know," "recognize," or "understand" individual words and be able to "vocalize" words, yet not comprehend what a group of words or the sentence(s) is saying. The teacher may apply a word recognition test, a vocabulary test, or some other tests to a mathematics passage and the results may indicate the reading level is very low - at least two or three years below the grade level (age level) of the students for which the book is designed. In many instances the students may be reading in other content fields at a much higher reading level. Yet in the mathematics content area students may not be able to get the meaning from the passage. The question, of course, is *why*?

One author states that mathematics passages often:

1. are conceptually packed;
2. have a high content density factor;
3. require eye movements other than left to right (require vertical movement, regressive eye movement, circular eye movement, from word to chart to word movement);
4. require a rate adjustment;

5. require multiple readings (to grasp the total idea, to note the sequence or order, to relate two or more significant ideas, to find key questions, to determine the operation or process necessary, to conceptualize or generalize);
6. use symbolic devices such as graphs, charts, diagrams and mathematical symbols;
7. are heavily laden with their own technical language which is very precise; often common words are used with special exact meanings, for example, function. (Earp, 1970)

Reasons for Problems in Reading

Mathematics is an abstract subject. It is usually presented in a very symbolic and generalized form. Its language is highly specific. Reading in mathematics often requires a higher level of cognitive and conceptual development than the students have achieved.

A logical framework for a model of cognitive development is necessary. The model is adapted from Piaget. Proponents of Piaget generally agree that a child, in developing a concept, goes through certain stages. These stages are not discreet but rather blend or meld into one another to form a continuum. It should be noted that:

1. The order of the stages is fixed.
2. The rate of progression through the stages is not fixed - nor can they be tied to any chronological ages. Where this is done, it is only for referencing.
3. The movement along the continuum can be altered by certain factors - teaching, environment, maturation, et cetera.

The Stages

A breakdown of the stages follows along with a general description of each. It is not intended to include the full Piagetian description.

1. SENSORI - MOTOR STAGE (from 0 to 2 years)
   - preverbal - though language does start to develop
   - direct motor action
   - begins to learn to coordinate perceptive and motor functions - see things with reference to what they do with it

2. PREOPERATIONAL STAGE (2 to 7 years)
   - objects take on qualitative features
     - permanent
     - non-permanent
   - tends to be egocentric
     - tendency to see things from his own point of view
     - tendency to see one feature (elation) to the exclusion of others

3. CONCRETE OPERATIONAL STAGE (7 to 12 years)

- the child now focuses on more than one attribute at a time
- begins to see other person's point of view
- the classic operations (combinativity, reversibility, commutativity, identity, associativity) are developed

The development of operations is not a unitary development. To illustrate, the stages in development of conservation are: conservation of discreet quantities, continuous quantity, weight and conservation of volume, in that order.

4. FORMAL OPERATION (12 onward)

- characterized by full, logical thinking
- can now deal with statements that are not known or supposed to be true:

  if....then
  either....or
  either or both....or
  $a>b,\ b>c \rightarrow a>c$

An example of how a concept would develop or the manifestations of the development of a concept follow.

Given a set of beads:

- Sensori-motor	- hit, throw - only present when seen	
- Preoperational	- red, round, thick yellow, square, thin	one feature at a time used for classification
- Concrete operational	- red and round and thin - yellow and round and thick	two or more features at a time used for classification
- Formal operational	- either the red or the square or the thick ones - neither the red nor the round ones	

In summary, Piaget maintains a child moves along a continuum - the continuum representing the blending of the stages of development for each concept.

Implications for Reading

While it is known that some students may enter the formal operational stage at the age of twelve, many will not do so until the late teens and some never will. It has been reported from one piece of research that 82 percent of the eighth and ninth grade students are still in the concrete operational stage while 50 percent of the university freshman classes are still in the concrete generalization stage (research title unknown). Yet many of the mathematical terms are used as though every child is in the formal operational stage. What does all this mean? What is to be done about it?

There are two things we must do:

1. Diagnose the weaknesses more specifically than in the past.
2. Devise ways and means (a) to bring about greater cognitive development, (b) to build more word and word group associations to facilitate greater cognitive development.

Diagnosing Weaknesses

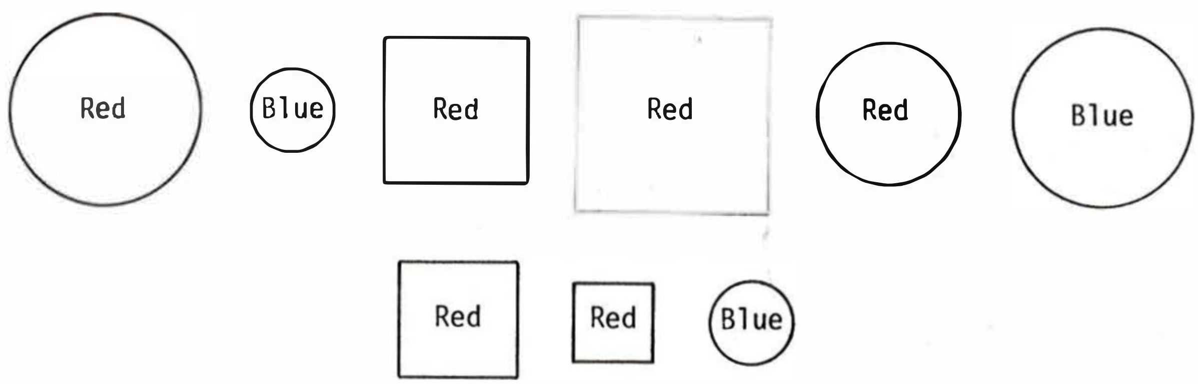
First is the diagnosis of the lack of interpretation of the printed symbol due to a lack of cognitive development in the conceptual area represented by the symbols.

The symbols may be words or mathematical symbols.

WORDS

Example (a):

Place in front of a student (age 5 to 9) red squares and blue squares and circles.



Ask the question: Are all the squares red?

The reply typical of a 5-to 6-year-old is "No." When pressed, the child may reply "No, because there are some red circles too."

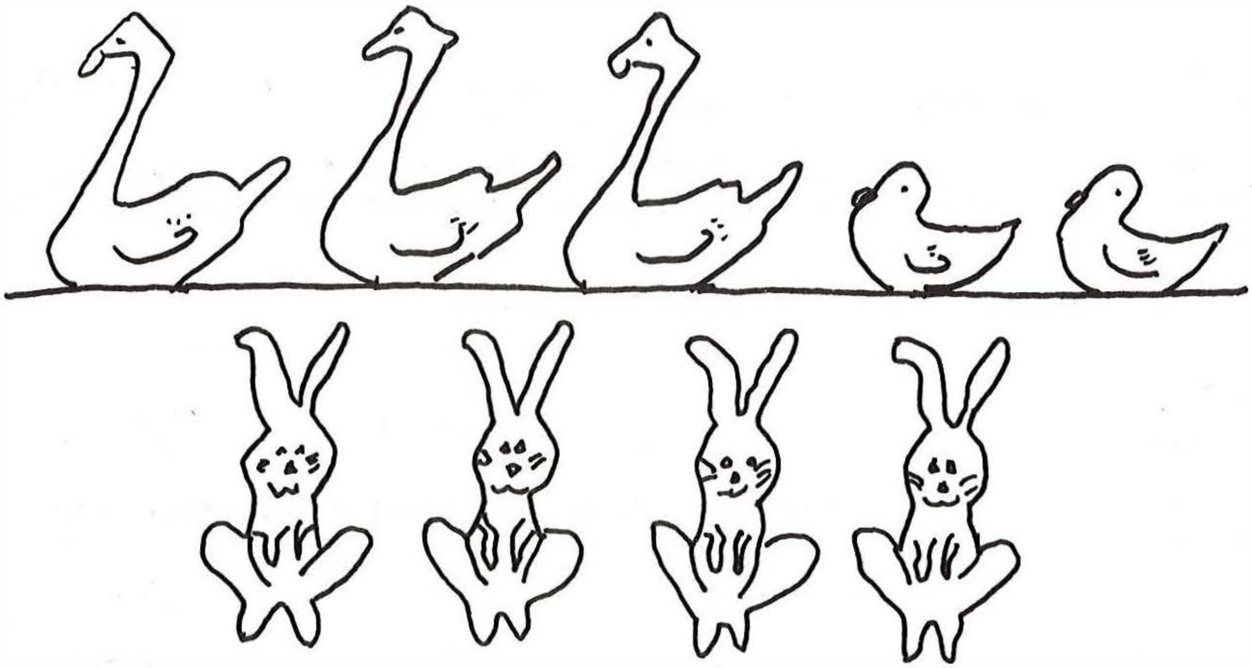
Let us consider extending this. Take the two statements:

Dogs are animals. (Squares are rectangles.)
Animals are dogs. (Rectangles are squares.)

What do we mean? In one case we mean all dogs are animals, and in the other we mean some animals are dogs. We expect children to make these subtle interpretations. (Duckworth, 1973)

Example (b):

Place in front of students drawings or photographs of geese, ducks, and rabbits.



Ask the students (age 5-9) these questions:

Is every goose some kind of bird (animal)? Why?
Are all geese some kind of bird (animal)? Why?

Then:

Is every bird (animal) some kind of goose? Why?
Are all birds (animals) some kind of geese? Why?

Typical replies indicate that students realize geese are birds and geese are animals. But they fail to realize that all birds (animals) are not geese. Yet when pressed they will identify a bird (animal) that is not a goose.

With older children similar situations can be structured:

Is every triangle some kind of polygon? Why?
Is every polygon some kind of triangle? Why?

Is every natural number some kind of a rational number? Why?
Is every rational number some kind of natural number? Why?

The concept underlying the two words "some" and "all" is that of inclusion. It is not an easy mathematical concept. It needs to be developed over a long period of time starting during the concrete operational state.

Example (c):

Place pictures of three geese, two ducks, and four rabbits in front of the students.

Then ask these questions:

Are there more geese or are there more ducks? Why?
Are there more geese or are there more birds? Why?
Are there more geese or are there more animals? Why?

Typical replies indicate that some students will not be able to accept the two classifications - geese and birds. They reply: "There are more geese because there are only two ducks."

Interviewer: "What did I ask you?"

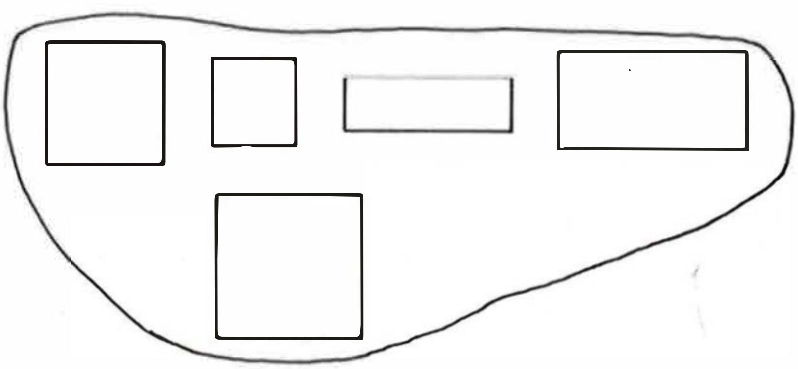
Student: "Are there more geese or more ducks?"

Interviewer: "Are there more geese or are there more birds?"

Student: "There are more geese because there are only two ducks." (Duckworth, 1973)

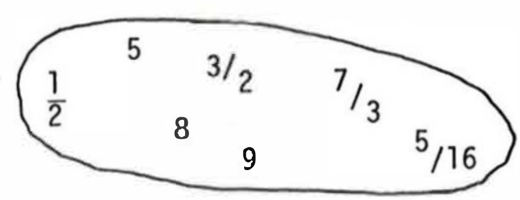
Inclusion is reflected in mathematics through the grades.

1.



Are there more squares or more rectangles in the set shown?

2.



Are there more natural or more rational numbers in the set shown?

Other words used casually in mathematics which students are expected to automatically have a grasp of are those which indicate relationships.

Example:

Each student is given these incomplete sentences and asked to complete them (typical reply in brackets).

Peter went away even though....(he went to the country).
It's not yet night even though....(it's still day).
The man fell from his bicycle because....(he broke his arm).
Fernand lost his pen so....(he found it again).
I did an errand yesterday because....(I went on my bike).

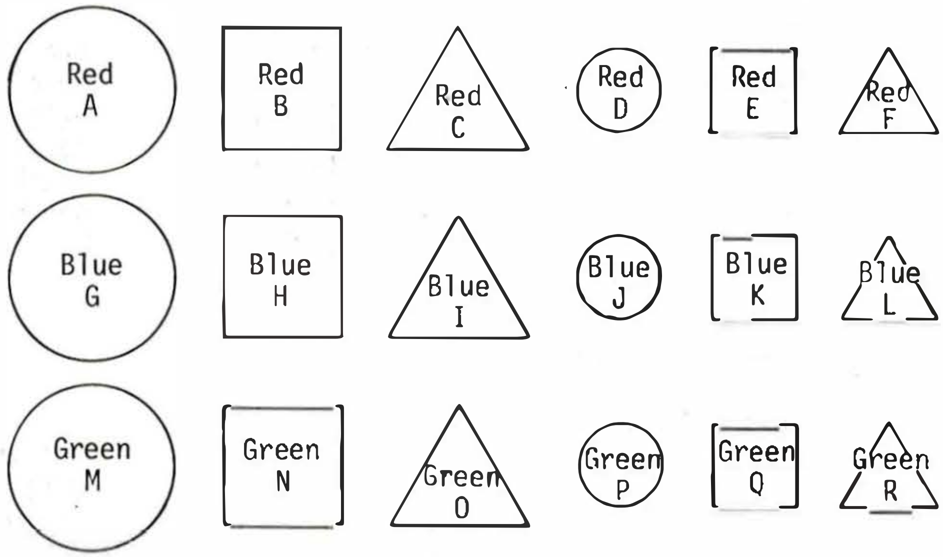
Instances where we use these terms in mathematics are:

8/3 is not an answer even though 8/3 < 10.
8/3 is not an answer because it is not a counting number.
8/3 is not an answer, so we do not include it in the solution set.
(Duckworth, 1973)

There is a tendency in mathematics to put simple words together into very difficult questions.

Example:

Each student is shown a set of shapes consisting of large triangles, circles, and squares, each in red, blue, and green. Small ones of each color and shape are also in the set. The shapes are lettered A to R. (Simpler versions can be given to younger children.)



Questions asked are of this type:

1. (a) Write the letters of all the shapes that are not circles.
   (b) Write the letters of all the shapes that are not blue circles.
   (c) Write the letters of all the shapes that are not small green circles.
2. (a) Write the letters of all the shapes that are not both big and red.
   (b) Write the letters of all the shapes that are not both small and green.
3. (a) Write the letters of all the shapes that are neither big nor red.
   (b) Write the letters of all the shapes that are neither small nor green.

4. Write the letters of all the shapes that are neither small and red nor big and green.

5. Write the letters of all the shapes that are not yellow.

Preliminary research has indicated that many students in high school had problems with the questions 3(a) and 4. Of one sample of 318 students, only 21 percent of the Grade 10s, 34 percent of the Grade 11s, and 34 percent of the Grade 12s got all the questions correct. Nearly 80 percent of the Grade 10s and 65 percent of the Grade 11s and 12s had difficulty with question 4. Apparently students were unable to keep track of four attributes at one time. This is not a reading problem in the physical sense but rather a problem of lack of cognitive development to be able to properly respond to the written words. (Bliss, 1973)

A final example of simple words that contain a difficult concept is the if... then... sentences. We all know that much of the logic underlying mathematics hinges on this sentence structure.

Example:

A picture of two girls is shown to the students.

The following statement is presented:

"If Carol goes for a walk, then Jean always goes with her."

The following questions are asked:

(a) Would it be possible for Carol to go for a walk, and Jean to stay at home?

(b) Would it be possible for Carol to go for a walk and for Jean to go for a walk?

(c) Would it be possible for Carol to stay at home and for Jean to stay at home?

(d) Would it be possible for Jean to go for a walk, and for Carol to stay at home?

(e) Would it be possible for Carol to stay at home, and for Jean to go for a walk?

Test results indicate that many students in the secondary school have difficulty with questions c, d, and e. They usually reply "Jean and Carol must always go for a walk together." (Bliss, 1973)

MATHEMATICAL SYMBOLS

So far this paper has only presented a sample of the problems connected with words. There is the problem of interpreting symbols as well. It is a compound one: first, the symbol needs to be translated into words; second, the words need to be interpreted (symbol→word→concept). Many students become familiar enough with symbols to omit the word stage (symbol→concept), but often in gaining this familiarity the word stage is used. Many students do not progress to the symbol→concept stage.

Three instances with reference to mathematical symbols will be illustrated. The first involves = (equals). Steffe points out that young children he worked with would write 3 + 4 = 7 but would not write 7 = 3 + 4. The reason given by the children paraphrased here is that equals always follows an operation - a doing. Is this built into our teaching or is it inherent in the stages of a child's development? We are not certain. (Steffe, 1975)

The second example involves basic concepts of union and intersection of sets.

$\{2, 3, 4\} \cup \{2, 4, 5\} = \{2, 3, 4, 5\}$

$\{2, 3, 4\} \cap \{2, 4, 5\} = \{2, 4\}$

Using a test devised by Bliss (1973), we tested a sample of high school students and found many students had a computational facility with union and intersection, yet did not have a usable conceptual understanding of the two.

Finally, Collis (1971) has indicated that:

$8 \times 3 = 3 \times \Delta$ is of lower cognitive level than $7 - 4 = \Delta - 7$

$8 + 4 - 4 = \Delta$ is of lower level than $4283 + 517 - 517 = \Delta$

$a \div b = 2a \div \Delta$ requires a comparatively high level of cognitive development.

In summary, a few samples are presented which tend to indicate that simple words often convey high level concepts requiring high level cognitive development. These samples are only a few of the many we have. It is to be pointed out that there is a definite need to not only teach students words but to teach them the ideas being represented by the words. The teacher must not assume that because a student does not perform adequately in response to a passage in a mathematics book that it is a reading problem in the physical sense - it may be a cognitive development problem.

Overcoming Weaknesses

Since progress is being made in identifying the specific nature of some of the weaknesses, the remaining need - that of indicating ways and means to overcome the weaknesses - is now under consideration. There are ideas concerning this objective, but at this point there are no experimental results to back them up. What follows includes a couple of ideas related to what has been said.

Ways must be found to build word-meaning associations and especially word groups-meaning associations. Perhaps symbol groups-meaning associations will follow more naturally.

Example:

Read: $8^2 + b$

Four ways to read this are:

$8^2 + b$	"eight squared plus b"
$8^2 + b$	"the square of eight plus b"
$8^2 + b$	"b is added to the square of eight"
$8^2 + b$	"b is added to eight squared"

Many varied experiences will develop the proper associations with more success than using only one of the four. (Hater, Kane, and Byrne, 1974)

A problem that symbols present is that a student may not be able to pronounce the word(s) represented. There are no hints. He cannot use the rules of his phonetics class. To establish a phoneme-grapheme relation to $\int f(x)dx$, one has to recall the following words as spoken by the teacher: the integral of the function of x with respect to x. The student can at least read aloud the word form, but may have no clue as to how to read the symbolic form. Utilizing symbols only when necessary or only after students have adequate facility with them will aid the weak reader in mathematics. Drill in symbols is necessary.

Mathematics teachers, along with others, like to quote Lewis Carroll in *Through the Looking Glass* where Humpty-Dumpty says to Alice, "When I use a word, it means just what I choose it to mean - neither more nor less." In mathematics, words are used with special meanings ("we pay them extra," said Humpty-Dumpty). These special meanings often - though not always - are related to the common meaning. Take the word "associative" as in The Associative Property. We can illustrate the relation this way:

Mathematics	*Example*
$(8 + 9) + 3 = 8 + (9 + 3)$	Jerry and Harry joined Mary. Jerry joined Harry and Mary.

Summary

To sum up, we must provide students with a broader set of experiences centered about the concepts in which weaknesses in reading are reflected. These broader experiences will provide for greater cognitive development in the conceptual area, hence the words the students read will have more specific and deeper meanings. This aspect of teaching reading is more of a concern to the mathematics teacher, while the mechanics and other aspects of reading not mentioned here may be of equal concern to both the teacher of mathematics and the teacher of reading.

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