Problem Solving in Mathematics: Are Reading Skills Important?

by

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It is unlikely that a person can solve a verbal problem if he cannot read. It is also well known that there is a significant correlation between reading ability and problem solving. Yet there are at least two major sources of doubt about the contributing effects of general reading ability to problem solving ability. First, while reading ability may be highly correlated with problem solving, problem solving skill may be due not so much to reading itself but to general intelligence, which is highly correlated with reading ability. Second, it may be that no more than a minimal reading level is necessary as one of the pre-conditions for good problem solving ability.

Balow (1964) obtained evidence that computation ability is more closely related to problem solving ability than is reading ability. Balow used 1,400 Grade 6 students with measures of reading ability and computation ability. On a test of reasoning ability he found that both reading ability and computation ability were very highly related to reasoning ability ($p < .01$). However, when he controlled for intelligence using IQ as a covariate in the analysis, he found that the relation between reading ability and reasoning ability was drastically reduced. The effect of reading ability was still significant ($p < .05$), but with IQ controlled, computation ability was a much more important factor in problem solving than reading ability.

Recent British Columbia assessment programs in reading and mathematics also provide some evidence that general reading ability may not be as important a factor in problem solving as generally accepted. Evanechko et al (1977) reported that in Grade 4, girls were superior to boys in reading on all three domains studied: word identification, prose comprehension, and comprehension of functional materials. Yet in the summary report of the mathematics assessment, Robitaille and Sherrill (1977) reported that boys exceeded girls at the Grade 4 level, as well as at Grade 8 and 12 levels, on the higher domains of comprehension and applications where reading would be expected to exert greater influence than on the first domain of knowledge.

Szetela (1977) has obtained some evidence that a reading competency over and above some minimal competency may have little effect on problem solving success. He constructed tests containing nine problems each, three of which contained only a few words essential to solving the problem, three of which were written in ordinary language sentences, and three of which were written with sentences containing difficult words and non-essential words. Three different versions of the test were constructed so that every problem appeared in all three forms. Every student solved three problems of each type. Students in Grades 5 and 6 were given the same tests. Table 1 shows that the wording of the problems appears to have had little effect on the number of correct solutions obtained by both Grade 5 and 6 students. These results are consistent with the findings of Kulm (1973) who studied the validity of standard reading tests for use in assessing mathematics text readability. Kulm found that the percentage of difficult words which is the best predictor of ordinary English reading ability was not even among the five best predictors of understanding of elementary algebra material whether in explanatory text or illustrative material.

Table I

Number of Problems Solved by Students in Five Grade 5 and 6 Classes

Grade	*Number of Students*	*Number of Problems*	*Number of Low Wording Correct*	*Number of Normal Wording Correct*	*Number of Difficult Wording Correct*
6	87	783	137	137	132
5	51	459	56	54	60

Knifong and Holtan (1976) found that of 470 errors in solving problems from a standard test, 52 percent of the incorrect solutions were due to computation or clerical error. Of the remaining 48 percent of the incorrect solutions they make no conclusion, although their examination of other errors includes 18 percent of the problems which were omitted. They offer the possibilities that such problems may have been due to reading difficulties or to the fact that the more difficult problems were in the latter part of the test.

While the above evidence seems to relegate general reading ability to reduced importance in problem solving, mathematics reading requires special reading skills not learned or experienced by students in ordinary reading instruction. Earp (1971) notes that in mathematics simple words are often used to form complex statements. Such a statment as, "the square of a sum is not equal to the sum of the squares," is troublesome for even good readers. Earp suggests that in ordinary reading instruction too much effort is expended on increasing rather than adjusting reading speed. Henney (1971) remarks on the compactness of mathematical statements with every word significant, familiar words in unfamiliar context, and other words met only in mathematics. She adds that the reader tends to concentrate on numerals, not relations, and that different patterns of eye fixation are required for reading words and numerals.

Consider also that often students are required to read a few words or symbols, refer to a diagram, and read and refer again perhaps several times in a single sentence or brief paragraph. Students may be unable or unwilling to keep changing the focus of their attention while trying to assimilate both verbal and diagrammatic information. Such reading requires a degree of concentration well beyond what is necessary for typical narrative reading.

A study by Call and Wiggin (1966) is an interesting example of the possible fruitfulness of teaching special mathematics reading skills. Call, a mathematics teacher, and Wiggin, an English teacher specializing in reading who had never even studied second year algebra in high school, each taught a second year algebra class in linear equations and word problems. Call taught his class by the usual methods, and Wiggin taught his class with instruction in such special skills as seeing relationships. Students in Wiggin's class exceeded the performance of those in Call's class with an average of 1.8 problems correct as compared to 0.6 problems correct. When results were analyzed with reading ability, verbal ability, quantitative ability, and IQ controlled, all comparisons favored the group taught with special mathematics reading skills.

One of the chief sources of difficulty in reading mathematics is students' uneasiness and unfamiliarity with mathematical symbols. Teachers must provide enough examples in different contexts together with sufficient practice with the use of symbols to ensure that the meanings of the symbols are as clear as ordinary vocabulary words. Even a familiar symbol like "=" may be misunderstood in some situations. Often the symbol is used as a substitute for the word, "equivalent." This use of the symbol may be practical in most cases but sometimes it may obscure some important conceptual differences. The interpretation of 1/3 is not the same as that of 6/18. A more important obliteration of conceptual differences occurs when one writes that $3/4 = 3 \div 4$. As concepts of a fraction, these two situations are nevertheless very different. Indeed mathematics is remarkable for such vastly different interpretations of certain concepts. Students who learn that pi is the ratio of the measures of the circumference and diameter of a circle later see pi as the measure in radians of a straight angle, and perhaps later still see pi as the sum of an infinite series. In expressions like $3x + 2x = 5x$, the symbol "=" has a different meaning than in the sentence $3x = 6$. In the first case the statement is an identity true for all real numbers, while in the second statement only one number makes the sentence true. Teachers and textbooks sometimes misuse the symbol "=" when they mean "approximately equals." Students who are taught that pi = 22/7 may well be confused when they learn later that pi is an irrational number. These examples suggest that teachers need to be alert to the various interpretations and meanings of symbols and words in mathematics so that they may more clearly introduce, explain, and exemplify new symbols and concepts and provide appropriate practice. Such care may reduce reading difficulties in problems as well as difficulties in concept attainment.

The main reason why students learn to punctuate sentences is to make the intended meaning clear. The sentence "Betty," said Sam, "is a clod," would take on a totally different meaning if the sentence were not punctuated. Similarly, the mathematical sentence, $2 \times (7 + 5) = 24$ without the bracket punctuation would have a different meaning and the statement would in fact be false. Algebra teachers are all too familiar with students who regard $2x^3$ as an equivalent form of $(2x)^3$. Such errors might be reduced by more time and attention to correct mathematical punctuation.

To help students read and interpret graphs and tables correctly, more emphasis might be given to having students obtain and record data instead of making use of artificial textbook data. Students who are given practice in graphing or charting their own data may learn the importance of choosing appropriate scales, may more easily observe relations between two variables, and may be better able to perform interpolation and extrapolation in graphs. Such experiences using student data may help to overcome negative student reactions when they see already-made graphs and tables with numbers and symbols which are visually overwhelming. A table of square roots or trigonometric function values is sometimes difficult to read because of such stunning visual impact. With the availability of mini-calculators, it is now possible for students to construct their own short trigonometric tables from which they can acquire understanding that will help them use the conventional textbook tables.

In order to help students overcome the habit of reading problems too quickly and reading them only once, mathematics teachers need help from the reading teachers. If students are given instruction and practice in reading to select the main idea, observe important relations, restate sentences and identify superfluous words in their reading classes, the mathematics teachers may have a better chance of promoting more careful reading and rereading in their own classes.

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