Reading in Arithmetic – Mathematics in Reading

by

John V. Trivett

John V. Trivett is a Professor of Education at Simon Fraser University, Vancouver. This paper was initially presented at the Third Transmountain Regional Conference, International Reading Association, Vancouver, 1977.

Our school traditions include the views that arithmetic and mathematics lessons are separate from those in reading and language arts. In universities, any intellectual collaboration between mathematics and language departments is rare, yet there is increasing awareness that much can be gained in the study of how language and mathematics can illuminate each other. This could have a significant effect on teachers of elementary grades in understanding the basics and in accomplishing greater efficiency in the teaching of the three R's.

Arithmetic and reading/writing both involve symbol systems. Children are expected to read arithmetic books, even though a reading lesson may have ended and a math lesson begun. They are asked to write number names which, in fact, is more common than any other activity in such lessons.

To the symbols, meanings are attached. The symbols have no meaning in themselves; they are arbitrary from the point of view of the learner. The symbols chosen could be otherwise, and in all languages except the learner's own, they are otherwise. Teachers know this well in language lessons, for they take the trouble to deal with the meanings which are absent when their students meet strange words.

Oddly enough, the school situation is such that meanings and spoken symbols are known to a very considerable degree in the non-arithmetic language by the time children reach school, but they have little mastery in arithmetic or mathematical language coming from home. It is rare that the kindergarten and other primary grades fill this gap by developing other than written language in arithmetic, keeping only a few number names, a few operations, and little else. One can wonder why this apparent difference.

Of our meanings to words, either spoken or written, there are both private and public components. The latter enables us to agree about word meaning, but each of us has private attachments not always significant or capable of isolated description. Nevertheless, there is always the risk that the private meaning can be very important. In extreme cases we can think of personal names or words which commonly are associated with feelings or abstractions such as beauty, democracy, or Canada. On closer examination, however, the dual attributive meanings can extend to most words.

Even though correct meanings are available to children, they also attach their own meanings to words which are not used in their first language. They do this far more so than is commonly realized. The meanings may certainly not be those which parents or teachers wish, but there is, especially these days, rarely a paucity of meaning. Perhaps this is one advantage of hundreds of hours of TV-watching.

Not only are symbols ambiguous in their private and public components, but also frequently in the accepted public interpretations. In English (and when we say English, we can refer to many, if not all, of the languages first met as one's native tongue) the words "refuse" and "wind" and the set of letters m-e-a-n cannot be read correctly until the context is known. In arithmetic-English, the symbol X can be read as "times," "multiply," "of" and "groups of," apart from the use of the same mark to indicate a wrong answer, a traffic signal for cross-roads, or a kiss!

The letter *a* in English words can be read in nine different ways, the letter *e* in seven, and so on. The fraction 2/3 in school is regularly read as "two-thirds," "2 over 3," and "two, three." The reaction to seeing the symbol "-" brings forth the responses "minus," "subtract," "take away," or "from."

These examples suggest different ways in which our symbols are frequently read or spoken. Other symbols are always read the same way, but are associated with different meanings. Does the word "dust" mean to put on particles as with a crop-duster, or remove them as with "dust the furniture"? Is a "rubber" an object used for erasing, a boot worn in wet weather, or a cloth used for dusting? Examples in non-arithmetic language are legion. In arithmetic, "6" is easy enough to read and say and write, but it will be continued to be used in a child's school life as a cardinal, ordinal, natural, or counting number; as a rational number, an integer, or a real number.

The multiplication sign, too, even though we may have decided to read and say it in only one way, (such as "times"), will have its meaning changed for 2/3 x 5/7 as compared to that probably attached 2 x 5. If this change of meaning does not occur, the fractional multiplication will appear meaningless, which is, in fact, only too common. Could this inability to comprehend several meanings be the reason that work with fractions in our schools is rarely mastered or understood?

In English there are words, and in arithmetic-English there are also words. 5 is a word even though it is also called a numeral; so is 378. Both words can be seen as nouns. Table is a noun in English (E); so is "cloth." Tablecloth is a compound noun. In arithmetic-English (AE), 3 + 2, 6 - 1, 1/3 x 12, 379 - 1, 380 - 1 - 1 are compound nouns.

Compound nouns can be short or long like "Newfoundland," or like

1 + 2 + 0 + 1 + 0 + 2 - 2 + 1	300 + 70 + 8
1 + (1/2 x 4) + 0 + 1 + 0 + 2 - 2 + (1/4 x 4)	300 + 70 + 9 - 1
	100+100+100+ (2 x 35) +9-1

The observation that 380-1-1 can be seen as a compound noun equivalent to 378 reminds us that synonyms exist in both arithmetic and non-arithmetic language. We can call 378 the standard name for a number and 380-1-1 as one of the corresponding non-standard names. To every standard name there is an infinity of non-standard names. We can say they are equivalent in the sense that any one of them is legitimately a substitute for the standard and for every one of the other non-standards. Indeed there will be contexts where many of them will not be appropriate, but they are still equivalents.

Most arithmetic books use non-standard and standard names all the time. It seems to be a main emphasis to mix them, although traditionally the direction has been one-way: students are presented with some non-standards and are required to find the correct standards. These are called the "answers."

The words are used in sentences which can be closed, as for example 4+5=11, even though this sentence is usually held to be false. 4+5=9 is also closed, but true. At other times, sentences are open in the sense that one cannot say whether they are true or false: 4+ =12, or 4+ ☐ =12. These are the preoccupation of many texts, not to mention classroom worksheets. The students' main job is to fill the open sentences.

Rarely are children in arithmetic asked to create false sentences, although it happens in other language lessons constantly. These fall under the headings of "myth," "imaginative narrative," or maybe even honest mistruths! On the whole, we do not worry unduly whether or not the students will fall into bad habits by such non-truths. But with arithmetic, the impression still prevails that mistakes must immediately be corrected, for if otherwise left unnoticed, it would be far more attractive to the learner and be copied henceforth assiduously!

There are even sentences which can be true, false, or open. 4+6=12 is false in Base X (X now means "ten"), true in Base VIII, and open in Base V (the 6 being used not as a permitted numeral, but as an unknown or a variable).

More can be said about the similarities between English and arithmetic-English. We have mentioned sentences in both and wish only to note that 4+6=10 is a sentence in which = denotes the verb, 4+6 is the subject noun, and =10 is the predicate or the complement of the subject. It is sufficient for now to leave the correspondences there and add only that other language arts headings are also appropriate in arithmetic and mathematics, including punctuation, spelling, narrative, rhymes, rhythms, silent letters and silent digits, short forms, and sounds which are read but do not correspond to symbols actually written.

I now wish to allege that the essence, the main purpose, and the overwhelming use of arithmetic in our schools is the substitution of words for words. Computation is the process by which we produce synonyms or acceptable substitutes. Consider typical "problems" from texts:

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  374       6000       287
+ 869     -  472     x  36        163 ÷ 21
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When we work these, what do we do other than substitute names for names? For 4+9, we substitute 13, whether our practice is to put down the 3 and carry the 1 or otherwise. For 6+7, we also use 13, though more precisely we mean 6 ones + 7 ones is 13 ones.

For the subtraction we can, of course, repeat the pattern "2 from 0 won't go, borrow ten, et cetera," seeking all the time substitutes for some of the simple nouns we select from the given compounds. Alternatively, we can substitute for the compound 6000-472 the equivalent of 5999-471 which may assist us in quick transformation to the standard form of 5528.

Similarly, for the multiplication and division, we substitute and substitute, using whatever strategies we have at our disposal at the time of thinking about it. Finally, if that is required, we reach the standard name for the number which was presented as the "problem":

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   287          287 x 36 -- 287 x 9 x 4
X   36                   -- 287 x (10-1) x 4
  1722                   -- 2870
  8610                       287
 10332                      2583
                           10332
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163 ÷ 21 = 1 142/21 = 2 121/21 = 3 100/21 = ... = 7 16/21

Every time there is a sequence of substitutions, words for words.

How do learners learn to substitute legitimately? To answer this, we can examine the process of how this happens in the non-arithmetic language. There is a complexity, which we cannot go into here, as to what goes on in the learning of language. A couple of observations can be made. The first is that words must be rote learned in the sense that it is impossible to discover words without hearing that certain sets of sounds and symbols are being used by people who speak the language being studied. One has to accept that certain sets of sounds are English because that was decided before one came along. When a child learns that a name for a piece of furniture is "table," first spoken and later written, it is not because the sound or the sight of the word is like a table. The sounds and signs used are arbitrary and are merely agreed upon by long usage.

In arithmetic 4+8 does not look or sound like that which it represents, and it certainly does not look or sound like 12. Even if a child knows that 4+8=12, it would be impossible for him to know whether or not 4+9=13 is true or false, unless he knows something else. Adults know too well that this is true and perhaps miss the fact that learners have to continually face new words of a new language with all the arbitrariness.

However, mixed with the arbitrariness is a more important source of power - that of detecting the patterns of structure which we can use once we have accepted parts of language not discoverable. The awareness of patterns can be used with a vast array of similar but not identical instances. All of us, adults and children, in our speech and later in our writing, continually create new sentences, and new stories using old words and the structure of the language already being used. We change the order of words, phrases, sentences and paragraphs; we alter the stress; we color by clever juxtapositions; and we invent new compounds or use old compounds in new contexts. All of this can be done in arithmetic language too.

If we know, by some means or other - and we do not wish to imply that *any* means will do - that 4+8=8+4 (whether or not we know that the standard form for both 4+8 and 8+4 is 12), then it is highly probable that we know that any expression using two number names and a plus sign can be reversed to produce an equivalent. This is surely powerful, if not immediately, then potentially. For we can make up, and so can first grade children, endless examples and have the confidence of correct procedure. Children have an enormous vocabulary of number names, even though some may be mispronounced and frequently misspelled, so they can be tapped for practice.

4+100009 = 100009+4   12345+75843 = 75843+12345

Such are accessible, regardless of the size of the numbers and of *correct* reading. At least they can be written and this is generally more than we challenge primary children to do.

Further, if we know that, for example,

8+4=8+4+1-1=8+4+2-2=8+4+3-3=...and so on

(and this pattern can be learned easily by playing with one's fingers), then we have another pattern which *can* be seen and *can* be heard and is not arbitrary. Both examples indicate that endless examples are attainable, which suggests that the power of algebraic awareness is ready to be capitalized upon in young students. This is where the mathematics really begins to enter, though it was latent all the time.

Mathematics is concerned with patterns, structure, connections, and interrelationships, not just with the enunciation of some simple nouns to name abstract entities. Available for our substitution strategies in computation are the essences of what we have always considered as essentials: place value, symmetry, factors and multiples, fractions and their operations, decimals and percentages - all of them concerning abstract patterning. When the ideas, the concepts and the awareness of such topics are mastered, we have increased powers to use them, not only as substitutional computation, but also as further understandings of what mathematics holds in store for humans.

Patterns are also the stronger aid in non-arithmetic English and many parents have noticed that their young children, working on learning their first language, seem to grasp aspects of the structure before being concerned about acquiring extensive vocabulary. They reverse phrases, change the order of words in sentences, and transform one pronoun into others, although they find that the patterns they try are not upheld by other speakers of the language. Regularity is spotty and gradually "bringed" becomes "brought," self-invented words are abandoned, and mispronunciations are self-corrected even though patterns have been attempted. Children know and experiment with reversals of words, but discover that the game does not provide much success since most words do not reverse to recognizable English words and certainly not with equivalent meanings. Words of one, two, and three or more syllables can and are mastered; substitute ways of expressing thoughts are frequently used even if it means a more complex change of order or change of words. Patterns are regularly sought and used, and we could go on instancing many more: the conventions of reading from left to right in some languages, or right to left in others; the spacing patterns in sound, or on paper between words, sentences, and paragraphs; tenses; suffixes and prefixes; and the use of pronouns - all are built on patterns and structure.

Particularly in English, there are standard and nonstandard forms of which different contexts become unacceptable. At home Mr. Smith is called Daddy or Jim; at work he probably prefers the former as the standard. There is so much pattern that we could say the grammar of a language is the study and use of the body of conventions and non-conventions and their patterns which are permissible.

Would it not therefore be a wise component of teaching to use the pattern and rote approach in English and in arithmetic-English? By the time children enter kindergarten they have mastered a great deal of their native language in the spoken form. Yet our tradition is to keep the students silent; most certainly the children's conversation at home has been in non-arithmetic English (apart from a few counting words). Could we not feel sure that had they *conversed* in meaningful situations with the arithmetic-language, it would be just as easy for them as with non-arithmetic language? Suppose kindergarten and other primary classes emphasized activities which promoted discussion, conversation, argument, "showing and telling" about mathematical ideas and models and their applications. Would this not be an essential make-up for the lack of arithmetic-language presently existing in our homes? Instead, many teachers require the written expression *before* the spoken and even before meaning has been acquired!

It is common sense and wisdom indeed to seriously ask ourselves whether or not we could teach arithmetic using the similarities it has with language and language learning and to teach reading more efficiently through the awareness of the necessary interrelationships studied otherwise under the name of mathematics. Teachers could involve their students in games, work, and studies which persuade the children first to talk about the relevant ideas and later to choose or accept written symbols for what they already understand and can talk about. Later still, movement can rightly be made to more formality in accord with the body of mathematical and other knowledge handed down to us from the past.

Instead of children being required only to give "answers" to bookset "problems," we can ask them to continually evolve their own substitutes for the expression they begin with. They can invent one, two, three, and more syllable arithmetic words (number names), and study their interrelationships. Punctuation in arithmetic can be varied purposefully to examine the consequences resulting when students themselves attempt to gain clarity from the confusion which exists without punctuation. For example, punctuate 2+6-3x7 in different ways.

In language arts lessons, we sometimes want children to study handwriting for the purpose of improving legibility. We should also do this when they write number words and sentences, without insisting that attention be paid to other attributes such as truth or falsity, punctuation, or convention.

Imaginative work in English and in arithmetic can be the emphasis in some lessons. The value of such in English is accepted by most of us, but this is not so with mathematics. Too frequently mathematics is seen as a non-creative and unimaginative activity for the majority of humans! Yet what an opportunity for the treatment of work with different bases on different planets, within the universe (each of which uses a different language) allowing, as previously quoted, that 4+6=12 can be true, false, or open!

Arithmetic is lettered with opportunities for language analogy. Teachers can ask, "If we do it in language arts, or in reading, writing or spelling, could it possibly be done in arithmetic?" We talk of the *three* R's, and practice them disjointly. I suggest we look at the 3-in-1 possibilities, with similarities perhaps more important than the differences.

Can we capitalize more effectively, efficiently, and joyfully on the powers existing in children before they meet the educational view that intellectual life must be broken into different subjects? If we begin to consider this possibility, we shall help develop in students a new kind of educated and educating being. At the same time, teachers will not need to agonize as to whether or not they can do one R but not another. All of what we do will be integrated; individual differences will be honored both in subjects and in people; and there will be no more highly honored profession than those who profess and practice what is needed to help children grow maximally from conception to birth.