Why Johnny Can't Visualize — The Failures of the Behaviorists

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Introduction

The following occurrences can be recalled by most college mathematics instructors: Trigonometry students are unable to set up the problem about telephone poles casting shadows down a hillside; Calculus II students find the riddle about drilling a hole through the center of a ball bearing incomprehensible; Algebra students find it difficult to derive an equation from the riddle about rowing with and against the current; Calculus III students are unable to get the proper limits of integration when changing from rectangular to polar coordinates. The instructor usually comes to their rescue, sketches out a picture that represents the situation, labels the parts of the drawing and sets up the appropriate equation. The student usually makes a comment such as: "Why couldn't I set that up?" "How simple it is when you do it!" "How did you know how to draw that diagram?" or "I don't understand." The next problem attempted is equally as baffling to the student and the patient tutor may spend a semi-infinite amount of time sketching out the diagrams that are implicit in the applied exercises. Why is it that so many students are unable to come up with any diagram that even remotely resembles the situation described in the problem? The student is unable to associate any visualization with the data related in the "story problem." This phenomenon we will refer to as "poor visualization."

Mathematics instructors refer to poor visualization by many different phrases. Some examples are: "The student can't read." "He is unable to understand." "He doesn't put in any effort." "He needs to be spoon fed." "He is immature." "He doesn't care." or "He is plain stupid." These remarks indicate a lack of understanding of the situation and are brutually unfair and insensitive to the student afflicted with poor visualizations. Certain equally brutal remedies are advanced, such as: "Fail the lot of them." "Give them more to do." "Have them hire a tutor." "Stay with it until they get it." or "Solve by the step process, first..." Most instructors find some way to avoid coming to grips with the situation, such as: "Skip the story problems." "Ask only about basics." or "Stress the formulas." If some instructors persist in trying to remedy the situation by some means or seek to punish the students for their inability, they are quite often reminded (perhaps by the Dean of Engineering) that "Our students don't need this theoretical math. What we want is mastery of the basic formulas." or (perhaps by the Department Chairman) to "Avoid these trivial applications. We waste too much time because the students are not able to set up the problems." One student was even heard to complain to a faculty member, "I have always had trouble with story problems. Besides, I have come here to learn, not to think."

Poor visualization is not new, but an increasing number of students seem to be afflicted with this disease. Why is it some astute students have developed good visualization and others are visually bankrupt? It is the purpose of this discussion to identify some of those facets of educational development that contribute to poor visualization.

Ingredients for Good Visualization

In order to have a good visualization, a person must have a sufficient "memory bank" of experiences to draw from and some practice in forming visualizations. The remembered experiences have to be of sufficient quality to warrant a vivid accurate recollection. These experiences are best realized if they consist of a concrete nature rather than an abstract one. The person should have been an active participant rather than a spectator. Experiences are best remembered if they produce a pleasurable reaction as opposed to a frightening, baffling, or arduous reaction. In addition, experiences that are relevant, rather than those artifically contrived or intricate, yield a better chance of creating a visualization.

The problem posed must be appropriate to the developmental level of the student. Consistent experiences with problems beyond the student's level of abstraction cause the student to believe the attempts are futile. The problems should be clearly stated and as explicit as possible. The more closely related to the student's interest and experience, the more likely a successful visualization of the problem will occur.

In addition to possessing the necessary background and having a good problem, the student should have an adequate reason for wanting to visualize the problem. In short, what reward can the student expect for attempting to visualize the problem? Direct rewards such as grades, acceptance of peers and/or instructor, or perhaps even the inner experience of intellectual contentment are important. Note here that we stress a visualization of the problem, not necessarily the ability to solve the problem as visualized. Instructors could help encourage students to visualize by being willing to mete out great amounts of partial credit for visualizations and even attempted visualizations.

Encouragements for Poor Visualization

Very early in life a child is encouraged to avoid visualization. The typical three-year-old is anxious to please the parents. Eating and toilet habits are beginning to conform to the ones desired by the parents. Most parents take pleasure in showing off the superior intellect of their child. The child is often encouraged to say verbal expressions that sound like "one, two, three," et cetera. The child has learned a verbal pattern that is encouraged and rewarded. He has not learned to count! Counting implies enumerating objects, but the child does not relate the sounds he is making to the counting of objects. This is usually the first "mathematical fraud" performed by the child. The child gives the adult the response desired, passes it off as mathematical understanding and receives a reward for "faking" the knowledge.

To cite a few other instances where a preschool child is encouraged to be a "mathematical fraud," consider the poem "1 and 1 is 2, 2 and 2 is 4, 4 and 4 is 8," et cetera, or counting backwards a la Sesame Street. Indeed we have heard that more preschool children can count backwards from ten than can count backwards from eight. During this period the child begins to be trained in answer emphasis. When asked to respond to a problem like "What is 3 and 3?" a wrong answer, say 4, might be given. When that answer is not accepted, the child may begin to rapidly give out random answers in hopes of satisfying the questioner. For example, the responses might come 7, 2, 11, 8. This is one of the first indications that the child is beginning to realize that the answer is to be rewarded and not the reasoning that produces the right answer.

When the child enters the school system, a greater variety of adult forces begin to come into play. One which occurs very often is the rush to symbols and their manipulation. Many learning theorists claim that the child should still be operating at the concrete level well into first grade, yet we find many teachers working on number concepts without the benefit of manipulatives. A great deal of pressure is put on the child to be an answer producer. The answers are the rote memorized responses to verbal sayings and are quite divorced from the reality of the situation. Indeed, the ones most in need of the concrete experience are penalized for being "slow." (These are usually boys.) The child willing to play games with symbols that have no reality is highly rewarded and praised. (These are usually girls.) Many a time a teacher can be observed doting over the child who answers quickly and uses no manipulatives. The comment often goes something like "Sally can do the problems without the beans!" Many of these children, thought to be advanced by their teachers, are initiating the course that will eventually crumble when visualization skills become necessary.

Even if the child survives the first grade with good concrete experiences and a fair ability to produce visualizations, the pressures will continually try to drive the child off course. Second grade teachers have been heard to comment, for example: "Put down the 3 and carry the 1." "Cross out the 3, change it to 2, and put the 1 over by the 5." and "If you can only answer faster you will get into my three-minute club."

The first two of these comments concern algorithms and are particularly conducive to memorization of symbol manipulation rather than reasoning and visualization. The last comment is referring to memorization of basic addition concepts where now the student is virtually forced to respond without thinking, that is, make a conditioned response. Again, those performing the least desirable trait, mere memorization, are likely to be the ones rewarded while the visualizers and the reasoners receive little recognition for their efforts. Comments of third grade teachers are similar: "Multiply the 6 by the 7 and get 42, put down the 2, carry the 4, 6 times 3 is 18, add the 4 getting 22, put that down, and the answer is 222." "We will keep doing this page until we can all get it in less than two minutes." "Always in these story problems subtract the smaller one from the larger one." At this level, the teacher is running into the visualization problem. The last comment was a signal that the teacher was no longer going to ask the child to read the problem and figure some method of solving via a visualization but would accept an answer produced by any means. The suggestion of a rule for all problems on that page completely discourages the student from getting a clear understanding of the problem. A fourth grade teacher instructs: "Always divide the larger number by the smaller." When it says 'more', you subtract." and "Cross out the 4, put a 9 above the 0 and a 1 by the 3." A fifth grade teacher sets the answer-producing strategies: "Divide the numerator and denominator by 3." "Don't ask how many 29s in 8763 but ask how many 3s in 8." or "Put the 2 under the 8 of the multiplier and carry the 3." A sixth grade teacher encourages: "Don't pay any attention to 'ladder' division. This short-cut will work better." "Invert the divisor and multiply." or "Slide the decimal point over two places to the right and add the % sign." A seventh grade teacher shares the secret of successful computation: "Divide the 'is' by the 'of' and you will get the right answer." Algebra instructors suggest: "Take the 5 to the other side and subtract." "Line up the formula of 'rate x time = distance' and then plug in the values." or "Memorize the quadratic formula then you won't have to try to factor." Some high school teachers say: "There is one way to work these problems. The first step is ....., second step ....., third step .....," et cetera.

All of the above expressions are encouragements for the child to avoid thinking about the problem, to avoid coming up with a vivid visualization, and instead to come up quickly and somewhat magically with the proper answer. Succumbing to these suggestions leads the child into a fraudulent behavior of generating answers for problems he does not understand.

We are not against memorization of facts, algorithms, and formulas but we are against divorcing this memorization from the essence of mathematical development. One of the greatest encouragers for avoiding visualization is the standardized time frame test. The plight of the fifth grade teacher on Long Island was described as follows:

> I know our textbook series stresses the "ladder" method of long division but it is too slow to use on the standardized tests (PEP) given to the children. We have decided at this school to teach the traditional algorithm rather than bother with the "ladder" method.

Here the format of the test has dictated a curriculum change which opts for the expediency of producing answers. This curriculum change can only help to convince the child that visualization is unimportant. We doubt the wisdom of any curriculum change that uses as its main objectives producing answers on standardized tests.

Standardized tests have in common the feature of aiming at superficial knowledge and never digging into the depth at which a student may actually comprehend the concepts. It is a shame that society places such a high value on the correct answers generated by these low level cognitive skills. We firmly believe that one of the best climates for improvement of real education would be one free of standardized time frame tests.

Forces Opposed to Improvement

We believe that the behavioral psychologists are at the root of this problem and are responsible for maintaining this answer-oriented and antivisualization strategy. The bill of goods they have sold the school administration and teachers has the schools virtually concerned with pretests, post-tests, recording the level of skills, moving down the prescribed path, evaluating, rewarding and charting. They pay little attention to individual human needs, modality preferences, esthetics, problem solving, individual investigations, or in short, all the aspects that are uniquely human. With the school administration and teachers dangling on the string of the behaviorists we see the producers of textbooks rushing to create materials that can be used by the teachers. One representative of a publisher informed us that they now have revised their texts so you can avoid the following: missing addends, brain teasers, ladder method, stacked divisors, regrouping and metrics. Why? Because the teachers requested that these difficult topics be avoided. The student can't work them anyway so they may as well use a method that will be successful. Notice that all of the topics to be avoided are building blocks for good visualization.

Partial Solutions for Improving Visualization Skills

What would happen if you confronted an integral calculus class with the following instructions: "Below are five definite integrals. Write up two different real life practical problems which would eventually use the first integral in the solution of the problem. Form two problems for the second integral, et cetera." Might we have a better picture of whether or not the student understood integral calculus by his responses rather than by asking him just to evaluate these integrals?

Could we do something similar with fifth graders who have been given seven long division problems? Could we ask them to build story problems in which the division problems would be an intermediate step to the answer?

Could we ask first graders to tell stories about "3 plus 5" or "8 minus 3"? Could we ask preschoolers to count the objects won in games and compare that number with the objects won by their competitor?

Numerous analagous examples can be generated for every conceivable case. At the very least, problems of this type demonstrate to the student that there is value in visualization of the problem as well as in generating the answer.

Remarks

Since mathematical skills, as measured by instruments designed by the behaviorists, have declined dramatically, we can conclude that the behaviorists' philosophy of teaching mathematics has failed. They have practically eliminated visualization skills from the educational process. Yet even with their continuing failures they somehow dominate school administrators, influence most teachers, and are able to dictate the textbooks that are available for mathematics in the schools. Can we not call them to task for their past failures and ban them from influencing the mathematical development of children? Probably not. Their vested interest is too strong. Perhaps the few visualizers that survived and those that will continue to survive, despite the efforts of the behaviorists, will lead the way in providing opposition and alternatives for the mathematical education of our youth. The problem is always that people of vision have many demands on their time. The priority of the education of our youth may not be high enough to warrant their attention. It should be.