A Directed Reading Procedure for Mathematics

by

Walter J. Lamberg *and* ***Charles E. Lamb***
The University of Texas
Austin, Texas

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Educators have been examining the relationships between reading and mathematics for several years. Evidence clearly supports the idea that there exists an important connection between the two. A student's success in mathematics learning seems to depend heavily upon his proficiency as a reader; a student having problems in the mathematics classroom may be having difficulties with reading skills, especially those pertaining to mathematics (Leary, 1948; Call and Wiggin, 1966; Lerch, 1967; Aiken, 1972).

The purpose of this article is to present a *directed reading procedure* which can help students who are experiencing difficulty meeting the demands of reading in mathematics. The procedure is an adaptation of a study-skill technique found effective in helping students improve reading, in general. Readers may be familiar with the variations of the technique, or the original version, SQ3R (Survey, Question, Read, Recite, Review) (Robinson, 1946).

To demonstrate the need for the procedure, a brief discussion of reading problems in mathematics will be presented. The main point of this discussion is that there is no one reading problem in mathematics, but rather there are many possible specific problems students may experience. These problems can be categorized as those having to do with (1) symbols, (2) symbols in context of formulas and mathematics sentences, (3) vocabulary, and (4) comprehension.*

*A more comprehensive discussion of language factors in mathematics along with a review of recent research can be found in L.R. Aiken, "Language Factors in Learning Mathematics," *Review of Educational Research*, 2, pp.359-85, Summer 1972.

In ordinary reading, the symbols the student must learn to recognize are the letters of the alphabet. In mathematics, the symbols may be more complex. The student must recognize and understand (1) numerals, for example, 2 or 3; (2) numerals which indicate operations, for example, 10^2; and (3) other symbols, such as X, -, ÷, +, which indicate operations.

The student must recognize and understand combinations of these symbols. These symbols are combined in a way that is analogous to combining letters to make words. Single numerals are combined to represent larger numbers such as 326. In certain cases, letters and numerals are combined, for example, $22x = 44$. Furthermore, the student must be able to understand these symbols and symbol combinations when they appear in complete patterns such as formulas and mathematical sentences. ($22x + 4 = 26$). Reading these patterns is analogous to reading of words in meaningful groups (phrases, clauses, and sentences). A student who cannot accurately and easily recognize these many symbols has a reading problem, that is, he cannot deal with the symbolic language of mathematics.

Vocabulary is a potential source of many problems. A student may have difficulties with any one or more of the following: (1) simple words which have a special mathematical meaning - root, point, slope; (2) simple words or phrases which have a special importance in mathematics - how many, how many more; (3) technical terminology, that is, words which are new to the student and which represent new concepts - cosine, tangent, polygon; (4) abbreviations such as in. for inches.

Finally, the student may have difficulties with comprehension, that is, understanding all the symbols and words when they are used in (1) formulas and statements, (2) problems, (3) instructions for problems, and (4) explanations of concepts, generalizations, principles and operations. Most reading educators see comprehension as a complex skill area rather than single skill. The complexity of comprehension is apparent in the reading of mathematics. The following is intended to illustrate this complexity: (1) the student must be able to locate and retain details, ideas, and relationships; (2) he must be able to translate details, ideas, and relationships into numerals and other symbols and vice versa; (3) the student must be able to interpret details, ideas, and relationships that are not explicitly presented; (4) he must be able to apply what he reads to the solving of problems; (5) and he must be able to read for evaluations, that is, he must be able to evaluate his own solutions for accuracy.

The directed reading procedure to be presented can help students meet the difficulties noted above.

The procedure is similar to other directed-reading procedures or activities (Catterson, 1971; Robinson, 1972; Shepherd, 1973; Maffei, 1973); all have the element of direction as a key feature. They consist of a set of questions or instructions which direct the student while he is reading, as opposed to a set of instructions or suggestions given before or after the reading has been done. The instructions are based on an identification of the tasks believed to be important in successfully reading material of particular content or reading for a particular purpose.

The procedure we present has certain advantages not found in others. It was designed to be applicable to all mathematics problems. It is more thorough; it includes steps for self-evaluation, revision, and reflection. It calls for observable or overt responses and, as a result, insures (1) that students will, in fact, make the responses to each step, (2) that teachers will be able to observe all the responses, and (3) that students can provide themselves with feedback.

The procedure also serves as a diagnostic tool. It provides the means by which the teacher can identify the specific difficulties a student is having, by giving the teacher the opportunity to observe the responses students make while reading and working through a problem. If a student does have difficulty, the teacher has more than just the incorrect answer to consider. The teacher can see whether or not the student is locating and understanding the main ideas, important details, and symbols presented in the problem itself, or in the instructions for, or explanation of, the problem.

The procedure can be followed each time a new skill, concept, and/or type of problem is introduced. The student responds overtly - in writing - to each step. If he can do one problem successfully, he can use a more efficient method for subsequent problems of the same type. The procedure has 10 steps.

1. *Read the problem quickly to obtain a general idea of its content. Write a statement of the general idea.* In the first step, the student is to gain a general idea of the material. The student is providing himself with an "advance organizer," (Ausubel, 1960) to aid him in the subsequent processing of the details. Research has demonstrated that advance organizers do increase comprehension of both ideas and details (Rickards, 1975). If the type of problem or some symbol or some words are new to the student, comprehension will probably not be aided by a very slow, careful reading, initially, because the student will overload himself with specifics that have no meaning for him.

2. *Reread the problem more slowly to answer the question. What specific information is given in the problem? List the details (pieces of information).* With the aid of the advance organizer, the student can begin processing the details. By writing them down, he does not have to rely on retention of what may be, at this point, not completely meaningful, specific information.

3. *What does the information mean? Translate, in writing, the numbers/symbols into words or objects, or the words into numbers or objects, or the objects into numbers or words. Reread the problem if necessary.* In the previous step, the student simply locates the information. In this step, by translating, he demonstrates understanding of the information.

4. *Show how the pieces of information fit together.* (The student might draw lines, arrows, diagrams, et cetera, to indicate relationships.) *Reread the problem if necessary.* A still higher level of understanding is demonstrated by the ability to indicate relationships, and in mathematics, the precise way details are related is a critical matter.

5. *What procedure should be used to solve this problem? Write (in outline form) the procedure.* This step insures that the student does attend to the instructions, rereading them if necessary. If the procedure is implied in the problem, this step insures the student has made the necessary inference.

This step further insures that the student relates the problem to previous instruction and reading. In a sense, he must answer the question: "Have I seen this problem before?"

6. *Use the procedure to solve the problem. Write the answer.* This step requires the student to apply what he has read.

7. *Check off each of your responses with an answer key.* The teacher provides an answer key which includes appropriate responses to all the steps.

8. *Count the number of steps you completed correctly. Write the number.* This step insures that the self-evaluation is done and provides objective feedback.

9. *Revise any incorrect responses.* Ask for help if needed. This step insures that the student learns from any errors.

10. *Reread the problem. Then briefly summarize what you have gained from working the problem.* This step insures reflection on what the student has done. It aids him to synthesize his experiences and, perhaps, make discoveries that will be of use in further reading and problem solving.

Rereading is called for in some of the steps. The necessity of rereading, as well as the acceptance of that necessity, seems of great importance to successful reading in mathematics. It should be noted that each rereading is done for specific purposes to answer particular questions.

This procedure parallels a common approach to problem solving proposed by Polya and others (Polya, 1957): (a) identify the problem to be solved - understand it, (b) devise a plan for solving the problem, (c) try out the plan, revising as necessary, (d) look back - reflect on new learnings. Steps 1-4 in the reading procedure are analogous to step (a) directly above. Step 5 parallels step (b), while steps 6-9 go with step (c) and step 10 parallels step (d).

The following presents examples of the use of the procedure for different areas of mathematics. Figure 1 shows an addition problem. A potential reading problem would be incorrectly recognizing the relationship between numbers. Figure 2 shows a trigonometry problem. Here the student is given instructions for a number of similar exercises. Potential reading difficulties would be in understanding the instructions, recognizing the many different symbols used, and understanding and applying the formulas. In the example, the student made an error. Figure 3 shows a problem requiring use of a graph. Two potential problems would be in comprehending the graph (the "pie") and recognizing the symbol for percentage (%).

Conclusion

A number of educators have developed and recommended directed reading procedures as a partial solution to potential reading difficulties in mathematics. The more thorough the procedure, the more helpful it should be, given the complexity of reading in mathematics. Teachers can help students with the reading skills required in mathematics if they know the specific difficulties the students are having.

Figure 1

Find the sum
$$\begin{array}{r} 350 \\ \underline{+345} \end{array}$$

1. Finding the sum of two 3-digit numbers.

2. 350
   345
   \+

3. three hundreds, five tens, zero ones
   three hundreds, four tens, five ones
   add

4. $\begin{array}{r} 3\ 5\ 0 \\ \underline{+\ 3\ 4\ 5} \end{array}$

5. Add the ones, then add the tens, and then add the hundreds

6. 695

7. Check steps 1-6

8. Count correct steps - 6

9. Correct if necessary. Discuss with teacher if unsure of any procedure.

10. Looking back - for example, procedures apply to other addition problems such as 4-digit problems, et cetera.

Figure 2

Let A, B, and C designate the vertices of a triangle; $\propto$, R, and ∂ designate the measures of the corresponding angles; and a, b, and c designate the length of the corresponding sides.

1. Find c, given that $b = 4$, $a = 3$, $\partial = 150°$

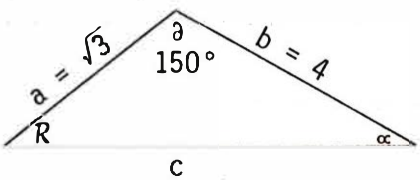
2. Find c, given that $a = 3$, $b = 7$, $\partial = 40°$.

   Note that student is given instructions for a series of similar exercises.

3. (a) Find the missing side of a triangle.

   (b) $c =$ unknown, $b = 4$, $a = \sqrt{3}$, $\partial = 150°$

   (c) Have two sides and the included angle; need to find the missing side.

   (d)
   

   (e) Use the law of cosines -

   $c^2 = a^2 + b^2 - 2ab\cos\partial$

   (f) $c^2 = (\sqrt{3})^2 + (4)^2 - 2(4)(\frac{\sqrt{3}}{2})$

   $= 3 + 16 - 12 = 6 => c \approx 2.45$

(g) Lost sign in step (f)

(h) Steps (a) to (f) correct

(i) $c^2 = 3 + 16 + 12 = 31 => c \approx 5.57$
Missed sign - careful of signs in quadrants

(j) Use law of cosines in variable forms

Note that the breakdown of a problem into parts may vary from teacher to teacher and from student to student.

Figure 3

1. Family budget - housing food

2. Rent 25%
   Savings 10%
   Food 15%
   Not housing food
   Clothing, etc. 35%
   Recreation, etc. 15%
   Average family's dollars

3. Housing - Food / Rest

4. See Figure 4

5. Add Housing % to Food %
   Subtract from 100%

6. 25% + 15% = 40%
   100% - 40% = 60%

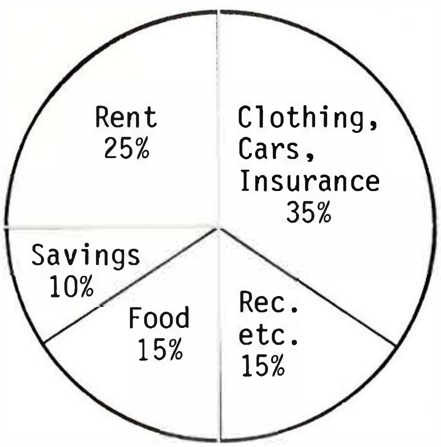
7. Check

8. 6 correct

9. Revise if necessary - with teacher's help. Teacher could use this opportunity to polish format, et cetera.

10. Look back - general graph reading, et cetera.

Figure 4



Average family's dollar

How much of the family's budget does not go to housing-food?

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