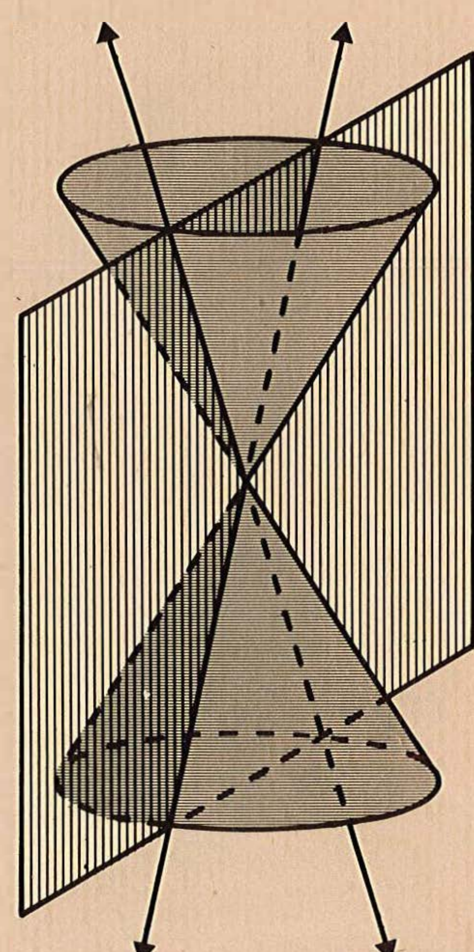
# MATH Monograph No. 6

Editor: John Percevault

December 1980

# Reading in Mathematics



Let $x = .\overline{a_1a_2 \ldots a_n}$; then $10^n \cdot x = a_1a_2 \ldots a_n.\overline{a_1a_2 \ldots a_n}$

$$\begin{aligned} 10^n \cdot x &= a_1a_2 \ldots a_n.\overline{a_1a_2 \ldots a_n} \\ x &= \phantom{a_1a_2 \ldots a_n}.\overline{a_1a_2 \ldots a_n} \\ 10^n \cdot x - x &= a_1a_2 \ldots a_n \\ (10^n - 1)x &= a_1a_2 \ldots a_n \\ x &= \frac{a_1a_2 \ldots a_n}{10^n - 1} \end{aligned}$$

Thus, $.\overline{a_1a_2 \ldots a_n} = \dfrac{a_1a_2 \ldots a_n}{10^n - 1}$

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## Contents



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# Editor's Comments

Reading is an essential skill in all subject areas - a skill which should be developed in those subjects. Can we, the teachers of mathematics, assume that skills developed in reading automatically transfer to the reading of mathematics? Or should we teach to ensure the transfer of reading skills?

This edition of the *Monograph* examines the question of reading in mathematics. Dr. Ahrendt examines the role of reading in the content areas in general.

The contributions of the authors in Section II survey the literature on reading in mathematics. The comprehensive paper of Kirkpatrick and Makar identifies reading skills required in teaching mathematics. Professor Froese's article continues the analysis. Bye links reading to Piaget's developmental stages and suggests that language development is an important requisite to reading. Dr. Szetela examines the role of reading in problem solving while Trivett identifies the problem of reading symbols - the ideograms of mathematics.

Section III presents some practical considerations for improving reading skills. Jordan argues that forces in implementing a mathematics curriculum thwart the child's ability to visualize and argues that problems should be practical and related to the child's experience. Gerald Coombs shows the application of skills taught in a reading program to reading mathematics. The article by Lamberg and Lamb applies the directed reading procedure to reading mathematics. The articles by Koster and Schill offer definite suggestions for improving the teaching of reading in the context of mathematics.

Section four presents specific ideas for teaching reading skills in mathematics. Five specific strategies, with accompanying exercises, are included in the article by Smith and Nielsen. Appreciation is extended to Marshall Bye for providing the section of teaching ideas compiled by the Calgary Board of Education. The next article, "Vocabulary and Symbols," was extracted from a Florida Department of Education publication sponsored under the Right to Read Program. Mary Wilmot's suggestions on mathematics assignment cards present a rich source of activities for the development of mathematics vocabulary.

The final section is a very complete bibliography prepared by Tricia Waddell and Rob Cowie. These former students, now teachers, aided in and encouraged the editor in the development of this *Monograph*.

A further note of appreciation and recognition must be extended to Tricia Waddell in her capacity as associate editor.

*John Percevault*

# Part I

## Reading in the Content Areas

# The Role of the Content Area Teacher and the Teaching of Reading

***by***

***Kenneth H. Ahrendt***

*Dr. Ahrendt is an Associate Professor of Reading Education, Oregon State University, Corvallis.* *This paper was presented to the Third Transmountain Conference, International Reading Association, Vancouver, 1977.*

In order to set the scene for the role of the content area teacher and the teaching of reading, we must examine some basic facts upon which this notion is based - that content area teachers teach reading.

First and foremost we must keep in mind that reading is not a separate subject to be taught at a particular time and in a particular place, but is a process which must permeate each subject in the total curriculum. If we examine this statement critically, reading is not a body of knowledge with its own content but rather an ongoing, developmental process that begins when we learn and ends when we die.

Reading is one of the most complex tasks man has devised for himself. Because man's quest for new learning is unquenchable, the reading process is never stationary. It is rather preposterous to assume that even the brightest child, given the finest instruction at the elementary level, can develop within a six-year period the reading skills needed to assimilate the vast amount of knowledge presented at the secondary school level.

The importance which our society attaches to formal schooling is reflected in the fact that today more Canadians are attending school, and on the average are remaining longer than before. Furthermore, research has clearly demonstrated a positive relationship between skill in reading and success in content subjects. Yet, thousands of Canadian children are frustrated in their secondary schooling because there is a definite lag in the development of the general or specific reading skills that could contribute to successful growth in the secondary school.

It is suggested that a probable cause, and one that is supported by research, for the poor reading performance of many secondary students is that a large proportion of those students who are not reading as well as their general ability indicates they should is due to the fact that (1) secondary readers have never been taught to prepare themselves to read, or (2) have been unable to transfer this preparation skill to new reading assignments. We know that most poor readers are passive readers; that is, readers who expect the writer to be responsible for capturing and building their interest, explaining his terminology, and so presenting his material that they need merely to gaze at it to grasp its meaning.

Reading is not a generalized skill that, once developed, can be applied in a special field. Rather, reading involves the ability to interpret this or that particular area of experience. Basic instruction, no matter how excellent it is, is not enough. Reading abilities must be developed in the areas where they can be used. No teacher can safely assume that the basic reading abilities and skills a student has achieved in the elementary school will automatically transfer to the special fields of study offered in the junior and senior high school.

The nature of experience the students have with reading in the various content areas will determine their growth in reading ability and the development of a positive attitude toward both reading and learning. Therefore, it becomes the concern of the content area teacher to assume the responsibility for teaching the basic skills which are necessary for achieving success in his content to the students in his classroom. Any teacher who uses printed matter as a vehicle of instruction assumes the responsibility to teach students how to use this material effectively and efficiently for maximum learning.

## Reading and the Content Teacher

There is much confusion about the responsibility content teachers have for teaching reading skills. The most satisfactory way to define teaching reading through content is to compare the approach of the reading teacher with that of the content teacher. Each has a specific set; each has a curriculum to teach.

Although the reading teacher's responsibilities include the improvement of reading for all students, he is frequently called upon to help students who are retarded in reading skills. His basic approach is commonly one of diagnosis and prescription. The reading materials he uses may not come from any of the content areas, since his concern is to provide material which will interest the student and at the same time be appropriate to practice the specific reading skills to be developed.

The content teacher has a defined field of information and ideas which he presents in a variety of ways to students. He is usually concerned more with the subject itself than with the development of any specific reading skills.

The two roles do not overlap. The content area teacher is not basically a teacher of reading. He is an expert in his field of study, for example, English, social studies, mathematics, science, business education, health, home economics, industrial arts, agricultural education, electronics, music and art. As a specialist his main concern is to help students learn his subject - to acquire the important skills, to comprehend the essential content, and to use such skills and knowledge to answer specific questions or in solving problems. He knows the technical vocabulary that must be learned, the sequence of learning, and the concepts the student must develop in order to have a complete and accurate picture of the subject matter being studied.

One reason why content area teachers have failed to embrace the notion that "all teachers are teachers of reading" is that they have not in the past been clear as to why they should be. They do not see reading related to the basic purpose of the content field in such a fashion as to make clear the significance of reading problems, nor what they, as content teachers, may do to assist in solving such problems.

No matter how poorly or how well high school students read, every high school teacher can help them to read with better understanding of the textbook and other materials that they are required to read in his or her course. No matter whether previous teachers did a poor or good job of training students to read, this teacher can aid his students to develop reading skills, habits, and attitudes that are necessary not only to pass his course but also to achieve more from it.

The reluctance of junior and senior high school instructors to teach content area reading usually stems from the simple fact that they have had little or no training in the teaching of reading in the content area. Content teachers must be convinced that reading skills, or maybe we should call them learning skills, are a valid part of their curriculum. These teachers need instruction and help in how to teach these skills. We must take the attitude that all teachers are responsible for reading and learning skills required in their particular content areas.

A major area of need is for the content teachers to understand the skill prerequisites for learning the tasks they assign students. Most content teachers have never paused to consider the many learning and study skills required for a student to complete a simple-sounding assignment such as "read and outline the next chapter." If the teacher can come to realize the complexity of such tasks and then learn to provide instruction to enable students to carry out all aspects of the task, better learning of the content can be achieved.

All content teachers use reading in their classrooms. However, what is important for them to realize is that knowledge of specific content is of far less importance to the student than strong, positive attitudes toward learning and toward mastery of skills that will make it possible for him to go on learning as long as he lives.

Therefore, one of the major responsibilities of every secondary content teacher is to teach students, in all his classes, efficient ways to read and study better in his particular field. These ways may differ considerably from efficient ways to read and study in some other field.

The most important factor is to present to the content area teacher the strategies and skills necessary to teach reading in his content. However, this must be done in such a way that these skills and strategies are incorporated into the ongoing process of presenting the content to students. It is unreasonable to expect a content teacher to become receptive to the idea of teaching vocabulary, study skills, and comprehension in his content as he is asked to take time from teaching content. These skills must be integrated into the teaching strategies used by the content teacher as he moves through his course.

Some subjects have special skills which are pertinent to that particular content area due to its composition and nature. Industrial Arts is replete with visual aids and illustrations which must be read and understood by the student. Mathematics has a language of symbols which must be translated by the student. Literature has many different skills of reading which change with each genre used.

Some basic skills to the successful reading of all printed materials require that the printed word must be decoded into speech either audibly or inaudibly, and the level of understanding attained in order that the purposes for reading be accomplished. Here is a major and most important skill a content area teacher can impart to his students - purpose for reading any selection, book, chapter, article or whatever. If we would take a few minutes to review the material we want students to read and then structure purpose questions as a guide to the student's reading of the passage, we are teaching a study skill and an invaluable learning tool. With purpose the student can read for answers, and comprehension takes place. The total time spent by the content teacher in preparation - 15 minutes at the most. The student benefits because he can approach his reading task with direction and a guide for gaining the information you want him to have. Always remember the purpose for learning any content area should be made clear at the beginning of the learning activity. Let students know what they are going to learn, how they should learn, and why they should learn. Giving the student a purpose for learning narrows his attention and allows him to focus on smaller areas of the learning activity with greater intensity.

## Goals for Reading

1. Reading to answer a specific question(s) raised by the teacher.
2. Reading to identify details in a selection.
3. Reading to collect all information relative to a specific question.
4. Reading to obtain directions.
5. Reading to discover the sequence of events.

Many other goals or purposes for reading can be identified by a teacher.

Every content teacher will readily admit that a student's knowledge of vocabulary is of paramount importance if the student is to master the content. Here we refer to vocabulary as the language of the content used to communicate ideas. Both the teacher and the student use this vocabulary to communicate ideas pertinent to the content subject; and the student uses this vocabulary to think ABSTRACTLY ABOUT THE SUBJECT. Ability to communicate in the language of a subject requires that one have facility with that language; hence the need for emphasis on vocabulary development in each subject.

It is important that when the content teacher plans his teaching strategies he must include in these strategies some method of introducing new vocabulary words and making his students aware of these words and their meaning. Words to be emphasized in vocabulary development by the content teacher rise out of what is being studied. Words to be studied and emphasized come from the textbook used, and when words emphasized by the teacher correspond to those which the students encounter in their texts, greater communication takes place. Teachers should explain technical vocabulary they will use orally in class during discussion and lecture. It is known that if a student does not recognize a vocabulary word in his reading he will many times ignore the word. Students frequently fail to understand what is read because the vocabulary is not within their experience. From words come meaning. The word itself has no meaning. The student will recognize the unfamiliar word and be fully aware that he doesn't know the meaning. The reader brings meaning to the word through his background of experience and the concepts he has developed during the course of his education and his lifetime.

Students should be taught early that words have no inherent meaning. Many adults, including some teachers, have never learned that words do not have meaning, and they add to a student's confusion about words and meaning by saying, "look at the word, tell me the meaning." If words had meaning, one would only need to hear and pronounce a word and meaning would emerge.

Words are no more than agreed-upon symbols that represent certain concepts. The meaning a particular word has for the student may be quite different than the meaning the word has for the teacher. The meaning of words resides within each person and is derived from his experience with it. Because no two people ever have the same experiences, they do not have the same meaning for a word nor do they attach the same connotations to its use. Thus a student may think that a *root* is the part of a plant that grows below the ground, but this is not certainly the meaning or concept a mathematics teacher would have for the same word. Human experiences, however, are sufficiently similar within a certain culture to make communication with words a reality.

Because a student can pronounce a word, and because he knows a meaning or two for that word, he may think he knows the word every time he sees or hears it. This simply is not true. The same word in our language can have several meanings depending upon the context in which we find the word. Common words may be used in new and strange combinations requiring the reader to adjust his understanding of them. The perpetually interesting thing about words is the ease with which they may be put into new, fresh, and startling combinations. These new combinations make the real difference between dull and imaginative writing. They also make the difference between simple and complex reading.

Most of our common, everyday words have more than one common meaning. These different meanings, however, are not revealed by the word when it stands alone. Often sentences in which the word appears decide the meaning of the word. The sentence surrounds the word like a mould, and shapes its exact meaning. The sentence is called the context of the word.

Out of context, a word is like a fish out of water; it is without sense or life. For example, the familiar word "take" has no meaning when printed alone on the page; it is a word out of context. But as soon as it is placed within a sentence, "Tom has to *take* a bus to get to school," "take" assumes the meaning of "use." In another sentence, "This bus will *take* you to the city," take has the meaning of "carry." See the difference in the meaning of the same word - take.

1. Peter plans to take golf sessions.

2. John will take a wife soon.

3. Prime Minister Jones will take the oath of office tomorrow.

4. Can you take the heat?

5. Mary's vaccination didn't take.

We, as teachers, cannot take it for granted that a student will automatically know the meaning of the word as the context changes. When we learn the meaning of a word in one context, we form a partial concept. Unless we add to or change that concept, the meaning we bring to the word is only that meaning which we have developed. We expand our concept when we are exposed to additional information or, in other words, we determine that in a given context a word can have more than one meaning. Word meanings do not automatically transfer from one situation to another. The meaning must be developed in the context in which it is found. What a context may reveal to a particular reader will depend upon his previous experience. It is unfortunately true that some words exist for most of us as words - spoken or printed symbols having only the vaguest of meaning for us. We have not tried to tie these words into our personal experience. We have not applied them as labels to physical objects or to the observable qualities of physical objects or to the behavior of persons, places, or things.

A word may represent a concept, but the concept resides within the student, not the word. No one can give a student a concept. One can teach him to pronounce a word, provide him with a definition of the word, and arrange activities in which the word is used, but he, and he alone, must develop his concepts. The teacher's job is to arrange activities that involve the student in direct and vicarious experiences that lead him to develop concepts himself.

The two strategies mentioned in this article are but only two of the many ways in which a content area teacher can improve the learning and/or reading skills of secondary school students. These strategies do not demand that the content area teacher be a reading specialist but that they include in their teaching techniques purpose for reading and use of the language or vocabulary of the content. They are the experts in this area and can give students a vast amount of assistance in learning by sharing their expertise.

# Part II

## Survey Papers

# Reading in Mathematics

**by**

**Joan Kirkpatrick** and **Mike Makar**

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*Mike Makar is a mathematics teacher at Ross Sheppard Composite High School in Edmonton, and is presently working on his master's degree in elementary reading at The University of Alberta.*

An examination of curriculum outlines leads one to the conclusion that reading in the content fields is one of the most neglected aspects of public school education. This can probably be attributed to the fact that traditionally it was assumed that if a student was taught to read (that is, narrative material), there would be more or less automatic carry-over from this general reading instruction to reading in the content fields. The assumption resulted in the teaching of reading during one specific period and content field material during other specific periods. Thus, little or no consideration was given to the specialized reading skills required within each content field. Gradually it was realized that too much carry-over was assumed, and that general reading skills were not automatically applied successfully to reading content area materials.

Despite the recognition that reading programs, of necessity, must teach the students to read in the content areas, instruction is still to a great extent concentrated on narrative (basal) type of reading material. This, according to Jackson (1968), can be largely attributed to the fact that the reading program is limited by time, and as such, teaching for the transfer of training to reading other types of material remains as the basis of instruction in reading.

In the content area of mathematics, the current trend at all grade levels places emphasis on teaching basic mathematical ideas that structure the discipline in addition to routine skills. This requires increased intellectual rigor and as a result places greater demands upon the reading ability of the students.

## The Nature of Reading in Mathematics

Reading in mathematics is very different from reading narrative material. While it would be agreed that general reading ability is operative in reading mathematics, there are also specific, unique skills necessary for successful reading of mathematics.

## General Reading Skills

Although the research is inconclusive, there appears to be general agreement between mathematics educators and reading educators on the skills of reading mathematics that are common to many content areas. These common skills are listed below, with numerals referring to the references cited:

1. Noting details (4,10,28,32,33,34,35)
2. Following directions (4,10,26,32,33,35)
3. Organizing and relating facts (4,10,29,32,33,34,35,43)
4. Judging the relevancy of information (4,10,32,35,43)
5. Recalling important facts (26,28,33)
6. Locating information (26,29,33,34)
7. Forming visual impressions (4,43)
8. Reading graphic materials (26,29,32,38)

## Specific Reading Skills

Research seems to indicate rather clearly that specific reading abilities must be developed in content areas where they are to be used. In the literature reviewed there is a general consensus of opinion between mathematics educators and reading educators regarding what the specific reading skills unique to mathematics entail. These skills are briefly discussed below, under five major headings with numerals indicating references.

ADJUSTMENT TO VOCABULARY

1. Words specific to mathematics
   (1,3,4,5,8,10,11,18,19,20,24,26,28,32,34,35,43)

Before a student can read mathematics, he must learn the language of mathematics. A very careful definition of words is necessary because it is often individual words which give meaning to what is being read. Thus, understanding every word becomes critical when a single word may influence the reading of an entire problem or example. Mathematics has a technical vocabulary characterized by a high degree of precision in meaning. Such words as perimeter, quotient, numerator, subtrahend, exponent, diameter, denominator, and decimal point serve as examples of the many words that are rarely met, with the same meanings, outside of mathematics. In many cases, these words are abstract and backgrounds of experience in the real world have to be carefully laid so the student does not merely verbalize, but understands as well.

2. Words with special meanings in mathematics
   (1,3,4,5,8,10,18,19,20,24,26,28,32,34,35,43)

Also part of the highly technical vocabulary of mathematics are common words and phrases that have special meanings in mathematics which are different from the common meanings the student knows. Meanings such as mixed in improper fraction; words such as root, table, and, and between are all examples of such specialized interpretations.

VOCABULARY OF SYMBOLISM
(1,3,4,5,9,10,23,26,28,32,33,35,40,43)

In order to interpret the shorthand of mathematics, the symbols used must be recognized and must have meaning. That is, the student must be able to read a new symbolic system, different from the symoblic system of words. There are symbols for numbers: 1,35,240; for geometric figures: <AB, $\overrightarrow{CD}$; for operations: +, -, x, ÷; for relations: =, <, >; and highly specialized uses of common symbols such as parentheses and brackets. To add to the difficulty, the symbols and their meanings are abstract, that is, are considered apart from any application to a particular object.

Not only does the student need to know the symbols, he also needs to know the pattern which governs their arrangement. For example, to read a numeral, each digit must be observed and read according to its positional value. This value is derived from the plan of the decimal system based on grouping by tens and assigning a value (place value) to each position. This is basically a simple scheme which applied repeatedly provides numerals for any number considered. However, there is not always a pattern to follow. The names of the numbers the student first learns may be confusing in that there is no pattern to follow for awhile. For example, ten suggests no cue relationship to either nine or eleven; twelve gives no clue as to the nature of the next name, thirteen. The three and five in thirteen and fifteen have strange names. The decades - twenty, thirty, forty, et cetera - do not follow the names originally assigned to two, three, or four. It is not until sixty that the numerals are named by the names originally assigned.

Another form of mathematical shorthand is the use of abbreviations, particularly with measures. Again, there appears no pattern to follow, as witness lb. for pound, min. for minute, and ft. for foot. Yet, these symbols must be learned in order to read mathematics.

Because of the hierarchical structure of mathematics, it is not surprising that knowledge of symbols is rapidly extended to knowledge of combinations of symbols. Those combinations which use only symbols to convey complete ideas are commonly called mathematical sentences, and represent a new form of sentence structure for the student. These generally begin as equations: $4 + 5 = 9$; then a variable is introduced: $4 + n = 9$; and a new use for a common symbol, a letter, must be learned. Another combination of symbols; one step further on the hierarchy is the algorithm: the form used in writing the symbols for computation purposes. Reading these computational procedures is a very specialized type of reading in which the student not only must know the symbols but must know the basis for each step in the computation.

ADJUSTED RATE OF READING
(1,3,4,5,26,28,32,34,35,39)

Mathematics must be read slowly, deliberately, carefully, and with intense concentration. The number of pages of mathematics that a student reads each day is small when compared to the number of pages he reads in other content areas, or for pleasure. This is due to the nature of the mathematical material. It is concise, contains more ideas per line and page than most other writing, and is written at a highly abstract level. That is, it is concerned with ideas and symbols rather than with actual objects. Rereading and vertical reading are often required, and particular attention must be paid to individual words. Thus, the student must make a definite adjustment in his rate and style of reading.

## READING CHARTS, GRAPHS, AND TABLES
(1,4,9,26,32,34)

Although suggested by many writers as a special skill in reading mathematics, this might be interpreted to mean that it is a reading skill to be taught in mathematics classes because of the quantitative nature of the data involved. Certainly the ability to get information from these sources is widely used in other content areas. In mathematics, the skill of reading charts, graphs, and tables could be extended to include their construction as well.

## READING IN VERBAL PROBLEM SOLVING
(3,4,5,10,26,28,31,32,34,35,43)

Verbal word problems are often considered *the* reading material of mathematics. The above sections should indicate that this is not the case. However, it is in the problem solving phase of mathematics that reading skills have their major application, because this is considered the central, crucial area of mathematics - "the very essence of mathematical behavior." The skills of reading most often related to problem solving are the literal, interpretive, and critical comprehension skills. Because the position of solving problems is uppermost in the hierarchy of mathematical skills, all the specialized reading skills discussed in the above sections also apply to problem solving.

## CHARACTERISTICS OF MATHEMATICS READING MATERIALS
(3,4,5,14,16,21,26,28,30,34,35,36,43)

There is available at all school levels an ever-increasing array of mathematics materials which require reading. This is particularly evident with the advent of individualized programs and published enrichment materials. However, the "traditional" textbook is still considered by many teachers as *the* course of study. This makes the selection of mathematics textbooks very cruicial for the development of an effective mathematics program. Yet, textbook selection committees often completely ignore readability of textbooks, and make the assumption that a mathematics textbook written for a specific grade level will be able to be read by all pupils in that grade. The research on textbook readability does not confirm this assumption.

The mathematics materials requiring reading are unique in that they involve content which is markedly different from any other materials the student must read. The vocabulary, both words and symbols, has been discussed previously. The typical textbook material has been described as lacking continuity, being very terse and concise, having little contextual relationship, and mixing technical vocabulary and vernacular vocabulary with the symbols of mathematics. Thus, in reading such materials, many of the habits or techniques the student has learned that are suitable for reading narrative materials must be extensively revised.

## Research on Reading in Mathematics

Research on the role of reading abilities in mathematics may be classified into five categories, which are overlapping to different degrees: (1) mathematics and general reading ability, (2) mathematics and vocabulary, (3) mathematics and specific reading skills, (4) verbal problem solving and reading ability, and (5) readability of mathematics textbooks. Several studies in each of these categories are summarized in the sections which follow.

### Mathematics and General Reading Ability

Studies may be found dating as far back as the 1910s. Monroe (1918) found that the same problem could be stated verbally 28 different ways in arithmetic textbooks; obviously reading was involved in these different statements. Stevens (1932) concluded that ability in the fundamental operations of mathematics was more closely correlated with ability in problem solving than with general reading ability. Coffing (1941) found no relationship between silent reading ability and ability in mathematics. Mortion (1953) reported skill in problem solving correlated highly with skill in the fundamental operations and with intelligence, but showed a low, though positive, correlation with general reading speed. Balow (1964) used sixth grade students in an effort to determine, among other things, whether or not general reading ability was significantly associated with problem solving ability. He concluded that general reading ability does have an effect on ability to solve problems, noting that this differs from the findings of most of the previous studies in this area. He suggested the reason for this was that rather than using two groups, good and poor readers, he used pupils with a total range of reading ability. He concluded that his findings "point out the importance of considering children's reading ability... when teaching problem-solving skills." (Balow, 1964)

All of these results necessarily depend upon the reading and mathematics tests used. Studies in this area seem to indicate some positive correlation between general reading ability and mathematics. However, the correlations are not particularly high; this is to be expected because general reading scores very often include tests of paragraph reading, reading literary materials for main ideas, and general vocabulary items.

### Mathematics and Vocabulary

Foran (1933) found that technical terms and other unfamiliar words interfered greatly with performance in problem solving, at different age and grade levels. Eagle (1948), Johnson (1944) and Johnston (1949) all reported that mathematics vocabulary was closely related to achievement in mathematics and could even be considered a main factor. Johnson (1952) concluded that a program of word enrichment was necessary for the understanding of mathematics textbooks in Grade 5. As a result of an experiment to determine the effects of a systematic, direct study of mathematical vocabulary on fifth graders' achievement in problem solving, Vanderlinde (1964) recommended incorporating vocabulary direct-study techniques into the mathematics curriculum. Lyda and Duncan (1967) identified and studied directly, with second grade pupils, 178 terms considered to have quantitative meaning. They concluded that a significant growth in problem-solving ability resulted from this direct study of quantitative vocabulary.

In contrast to the studies on general reading ability, research in the area of vocabulary indicates a consistently positive and strong connection between mathematics and vocabulary, especially mathematics vocabulary.

## Mathematics and Specific Reading Skills

Lessenger (1925) found that specific instruction in reading the signs of operation had favorable effects on mathematics computation scores. In a study involving general language ability, vocabulary, and specific reading skills, Hansen (1944) found significant differences between good and poor problem solvers. Two studies which appear to report conflicting results are those of Treacy and Fay. Treacy (1944) showed clear evidence in favor of specific instruction in reading skills for mathematics; Fay (1950) found that arithmetic achievement was not specifically related to any group of reading abilities. The apparent conflict is explained by Russell (1960) as stemming from the fact that the reading abilities which Treacy and Fay tested were not the same. Koenker (1941) found no differences in reading comprehension ability between sixth grade students classed as good or poor according to their ability to handle the long division algorithm. Coulter (1965) concluded that pupils receiving special skills instruction relating to vocabulary, literal interpretation of problems, and selection of the proper solution process, appeared to gain in both reading and mathematics performance. Gilmary (1967) reported that students receiving help in both arithmetic and reading achieved higher than those receiving help in arithmetic only. The reading skills stressed were reported as having significant transfer value for the arithmetic classes.

The research in this area seems to indicate a need for direct teaching of certain skills and abilities applicable to reading in mathematics. Generally, the reading skills stressed in the studies were reading symbols, gaining meanings from symbols, transforming and applying symbols, and those reading skills most closely related to quantitative thinking.

## Verbal Problem Solving and Reading

Many of the above studies overlap this area and will not be referred to in this section. Problem solving is included as a spearate section because the solving of verbal or word problems is a most important part of elementary school mathematics and is the most obvious area of relationship between reading and mathematics. In the field of mathematics education, there is no area which has received greater attention than problem solving: the very essence of mathematical behavior is the solving of problems. For over fifty years researchers have been inquiring into the improvement of instruction in this area, yet we still know very little about this complex mental process, problem solving.

Many lists and summaries of research on problem solving have been compiled, one of which is a selected list of research sources done by Riedesel (1969) for

*The Arithmetic Teacher.* The summaries indicate that there is some agreement that problem solving involves "a group of skilled and interrelated activities marked by relational thinking in a variety of patterns." (Spencer, 1960) There also seems to be general agreement that problem-solving ability is not related to any one specific reading skill, but rather is related to reading abilities such as word recognition and word meaning skills for general and mathematical vocabularies, the ability to grasp quantitative relationships and to draw inferences, and the ability to integrate ideas.

## Readability of Mathematics Textbooks

Heddens and Smith (1964) examined the readability of five commercially-available series of mathematics textbooks for Grades 1-6. They concluded that the readability of the selected texts was generally above the assigned grade level, although there was considerable variation within each textbook and among the texts of each series.

Stauffer (1966) compared vocabularies in primary grade basal readers and textbooks of three content areas, including mathematics. The results indicated very little overlap of vocabularies between the basal reader and the content texts, and a lack of uniform vocabulary usage in the content texts. The recommendation was made for a program of word attack skills emphasizing meaning in each of the content areas.

Kerfoot (1961) examined the vocabulary in six arithmetic textbook series for Grades 1 and 2. He compiled a list of 49 basic words for Grade 1 and 370 basic words for Grade 2. Of the Grade 2 words, 62 did not appear on either the Gates List of Vocabulary for Primary Grades or the Dale List of 769 Easy Words. This would indicate that the 62 basic mathematics words should be taught.

Reed (1968) analyzed the vocabulary of California state-adopted mathematics textbooks for Grades 1 through 3. She found little agreement between the vocabularies of the state-adopted texts in mathematics and reading.

Hill (1967) cites research indicating the density of concepts, the difficult vocabulary, and the generalized impersonal style of presentation as criticisms of content area textbooks, including mathematics textbooks. Evidence reported tends to confirm that content textbooks may be a hinderance to the pupil. The suggestion is made that successful mastery of content textbooks depends on a systematic program of instruction in comprehension study skills.

## Summary and Conclusions

Reading in mathematics is highly specialized. Improvement in reading of mathematics takes place when instruction and practice are provided to strengthen the unique skills needed. These skills center around the highly technical word and symbol vocabulary, the rate and style of reading, the reading of word problems, and reading data presented in charts and graphs.

Teachers of mathematics must accept a large part of the responsibility for teaching pupils the special reading skills necessary for understanding the basic processes and quantitative relationships of mathematics. This is not to imply the old cliché that "every teacher must be a reading teacher." Rather, all teachers should become more proficient teachers in their own areas of specialization, through teaching *of* their subject area material rather than teaching reading *in* their subject field. The person best equipped to teach reading of mathematics is the mathematics teacher, who should see the teaching of mathematical reading as an integral part of the learning of mathematics. To do this, however, mathematics teachers should seek assistance from reading specialists.

The fact that steps must be taken to make mathematics teachers more aware of the importance of teaching reading of mathematics is apparent from a review of the related literature. The vast majority of the writing dealing with reading of mathematics appears in the field of reading education. There is comparatively little attention given to this problem in mathematics education literature. It would appear that at least some members of the mathematics education community should turn their attention to this problem. Two areas of concern are evident: making mathematics teachers aware of their responsibility for teaching pupils the special skills necessary, and making specific suggestions as to how these skills can most effectively be taught.

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# Reading in Mathematics

**by**

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## Introduction

ARITHMETIC is where numbers fly like pigeons in and out of your head.

ARITHMETIC is numbers you squeeze from your head to your hand to your pencil to your paper till you get the answer.
(from "Arithmetic" by Carl Sandburg)

READING is externally guided thinking.
(from *Cognitive Psychology* by Ulric Neisser, 1967)

READING is reasoning.
(reprinted in *Reading Research Quarterly* by E.L. Thorndike, 1917)

READING is thinking with someone else's head instead of one's own.

Mathematics (or arithmetic), poetry, and reading involve complex cognitive processes. To understand them is an awesome task. We achieve it only in degrees.

Mathematics and reading, as suggested by the definitions, are in fact closely related. Pitts (1952) found a highly significant relationship (.53) between functional competence in mathematics and reading grade level. In fact the relationship was higher than between math and IQ (.46).

Krantz (1957) in trying to predict success in the content areas in senior secondary schools (from measures taken in the seventh grade) found that reading *vocabulary* was more closely related to all content areas on the ninth grade level than any other measured ability. Reading *comprehension* was the best predictor for the eleventh grade. *Study skills* was the third most persistent predictor.

These are some of the reasons, then, that I have selected this particular approach (that is, to consider vocabulary, comprehension, and study skills) to reading in the field of mathematics. The following outline (moving from Testing to teaching) will provide a preview of my presentation.

testing

| *Vocabulary* | *Comprehension* | *Study Skills* |
|---|---|---|
| Vocab-Concept | Readability | Magic number |
| General-Specific | Concept load | 7 ± 2 |
| Analysis of | Concept | Flexibility |
| basic list | development | Model |
| Invariant word | Advance | Approach |
| parts | organizer | Book parts |
| Invariant | Reasoning | Making |
| prefixes | guides | Assignments |

teaching

## Diagnosing

To identify functional reading problems and where within the reading process the problem occurs, a number of formal and informal evaluative procedures may be used.

First, standardized reading test results from the student's permanent record should be collected. This will quickly reveal the range of reading competence within a class - generally a very wide range. A study was done using seventh and eighth graders of two schools in Winnipeg. The study examined the ranges of difficulties encountered in the students' reading competence. It was found that the seventh graders ranged from below a third grade reading ability to college level with a mean of 7.9. The eighth graders ranged approximately the same but the mean was slightly higher at 8.2.

Second, one may glean some indications of problem areas from reading test data. If rate scores are available, two sub-problems will be identifiable: (1) readers exhibiting high rate with low comprehension, and (2) readers exhibiting low rate with low comprehension.

Third, vocabulary scores below grade placement will probably interfere with the reading of expository materials and with verbal problems.

Thomas and Robinson (1972) suggest, however, that standardized tests do not answer the following questions: What are the special reading competencies for mathematics? How strong are mathematics vocabularies? These authors suggest the following technique:

1. Select a passage or section which could be handled by the average student.

2. Instruct students to read the passage as if they were being tested on it. Inform them that questions will follow and to use paper and pencil to make notes if they wish.

3. Give a specified time period within which to work, perhaps 25 minutes. Indicate the point to which the student has worked.

4. Analyze these papers for ability to handle technical terms, key concepts, and diagrams. Does the student use a scratch pad actively? Does he make sketches? Does he fill in the "inner steps" of explanations that the author assumes?

   An analysis chart may be readily compiled to pinpoint areas of weaknesses.

Another informal measure suggested by Muelder (1966) is to assign a 100 percent reading assignment in the mathematics class. This is followed by a quiz covering the following items which are discussed when giving the assignment:

1. Note the author's style (as compared to other subjects), the density of ideas, and the high level of abstraction - for example, square root.

2. Read through the assignment at moderate speed, without stops, to get the general idea.

3. Reread with pencil in hand. Check the author's figuring.

4. Notice positive and negative examples. Supplement with own examples.

5. Is the description clear enough to allow drawing a sketch?

6. Question (mentally) the author.

7. Why does it make sense? How could I explain it to someone else?

8. Maintain a glossary (read, say, write).

9. Be prepared to use the same words in different ways (specialized sense).

10. Feel free to read out loud parts causing difficulty. We all do this when interference reaches a certain level of intolerance.

A further prognostic device has been suggested by Aukerman (1972). Four columns of mathematical terms are presented to the student which he is asked to read orally from left to right. Any hesitation, repetitions, regressions, uncertainty, gross mispronunciation, or incorrect stress is to be recorded. A performance of less than one word per two seconds indicates general vocabulary problems.

Example (Form A, p.216):

| | | | |
|---|---|---|---|
| horizontal | conclusions | numerical | equivalent |
| vertical | polygon | measurement | perpendicular |

The teacher might well prepare a similar test by randomly selecting vocabulary from the index of the particular textbook in use.

## Vocabulary

Having considered some procedures for diagnosing mathematics reading difficulties, let us consider the roots of the difficulties.

Earp (1967) has suggested that in mathematics we are frequently dealing with two or three sets of symbolic meaning within the same line of text. That is, traditional English orthography is interspersed with Hindu-Arabic numerals and mathematical symbols with specialized meanings.

Example: How is this expression read?

$\{\ g \in W \mid g \text{ is between 5 and 15 }\}$

It has also been pointed out by Aukerman (1972) that mathematical language has little redundancy, fewer contextual clues, and more symbolization and abbreviations than many other forms of writing. He further identified three types of mathematics vocabulary: (1) words not unique to math but difficult, (2) words in general usage but with specialized meaning in math, and (3) technical terms peculiar to math.

Since a one-to-one relationship does not necessarily exist between a word (printed or spoken) and a concept, principle, or other referent, several mediation processes are necessary in order to understand the meaning. Schnepf and Meyer (1971) have diagrammed the above relationship like this:

{ Stimuli → Percept → Spoken word → Printed word }

Concept

In fact considerable evidence exists, according to DeCecco (1968), that the learning of certain names or labels (as verbal mediators) facilitates the student's learning of a concept. Generally the class labels (as opposed to the specific object, for example, tree - fir) have been more facilitative. DeCecco's nine steps to *concept* teaching can be most valuable:

1. Describe the level of performance expected of the student after he has learned the concept.

2. Reduce the number of attributes to be learned in complex concepts and make important ones dominant.

3. Provide the student with useful verbal mediators.

4. Provide positive and negative examples of the concept.

5. Present the examples in close succession (continuity).

6. Present a new positive example of the concept and ask the student to identify it.

7. Verify the student's learning of the concept.

8. Require the student to define the concept (note, however, that concept learn- is prelingual or alingual).

9. Provide occasions for student responses and the reinforcement of these responses.

A facet of vocabulary already alluded to is the specialized nature of certain mathematical terms. Not only is it necessary for the student to have the proper conceptual background but he must also recognize whether it is a cognate (having the same technical and general meaning) or not.

In order to assist the student in handling a heavy vocabulary load, the teacher may wish to capitalize on the analogies with grammatical terms suggested by Aukerman (1972):

| *Grammar* | *Mathematics* |
|---|---|
| noun function | numeral, variable, point |
| verb function | equality (=), size ( ), congruence (=) |
| punctuation | parentheses, braces |
| abbreviations | + |

The difficulty of dealing with symbolism cannot be overstated. The example already stated underlines the point that several symbol systems are required to read such a statement.

indicates any element an element of whole numbers such that "sets"

Read: The set of all whole numbers such that g is between 5 and 15.

In order to give maximum assistance to students in learning vocabulary the teacher may wish to utilize the Vocabulary Analysis Chart adapted from Herber as illustrated on the following page.

The vocabulary words from a given selection are written in a list in the left-hand column. To the right of these words are various subsections into which vocabulary can be categorized such as whether or not the words have been defined and/or explained, if the word has been illustrated, the key-word parts involved, the most efficient word meaning skill, the key concept involved, and the desired level of understanding. The teacher can fill out this chart and from here determine the difficulty of the selection and the degree of assistance the children may need.

Leading directly to difficulties in comprehension of mathematical reading is the shorthand of mathematics. George Boole, a nineteenth century English mathematician (*Modern Mathematics 9*, p.54) applied algebraic symbols to logic - a procedure now commonly used.

Example: "men minus Asiatics, which are both white" becomes z (x-y) where z means white, x means men, and y means Asiatics. This may also be written z x - z y.

## Vocabulary Analysis Chart

Selection: ______________________________

Part of Selection: ______________ Grade: ______ Teacher: __________

Analysis Page: ______

| | 1-A | 1-B | 1-C | | 1-D | 2 | | | 3 | |
|---|---|---|---|---|---|---|---|---|---|---|
| Vocabulary | Defined and/or Explained | Illustrated | Key Word Parts Involved | Most Efficient Word Meaning Skill (i.e., c/s/d) | Key Concept | Relative Value | Students' Competence | Reinforcement through Context | Desired Level of Understanding | Students' Level of Understanding |
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## Comprehension

The readability of a textbook or passage refers to the ease or difficulty with which it may be read. A whole host of factors affecting readability have been identified by Smith and Dechant (1961):

| | |
|---|---|
| word length | sentence length |
| percent of different words | simple or complex sentences |
| personal reference | density of facts |
| number of syllables | number of illustrations |
| number of pronouns | interest and purpose |
| number of affixes | organization and format |
| prepositional phrases | interrelationship of ideas |
| number of difficult words | clauses |
| abstractness of words | |

Readability can easily be varied to cater to the needs of students. Much progress has been made in this regard, the SMSG materials being an example. These materials, *Secondary School Mathematics* published by Byrne Publishing of New York in 1972, have three versions - a Blue Version for the above-average reader (top 25 percent), a Green Version for the average and low-average, and a Gold Version for the low achievers.

A practical example is found in *Modern Mathematics 9* (Thomas Nelson, 1970) on page 21, the last page of the first chapter. While this passage is probably meant for independent reading, its average sentence length is 18 words and it contains 32 three-syllable and longer words. To translate these figures into a more meaningful grade equivalent score, one may use one of the many readability formulas - the FOG Index (Technique of Clear Writing, 1968).

1. Take approximately a hundred-word selection from the material you are working with.
2. Count the number of three or more syllable words found in the approximate hundred-word selection omitting any words made into three-syllable words by adding *ing* or *ed* or by being compound words (for example, bookkeeper).
3. Count the number of sentences in the hundred-word selection.
4. Determine the average sentence length by dividing the total number of words in the selection by the total number of sentences in the selection.
5. Then add the total number of three or more syllable words in the approximate hundred-word selection to the average sentence length which you calculated in number four. This result is then multiplied by .4 and the end product is the readability level of the material.

In this selection there are 180 words, 30 three or more syllable words (according to the rules), sentences averaging 18 words in length, and a readability of 19.2 - college level.

Another aspect of mathematics reading comprehension is the interpretation factor. Earp (1967) has estimated that approximately two-thirds of the differences in solving verbal problems were a result of different interpretation, not vocabulary. While this problem has not been carefully documented, it may be illustrated easily.

If I read to you the statement, "Find the square root of the sums of the squares of the differences from the mean and divide it by the number of students in the sample," you will not likely be able to process it unless you happen to be well acquainted with statistical calculations. Note how the symbolization clarifies the meaning to some extent:

$$SD = \sqrt{\frac{(X - M)^2}{N}}$$

Another example used by Earp (1967) is particularly interesting since it illustrates the value of "easy number" substitution: "The square of the sum of two numbers is equal to the square of the first number added to twice the product of the first and second numbers added to the square of the second number."

Example: $(2 + 3)^2 = 2^2 + 2\ (2X3) + 3^2$

or $(a + b)^2 = a^2 + a(aXb) + b^2$

Comprehension becomes a special problem when word or verbal problems are encountered. Generally the difficulty is due either to (1) low reading ability, (2) inadequate knowledge of mathematics shorthand or punctuation, (3) inadequate knowledge of the process, or (4) poorly worded problems.

Several procedures for overcoming these difficulties may be utilized.

Aukerman (1972) suggests a method of finding the "core" of verbal problems by (1) crossing out unnecessary words, without distorting the meaning, (2) requiring the student to paraphrase the core statement, and (3) classifying words according to their functional role; for example:

| *Math Individuals* | *Operators* |
|---|---|
| length | twice |
| width | is equal to |
| perimeter | plus |

Thomas and Robinson (1972) recommend the use of a "translation device," that is, writing out the statement in one long line on the board, then writing below it the equivalent mathematical symbolization.

Two more comprehension techniques follow. Advance organizers come from the theoretical work of Ausubel (DeCecco, 1968, p.448) and are "concepts or principles, the subsumers, introduced before the presentation of the main body of instructional material. They are chosen for their usefulness in explaining and organizing that material." The organizers provide a framework within which facts may be referenced.

Reasoning guides as proposed by Herber (1970, p.106) are the last comprehension strategy described here. The basic idea behind reasoning guides is that they will aid the student "in focusing on information that contributes to the organizational patterns which formulate the intrinsic and extrinsic concepts." The guides are "stimulators," not tests, and are discussed with the student.

Levels of difficulty according to some acceptable taxonomy are generally assigned to these exercises. Herber prefers to use the terms literal (*), interpretive (**), and applied (***) to classify the difficulty of exercises.

## Study Skills

The last major area of mathematical reading is the area designated as study skills, although this classification is by no means discrete from that of vocabulary and comprehension; it serves as an umbrella term.

Perhaps one of the governing constructs of all learning - short-term memory - is even more crucial to concepts packed with mathematics reading. George Miller, a psychologist and communications authority, has discovered the "magic number $7 \pm 2$" in a variety of learning tasks. Apparently our short-term memory (which allows information storage for up to one-fifth of a second without relearning) is able to process $7 \pm 2$ discrete bits of information simultaneously. When the limit of nine bits is reached no more information may be handled, except through recoding (concept formation). This may easily be illustrated by speaking a sentence slowly with one-second gaps between words. Those who can recall the sentence invariably will have consciously or subconsciously repeated the sentence adding one word at a time.

One study skill involved in reading mathematics is working with parallel structures, that is, the reader following both verbal context and worked-out example or he relates context to tabular material (Earp, 1967). The short-term memory is severely taxed. This is probably why several readings are often required before full comprehension occurs. Realizing this source of difficulty should aid the teacher in devising strategies to help students follow the parallel processing. Some recent textbooks provide colored symbols to aid the reader in moving from text to example.

Another study skill is reading flexibility, made by the reader to adapt his speed to the textual demand. Several reading rates have already been mentioned in the section on diagnosis. Generally speaking, mathematics materials are read at a rate slower than narrative materials. Oral reading generally has the side-effect of slowing down reading to less than 200 wpm as well as reinforcing aurally (through the listening channel) the visual stimulus. Earp (1967) suggests that much more varied eye movements are exhibited and that more right-to-left and regressive movements occur in oral reading.

A third study skill, the "model approach," has been suggested by Thomas and Robinson (1972) and combines a number of sound practices. The teacher leads students through these steps:

1. Pre-read and discuss major ideas (use subheads or read summary first)
2. Orally read key parts

3. Interrupt at crucial points to -
   draw diagrams
   do figuring
   examine tables or graphs
4. ask questions (use visual aids or colored words)
5. reread at a moderate rate
6. close books and take quiz

A number of well-known study techniques such as PQRST, SQ4R, and others may be adapted to certain types of mathematics reading as well.

A fourth study skill relates to the reading of graphs, charts, and tables. Graphs present relatively concrete data which are easily visualized and may become the basis of developing more abstract equations. Often, however, the specialized skill of reading graphs is assumed.

Aukerman (1972) suggested the following four steps in reading graphs, charts, and tables:

1. Read title. Know *what* is being compared to *what*.
2. Read labels and figures on graph. Read titles on axis.
3. Study graphs to make comparisons among different items on it. Which items give the most meaningful data?
4. Interpret significance of graph. Draw conclusions.

A further study skill is the efficient use of book parts, a skill so basic that it is commonly overlooked. To help students become proficient, functional exercises are recommended. The use of several of the following parts may be planned into each lesson (or may form the basis of the first class of the year):

1. table of contents and index
2. list of mathematical symbols
3. headings
4. italics, colored words, boxes
5. aids for pronunciation
6. chapter summaries
7. self-check tests
8. tables of squares, square roots
9. list of axioms, theorems, etc.
10. glossary

The final study aid discussed here is giving assignments. It is perhaps one of the most poorly-used techniques. Who has not heard, "Read pp.48-50 for next day"? The foremost fault in this command is that the student has no real purpose for reading - no conceptual framework in which to store information. Furthermore, if no purpose is set it becomes a *testing* rather than a *teaching* exercise. Therefore, it is necessary to *relate* the reading assignment to previous work, give guide questions or specific purpose, do follow-up to check on comprehension.

Lastly, idea books such as the following are helpful in sparking interest in methods:

Barnard, D. *A Book of Mathematical and Reasoning Problems: Fifty Brain Twisters.* Princeton, N.J.: Van Nostrand, 1963.

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Assignments can frequently be made challenging in the form of puzzles, problems, games, building models or manipulating materials. Perhaps, as Carl Sandburg wrote:

- Arithmetic is where numbers fly like pigeons in and out of your head.
- Arithmetic tells you how many you lose or win if you know how many you had before you lost or won.
- Arithmetic is seven eleven all good children go to heaven - or five six bundle of sticks.
- Arithmetic is numbers you squeeze from your head to your hand to your pencil to your paper till you get the answer.
- Arithmetic is where the answer is right and everything is nice and you can look out of the window and see the blue sky - or the answer is wrong and you have to start all over and try again and see how it comes out this time.
- If you take a number and double it and double it again and then double it a few more times, the number gets bigger and bigger and goes higher and higher and only arithmetic can tell you what the number is when you decide to quit doubling.
- Arithmetic is where you have to multiply - and you carry the multiplication table in your head and hope you won't lose it.
- If you have two animal crackers, one good and one bad, and you eat one, a striped zebra with streaks all over him eats the other, how many animal crackers will you have if somebody offers you five six seven and you say No No No and you say Nay Nay Nay and you say Nix Nix Nix?
- If you ask your mother for one fried egg for breakfast and she gives you two fried eggs and you eat both of them, who is better in arithmetic, you or your mother?

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# Reading in Mathematics and Cognitive Development

by

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## Introduction

There are many aspects to reading, including the following:

- word recognition,
- vocalizing the word (oral reading),
- understanding individual words,
- understanding groups of words,
- understanding sentences and groups of sentences,
- remembering.

This paper focuses on understanding. A student may "know," "recognize," or "understand" individual words and be able to "vocalize" words, yet not comprehend what a group of words or the sentence(s) is saying. The teacher may apply a word recognition test, a vocabulary test, or some other tests to a mathematics passage and the results may indicate the reading level is very low - at least two or three years below the grade level (age level) of the students for which the book is designed. In many instances the students may be reading in other content fields at a much higher reading level. Yet in the mathematics content area students may not be able to get the meaning from the passage. The question, of course, is *why*?

One author states that mathematics passages often:

1. are conceptually packed;
2. have a high content density factor;
3. require eye movements other than left to right (require vertical movement, regressive eye movement, circular eye movement, from word to chart to word movement);
4. require a rate adjustment;

5. require multiple readings (to grasp the total idea, to note the sequence or order, to relate two or more significant ideas, to find key questions, to determine the operation or process necessary, to conceptualize or generalize);

6. use symbolic devices such as graphs, charts, diagrams and mathematical symbols;

7. are heavily laden with their own technical language which is very precise; often common words are used with special exact meanings, for example, function. (Earp, 1970)

## Reasons for Problems in Reading

Mathematics is an abstract subject. It is usually presented in a very symbolic and generalized form. Its language is highly specific. Reading in mathematics often requires a higher level of cognitive and conceptual development than the students have achieved.

A logical framework for a model of cognitive development is necessary. The model is adapted from Piaget. Proponents of Piaget generally agree that a child, in developing a concept, goes through certain stages. These stages are not discreet but rather blend or meld into one another to form a continuum. It should be noted that:

1. The order of the stages is fixed.
2. The rate of progression through the stages is not fixed - nor can they be tied to any chronological ages. Where this is done, it is only for referencing.
3. The movement along the continuum can be altered by certain factors - teaching, environment, maturation, et cetera.

## The Stages

A breakdown of the stages follows along with a general description of each. It is not intended to include the full Piagetian description.

1. SENSORI - MOTOR STAGE (from 0 to 2 years)

   - preverbal - though language does start to develop
   - direct motor action
   - begins to learn to coordinate perceptive and motor functions - see things with reference to what they do with it

2. PREOPERATIONAL STAGE (2 to 7 years)

   - objects take on qualitative features
     - permanent
     - non-permanent
   - tends to be egocentric
     - tendency to see things from his own point of view
     - tendency to see one feature (elation) to the exclusion of others

3. CONCRETE OPERATIONAL STAGE (7 to 12 years)

- the child now focuses on more than one attribute at a time
- begins to see other person's point of view
- the classic operations (combinativity, reversibility, commutativity, identity, associativity) are developed

The development of operations is not a unitary development. To illustrate, the stages in development of conservation are: conservation of discreet quantities, continuous quantity, weight and conservation of volume, in that order.

4. FORMAL OPERATION (12 onward)

- characterized by full, logical thinking
- can now deal with statements that are not known or supposed to be true:

if....then<br>
either....or<br>
either or both....or<br>
$a>b,\ b>c \rightarrow a>c$

An example of how a concept would develop or the manifestations of the development of a concept follow.

Given a set of beads:

| | | |
|---|---|---|
| - Sensori-motor | - hit, throw<br>- only present when seen | |
| - Preoperational | - red, round, thick<br>yellow, square, thin | one feature at a time used for classification |
| - Concrete operational | - red and round and thin<br>- yellow and round and thick | two or more features at a time used for classification |
| - Formal operational | - either the red or the square or the thick ones<br>- neither the red nor the round ones | |

In summary, Piaget maintains a child moves along a continuum - the continuum representing the blending of the stages of development for each concept.

## Implications for Reading

While it is known that some students may enter the formal operational stage at the age of twelve, many will not do so until the late teens and some never will. It has been reported from one piece of research that 82 percent of the eighth and ninth grade students are still in the concrete operational stage while 50 percent of the university freshman classes are still in the concrete generalization stage (research title unknown). Yet many of the mathematical terms are used as though every child is in the formal operational stage. What does all this mean? What is to be done about it?

There are two things we must do:

1. Diagnose the weaknesses more specifically than in the past.
2. Devise ways and means (a) to bring about greater cognitive development, (b) to build more word and word group associations to facilitate greater cognitive development.

## Diagnosing Weaknesses

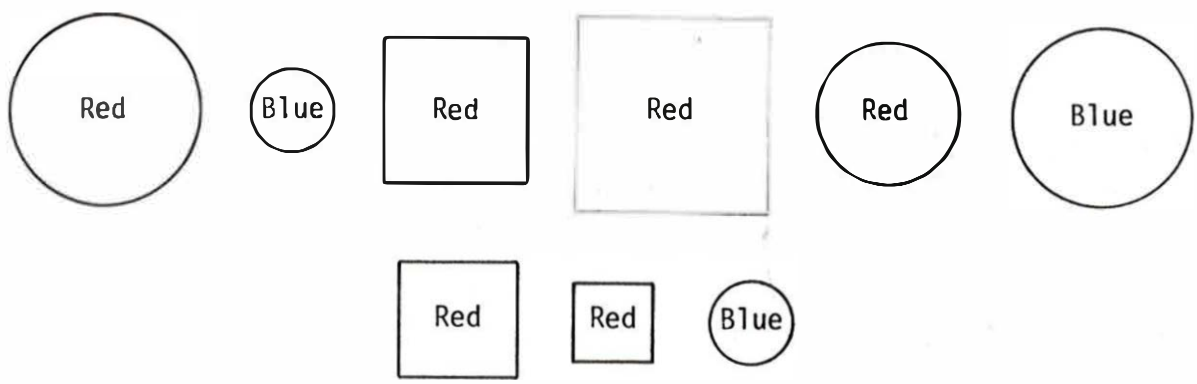
First is the diagnosis of the lack of interpretation of the printed symbol due to a lack of cognitive development in the conceptual area represented by the symbols.

The symbols may be words or mathematical symbols.

WORDS

Example (a):

Place in front of a student (age 5 to 9) red squares and blue squares and circles.




Ask the question: Are all the squares red?

The reply typical of a 5-to 6-year-old is "No." When pressed, the child may reply "No, because there are some red circles too."

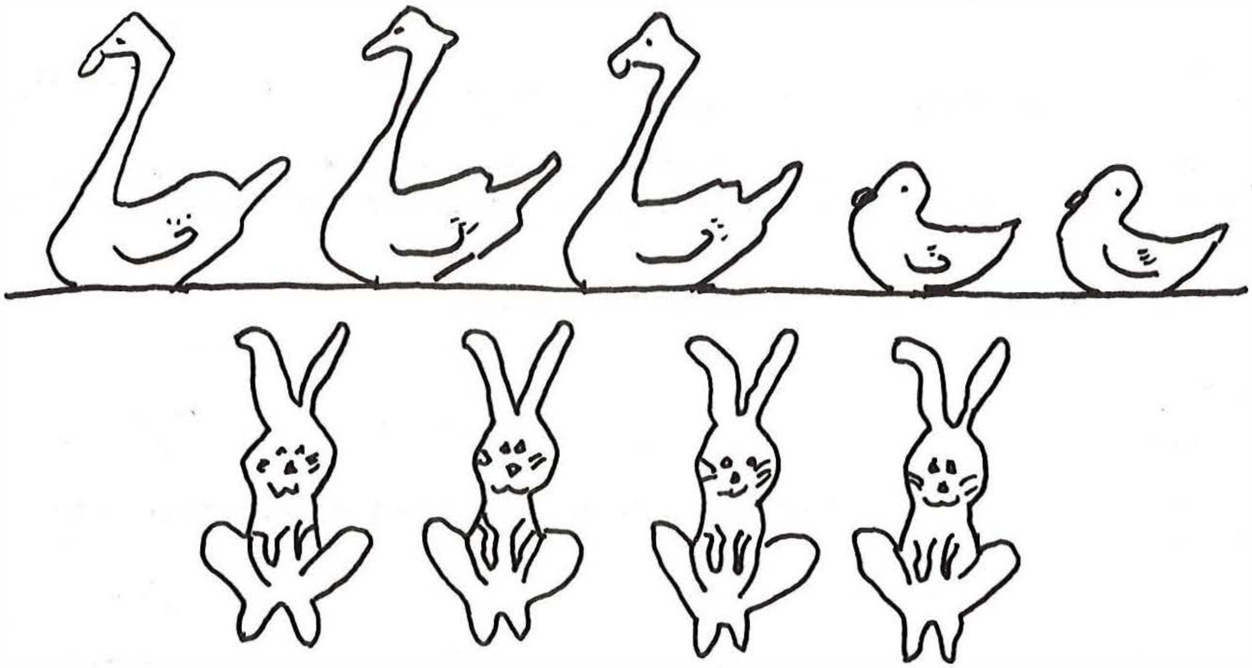
Let us consider extending this. Take the two statements:

Dogs are animals. (Squares are rectangles.)
Animals are dogs. (Rectangles are squares.)

What do we mean? In one case we mean all dogs are animals, and in the other we mean some animals are dogs. We expect children to make these subtle interpretations. (Duckworth, 1973)

Example (b):

Place in front of students drawings or photographs of geese, ducks, and rabbits.



Ask the students (age 5-9) these questions:

Is every goose some kind of bird (animal)? Why?
Are all geese some kind of bird (animal)? Why?

Then:

Is every bird (animal) some kind of goose? Why?
Are all birds (animals) some kind of geese? Why?

Typical replies indicate that students realize geese are birds and geese are animals. But they fail to realize that all birds (animals) are not geese. Yet when pressed they will identify a bird (animal) that is not a goose.

With older children similar situations can be structured:

Is every triangle some kind of polygon? Why?
Is every polygon some kind of triangle? Why?

Is every natural number some kind of a rational number? Why?
Is every rational number some kind of natural number? Why?

The concept underlying the two words "some" and "all" is that of inclusion. It is not an easy mathematical concept. It needs to be developed over a long period of time starting during the concrete operational state.

Example (c):

Place pictures of three geese, two ducks, and four rabbits in front of the students.

Then ask these questions:

Are there more geese or are there more ducks? Why?
Are there more geese or are there more birds? Why?
Are there more geese or are there more animals? Why?

Typical replies indicate that some students will not be able to accept the two classifications - geese and birds. They reply: "There are more geese because there are only two ducks."

Interviewer: "What did I ask you?"

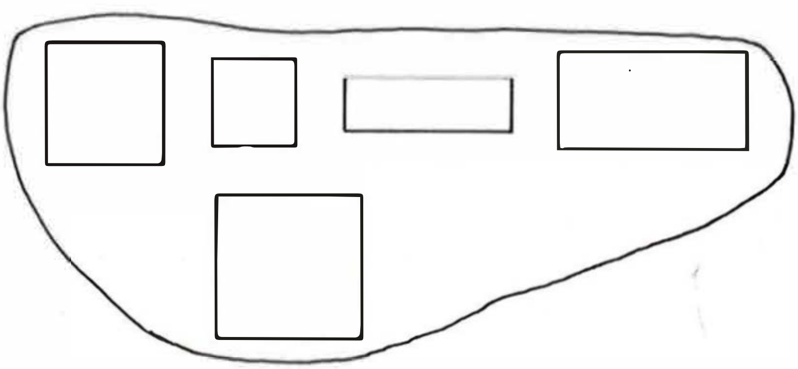
Student: "Are there more geese or more ducks?"

Interviewer: "Are there more geese or are there more birds?"

Student: "There are more geese because there are only two ducks." (Duckworth, 1973)

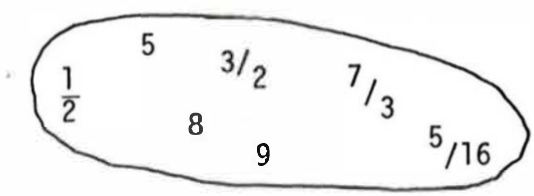
Inclusion is reflected in mathematics through the grades.

1.



Are there more squares or more rectangles in the set shown?

2.




Are there more natural or more rational numbers in the set shown?

Other words used casually in mathematics which students are expected to automatically have a grasp of are those which indicate relationships.

Example:

Each student is given these incomplete sentences and asked to complete them (typical reply in brackets).

Peter went away even though....(he went to the country).
It's not yet night even though....(it's still day).
The man fell from his bicycle because....(he broke his arm).
Fernand lost his pen so....(he found it again).
I did an errand yesterday because....(I went on my bike).

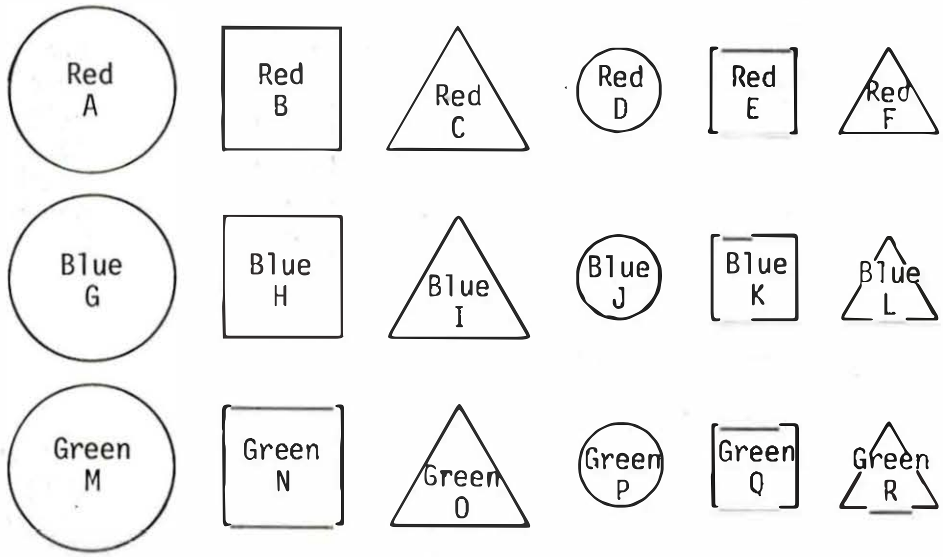
Instances where we use these terms in mathematics are:

8/3 is not an answer even though 8/3 < 10.
8/3 is not an answer because it is not a counting number.
8/3 is not an answer, so we do not include it in the solution set.
(Duckworth, 1973)

There is a tendency in mathematics to put simple words together into very difficult questions.

Example:

Each student is shown a set of shapes consisting of large triangles, circles, and squares, each in red, blue, and green. Small ones of each color and shape are also in the set. The shapes are lettered A to R. (Simpler versions can be given to younger children.)




Questions asked are of this type:

1. (a) Write the letters of all the shapes that are not circles.
   (b) Write the letters of all the shapes that are not blue circles.
   (c) Write the letters of all the shapes that are not small green circles.
2. (a) Write the letters of all the shapes that are not both big and red.
   (b) Write the letters of all the shapes that are not both small and green.
3. (a) Write the letters of all the shapes that are neither big nor red.
   (b) Write the letters of all the shapes that are neither small nor green.

4. Write the letters of all the shapes that are neither small and red nor big and green.

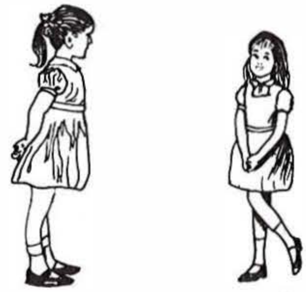
5. Write the letters of all the shapes that are not yellow.

Preliminary research has indicated that many students in high school had problems with the questions 3(a) and 4. Of one sample of 318 students, only 21 percent of the Grade 10s, 34 percent of the Grade 11s, and 34 percent of the Grade 12s got all the questions correct. Nearly 80 percent of the Grade 10s and 65 percent of the Grade 11s and 12s had difficulty with question 4. Apparently students were unable to keep track of four attributes at one time. This is not a reading problem in the physical sense but rather a problem of lack of cognitive development to be able to properly respond to the written words. (Bliss, 1973)

A final example of simple words that contain a difficult concept is the if... then... sentences. We all know that much of the logic underlying mathematics hinges on this sentence structure.

Example:

A picture of two girls is shown to the students.



The following statement is presented:

"If Carol goes for a walk, then Jean always goes with her."

The following questions are asked:

(a) Would it be possible for Carol to go for a walk, and Jean to stay at home?

(b) Would it be possible for Carol to go for a walk and for Jean to go for a walk?

(c) Would it be possible for Carol to stay at home and for Jean to stay at home?

(d) Would it be possible for Jean to go for a walk, and for Carol to stay at home?

(e) Would it be possible for Carol to stay at home, and for Jean to go for a walk?

Test results indicate that many students in the secondary school have difficulty with questions c, d, and e. They usually reply "Jean and Carol must always go for a walk together." (Bliss, 1973)

MATHEMATICAL SYMBOLS

So far this paper has only presented a sample of the problems connected with words. There is the problem of interpreting symbols as well. It is a compound one: first, the symbol needs to be translated into words; second, the words need to be interpreted (symbol→word→concept). Many students become familiar enough with symbols to omit the word stage (symbol→concept), but often in gaining this familiarity the word stage is used. Many students do not progress to the symbol→concept stage.

Three instances with reference to mathematical symbols will be illustrated. The first involves = (equals). Steffe points out that young children he worked with would write 3 + 4 = 7 but would not write 7 = 3 + 4. The reason given by the children paraphrased here is that equals always follows an operation - a doing. Is this built into our teaching or is it inherent in the stages of a child's development? We are not certain. (Steffe, 1975)

The second example involves basic concepts of union and intersection of sets.

$\{2, 3, 4\} \cup \{2, 4, 5\} = \{2, 3, 4, 5\}$
$\{2, 3, 4\} \cap \{2, 4, 5\} = \{2, 4\}$

Using a test devised by Bliss (1973), we tested a sample of high school students and found many students had a computational facility with union and intersection, yet did not have a usable conceptual understanding of the two.

Finally, Collis (1971) has indicated that:

$8 \times 3 = 3 \times \Delta$ is of lower cognitive level than $7 - 4 = \Delta - 7$

$8 + 4 - 4 = \Delta$ is of lower level than $4283 + 517 - 517 = \Delta$

$a \div b = 2a \div \Delta$ requires a comparatively high level of cognitive development.

In summary, a few samples are presented which tend to indicate that simple words often convey high level concepts requiring high level cognitive development. These samples are only a few of the many we have. It is to be pointed out that there is a definite need to not only teach students words but to teach them the ideas being represented by the words. The teacher must not assume that because a student does not perform adequately in response to a passage in a mathematics book that it is a reading problem in the physical sense - it may be a cognitive development problem.

## Overcoming Weaknesses

Since progress is being made in identifying the specific nature of some of the weaknesses, the remaining need - that of indicating ways and means to overcome the weaknesses - is now under consideration. There are ideas concerning this objective, but at this point there are no experimental results to back them up. What follows includes a couple of ideas related to what has been said.

Ways must be found to build word-meaning associations and especially word groups-meaning associations. Perhaps symbol groups-meaning associations will follow more naturally.

Example:

Read: $8^2 + b$

Four ways to read this are:

| | |
|---|---|
| $8^2 + b$ | "eight squared plus b" |
| $8^2 + b$ | "the square of eight plus b" |
| $8^2 + b$ | "b is added to the square of eight" |
| $8^2 + b$ | "b is added to eight squared" |

Many varied experiences will develop the proper associations with more success than using only one of the four. (Hater, Kane, and Byrne, 1974)

A problem that symbols present is that a student may not be able to pronounce the word(s) represented. There are no hints. He cannot use the rules of his phonetics class. To establish a phoneme-grapheme relation to $\int f(x)dx$, one has to recall the following words as spoken by the teacher: the integral of the function of x with respect to x. The student can at least read aloud the word form, but may have no clue as to how to read the symbolic form. Utilizing symbols only when necessary or only after students have adequate facility with them will aid the weak reader in mathematics. Drill in symbols is necessary.

Mathematics teachers, along with others, like to quote Lewis Carroll in *Through the Looking Glass* where Humpty-Dumpty says to Alice, "When I use a word, it means just what I choose it to mean - neither more nor less." In mathematics, words are used with special meanings ("we pay them extra," said Humpty-Dumpty). These special meanings often - though not always - are related to the common meaning. Take the word "associative" as in The Associative Property. We can illustrate the relation this way:

| *Mathematics* | *Example* |
|---|---|
| $(8 + 9) + 3 = 8 + (9 + 3)$ | Jerry and Harry joined Mary.<br>Jerry joined Harry and Mary. |

## Summary

To sum up, we must provide students with a broader set of experiences centered about the concepts in which weaknesses in reading are reflected. These broader experiences will provide for greater cognitive development in the conceptual area, hence the words the students read will have more specific and deeper meanings. This aspect of teaching reading is more of a concern to the mathematics teacher, while the mechanics and other aspects of reading not mentioned here may be of equal concern to both the teacher of mathematics and the teacher of reading.

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# Problem Solving in Mathematics: Are Reading Skills Important?

*by*

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It is unlikely that a person can solve a verbal problem if he cannot read. It is also well known that there is a significant correlation between reading ability and problem solving. Yet there are at least two major sources of doubt about the contributing effects of general reading ability to problem solving ability. First, while reading ability may be highly correlated with problem solving, problem solving skill may be due not so much to reading itself but to general intelligence, which is highly correlated with reading ability. Second, it may be that no more than a minimal reading level is necessary as one of the pre-conditions for good problem solving ability.

Balow (1964) obtained evidence that computation ability is more closely related to problem solving ability than is reading ability. Balow used 1,400 Grade 6 students with measures of reading ability and computation ability. On a test of reasoning ability he found that both reading ability and computation ability were very highly related to reasoning ability ($p < .01$). However, when he controlled for intelligence using IQ as a covariate in the analysis, he found that the relation between reading ability and reasoning ability was drastically reduced. The effect of reading ability was still significant ($p < .05$), but with IQ controlled, computation ability was a much more important factor in problem solving than reading ability.

Recent British Columbia assessment programs in reading and mathematics also provide some evidence that general reading ability may not be as important a factor in problem solving as generally accepted. Evanechko et al (1977) reported that in Grade 4, girls were superior to boys in reading on all three domains studied: word identification, prose comprehension, and comprehension of functional materials. Yet in the summary report of the mathematics assessment, Robitaille and Sherrill (1977) reported that boys exceeded girls at the Grade 4 level, as well as at Grade 8 and 12 levels, on the higher domains of comprehension and applications where reading would be expected to exert greater influence than on the first domain of knowledge.

Szetela (1977) has obtained some evidence that a reading competency over and above some minimal competency may have little effect on problem solving success. He constructed tests containing nine problems each, three of which contained only a few words essential to solving the problem, three of which were written in ordinary language sentences, and three of which were written with sentences containing difficult words and non-essential words. Three different versions of the test were constructed so that every problem appeared in all three forms. Every student solved three problems of each type. Students in Grades 5 and 6 were given the same tests. Table 1 shows that the wording of the problems appears to have had little effect on the number of correct solutions obtained by both Grade 5 and 6 students. These results are consistent with the findings of Kulm (1973) who studied the validity of standard reading tests for use in assessing mathematics text readability. Kulm found that the percentage of difficult words which is the best predictor of ordinary English reading ability was not even among the five best predictors of understanding of elementary algebra material whether in explanatory text or illustrative material.

**Table I**

Number of Problems Solved by Students in Five Grade 5 and 6 Classes

| *Grade* | *Number of Students* | *Number of Problems* | *Number of Low Wording Correct* | *Number of Normal Wording Correct* | *Number of Difficult Wording Correct* |
|---|---|---|---|---|---|
| 6 | 87 | 783 | 137 | 137 | 132 |
| 5 | 51 | 459 | 56 | 54 | 60 |

Knifong and Holtan (1976) found that of 470 errors in solving problems from a standard test, 52 percent of the incorrect solutions were due to computation or clerical error. Of the remaining 48 percent of the incorrect solutions they make no conclusion, although their examination of other errors includes 18 percent of the problems which were omitted. They offer the possibilities that such problems may have been due to reading difficulties or to the fact that the more difficult problems were in the latter part of the test.

While the above evidence seems to relegate general reading ability to reduced importance in problem solving, mathematics reading requires special reading skills not learned or experienced by students in ordinary reading instruction. Earp (1971) notes that in mathematics simple words are often used to form complex statements. Such a statment as, "the square of a sum is not equal to the sum of the squares," is troublesome for even good readers. Earp suggests that in ordinary reading instruction too much effort is expended on increasing rather than adjusting reading speed. Henney (1971) remarks on the compactness of mathematical statements with every word significant, familiar words in unfamiliar context, and other words met only in mathematics. She adds that the reader tends to concentrate on numerals, not relations, and that different patterns of eye fixation are required for reading words and numerals.

Consider also that often students are required to read a few words or symbols, refer to a diagram, and read and refer again perhaps several times in a single sentence or brief paragraph. Students may be unable or unwilling to keep changing the focus of their attention while trying to assimilate both verbal and diagrammatic information. Such reading requires a degree of concentration well beyond what is necessary for typical narrative reading.

A study by Call and Wiggin (1966) is an interesting example of the possible fruitfulness of teaching special mathematics reading skills. Call, a mathematics teacher, and Wiggin, an English teacher specializing in reading who had never even studied second year algebra in high school, each taught a second year algebra class in linear equations and word problems. Call taught his class by the usual methods, and Wiggin taught his class with instruction in such special skills as seeing relationships. Students in Wiggin's class exceeded the performance of those in Call's class with an average of 1.8 problems correct as compared to 0.6 problems correct. When results were analyzed with reading ability, verbal ability, quantitative ability, and IQ controlled, all comparisons favored the group taught with special mathematics reading skills.

One of the chief sources of difficulty in reading mathematics is students' uneasiness and unfamiliarity with mathematical symbols. Teachers must provide enough examples in different contexts together with sufficient practice with the use of symbols to ensure that the meanings of the symbols are as clear as ordinary vocabulary words. Even a familiar symbol like "=" may be misunderstood in some situations. Often the symbol is used as a substitute for the word, "equivalent." This use of the symbol may be practical in most cases but sometimes it may obscure some important conceptual differences. The interpretation of 1/3 is not the same as that of 6/18. A more important obliteration of conceptual differences occurs when one writes that $3/4 = 3 \div 4$. As concepts of a fraction, these two situations are nevertheless very different. Indeed mathematics is remarkable for such vastly different interpretations of certain concepts. Students who learn that pi is the ratio of the measures of the circumference and diameter of a circle later see pi as the measure in radians of a straight angle, and perhaps later still see pi as the sum of an infinite series. In expressions like $3x + 2x = 5x$, the symbol "=" has a different meaning than in the sentence $3x = 6$. In the first case the statement is an identity true for all real numbers, while in the second statement only one number makes the sentence true. Teachers and textbooks sometimes misuse the symbol "=" when they mean "approximately equals." Students who are taught that pi = 22/7 may well be confused when they learn later that pi is an irrational number. These examples suggest that teachers need to be alert to the various interpretations and meanings of symbols and words in mathematics so that they may more clearly introduce, explain, and exemplify new symbols and concepts and provide appropriate practice. Such care may reduce reading difficulties in problems as well as difficulties in concept attainment.

The main reason why students learn to punctuate sentences is to make the intended meaning clear. The sentence "Betty," said Sam, "is a clod," would take on a totally different meaning if the sentence were not punctuated. Similarly, the mathematical sentence, $2 \times (7 + 5) = 24$ without the bracket punctuation would have a different meaning and the statement would in fact be false. Algebra teachers are all too familiar with students who regard $2x^3$ as an equivalent form of $(2x)^3$. Such errors might be reduced by more time and attention to correct mathematical punctuation.

To help students read and interpret graphs and tables correctly, more emphasis might be given to having students obtain and record data instead of making use of artificial textbook data. Students who are given practice in graphing or charting their own data may learn the importance of choosing appropriate scales, may more easily observe relations between two variables, and may be better able to perform interpolation and extrapolation in graphs. Such experiences using student data may help to overcome negative student reactions when they see already-made graphs and tables with numbers and symbols which are visually overwhelming. A table of square roots or trigonometric function values is sometimes difficult to read because of such stunning visual impact. With the availability of mini-calculators, it is now possible for students to construct their own short trigonometric tables from which they can acquire understanding that will help them use the conventional textbook tables.

In order to help students overcome the habit of reading problems too quickly and reading them only once, mathematics teachers need help from the reading teachers. If students are given instruction and practice in reading to select the main idea, observe important relations, restate sentences and identify superfluous words in their reading classes, the mathematics teachers may have a better chance of promoting more careful reading and rereading in their own classes.

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# Reading in Arithmetic – Mathematics in Reading

*by*

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Our school traditions include the views that arithmetic and mathematics lessons are separate from those in reading and language arts. In universities, any intellectual collaboration between mathematics and language departments is rare, yet there is increasing awareness that much can be gained in the study of how language and mathematics can illuminate each other. This could have a significant effect on teachers of elementary grades in understanding the basics and in accomplishing greater efficiency in the teaching of the three R's.

Arithmetic and reading/writing both involve symbol systems. Children are expected to read arithmetic books, even though a reading lesson may have ended and a math lesson begun. They are asked to write number names which, in fact, is more common than any other activity in such lessons.

To the symbols, meanings are attached. The symbols have no meaning in themselves; they are arbitrary from the point of view of the learner. The symbols chosen could be otherwise, and in all languages except the learner's own, they are otherwise. Teachers know this well in language lessons, for they take the trouble to deal with the meanings which are absent when their students meet strange words.

Oddly enough, the school situation is such that meanings and spoken symbols are known to a very considerable degree in the non-arithmetic language by the time children reach school, but they have little mastery in arithmetic or mathematical language coming from home. It is rare that the kindergarten and other primary grades fill this gap by developing other than written language in arithmetic, keeping only a few number names, a few operations, and little else. One can wonder why this apparent difference.

Of our meanings to words, either spoken or written, there are both private and public components. The latter enables us to agree about word meaning, but each of us has private attachments not always significant or capable of isolated description. Nevertheless, there is always the risk that the private meaning can be very important. In extreme cases we can think of personal names or words which commonly are associated with feelings or abstractions such as beauty, democracy, or Canada. On closer examination, however, the dual attributive meanings can extend to most words.

Even though correct meanings are available to children, they also attach their own meanings to words which are not used in their first language. They do this far more so than is commonly realized. The meanings may certainly not be those which parents or teachers wish, but there is, especially these days, rarely a paucity of meaning. Perhaps this is one advantage of hundreds of hours of TV-watching.

Not only are symbols ambiguous in their private and public components, but also frequently in the accepted public interpretations. In English (and when we say English, we can refer to many, if not all, of the languages first met as one's native tongue) the words "refuse" and "wind" and the set of letters m-e-a-n cannot be read correctly until the context is known. In arithmetic-English, the symbol X can be read as "times," "multiply," "of" and "groups of," apart from the use of the same mark to indicate a wrong answer, a traffic signal for cross-roads, or a kiss!

The letter *a* in English words can be read in nine different ways, the letter *e* in seven, and so on. The fraction 2/3 in school is regularly read as "two-thirds," "2 over 3," and "two, three." The reaction to seeing the symbol "-" brings forth the responses "minus," "subtract," "take away," or "from."

These examples suggest different ways in which our symbols are frequently read or spoken. Other symbols are always read the same way, but are associated with different meanings. Does the word "dust" mean to put on particles as with a crop-duster, or remove them as with "dust the furniture"? Is a "rubber" an object used for erasing, a boot worn in wet weather, or a cloth used for dusting? Examples in non-arithmetic language are legion. In arithmetic, "6" is easy enough to read and say and write, but it will be continued to be used in a child's school life as a cardinal, ordinal, natural, or counting number; as a rational number, an integer, or a real number.

The multiplication sign, too, even though we may have decided to read and say it in only one way, (such as "times"), will have its meaning changed for 2/3 x 5/7 as compared to that probably attached 2 x 5. If this change of meaning does not occur, the fractional multiplication will appear meaningless, which is, in fact, only too common. Could this inability to comprehend several meanings be the reason that work with fractions in our schools is rarely mastered or understood?

In English there are words, and in arithmetic-English there are also words. 5 is a word even though it is also called a numeral; so is 378. Both words can be seen as nouns. Table is a noun in English (E); so is "cloth." Tablecloth is a compound noun. In arithmetic-English (AE), 3 + 2, 6 - 1, 1/3 x 12, 379 - 1, 380 - 1 - 1 are compound nouns.

Compound nouns can be short or long like "Newfoundland," or like

| | |
|---|---|
| 1 + 2 + 0 + 1 + 0 + 2 - 2 + 1 | 300 + 70 + 8 |
| 1 + (1/2 x 4) + 0 + 1 + 0 + 2 - 2 + (1/4 x 4) | 300 + 70 + 9 - 1 |
| | 100+100+100+ (2 x 35) +9-1 |

The observation that 380-1-1 can be seen as a compound noun equivalent to 378 reminds us that synonyms exist in both arithmetic and non-arithmetic language. We can call 378 the standard name for a number and 380-1-1 as one of the corresponding non-standard names. To every standard name there is an infinity of non-standard names. We can say they are equivalent in the sense that any one of them is legitimately a substitute for the standard and for every one of the other non-standards. Indeed there will be contexts where many of them will not be appropriate, but they are still equivalents.

Most arithmetic books use non-standard and standard names all the time. It seems to be a main emphasis to mix them, although traditionally the direction has been one-way: students are presented with some non-standards and are required to find the correct standards. These are called the "answers."

The words are used in sentences which can be closed, as for example 4+5=11, even though this sentence is usually held to be false. 4+5=9 is also closed, but true. At other times, sentences are open in the sense that one cannot say whether they are true or false: 4+ =12, or 4+ $\boxed{\phantom{0}}$ =12. These are the preoccupation of many texts, not to mention classroom worksheets. The students' main job is to fill the open sentences.

Rarely are children in arithmetic asked to create false sentences, although it happens in other language lessons constantly. These fall under the headings of "myth," "imaginative narrative," or maybe even honest mistruths! On the whole, we do not worry unduly whether or not the students will fall into bad habits by such non-truths. But with arithmetic, the impression still prevails that mistakes must immediately be corrected, for if otherwise left unnoticed, it would be far more attractive to the learner and be copied henceforth assiduously!

There are even sentences which can be true, false, or open. 4+6=12 is false in Base X (X now means "ten"), true in Base VIII, and open in Base V (the 6 being used not as a permitted numeral, but as an unknown or a variable).

More can be said about the similarities between English and arithmetic-English. We have mentioned sentences in both and wish only to note that 4+6=10 is a sentence in which = denotes the verb, 4+6 is the subject noun, and =10 is the predicate or the complement of the subject. It is sufficient for now to leave the correspondences there and add only that other language arts headings are also appropriate in arithmetic and mathematics, including punctuation, spelling, narrative, rhymes, rhythms, silent letters and silent digits, short forms, and sounds which are read but do not correspond to symbols actually written.

I now wish to allege that the essence, the main purpose, and the overwhelming use of arithmetic in our schools is the substitution of words for words. Computation is the process by which we produce synonyms or acceptable substitutes. Consider typical "problems" from texts:

$$\begin{array}{r} 374 \\ +\ \underline{869} \end{array} \qquad \begin{array}{r} 6000 \\ -\ \underline{472} \end{array} \qquad \begin{array}{r} 287 \\ \times\ \underline{36} \end{array} \qquad 163 \div 21$$

When we work these, what do we do other than substitute names for names? For 4+9, we substitute 13, whether our practice is to put down the 3 and carry the 1 or otherwise. For 6+7, we also use 13, though more precisely we mean 6 ones + 7 ones is 13 ones.

For the subtraction we can, of course, repeat the pattern "2 from 0 won't go, borrow ten, et cetera," seeking all the time substitutes for some of the simple nouns we select from the given compounds. Alternatively, we can substitute for the compound 6000-472 the equivalent of 5999-471 which may assist us in quick transformation to the standard form of 5528.

Similarly, for the multiplication and division, we substitute and substitute, using whatever strategies we have at our disposal at the time of thinking about it. Finally, if that is required, we reach the standard name for the number which was presented as the "problem":

$$\begin{array}{r} 287 \\ \text{X}\ \ \underline{36} \\ 1722 \\ \underline{8610} \\ 10332 \end{array} \qquad\qquad \begin{array}{rl} 287 \times 36 & \text{--}\ 287 \times 9 \times 4 \\ & \text{--}\ 287 \times (10-1) \times 4 \\ & \text{--}\ 2870 \\ & \quad\ \ \underline{287} \\ & \quad\ \ 2583 \\ & \quad 10332 \end{array}$$

$163 \div 21 = 1\ 142/21 = 2\ 121/21 = 3\ 100/21 = \ldots = 7\ 16/21$

Every time there is a sequence of substitutions, words for words.

How do learners learn to substitute legitimately? To answer this, we can examine the process of how this happens in the non-arithmetic language. There is a complexity, which we cannot go into here, as to what goes on in the learning of language. A couple of observations can be made. The first is that words must be rote learned in the sense that it is impossible to discover words without hearing that certain sets of sounds and symbols are being used by people who speak the language being studied. One has to accept that certain sets of sounds are English because that was decided before one came along. When a child learns that a name for a piece of furniture is "table," first spoken and later written, it is not because the sound or the sight of the word is like a table. The sounds and signs used are arbitrary and are merely agreed upon by long usage.

In arithmetic 4+8 does not look or sound like that which it represents, and it certainly does not look or sound like 12. Even if a child knows that 4+8=12, it would be impossible for him to know whether or not 4+9=13 is true or false, unless he knows something else. Adults know too well that this is true and perhaps miss the fact that learners have to continually face new words of a new language with all the arbitrariness.

However, mixed with the arbitrariness is a more important source of power - that of detecting the patterns of structure which we can use once we have accepted parts of language not discoverable. The awareness of patterns can be used with a vast array of similar but not identical instances. All of us, adults and children, in our speech and later in our writing, continually create new sentences, and new stories using old words and the structure of the language already being used. We change the order of words, phrases, sentences and paragraphs; we alter the stress; we color by clever juxtapositions; and we invent new compounds or use old compounds in new contexts. All of this can be done in arithmetic language too.

If we know, by some means or other - and we do not wish to imply that *any* means will do - that 4+8=8+4 (whether or not we know that the standard form for both 4+8 and 8+4 is 12), then it is highly probable that we know that any expression using two number names and a plus sign can be reversed to produce an equivalent. This is surely powerful, if not immediately, then potentially. For we can make up, and so can first grade children, endless examples and have the confidence of correct procedure. Children have an enormous vocabulary of number names, even though some may be mispronounced and frequently misspelled, so they can be tapped for practice.

4+100009 = 100009+4    12345+75843 = 75843+12345

Such are accessible, regardless of the size of the numbers and of *correct* reading. At least they can be written and this is generally more than we challenge primary children to do.

Further, if we know that, for example,

8+4=8+4+1-1=8+4+2-2=8+4+3-3=...and so on

(and this pattern can be learned easily by playing with one's fingers), then we have another pattern which *can* be seen and *can* be heard and is not arbitrary. Both examples indicate that endless examples are attainable, which suggests that the power of algebraic awareness is ready to be capitalized upon in young students. This is where the mathematics really begins to enter, though it was latent all the time.

Mathematics is concerned with patterns, structure, connections, and interrelationships, not just with the enunciation of some simple nouns to name abstract entities. Available for our substitution strategies in computation are the essences of what we have always considered as essentials: place value, symmetry, factors and multiples, fractions and their operations, decimals and percentages - all of them concerning abstract patterning. When the ideas, the concepts and the awareness of such topics are mastered, we have increased powers to use them, not only as substitutional computation, but also as further understandings of what mathematics holds in store for humans.

Patterns are also the stronger aid in non-arithmetic English and many parents have noticed that their young children, working on learning their first language, seem to grasp aspects of the structure before being concerned about acquiring extensive vocabulary. They reverse phrases, change the order of words in sentences, and transform one pronoun into others, although they find that the patterns they try are not upheld by other speakers of the language. Regularity is spotty and gradually "bringed" becomes "brought," self-invented words are abandoned, and mispronunciations are self-corrected even though patterns have been attempted. Children know and experiment with reversals of words, but discover that the game does not provide much success since most words do not reverse to recognizable English words and certainly not with equivalent meanings. Words of one, two, and three or more syllables can and are mastered; substitute ways of expressing thoughts are frequently used even if it means a more complex change of order or change of words. Patterns are regularly sought and used, and we could go on instancing many more: the conventions of reading from left to right in some languages, or right to left in others; the spacing patterns in sound, or on paper between words, sentences, and paragraphs; tenses; suffixes and prefixes; and the use of pronouns - all are built on patterns and structure.

Particularly in English, there are standard and nonstandard forms of which different contexts become unacceptable. At home Mr. Smith is called Daddy or Jim; at work he probably prefers the former as the standard. There is so much pattern that we could say the grammar of a language is the study and use of the body of conventions and non-conventions and their patterns which are permissible.

Would it not therefore be a wise component of teaching to use the pattern and rote approach in English and in arithmetic-English? By the time children enter kindergarten they have mastered a great deal of their native language in the spoken form. Yet our tradition is to keep the students silent; most certainly the children's conversation at home has been in non-arithmetic English (apart from a few counting words). Could we not feel sure that had they *conversed* in meaningful situations with the arithmetic-language, it would be just as easy for them as with non-arithmetic language? Suppose kindergarten and other primary classes emphasized activities which promoted discussion, conversation, argument, "showing and telling" about mathematical ideas and models and their applications. Would this not be an essential make-up for the lack of arithmetic-language presently existing in our homes? Instead, many teachers require the written expression *before* the spoken and even before meaning has been acquired!

It is common sense and wisdom indeed to seriously ask ourselves whether or not we could teach arithmetic using the similarities it has with language and language learning and to teach reading more efficiently through the awareness of the necessary interrelationships studied otherwise under the name of mathematics. Teachers could involve their students in games, work, and studies which persuade the children first to talk about the relevant ideas and later to choose or accept written symbols for what they already understand and can talk about. Later still, movement can rightly be made to more formality in accord with the body of mathematical and other knowledge handed down to us from the past.

Instead of children being required only to give "answers" to bookset "problems," we can ask them to continually evolve their own substitutes for the expression they begin with. They can invent one, two, three, and more syllable arithmetic words (number names), and study their interrelationships. Punctuation in arithmetic can be varied purposefully to examine the consequences resulting when students themselves attempt to gain clarity from the confusion which exists without punctuation. For example, punctuate 2+6-3x7 in different ways.

In language arts lessons, we sometimes want children to study handwriting for the purpose of improving legibility. We should also do this when they write number words and sentences, without insisting that attention be paid to other attributes such as truth or falsity, punctuation, or convention.

Imaginative work in English and in arithmetic can be the emphasis in some lessons. The value of such in English is accepted by most of us, but this is not so with mathematics. Too frequently mathematics is seen as a non-creative and unimaginative activity for the majority of humans! Yet what an opportunity for the treatment of work with different bases on different planets, within the universe (each of which uses a different language) allowing, as previously quoted, that 4+6=12 can be true, false, or open!

Arithmetic is lettered with opportunities for language analogy. Teachers can ask, "If we do it in language arts, or in reading, writing or spelling, could it possibly be done in arithmetic?" We talk of the *three* R's, and practice them disjointly. I suggest we look at the 3-in-1 possibilities, with similarities perhaps more important than the differences.

Can we capitalize more effectively, efficiently, and joyfully on the powers existing in children before they meet the educational view that intellectual life must be broken into different subjects? If we begin to consider this possibility, we shall help develop in students a new kind of educated and educating being. At the same time, teachers will not need to agonize as to whether or not they can do one R but not another. All of what we do will be integrated; individual differences will be honored both in subjects and in people; and there will be no more highly honored profession than those who profess and practice what is needed to help children grow maximally from conception to birth.

# Part III

## Practical Considerations

# Why Johnny Can't Visualize – The Failures of the Behaviorists

*by*

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## Introduction

The following occurrences can be recalled by most college mathematics instructors: Trigonometry students are unable to set up the problem about telephone poles casting shadows down a hillside; Calculus II students find the riddle about drilling a hole through the center of a ball bearing incomprehensible; Algebra students find it difficult to derive an equation from the riddle about rowing with and against the current; Calculus III students are unable to get the proper limits of integration when changing from rectangular to polar coordinates. The instructor usually comes to their rescue, sketches out a picture that represents the situation, labels the parts of the drawing and sets up the appropriate equation. The student usually makes a comment such as: "Why couldn't I set that up?" "How simple it is when you do it!" "How did you know how to draw that diagram?" or "I don't understand." The next problem attempted is equally as baffling to the student and the patient tutor may spend a semi-infinite amount of time sketching out the diagrams that are implicit in the applied exercises. Why is it that so many students are unable to come up with any diagram that even remotely resembles the situation described in the problem? The student is unable to associate any visualization with the data related in the "story problem." This phenomenon we will refer to as "poor visualization."

Mathematics instructors refer to poor visualization by many different phrases. Some examples are: "The student can't read." "He is unable to understand." "He doesn't put in any effort." "He needs to be spoon fed." "He is immature." "He doesn't care." or "He is plain stupid." These remarks indicate a lack of understanding of the situation and are brutually unfair and insensitive to the student afflicted with poor visualizations. Certain equally brutal remedies are advanced, such as: "Fail the lot of them." "Give them more to do." "Have them hire a tutor." "Stay with it until they get it." or "Solve by the step process, first..." Most instructors find some way to avoid coming to grips with the situation, such as: "Skip the story problems." "Ask only about basics." or "Stress the formulas." If some instructors persist in trying to remedy the situation by some means or seek to punish the students for their inability, they are quite often reminded (perhaps by the Dean of Engineering) that "Our students don't need this theoretical math. What we want is mastery of the basic formulas." or (perhaps by the Department Chairman) to "Avoid these trivial applications. We waste too much time because the students are not able to set up the problems." One student was even heard to complain to a faculty member, "I have always had trouble with story problems. Besides, I have come here to learn, not to think."

Poor visualization is not new, but an increasing number of students seem to be afflicted with this disease. Why is it some astute students have developed good visualization and others are visually bankrupt? It is the purpose of this discussion to identify some of those facets of educational development that contribute to poor visualization.

## Ingredients for Good Visualization

In order to have a good visualization, a person must have a sufficient "memory bank" of experiences to draw from and some practice in forming visualizations. The remembered experiences have to be of sufficient quality to warrant a vivid accurate recollection. These experiences are best realized if they consist of a concrete nature rather than an abstract one. The person should have been an active participant rather than a spectator. Experiences are best remembered if they produce a pleasurable reaction as opposed to a frightening, baffling, or arduous reaction. In addition, experiences that are relevant, rather than those artifically contrived or intricate, yield a better chance of creating a visualization.

The problem posed must be appropriate to the developmental level of the student. Consistent experiences with problems beyond the student's level of abstraction cause the student to believe the attempts are futile. The problems should be clearly stated and as explicit as possible. The more closely related to the student's interest and experience, the more likely a successful visualization of the problem will occur.

In addition to possessing the necessary background and having a good problem, the student should have an adequate reason for wanting to visualize the problem. In short, what reward can the student expect for attempting to visualize the problem? Direct rewards such as grades, acceptance of peers and/or instructor, or perhaps even the inner experience of intellectual contentment are important. Note here that we stress a visualization of the problem, not necessarily the ability to solve the problem as visualized. Instructors could help encourage students to visualize by being willing to mete out great amounts of partial credit for visualizations and even attempted visualizations.

## Encouragements for Poor Visualization

Very early in life a child is encouraged to avoid visualization. The typical three-year-old is anxious to please the parents. Eating and toilet habits are beginning to conform to the ones desired by the parents. Most parents take pleasure in showing off the superior intellect of their child. The child is often encouraged to say verbal expressions that sound like "one, two, three," et cetera. The child has learned a verbal pattern that is encouraged and rewarded. He has not learned to count! Counting implies enumerating objects, but the child does not relate the sounds he is making to the counting of objects. This is usually the first "mathematical fraud" performed by the child. The child gives the adult the response desired, passes it off as mathematical understanding and receives a reward for "faking" the knowledge.

To cite a few other instances where a preschool child is encouraged to be a "mathematical fraud," consider the poem "1 and 1 is 2, 2 and 2 is 4, 4 and 4 is 8," et cetera, or counting backwards a la Sesame Street. Indeed we have heard that more preschool children can count backwards from ten than can count backwards from eight. During this period the child begins to be trained in answer emphasis. When asked to respond to a problem like "What is 3 and 3?" a wrong answer, say 4, might be given. When that answer is not accepted, the child may begin to rapidly give out random answers in hopes of satisfying the questioner. For example, the responses might come 7, 2, 11, 8. This is one of the first indications that the child is beginning to realize that the answer is to be rewarded and not the reasoning that produces the right answer.

When the child enters the school system, a greater variety of adult forces begin to come into play. One which occurs very often is the rush to symbols and their manipulation. Many learning theorists claim that the child should still be operating at the concrete level well into first grade, yet we find many teachers working on number concepts without the benefit of manipulatives. A great deal of pressure is put on the child to be an answer producer. The answers are the rote memorized responses to verbal sayings and are quite divorced from the reality of the situation. Indeed, the ones most in need of the concrete experience are penalized for being "slow." (These are usually boys.) The child willing to play games with symbols that have no reality is highly rewarded and praised. (These are usually girls.) Many a time a teacher can be observed doting over the child who answers quickly and uses no manipulatives. The comment often goes something like "Sally can do the problems without the beans!" Many of these children, thought to be advanced by their teachers, are initiating the course that will eventually crumble when visualization skills become necessary.

Even if the child survives the first grade with good concrete experiences and a fair ability to produce visualizations, the pressures will continually try to drive the child off course. Second grade teachers have been heard to comment, for example: "Put down the 3 and carry the 1." "Cross out the 3, change it to 2, and put the 1 over by the 5." and "If you can only answer faster you will get into my three-minute club."

The first two of these comments concern algorithms and are particularly conducive to memorization of symbol manipulation rather than reasoning and visualization. The last comment is referring to memorization of basic addition concepts where now the student is virtually forced to respond without thinking, that is, make a conditioned response. Again, those performing the least desirable trait, mere memorization, are likely to be the ones rewarded while the visualizers and the reasoners receive little recognition for their efforts. Comments of third grade teachers are similar: "Multiply the 6 by the 7 and get 42, put down the 2, carry the 4, 6 times 3 is 18, add the 4 getting 22, put that down, and the answer is 222." "We will keep doing this page until we can all get it in less than two minutes." "Always in these story problems subtract the smaller one from the larger one." At this level, the teacher is running into the visualization problem. The last comment was a signal that the teacher was no longer going to ask the child to read the problem and figure some method of solving via a visualization but would accept an answer produced by any means. The suggestion of a rule for all problems on that page completely discourages the student from getting a clear understanding of the problem. A fourth grade teacher instructs: "Always divide the larger number by the smaller." When it says 'more', you subtract." and "Cross out the 4, put a 9 above the 0 and a 1 by the 3." A fifth grade teacher sets the answer-producing strategies: "Divide the numerator and denominator by 3." "Don't ask how many 29s in 8763 but ask how many 3s in 8." or "Put the 2 under the 8 of the multiplier and carry the 3." A sixth grade teacher encourages: "Don't pay any attention to 'ladder' division. This short-cut will work better." "Invert the divisor and multiply." or "Slide the decimal point over two places to the right and add the % sign." A seventh grade teacher shares the secret of successful computation: "Divide the 'is' by the 'of' and you will get the right answer." Algebra instructors suggest: "Take the 5 to the other side and subtract." "Line up the formula of 'rate x time = distance' and then plug in the values." or "Memorize the quadratic formula then you won't have to try to factor." Some high school teachers say: "There is one way to work these problems. The first step is ....., second step ....., third step .....," et cetera.

All of the above expressions are encouragements for the child to avoid thinking about the problem, to avoid coming up with a vivid visualization, and instead to come up quickly and somewhat magically with the proper answer. Succumbing to these suggestions leads the child into a fraudulent behavior of generating answers for problems he does not understand.

We are not against memorization of facts, algorithms, and formulas but we are against divorcing this memorization from the essence of mathematical development. One of the greatest encouragers for avoiding visualization is the standardized time frame test. The plight of the fifth grade teacher on Long Island was described as follows:

> I know our textbook series stresses the "ladder" method of long division but it is too slow to use on the standardized tests (PEP) given to the children. We have decided at this school to teach the traditional algorithm rather than bother with the "ladder" method.

Here the format of the test has dictated a curriculum change which opts for the expediency of producing answers. This curriculum change can only help to convince the child that visualization is unimportant. We doubt the wisdom of any curriculum change that uses as its main objectives producing answers on standardized tests.

Standardized tests have in common the feature of aiming at superficial knowledge and never digging into the depth at which a student may actually comprehend the concepts. It is a shame that society places such a high value on the correct answers generated by these low level cognitive skills. We firmly believe that one of the best climates for improvement of real education would be one free of standardized time frame tests.

## Forces Opposed to Improvement

We believe that the behavioral psychologists are at the root of this problem and are responsible for maintaining this answer-oriented and antivisualization strategy. The bill of goods they have sold the school administration and teachers has the schools virtually concerned with pretests, post-tests, recording the level of skills, moving down the prescribed path, evaluating, rewarding and charting. They pay little attention to individual human needs, modality preferences, esthetics, problem solving, individual investigations, or in short, all the aspects that are uniquely human. With the school administration and teachers dangling on the string of the behaviorists we see the producers of textbooks rushing to create materials that can be used by the teachers. One representative of a publisher informed us that they now have revised their texts so you can avoid the following: missing addends, brain teasers, ladder method, stacked divisors, regrouping and metrics. Why? Because the teachers requested that these difficult topics be avoided. The student can't work them anyway so they may as well use a method that will be successful. Notice that all of the topics to be avoided are building blocks for good visualization.

## Partial Solutions for Improving Visualization Skills

What would happen if you confronted an integral calculus class with the following instructions: "Below are five definite integrals. Write up two different real life practical problems which would eventually use the first integral in the solution of the problem. Form two problems for the second integral, et cetera." Might we have a better picture of whether or not the student understood integral calculus by his responses rather than by asking him just to evaluate these integrals?

Could we do something similar with fifth graders who have been given seven long division problems? Could we ask them to build story problems in which the division problems would be an intermediate step to the answer?

Could we ask first graders to tell stories about "3 plus 5" or "8 minus 3"? Could we ask preschoolers to count the objects won in games and compare that number with the objects won by their competitor?

Numerous analagous examples can be generated for every conceivable case. At the very least, problems of this type demonstrate to the student that there is value in visualization of the problem as well as in generating the answer.

## Remarks

Since mathematical skills, as measured by instruments designed by the behaviorists, have declined dramatically, we can conclude that the behaviorists' philosophy of teaching mathematics has failed. They have practically eliminated visualization skills from the educational process. Yet even with their continuing failures they somehow dominate school administrators, influence most teachers, and are able to dictate the textbooks that are available for mathematics in the schools. Can we not call them to task for their past failures and ban them from influencing the mathematical development of children? Probably not. Their vested interest is too strong. Perhaps the few visualizers that survived and those that will continue to survive, despite the efforts of the behaviorists, will lead the way in providing opposition and alternatives for the mathematical education of our youth. The problem is always that people of vision have many demands on their time. The priority of the education of our youth may not be high enough to warrant their attention. It should be.

# Reading in Mathematics

**by**

**Gerald Coombs**

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Students, please answer the following questions: "Jane has 50 tomato plants. If she plants them in four rows, how many will there be in each row? Will there be any plants left over?" This is a common type question in elementary school math classes. Did you get 12 in each row and 2 left over? Yes, that was the answer in the teacher's book but it is not right! (The right answer is given later.)

One of the important areas of education considered by several school boards in the last couple of years in in-service workshops has been "Reading in the Content Areas." This has probably been because of the general complaint by teachers that "Johnny can't read" and as a result he seems to have trouble in other subject areas. The feeling seems to be that if reading is causing trouble in a particular subject, the teachers involved with that content area should make some attempt to teach the applicable reading skills.

Part of the problem, however, is that teachers in many subject areas have not had any formal training in teaching reading and as a result often feel insecure in dealing with it.

In this context, I will try to give a brief outline of general reading skills, the reading skills that relate in particular to mathematics, and discuss how some of these may be identified and used in the teaching of mathematics. I hope this attempt will help in some small way with the problem.

## Developmental Reading Program

Most reading specialists agree that the following components are necessary for an effective developmental reading program:

1. development of the basic reading and study skills,
2. mastery of the specialized reading and study skills in the content subjects,

3. opportunity to do reading for purposeful development,
4. experience in reading for pleasure.

## General Reading Skills

The general reading skills which have been identified as common to all academic subjects are:

1. reading for the main idea,
*2. knowledge of advanced and specialized vocabulary,
3. utilization of contextual clues for meaning,
4. proficiency in structural analysis,
*5. ability to adjust rate and technique to purpose,
*6. skill in analytical and critical reading.

The degree to which a skill is taught and the method of instruction used will vary from subject to subject. In the preceding paragraphs I have indicated (*) those areas where I feel we can do most in mathematics.

## Reading Skills that Relate Specifically to Mathematics

1. Understanding the specialized vocabulary.
   (a) technical mathematical terms,
   (b) alphabetical, operational, grouping and relationship symbols,
   (c) roots, prefixes, and suffixes that aid in understanding mathematical terms,
   (d) also, common words that have special mathematical meanings.

2. Reading and interpreting verbal problems.
   (a) evolving procedures for problem solving,
   (b) distinguishing between the important and irrelevant information in problems,
   (c) apply graphic representations to problems.

3. Recognizing relationships.

4. Recognizing equations as expressive in a manner similar to sentences in regular prose writing.

5. Acquiring meaning from the statements of rules and definitions so that they may be used with understanding.

6. Organizing details and processes to find solutions.

7. Checking to verify solutions and/or to locate errors.

What implications do these reading skills have for us in the teaching of mathematics in the classroom?

1. Understanding the specialized vocabulary.

How much emphasis do we place on vocabulary development and what approaches do we take toward teaching it? Do we use roots of words in a meaningful way to show interrelationships between concepts? Is there enough consideration given to the distinct mathematical meaning of common words? What does a student really understand by: negative (especially if he is involved in photography), product (if he has been shopping the evening before), dividend (if his father is involved with stocks, et cetera), similarly with: complementary, scale, yard, set, partition; the list is endless.

There are obviously hundreds of technical terms within the total scope of the field of mathematics. If vocabulary study is conducted, in most cases, it will be with technical terms. Consider the terms a Grade 9 geometry student must have mastery of before he can attempt to understand: Jurgensen, p.56, Theorem 2-2: "If the exterior sides of two adjacent angles are opposite rays then the angles are supplementary." An understanding of the terminology of mathematics is essential for mastery. A few suggestions may prove helpful in teaching the mathematics vocabulary.

(a) Make few assumptions that the students are familiar with terms.

(b) Be sure all words are correctly pronounced and spelled.

Language accuracy should be given the same degree of importance as computational accuracy.

(c) Use specialized terms when called for.

(d) Test for growth in vocabulary, as well as reasoning and computation, regularly.

2. Reading and interpreting verbal problems.

This is the skill given most emphasis in the literature, but I feel it is the area in which mathematics teachers are strongest. Procedures have been evolved and used for attacking this kind of problem. A common procedure is the Ladder:

7. Check
6. Solution
5. Estimated
4. Processes
3. Given
2. Required
1. Read!

We can use verbal problems to determine other than numerical answers. There are many such problems that can prove useful in the mathematics and/or language arts classroom.

Consider as an example:

"Each spring the school held a marble shooting contest, with prizes for the winners. One of the rules for entering the contest was that each person must have a minimum of 50 marbles. Jack, Harry, and Bill were three friends who wanted to enter. Jack had 75 marbles, Harry had 50 marbles, but Bill had only 40 marbles. The boys realized that one of them could not enter the contest. They decided to combine their marbles and share them equally to see if they would each have enough marbles to become contestants.

1. What does the word combine mean in this problem?
2. What does the word minimum mean in this problem?
3. Could a person enter the contest if he had fewer than 50 marbles?
4. Could a person enter the contest if he had more than 50 marbles?
5. After combining their marbles, what was the total number the boys had?
6. How many marbles did each boy have after they shared the marbles?
7. Which boy lost the most marbles by sharing them?
8. Which boy gained the most marbles by sharing them?
9. Could all three boys enter the contest?
10. Do you think these boys were good friends. Why or why not?

There are still some who feel, however, that Johnny cannot read so we will omit this topic. It is when this approach is duplicated intermittently or completely through the system from primary to high school that we as mathematics teachers have failed to do our part in the total education process!

3. Acquiring meaning from the statements of rules and definitions so that they may be used with understanding.

How often are students required to memorize a statement as compared to the number of times they are encouraged to write it out in their own language? How much discussion is generally involved with new rules and definitions? Do we have time to do this, some ask? Can we afford not to? Perhaps consideration might be given to the idea that a longer time spent in introducing these ideas might actually shorten the time needed later when they are to be used, and more importantly, that students would have a better understanding in dealing with the problems related to the rules and definitions.

Along a similar line, I would suggest that you look at the other specific study skills and ask how you are dealing with these. If you have not been aware of these specific skills, maybe some consideration might be given to them in your future teaching of mathematics.

It was stated previously, in the outline of general reading skills, that the degree to which a skill is taught and the method of instruction used will vary from subject to subject. Three of these (2, 5, 6) were indicated as of particular significance in mathematics. The first of these (2. Knowledge of advanced and specialized vocabulary) has already been dealt with.

Of importance also is - 5. The ability to adjust rate and technique to purpose. We should make a point of emphasizing this aspect of reading mathematics. It is obvious to us, as teachers, that in reading mathematics material a very careful, deliberate, and often slow approach must be taken. This may not, however, be obvious to the student. Do we spend time in teaching Johnny "how" to read the material we assign? The value of slow, careful reading should be emphasized. When a student has an incorrect answer to a problem, the reteaching is often directed toward the relationships between the numbers involved (and many times it should be), but how much consideration is given to the student's understanding of what he has read?

The following is an example taken from ESM Book 6, p.80: "When the product of two whole numbers is 36, we say that each of the numbers is a factor of 36. You have learned that the product divided by one factor gives the other factor."

Although this is a simple statement to us as mathematics teachers, when read by a Grade 6 student, especially without the mathematical reading approach, it may be quite complex.

The last - 6. Skill in analytical and critical reading - is an area of importance, to which generally very little consideration has been given. How, you might ask, can we aid in critical reading in mathematics? Are students encouraged to look for imprecise statements, or for that matter, is it even mentioned to them? As an example of the imprecise statement, reread the first paragraph in this article!

This problem was written to reinforce understanding of division of whole numbers with a remainder. Therefore, any reader would know that he should have a remainder since the entire lesson deals with remainders. However, a critical reader would immediately answer that there would be no remainder, since the statement says Jane plants them (all 50) in four rows. If she plants them, there will not be any left over. Yet the teacher's answer sheet showed a remainder of two in the correct solution. The good reader will also penalize himself if he does not assume that Jane planted an "equal number" of tomato plants in each of the four rows, although there is no good reason to believe that all rows had an equal number of plants. Therefore, the answer to the question of how many will be in each row can be "one or more." The writer probably intended that the problem read:

> Jane has 50 tomato plants. If she plants an equal number of them in each of four rows, how many plants will there be in each row? How many will be left over, if any?

The student must now divide 50 by four and should arrive at the correct solution of 12 plants in each row with 2 plants left over.

We should point out to students various situations like these and you will soon find they will be reading their mathematics problems very critically!

The story is told of a school teacher who decided upon due reflection to give full credit to a pupil for his answer to an arithmetic problem.

*Question:* If your father sold fifteen hundred bushels of grain for $2.00 per bushel, what would he get?

*Answer:* A new car.

From the foregoing, one can see the importance of asking accurate questions.

Even when we ask accurate questions, however, we should be prepared for unexpected answers, as the following conversation indicates:

*Teacher:* How old were you on your last birthday?

*Junior:* Seven, ma'am!

*Teacher:* How old will you be on your next birthday?

*Junior:* Nine, ma'am!

*Teacher:* Nonsense, if you were seven on your last birthday, how can you be nine on your next birthday?

*Junior:* I'm eight today!

To quote from the writing of a one-time teaching colleague: "The fact that each content field requires specific reading skills uniquely related to the discourse of that field implies that each subject teacher must assume direct responsibility for developing those skills pertinent to his particular area of instruction. It is apparent that if students are to pursue effectively their content area subjects, provision must be made for developing those reading skills and abilities which are essential for adequate comprehension within each particular area of instruction. The fact that proficiency in one subject is not necessarily predictive of success in another subject presents further evidence to support the necessity for teaching applicable reading skills within each content area."

# Teaching Children How to Read Mathematics

***by***



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Reading the materials of elementary school mathematics is so specialized that a child must be taught how to do the reading. In mathematics, ideas are communicated by using a special language. Some sentences may be vocalized using the word attack skills needed for ordinary written English. Other sentences cannot be vocalized using these skills because they contain more than alphabetical signs. An example of a sentence containing a mixture of alphabetical and mathematical signs is given below.

Addition is associative on the set of whole numbers, that is,
$(2 + 3) + 4 = 2 + (3 + 4)$ since $5 + 4 = 2 + 7$.

Conversions of mathematics symbols such as 2, 3, 4, +, and = into words must be learned. The child must also know the meaning of other symbols, for example, [ ], X, >, <, $\neq$, $\sqrt{\ }$, $\overline{)\ }$, $\varepsilon$. In addition to learning the symbols, the child must learn many abbreviations, such as mo., t., min., hr., lb., doz., gal., and in. Learning the symbols and abbreviations constitutes a reading task. When a child is experiencing trouble, care should be taken in ascertaining whether it is because he does not know how to do the mathematical operation or does not know the symbolism of mathematics.

The language of mathematics contains words which are specific to mathematics, such as multiplicand, numerator, denominator, addend, perimeter, rectangle, rhombus and sphere. Words of this type are part of the reading equipment necessary in reading the usual mathematics book. The best way to teach such specialized terms is through their informal use. The teacher calls the concept by its specific name as he goes about his instructing. The language of mathematics also contains words which have one meaning in mathematics and another meaning which is not mathematical, for example, set, table, plane, point, power, yard, intersection, line and square. Words with more than one meaning cause problems for children because the non-mathematical meanings are used in everyday experience and are more familiar to them. Consequently, they have a tendency to resort to non-mathematical meanings even when reading in a mathematical context. If the meaning is to be clear in any given case, then the words having more than one meaning need to be studied in context. The teacher should anticipate difficulties and give the child the specialized interpretation.

The arrangement of words on a page in ordinary English is from left to right and is said to be linear. In mathematics, the arrangement of words and symbols is not necessarily linear. In the expression $5^2$, the motion from 5 to $^2$ is diagonal, $5^2 \nearrow$. In the expression 1/3 + 1/4, the ordering is shown by the following pattern: $\updownarrow \frac{1}{3} \nearrow + \updownarrow \frac{1}{4}$. In reading ordinary English, children learn to move smoothly from the end of one line of print to the beginning of the next. In mathematics, difficulty in reading is increased as the number of moves increases within a given line and in going from line to line. To help children read mathematical selections smoothly, pick out passages from the textbook. Allow each child to read a passage silently, then check by having each child read the passage aloud. This will help to identify trouble spots in reading order, and instructions can take place which will help clear up the difficulties.

One form of mathematical expression, the story problem, is especially difficult for children. Insofar as reading procedure is concerned, the following steps are widely prescribed on problem solving:

1. Rapidly read the problem to get a general impression; visualize the situation; ascertain what students are to find out.
2. Reread to get facts, isolating those facts which are pertinent to the solution.
3. Reread to help in planning the steps for solution. Some authorities have students state the situation in a mathematical sentence, while others have them estimate the answer and then perform the necessary numerical computation.
4. Read the problem again to check the procedure and to see if the solution is a tenable one.

Thus, several readings are entailed in properly approaching problem solving in mathematics.

The preceding paragraphs set forth some procedural suggestions for teaching reading in mathematics. All teachers should be reading teachers regardless of their content specialization. The reading teacher specialist can effectively work on content skills, but the only place to teach the reading pertinent to a content area is in the class dealing with the material. Teachers of self-contained classrooms in the elementary school must be committed to the fact that reading instruction must go on all day in every subject area.

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# A Directed Reading Procedure for Mathematics

***by***

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*This article first appeared in* The Illinois Mathematics Teacher, *Vol. 28, No. 5, November 1977, a publication of the Illinois Council of Teachers of Mathematics.*

Educators have been examining the relationships between reading and mathematics for several years. Evidence clearly supports the idea that there exists an important connection between the two. A student's success in mathematics learning seems to depend heavily upon his proficiency as a reader; a student having problems in the mathematics classroom may be having difficulties with reading skills, especially those pertaining to mathematics (Leary, 1948; Call and Wiggin, 1966; Lerch, 1967; Aiken, 1972).

The purpose of this article is to present a *directed reading procedure* which can help students who are experiencing difficulty meeting the demands of reading in mathematics. The procedure is an adaptation of a study-skill technique found effective in helping students improve reading, in general. Readers may be familiar with the variations of the technique, or the original version, SQ3R (Survey, Question, Read, Recite, Review) (Robinson, 1946).

To demonstrate the need for the procedure, a brief discussion of reading problems in mathematics will be presented. The main point of this discussion is that there is no one reading problem in mathematics, but rather there are many possible specific problems students may experience. These problems can be categorized as those having to do with (1) symbols, (2) symbols in context of formulas and mathematics sentences, (3) vocabulary, and (4) comprehension.*

*A more comprehensive discussion of language factors in mathematics along with a review of recent research can be found in L.R. Aiken, "Language Factors in Learning Mathematics," *Review of Educational Research*, 2, pp.359-85, Summer 1972.

In ordinary reading, the symbols the student must learn to recognize are the letters of the alphabet. In mathematics, the symbols may be more complex. The student must recognize and understand (1) numerals, for example, 2 or 3; (2) numerals which indicate operations, for example, $10^2$; and (3) other symbols, such as X, -, ÷, +, which indicate operations.

The student must recognize and understand combinations of these symbols. These symbols are combined in a way that is analogous to combining letters to make words. Single numerals are combined to represent larger numbers such as 326. In certain cases, letters and numerals are combined, for example, $22x = 44$. Furthermore, the student must be able to understand these symbols and symbol combinations when they appear in complete patterns such as formulas and mathematical sentences. ($22x + 4 = 26$). Reading these patterns is analogous to reading of words in meaningful groups (phrases, clauses, and sentences). A student who cannot accurately and easily recognize these many symbols has a reading problem, that is, he cannot deal with the symbolic language of mathematics.

Vocabulary is a potential source of many problems. A student may have difficulties with any one or more of the following: (1) simple words which have a special mathematical meaning - root, point, slope; (2) simple words or phrases which have a special importance in mathematics - how many, how many more; (3) technical terminology, that is, words which are new to the student and which represent new concepts - cosine, tangent, polygon; (4) abbreviations such as in. for inches.

Finally, the student may have difficulties with comprehension, that is, understanding all the symbols and words when they are used in (1) formulas and statements, (2) problems, (3) instructions for problems, and (4) explanations of concepts, generalizations, principles and operations. Most reading educators see comprehension as a complex skill area rather than single skill. The complexity of comprehension is apparent in the reading of mathematics. The following is intended to illustrate this complexity: (1) the student must be able to locate and retain details, ideas, and relationships; (2) he must be able to translate details, ideas, and relationships into numerals and other symbols and vice versa; (3) the student must be able to interpret details, ideas, and relationships that are not explicitly presented; (4) he must be able to apply what he reads to the solving of problems; (5) and he must be able to read for evaluations, that is, he must be able to evaluate his own solutions for accuracy.

The directed reading procedure to be presented can help students meet the difficulties noted above.

The procedure is similar to other directed-reading procedures or activities (Catterson, 1971; Robinson, 1972; Shepherd, 1973; Maffei, 1973); all have the element of direction as a key feature. They consist of a set of questions or instructions which direct the student while he is reading, as opposed to a set of instructions or suggestions given before or after the reading has been done. The instructions are based on an identification of the tasks believed to be important in successfully reading material of particular content or reading for a particular purpose.

The procedure we present has certain advantages not found in others. It was designed to be applicable to all mathematics problems. It is more thorough; it includes steps for self-evaluation, revision, and reflection. It calls for observable or overt responses and, as a result, insures (1) that students will, in fact, make the responses to each step, (2) that teachers will be able to observe all the responses, and (3) that students can provide themselves with feedback.

The procedure also serves as a diagnostic tool. It provides the means by which the teacher can identify the specific difficulties a student is having, by giving the teacher the opportunity to observe the responses students make while reading and working through a problem. If a student does have difficulty, the teacher has more than just the incorrect answer to consider. The teacher can see whether or not the student is locating and understanding the main ideas, important details, and symbols presented in the problem itself, or in the instructions for, or explanation of, the problem.

The procedure can be followed each time a new skill, concept, and/or type of problem is introduced. The student responds overtly - in writing - to each step. If he can do one problem successfully, he can use a more efficient method for subsequent problems of the same type. The procedure has 10 steps.

1. *Read the problem quickly to obtain a general idea of its content. Write a statement of the general idea.* In the first step, the student is to gain a general idea of the material. The student is providing himself with an "advance organizer," (Ausubel, 1960) to aid him in the subsequent processing of the details. Research has demonstrated that advance organizers do increase comprehension of both ideas and details (Rickards, 1975). If the type of problem or some symbol or some words are new to the student, comprehension will probably not be aided by a very slow, careful reading, initially, because the student will overload himself with specifics that have no meaning for him.

2. *Reread the problem more slowly to answer the question. What specific information is given in the problem? List the details (pieces of information).* With the aid of the advance organizer, the student can begin processing the details. By writing them down, he does not have to rely on retention of what may be, at this point, not completely meaningful, specific information.

3. *What does the information mean? Translate, in writing, the numbers/symbols into words or objects, or the words into numbers or objects, or the objects into numbers or words. Reread the problem if necessary.* In the previous step, the student simply locates the information. In this step, by translating, he demonstrates understanding of the information.

4. *Show how the pieces of information fit together.* (The student might draw lines, arrows, diagrams, et cetera, to indicate relationships.) *Reread the problem if necessary.* A still higher level of understanding is demonstrated by the ability to indicate relationships, and in mathematics, the precise way details are related is a critical matter.

5. *What procedure should be used to solve this problem? Write (in outline form) the procedure.* This step insures that the student does attend to the instructions, rereading them if necessary. If the procedure is implied in the problem, this step insures the student has made the necessary inference.

This step further insures that the student relates the problem to previous instruction and reading. In a sense, he must answer the question: "Have I seen this problem before?"

6. *Use the procedure to solve the problem. Write the answer.* This step requires the student to apply what he has read.

7. *Check off each of your responses with an answer key.* The teacher provides an answer key which includes appropriate responses to all the steps.

8. *Count the number of steps you completed correctly. Write the number.* This step insures that the self-evaluation is done and provides objective feedback.

9. *Revise any incorrect responses.* Ask for help if needed. This step insures that the student learns from any errors.

10. *Reread the problem. Then briefly summarize what you have gained from working the problem.* This step insures reflection on what the student has done. It aids him to synthesize his experiences and, perhaps, make discoveries that will be of use in further reading and problem solving.

Rereading is called for in some of the steps. The necessity of rereading, as well as the acceptance of that necessity, seems of great importance to successful reading in mathematics. It should be noted that each rereading is done for specific purposes to answer particular questions.

This procedure parallels a common approach to problem solving proposed by Polya and others (Polya, 1957): (a) identify the problem to be solved - understand it, (b) devise a plan for solving the problem, (c) try out the plan, revising as necessary, (d) look back - reflect on new learnings. Steps 1-4 in the reading procedure are analogous to step (a) directly above. Step 5 parallels step (b), while steps 6-9 go with step (c) and step 10 parallels step (d).

The following presents examples of the use of the procedure for different areas of mathematics. Figure 1 shows an addition problem. A potential reading problem would be incorrectly recognizing the relationship between numbers. Figure 2 shows a trigonometry problem. Here the student is given instructions for a number of similar exercises. Potential reading difficulties would be in understanding the instructions, recognizing the many different symbols used, and understanding and applying the formulas. In the example, the student made an error. Figure 3 shows a problem requiring use of a graph. Two potential problems would be in comprehending the graph (the "pie") and recognizing the symbol for percentage (%).

## Conclusion

A number of educators have developed and recommended directed reading procedures as a partial solution to potential reading difficulties in mathematics. The more thorough the procedure, the more helpful it should be, given the complexity of reading in mathematics. Teachers can help students with the reading skills required in mathematics if they know the specific difficulties the students are having.

### *Figure 1*

Find the sum 350
+345

1. Finding the sum of two 3-digit numbers.
2. 350
   345
   +
3. three hundreds, five tens, zero ones
   three hundreds, four tens, five ones
   add
4. 3 5 0
   + 3 4 5
5. Add the ones, then add the tens, and then add the hundreds
6. 695
7. Check steps 1-6
8. Count correct steps - 6
9. Correct if necessary. Discuss with teacher if unsure of any procedure.
10. Looking back - for example, procedures apply to other addition problems such as 4-digit problems, et cetera.

### *Figure 2*

Let A, B, and C designate the vertices of a triangle; $\alpha$, R, and ə designate the measures of the corresponding angles; and a, b, and c designate the length of the corresponding sides.

1. Find c, given that b = 4, a = 3, ə = 150°
2. Find c, given that a = 3, b = 7, ə = 40°.

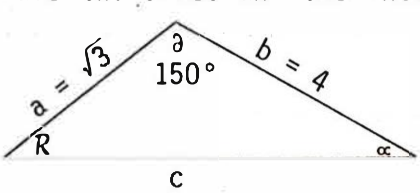
   Note that student is given instructions for a series of similar exercises.

3. (a) Find the missing side of a triangle.

   (b) c = unknown, b = 4, a = $\sqrt{3}$, ə = 150°

   (c) Have two sides and the included angle; need to find the missing side.

   (d)




   (e) Use the law of cosines -

   $c^2 = a^2 + b^2 - 2ab\cos ə$

   (f) $c^2 = (\sqrt{3})^2 + (4)^2 - 2(4)(\frac{\sqrt{3}}{2})$

   $= 3 + 16 - 12 = 6 => c \approx 2.45$

(g) Lost sign in step (f)

(h) Steps (a) to (f) correct

(i) $c^2 = 3 + 16 + 12 = 31 => c \approx 5.57$
Missed sign - careful of signs in quadrants

(j) Use law of cosines in variable forms

Note that the breakdown of a problem into parts may vary from teacher to teacher and from student to student.

***Figure 3***

1. Family budget - housing food

2. Rent 25%
   Savings 10%
   Food 15%
   Not housing food
   Clothing, etc. 35%
   Recreation, etc. 15%
   Average family's dollars

3. Housing - Food / Rest

4. See Figure 4

5. Add Housing % to Food %
   Subtract from 100%

6. 25% + 15% = 40%
   100% - 40% = 60%

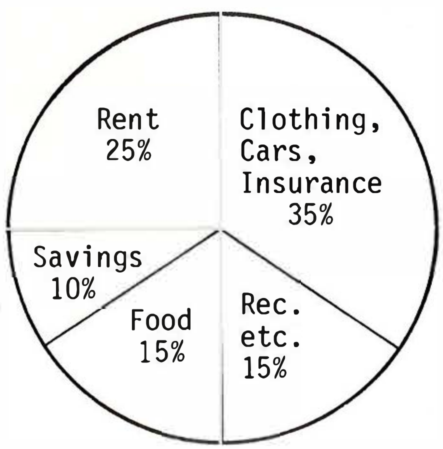
7. Check

8. 6 correct

9. Revise if necessary - with teacher's help. Teacher could use this opportunity to polish format, et cetera.

10. Look back - general graph reading, et cetera.

***Figure 4***




Average family's dollar

How much of the family's budget does not go to housing-food?

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# Teaching Reading Skills in the Mathematics Classroom

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There are two purposes to this article. The first is to provide a brief but generalized report on the problems of teaching reading in the mathematics class. The second is to provide a set of specific techniques from which the mathematics teacher may choose, to potentially improve the reading skills of students.

Reading is fundamental to all disciplines and it is especially important in mathematics, where words, symbols, and pictures are combined and condensed on the printed page. Reading instruction provides a unique set of difficulties for the mathematics teacher. Most mathematics teachers realize the importance of reading instruction, yet they also feel frustrated when attempting to improve students' reading comprehension. Let us examine some of the difficulties of teaching reading in a mathematics classroom.

## General Difficulties

For the average child in pre-algebra, a reading assignment may prove difficult because students encounter reading problems unique to mathematics.

For example, words in mathematics have very precise meanings. Some words signal a process (such as add, subtract, multiply or divide), while others increase the student's comprehension of the problem (with words such as before, increase, of, for, or compare). Few words are wasted, and the students must concentrate on reading and also on the conceptual relationships implied. Paragraphs often must be reread, and for some, this is a slow and difficult ordeal. In addition, some words used in mathematics which appear to be everyday words have special mathematical meanings. Examples are: radical, times, rational, mean, range, and (if one includes computer mathematics) bit, job, run and hardware.

The reading level required by mathematics books is usually high, even in junior high school texts.

But, with pressure to publish "modern" textbooks, emphasis upon content has caused the readability level of material to be too high.

Not only is the level too high, but there is also too great a range within the text. Some selections examined fell within the fourth-grade-and-below category while others would be appropriate for a college graduate student.[1]

For the average student of mathematics, some reading assignments prove difficult. For the poor reader, these assignments prove to be impossible. What the reader brings with him when he opens the text determines, to a large degree, what he takes from it. The mathematics teacher should be aware of special reading problems in mathematics and should prepare the student for meeting problems inherent to mathematics materials. The key is that the mathematics teacher must be aware:

1. The teacher should be aware that the student may have (or has had) a sense of frustration in reading mathematics. The student's past experience may have been discouraging, and the student must be given a feeling of success through slow but patient work in reading.

2. The teacher should be aware that teaching reading in the mathematics class can make a difference in the student's overall mathematical ability. The mathematics teacher can be an effective reading teacher.

    > Call and Wiggin pointed up the need for reading instruction in mathematics classes. They conducted a study to determine whether there is a correlation between a student's ability to solve word problems in second year algebra and the presence or absence of special reading instruction. The results, they said, indicated that the students receiving reading instruction in their classes did better in problem solving than those who did not.[2]

3. Finally, the teacher should be aware of specific techniques from which he may choose to meet the needs of the class, be it regents, non-regents, pre-algebra, eleventh year mathematics, or even calculus.

    Many suggestions for improving reading comprehension are included below.

## Specific Techniques for Improving Reading

Many mathematics teachers realize the importance of reading ability in mathematics and assign a passage from the text as part of a student's homework. For the poor reader, little is usually accomplished by this. The poor reader may

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[1]Frank Smith, "The Readability of Junior High School Mathematics Textbooks," *The Mathematics Teacher*, LXII, No. 4 (April 1969), p.290.

[2]David L. Shepherd, *Comprehensive High School Reading Methods* (Columbus, Ohio, 1973), p.252.

become discouraged and not do the reading. To ensure that the student "does" the assignment, the teacher may give a quiz the next day on this reading assignment. This quiz, however, may actually discourage the poor reader who tried to do the reading but who could not completely master it. The quiz is, therefore, "after the fact" where reading instruction is concerned. The student should be prepared ahead of time for all he will meet in the reading selection. Of course, the teacher should read and analyze the assignment the day before to anticipate difficulties.

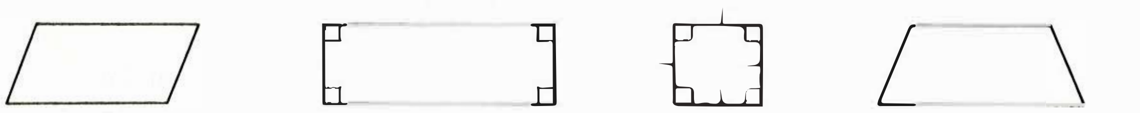
Here are some techniques for improving reading:

1. VOCABULARY

   An important aspect of the reading assignment is new vocabulary words. The teacher may introduce new vocabulary words by (a) writing the words on the board and going over their meaning, or (b) having the students keep a "mathematics dictionary" in their math notebooks. A corner of the board may be reserved for new vocabulary words each day. The teacher may also prepare word puzzles using math words listed in the student's notebook.

2. DIAGRAMS

   In many cases, one picture is worth a thousand words; and the teacher should refer to diagrams often in defining words.



Parallelogram　　Rectangle　　Square　　Isosceles Trapezoid

3. REWRITING

   Rewrite a paragraph using words the students may better understand. Even asking the students themselves to rewrite a paragraph, using their own words, may be helpful. The brighter students in the class may come up with a clear and mathematically acceptable interpretation of the paragraph. Such an assignment can be a good "Do Now" assignment. It may be helpful to mimeograph results for future reference.

4. DIRECTIONS

   Stress the importance of following directions. Read important instructions aloud and discuss the wording of test questions. Underline key words. Give an easy quiz in which following directions is important.

5. VERBALIZATION

   The teacher should try to develop improved questioning techniques. Ask the student to not simply work out the problem, but to explain what was done. Involve the class in such discussions. Ask questions such as "How did you get from step 2 to step 4?" rather than "What is the answer?"

   Ask students to write out the steps used in solving problems.

For example: If the length of a rectangle is 10 feet and the width is 5 feet, find the perimeter.
Reread the problem.

step 1 Find ______________________________

step 2 Given ______________________________

step 3 Formula ____________________________

step 4 Check ______________________________

6. DRILL AND REVIEW

Have students underline certain words, reread certain paragraphs in class, and review topics already covered, thereby keeping the material fresh in the students' minds.

7. OUTSIDE REPORTS

There are many interesting books on mathematics from mathematical recreations to mathematics in literature. Assign outside readings for extra credit. One very popular and enjoyable mathematics book is *Mathematics in Everyday Things*, by William Vergara.

It is helpful to show how mathematics can be applied to everyday situations.

8. CONTESTS AND GAMES

Contests may work especially well in motivating students. Contests are also very good on "short" school days or on a day just before a vacation. Scrambled letters, matching columns, fill in the blanks, code words, word association, and other word games may be appropriate.

9. QUESTION THE STUDENTS

Ask the students for reasons why they think verbal problems are difficult. List these reasons and discuss with your students techniques for overcoming difficulties.

The problems associated with teaching reading are very complex. There are many factors, physiological as well as psychological, which affect one's reading ability. Students with severe reading problems should be scheduled for special reading classes.

Presented here was a brief overview of what a mathematics teacher can do to help improve the reading comprehension of some students. Awareness of the problems and the knowledge of some techniques for helping students should be a part of the repertoire of any mathematics teacher.

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# Tips for Teachers

by

**M.J. Schill**

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Whether or not a teacher spends time instructing students in the broad area of reading in mathematics, inevitably some students still experience difficulties in one or more of the numerous specific areas. It is the responsibility of the teacher to diagnose in detail the area(s) in which the student is having problems. Then he must apply some concrete techniques to alleviate the difficulties.

The following is a list of suggestions that may be used by a teacher to aid in overcoming a student's problem(s).

1. TECHNICAL WORDS

These are often a problem for the child because he does not understand them or there are too many in a given passage.

If there are numerous technical terms to be encountered, either be sure that the student understands what their meaning is, or avoid that particular problem. The following is a method used to decide if the word problem should be eliminated: "If a given problem averages more than one unfamiliar word per 20 running words of print (5 percent unfamiliar; 95 percent familiar), it may be difficult for the average student to attack that problem."[1]

2. OCCASIONAL VOCABULARY WORDS

If there are one or two words in a problem that may create confusion, the meaning of such words should be clarified before the assignment is given.

3. WORDS WITH MULTIPLE MEANINGS

"The teacher should be aware of problems that contain words having one meaning in literature and another in arithmetic. He should be prepared to give a vocabulary lesson immediately prior to the arithmetic assignment."[2]

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[1]Leroy Barney, "Problems Associated with Reading of Mathematics," *The Arithmetic Teacher*, Volume 19 (February 1972), p.132.

[2]*Ibid.*

An idea that can be put to use in your classroom is to have each child keep a book in which new mathematical terms are recorded, as well as their meanings. This gives the child a reference which he can refer back to.

Another idea that could be used along with the above suggestion is to keep a math dictionary in the classroom which the students can refer to if they are not sure what a particular word means.

4. NARRATIVES WITH UNNECESSARY OR INSUFFICIENT DATA

Have the child read these aloud, and then discuss what is given and what is required, and from here whether there is unnecessary or insufficient data. A great deal of work usually needs to be done in this area, since many students fall prey to problems with unnecessary or insufficient data. These types of problems can be used "... with discretion to determine how well the child is able to use his reading skills at the interpretive or analytical level."[3]

5. STUDENT RESTATEMENT

Time should be allowed the students to orally restate a problem in their own words. This is a way for you, the teacher, to check whether the student understands the vocabulary and ideas presented, and gives the student a chance to introduce a little humor into the usually dry, humorless word problem.

6. PUNCTUATION

Occasionally punctuation marks in math books may represent uses different from those in an English text.

For example: The girl ran into the room and hollered at her dad,<br>"Take care of yourself and enjoy <u>...</u>"<br>(1, 2, 3, 4, <u>....</u>)

As the math teacher then, you must reteach and reapply the use of punctuation in mathematics.

7. As the class comes across an abbreviation, the meaning and its origin should be explained, that is, their Latin and Greek roots, for example, lb. and oz.

8. PREFIXES AND SUFFIXES

The teacher should teach the student the most-used prefixes and suffixes and what they mean, for example, poly - many, di - two.

9. READING SPEED

"A mathematics book should in general be read slowly, but there are times when more rapid reading is advisable. Children should be encouraged to combine both types of reading... The following steps may help children read... at the appropriate rates:

(a) Read the selection rapidly to determine what it is about.

(b) Reread slowly to see how ideas are related.

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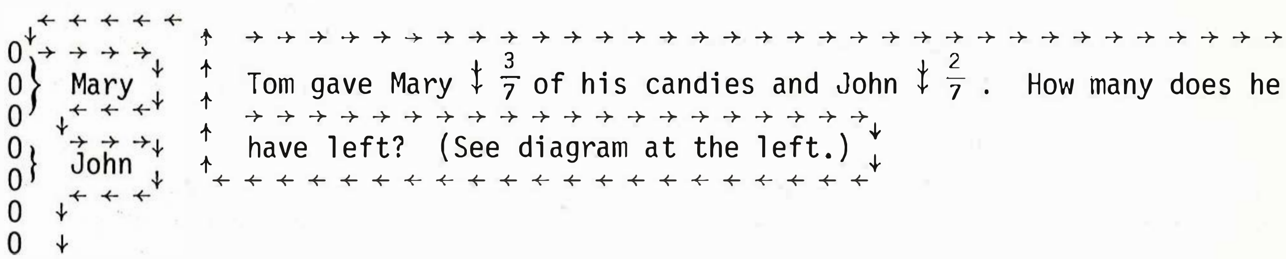
[3]Leroy Barney, "Problems Associated with Reading of Mathematics," *The Arithmetic Teacher*, Volume 19 (February 1972), p.132.

(c) Ask the question: Did you follow all the directions and answer all questions?"[4]

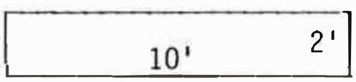
10. ORDERING SYMBOLS

In any English text, a person reads from left to right, horizontally across the page (→ → → →). In mathematics, however, a person may end up reading horizontally, vertically, from the top of a page and skip to the bottom and then back to the top of the page. A child must be taught how to read the page, and the order in which it is read. For example, one would read ↓ $\frac{3}{7}$, not ↑ $\frac{3}{7}$. An example of a question in a text which requires the child's course of reading to jump around is:




11. READING WITH PAPER AND PENCIL

A child should be taught to draw diagrams or charts as he reads the problem through. For example, if a child was reading a problem which stated that a yard was 10' x 2', he should make a diagram looking like this:




This pictorial representation is easier to work with for most students, rather than simply relying on mental image.

12. READING GRAPHIC MATERIALS

Often word problems will refer to graphs or charts. A child must be taught to read these properly. One method would be to have the child skim first for the topic, categories, and elements. Then a detailed study should follow, in order to ascertain the desired facts. It is a good idea to have the class make their own graphic materials in an area which interests them. Also, engage the class in games which require the children to fill in a grid by moving horizontally and vertically.

13. ORAL READING

"Oral reading of mathematics can provide many benefits. A few of these benefits are listed below.

(a) Students hear the sounds for symbols which have no phoneme-grapheme relations with spoken words.

(b) Students recognize that different verbalizations can occur for a set of symbols.

(c) Students see how the meaning of the page parallels the meaning of a lesson presented by a teacher.

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[4]Robert B. Kane, Mary Ann Byrne, and Mary Ann Hater, *Helping Children Read Mathematics* (New York: American Book Company, 1974), p.41.

(d) Students, of necessity, slow down their reading pace.

(e) Teachers can recognize whether an inability to solve a problem is due to weakness in perception or comprehension."[5]

14. NONVERBAL PROBLEMS

"(a) Nonverbal problems allow pupils to focus quickly on a problem situation without heavy reliance on advanced reading skills ...

(b) The nonverbal problem format is very flexible ...

(c) *Perhaps* the most potent advantage is that the teacher can provide the types of problem situations that come closest to the real-life experiences faced by pupils inside and outside the classroom.

(d) Nonverbal problems can be tailored to meet the needs of pupils who have special requirements ..."[6]

15. ESTIMATION

By teaching estimation, the student can compare his computed answer with the estimated answer to see whether he has arrived at an answer that is plausible.

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[5] Robert B. Kane, Mary Ann Byrne, and Mary Ann Hater, *Helping Children Read Mathematics* (New York: American Book Company, 1974), p.73.

[6] John W. Trueblood, "Promoting Problem Solving Skills Through Nonverbal Problems" *The Arithmetic Teacher*, Volume 16 (January 1969), p.8.

# Part IV

## Teaching Ideas

# Reading and Writing Arithmetic

**by**

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## Editorial Comment

The BCCC Formula -

This formula, presented on page 88, was developed by Mary Langer Smith and Karen Jacobsen Nielsen to aid in the instruction of reading, not only in mathematics but in all content areas. This formula deals basically with vocabulary development. An explanation follows along with a lesson plan and some working examples in mathematics.

The Four Vocabularies -

There are four "vocabularies" available to students and to people in general. The four - listening, speaking, reading and written - are acquired in a hierarchical sequence. The four vocabularies are explained in greater detail on page 90, along with a list of typical mathematical words found in elementary arithmetic texts which teachers can use for their own reference. Also included is a lesson plan dealing with mathematics vocabulary.

Math Word Puzzle -

Children enjoy puzzles. Therefore, on page 92, a puzzle is presented which will help the children develop their mathematical vocabularies by discovering the terms in the given array of letters.

Vocabulary Development Exercise -

On page 93 is another exercise useful in developing mathematical vocabulary. This exercise can be used with nearly any concept being taught in any subject area. Another lesson plan has been provided along with two worksheets, the first one being a classroom exercise and the second being a form of evaluation.

Cloze Procedure -

Another vocabulary development procedure, called the Cloze Procedure, can also be used in the subject of mathematics. It is discussed fully in a section beginning on page 94, which includes a sample exercise on the history of the metre. The full essay is provided, along with a sample in cloze form.

Key Words in Written Directions -

It is a good idea to give children some instruction on following directions. One excellent method is to look for key words within the directions and to know exactly what steps are to be taken from these key words. On page 97, we have several examples of various direction-giving key words and how they can be used in mathematics. Following this is an exercise which tests a student's ability to follow directions.

## Procedure for BCCC Formula

Formula -

B Beginning sound of first letter(s).
C Context - Read as far as needed to figure out the word.
C Construction - Divide the word into its parts.
C Check - Use dictionary or glossary.

Example to explain the formula -

| | | |
|---|---|---|
| B | Beginning Sound ........... | A consonant cluster must be sounded together. |
| C | Context .................. | Context is the only way to determine if the word is object' or ob'ject. |
| C | Construction ............. | Transportable trans/port/able (Teach roots and affixes to assist in identification.) |
| C | Check .................... | Use reference materials. |

Procedure -

Select a paragraph or selection which you wish the student to read. Underline selected words to be identified and defined according to context. Have the student do the following:

- Look at the underlined word(s). If it is not familiar, use the B in the formula.
- If the sound doesn't help, try the first C.
- Look back at the word. Did the way it was used in the sentence help decide what it was? If not, try the next C.
- If the beginning sound and context did not help, did the construction give any clues? If not, consult the dictionary or glossary.
- Sound out the beginning letter(s).

- Read the sentence for clues to the word. Context clues are the most important single aid for readers if they do not recognize a word at sight. Tell students to read to the end of the sentence or paragraph to see if they are able to identify the word. With attention focused on meaning from context, they will often discover what the word is.
- Divide the word into parts. Look for prefixes and suffixes. Identify the root word. Try to sound out the word by syllables.
- As a last resort, check the dictionary or glossary.

Sample Lesson -

Lesson title: Circle formulas

Lesson objective:
Using the circle formulas $C=\pi d$ and $A=\pi r^2$, students will solve story problems.

Reading skill objective:
Practice in using BCCC strategy on unknown words in story problems.

To achieve the reading objective the teacher will:
Review BCCC Strategy Chart on the bulletin board. Remind students of index and glossary in the text for help with definitions. Provide worksheet with underlined words divided into syllables.

To achieve the lesson objective the teacher will:
Provide a worksheet with underlined words and problems using the circle formulas. Review formulas and discuss any questions on substitution.

Evaluation -

Achievement of reading skill will be evaluated by:
Observation of student behavior during assignment and correcting worksheet.

Achievement of lesson objective will be evaluated by:
Correcting assignment.

Directions -

Use the BCCC formula on those underlined words. Write definitions you don't know in the chart provided below. Apply the proper circle formula to each question. Formulas are $C = \pi d$ and $A = \pi r^2$. $\pi = 3.14$ for our use.

1. The *diameter* of a circle is 5.5 centimetres. What is the area?
2. A circle has a *radius* of 9 units. What is the area?
3. A circle has a diameter of $1\frac{1}{4}$ units. What is the *circumference*?
4. The area of a *square* is 625 square feet. What is the largest circle that can be *inscribed* in the square?
5. A wheel is 24 inches in circumference. If this wheel is on a *vehicle* that travels one mile, how many *revolutions* will the wheel make?
6. Two *concentric* circles are drawn. The radius of the smaller is 8 units and the larger is 12 units. What is the *difference* in the circumference of the two?

| *Key Word* | *Definition* |
|---|---|
| 1. di am e ter | |
| 2. cent i me tre | |
| 3. ra di us | |
| 4. cir cum fer ence | |
| 5. square | |
| 6. in scribe | |
| 7. ve hi cle | |
| 8. rev o· lu tion | |
| 9. con cen tric | |
| 10. dif fer ence | |

## Process for Developing the Four Vocabularies

The natural order for the development of vocabulary at any age or level begins with *listening*. As the teacher talks, using special terms correctly, the students establish a relationship between the words and the concepts. After carefully guided concrete and semi-concrete experiences with the concepts, the students are encouraged to verbalize and demonstrate ideas, thus moving the words into the *speaking* vocabulary. It is critical that these steps be carefully followed to avoid failure when the terms are transferred into the *reading* vocabulary. Numerous experiences with identifying the written symbols or words places them into the reading vocabulary. The most advanced and therefore the most difficult is the *written* vocabulary. It synthesizes all the preceding skills.

Because various subjects employ certain words and symbols to convey meanings specific to them, there has always existed the problem of translating English expressions into technical language and vice versa. Our research has shown that the most difficult task for the student is posed by the special vocabulary and symbols which have one meaning in everyday life, but take on a different meaning in the context of the subject being studied. Therefore, every teacher must be concerned with teaching the specific meanings needed to comprehend and use the vocabulary of their discipline.

Knowledge of special vocabulary or symbols is cumulative. A weakness in either of these will result in incomplete knowledge of a concept. Since one concept is often built on another, the student must learn the language used by the teacher and the text and learn it thoroughly and carefully.

### "Must-Know" Mathematical Words

This list has been drawn from recent elementary arithmetic texts. Cardinal, ordinal, and measurement words have not been included but must be taught as needed.

It is recommended that the teacher survey the text in use and identify essential vocabulary beyond this list to be taught as needed.

above
add
addition
addend
alike
angle
arithmetic
around

backward
begin
below
between

center
chart
check
circle
clockwise
collection
column
common
compare
complete
connect
corner
count
correct
curve

decrease
definition
diagonal
difference
digit
dime
direction
distance
divide
division
divisible
dollar
double

each
equal
equation
error
estimate
example
even
exercise

factor
false
fewest
figure
final
fraction

graph
greater
greatest
group

half
height
hour

identical
inside

join

large
least
length
less
like
line
long
lowest

matching
measure
minus
missing
money
more
multiplication

nickel
number
numeral

odd
opposite
order
outside

pair
parallel
pattern
penny
place
plus
point
prime
problem
product

quarter

rectangle
regrouping
remainder
reverse
right
round
row
ruler

second
segment
sentence
set
short
shape
side
sign
similar
size
solve
square
straight
subtract
sum

table
take away
term
test
times
total
triangle
true

unequal
unit
unlike

value
vertical

weight
whole
width

Sample Lesson -

Lesson title: General Math Words

Lesson objective:
Math vocabulary drill for familiarity.

Reading skill objective:
1. Identify word parts (prefixes, suffixes, and roots).
2. Determine origin - Latin or Greek.

To achieve the reading objective, the teacher will:
Prepare the students the day before by discussing prefixes, suffixes, and roots of words of Latin and Greek origins.

To achieve the lesson objective, the teacher will:
Provide a puzzle containing math words, a sheet providing outline of work to be done and dictionaries for reference. Provide oral instruction of a general nature discussing what to look for in a word and how to use the dictionary for the purposes of this lesson.

Evaluation -

Achievement of reading skill will be evaluated by:
The student's ability to identify the word parts and meanings.

Achievement of lesson objective will be evaluated by:
Checking the worksheets provided in this lesson.

## Steps for Constructing a Word Search

1. Develop a list of words related to the topic being covered.
2. Make a grid large enough to fit all the words in.
3. Arrange the words in the grid horizontally, vertically, and diagonally, interlocking where possible.
4. Fill in the blank spaces with random letters. (Part or all of the word list may be provided to guide the student in the search.)

MATH WORD PUZZLE:

bisect
parallel
polygon
diagonal
factor
arc
trig
vertex
finite
proportion
absolute
log
origin
decimal
joint
integer
median
transversal
multiples

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | S | O | L | U | T | E | V | E | R | T | E | X |
| P | A | R | A | L | L | E | L | F | A | C | T | O | R |
| O | H | O | N | R | K | W | D | E | C | I | M | A | L |
| L | I | T | D | E | M | I | Q | C | Z | U | P | D | C |
| Y | N | A | F | E | R | A | S | U | W | E | M | M | I |
| G | F | R | U | F | E | C | J | I | T | R | U | D | R |
| O | I | E | R | W | A | T | O | L | B | O | L | A | C |
| N | N | M | D | L | S | W | I | A | B | P | T | E | U |
| L | I | U | E | W | R | Q | N | T | L | I | I | R | M |
| K | T | N | S | H | M | W | T | E | O | S | P | W | F |
| K | E | P | P | E | D | W | L | R | W | B | L | V | E |
| L | U | A | D | T | J | A | W | A | I | H | E | W | R |
| O | R | I | G | I | N | R | T | L | R | G | S | V | E |
| G | A | P | R | O | P | O | R | T | X | O | N | X | N |
| N | H | F | G | I | N | T | E | G | E | R | K | D | C |
| G | C | A | U | R | L | T | N | E | N | O | P | X | E |
| G | I | T | R | A | N | S | V | E | R | S | A | L | H |
| D | J | R | T | C | H | D | L | T | C | E | S | I | B |

Sample Lesson -

Lesson title: Geometry Vocabulary

Lesson objective:
To learn and show through demonstration some basic geometry terms.

Reading skill objective:
Using dictionary markings for word pronunciation and definitions, have students learn vocabulary.

To achieve the reading objective the teacher will:
Provide a worksheet with words and definitions for study (Worksheet I).

To achieve the lesson objective the teacher will:
Provide Worksheet II to demonstrate knowledge of vocabulary after students have studied the words.

Evaluation -

Achievement of reading skill will be evaluated by:
Observation of team work.

Achievement of lesson objective will be evaluated by:
Checking Worksheet II for correct answers.

GEOMETRY VOCABULARY WORKSHEET I:

Directions -

Study the pronunciation of the word. Use a dictionary if necessary. Say the word. Then read the meaning of the word carefully. Be sure you can use it. Work with a partner and take turns saying words out loud and reading the sentences.

1. When two straight lines meet, they form an *angle*.
2. The point at which two lines meet to form an angle is called the *vertex*.
3. A part of a curved line is an *arc*.
4. The distance from the center of a circle to any point in its *circumference* is the *radius*. *Radii* is the plural of radius.
5. A *sector* is the part of a circle bounded by an arc and two radii.
6. A line parallel to the horizon or perfectly level is *horizontal*.
7. A line which extends straight up and down is *vertical*.
8. An *oblique* line is neither vertical nor horizontal.
9. When two lines meet to form a square corner, each is *perpendicular* to the other.
10. A four-sided figure having equal angles and sides of equal length is a *square*.
11. A *triangle* is a three-sided figure.
12. A four-sided figure of which both pairs of opposite sides are parallel is called a *parallelogram*.

13. A four-sided figure of which only two sides are parallel is a *trapezoid.*
14. A *hexagon* is a plane figure with six sides.
15. Any plane figure enclosed by straight lines regardless of the number is a *polygon*.
16. The distance around any figure bounded by straight lines is the *perimeter*.
17. Equal in length, size, value, or amount makes things *equivalent*.

GEOMETRY VOCABULARY WORKSHEET II:

Directions -

See how well you know the meanings of the words you studied. Write or draw answers to the blanks below.

1. A figure with four equal sides is a ____________________.
2. The two circles are the same size. They are ______________________.
3. Susan measured the distance around the rectangle. She found its ______________________.
4. A plane figure that has six sides is known as a _____________________.
5. Draw a horizontal line:
6. Draw a vertical line:
7. Draw an oblique line:
8. Draw two lines that are perpendicular to each other:
9. Draw a parallelogram:
10. Draw a square:
11. Draw a triangle:
12. Draw two polygons:
13. Draw a trapezoid:
14. Draw an angle. Label the vertex and sides.
15. Draw a circle. Make a sector in it. Label the arc, the sector, and one of the radii.

## Procedures for Cloze

Teacher -

- Prepare a cloze passage deleting selected terms, that is... if for vocabulary words studied, choose nouns, verbs, or adverbs.
- Introduce practice exercises to class prior to use as a measure. This must be teacher-directed. Instruct the students in the steps to success. Show them how to use context to determine an acceptable word.

Instructions for Students -

- Read the entire cloze passage silently. This allows you to look at the passage as a whole.
- Reread the selection, writing in the word you think fits the blank.
- Try to evaluate why you have chosen a word for the blank. (If the teacher has asked you to give a reason for your choice, write that in.) Does the word sound right? If so, it is usually right.

Follow-up -

Compare your completed paragraph with the original.

Teacher Follow-up -

Discuss the completed activity with the class. This will allow everyone to hear alternatives. If the choices are semantically and syntactically correct, they should be accepted (except where you demand the exact response). Point out context clues to provide students with useful reading techniques.

Preparation -

Selection:
Prose with quoted words not exceeding 10 words per 300 words of text.

Length:
For *testing* comprehension and vocabulary, use long selections of from 250 to 300 words, deleting at least 50 words.
For *teaching* vocabulary and concepts in context, use short selections of 50 to 150 words with 10 to 20 percent deletion.

Word deletion:
Random or every nth words. Random word deletion is best for most reading comprehension exercises. Nouns, verbs, and adverbs are best for study of vocabulary in most content areas. Conjunctions, pronouns, articles, and verb auxiliaries are of questionable value.
For 10 percent deletion, delete every 10th word; for 20 percent, delete every 5th word.

Administration - Students read silently.

Ages, time required:
Intermediate and secondary students. Untimed.

Scoring -

Exact word synonym:
Essential for concept and vocabulary testing purposes. Allowed when using cloze to determine if the student can understand the written materials of your classroom.

Score:
A minimum score of 60 percent should be expected to assure the student's understanding of the material. Divide total words into correct words.

.60 = 60% comprehension cloze scores
50 total words / 30. correct words

## A Brief History of the Metre

Early units of measurement were based on the length of toes, hands, and other parts of the body. The king's measurements were very often used as the units of measure for everyone.

During the French Revolution which started in 1789, however, the fighting people cut off the king's head.

Then they wanted to do away with anything that had to do with the hated king, so the metric system was made. It wasn't easy to find a good standard unit of length, but finally French scientists agreed to use one ten-millionth part of the distance from the Equator to the North Pole. They used the line called a meridian that goes through Paris.

It took seven years to do the necessary measuring of the meridian, but the new unit was finally done in 1799. It was called a metre and is the same length used today.

The metric system was made the law in France in 1790, even before the measured metre was made. It didn't help the measurement problem, however. The common people were still very happy using the dead king's measurements and didn't want to change. Spies and police were used to make the people use the new system. To sell anything by the dozen or by 12s was a crime. Everything had to be bought by 10s. The people got so unhappy about the forced changes that in 1793, they cut off the head of the leader of the metric system. It didn't do any good, though. The metric system was there to stay and the people just learned to live with it after awhile.

This is a good lesson for the people in the United States. Even chopping off heads won't stop the metric system. No matter how hard we fight metrics, it is here to stay. We might as well learn to live with it sooner than later.

. . . . . . . . .

Early units of measurement were based on the length of toes, hands, and other parts of the body. The king's ______________ were very often used as the ___________ of measure for everyone.

During the French _______________ which started ___________ 1789, however, the fighting ___________ cut off the king's ________.

Then ___________ wanted to do ___________ with anything that ________ to do with the ___________ king, so the metric ___________ was made. It wasn't ___________ to find a good ______________ unit of length, but ___________ French scientists ______________ to use one ten-millionth ___________ of the ______________ from the Equator to the North Pole. ______________ used the line ___________ a meridian that _______________ through Paris.

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The __________ system was made the law in __________ in 1790, even before the measured __________ was made. It didn't help the measurement __________, however. The common __________ were still very happy ,______ the dead king's __________ and didn't want to __________. Spies and police were used to __________ the people use the __________ system. To sell anything by the __________ or 12s was a crime. __________ had to be bought by 10s. The __________ got so unhappy about the __________ changes that in 1793 they cut ______ the head of the __________ of the metric system. It didn't do any __________, though. The metric __________ was there to stay and the __________ just learned to live with it after awhile.

This is a __________ lesson for the __________ in the United __________. Even chopping off __________ won't stop the __________ system. No matter how __________ we fight metrics, it is ________ to stay. We might as well learn to live with it sooner than later.

## Key Words in Written Directions

Key Words - Examples of how they can be used in mathematics:

1. *Explain* the process of division of fractions.
   What is the process of division of fractions?
   How does the process of division of fractions work?
   Why does the process of division of fractions work?
2. *Describe* how to subtract signed numbers.
3. *Discuss* three different methods of finding square roots.
   Give the processes involved in guess and check, division-average method, and calculation methods of finding square root. Give one advantage and one disadvantage of each.
4. *Define* a prime number.
5. *Compare* a composite number with a prime number.
   In what two ways are they alike and in what two ways are they different?
6. *Enumerate* the steps in calculating square root.
7. *Prove* that $4 + 4 = 13_{five}$.
8. Outline the process or operation of long division.
9. Evaluate the idea of having rules to add signed numbers.
10. Summarize the three rules that can be applied to addition of signed numbers.
11. Simplify.

## An Activity

### Reading, Understanding, and Following Written Directions

Objective: To follow written directions.

Directions:

1. Read the directions that must be followed in order to complete the activity below.
2. Put all answers on another sheet of paper.
3. Read completely each step. Then do what it says.
4. If you are unable to complete the assignment by following these written directions, I will be happy to help you by giving oral directions. However, there is a limitation to these directions. I must be able to answer your questions with only "yes" or "no."

Steps:

1. For this investigation, cut three strips of paper two inches wide and eleven inches long.
2. Hold one end of a strip in one hand.
3. With your other hand, bring the two ends together.
4. Take one end, give it a half twist and tape the two ends together.
5. A band twisted this way is called a Moebius strip.
6. With your pencil, draw a line down the middle of the strip until you have gone its full length. Do you return to your starting point?
   (1) ______________________.
7. On the basis of this experiment, how many surfaces does the strip have?
   (2) ______________________.
8. Run your finger along one edge of the strip. How many edges can you trace?
   (3) ______________________.
9. If you were to cut with a pair of scissors along the line you have drawn and continue to cut until you returned to your starting point, will you get two strips? (4)______________________.
   Will this cut add another edge? (5) ______________________.
   Will this cut add another surface? (6) ______________________.
10. If you were to make two complete trips around the strip with one continuous cut, what would you predict will happen? (7) ______________________.
11. With your scissors, cut along the line you have drawn and continue to cut until you return to your starting point. What happened? (8) ______________.
    Does this agree with your prediction in answer 4? (9) ______________.
12. Make a second Moebius strip. Cut it parallel to an edge one-third of the way into the strip. Continue cutting until you are back to your starting place.
13. What is the result? (10) ______________________.

14. How do your loops compare in length and width? (11) ______________________.
15. How does the result compare with your guess in answer 7? (12) ___________.
16. Make a third Moebius strip.
17. Cut it one-fourth the distance from the edge until you return to the starting place.
18. What is the result? (13) ______________________.
19. How is it different from your other strips in length and width? (14) ____________________.
20. Without cutting another strip, what do you think the results would be if you cut it in fifths? (15) ____________________________.

# How to Use Textual Materials

*The following material was taken from a book called* Reading in Math and Science, *published by the Calgary Board of Education.*

All too often math students expect their teacher to "give them the course" by explaining every concept to them. With *adequate* guidance, many students can accept a share of the responsibility for their own learning. If students can learn to read effectively, they are moving toward mathematical maturity. If students were introduced early in junior high to effective reading techniques, perhaps they would feel more at ease with their texts. One of the objectives of math education should be to give students the techniques for reading materials in these fields. (Not only is the teacher helping students to higher achievement in math, but he is also equipping them for learning on their own in other disciplines.)

Math requires a completely different approach to reading since more information is contained per square centimetre, per sentence, per page, than in any other type of writing. Sentences and paragraphs are packed with concepts. In actual experiments, the slowest readers have often proved to be the best in math. Three pages of *new* reading in math may be equivalent to forty pages in a novel or twenty pages of social studies.

Two types of material follow:

1. Materials to familiarize students with their math or science text.
2. Materials to help students approach a new unit or chapter.

## General Lesson on Using a Math Text

Objective: To familiarize the student with his math text.

Name of Text ______________ Author ____________________

Copyright Year ____________ Publisher _________________

A. TABLE OF CONTENTS

This is found near the front of a book.

A book's table of contents is a list of the main topics and where they can be found in the text.

Find the table of contents in your text and answer the following questions:

1. How many chapters? ______________________

2. What is the main topic covered in chapter 7?

______________________

3. Name one of the subtopics listed under chapter 4.

______________________

4. On what page does this subtopic begin? ______________

5. On which page will you find the following?
   (a) Beginning of chapter 3 ______________
   (b) End of chapter 5 ______________
   (c) Table of squares ______________
   (d) Index ______________

B. INDEX

This is found at the end of a book. It lists all topics in alphabetical order and the pages where they can be found.

An index will allow you to find information about a certain topic without reading the entire chapter.

Find the index in your text and answer the following questions:

1. On what page is "decimal" first mentioned? ______________

2. On what pages would you find information on triangles?

______________________

3. Pick three new words that you are unfamiliar with, from the index. What is the first page where each is used in your book? Turn to that page and copy the first sentence in which the word is contained.
   (a) ______________________
   (b) ______________________
   (c) ______________________

C. TABLES

Frequently texts have tables at the end of the book with information used throughout the text.

1. Does your text have any tables? ______________________

2. If so, write the title and page of three tables.

___

___

___

D. OTHER INFORMATION

Near the end of the book you may find other information such as symbols, measures, formulas.

1. Does your textbook have this extra information? ___

2. Which ones? ___

E. ANSWERS

Some books have answers to exercises.

1. Does your textbook have answers to *all* questions or just selected answers? ___

2. If it contains only selected answers, what do you think is the reason for this?

___

3. Look up one question in chapter 5. Does it have the answer in the back of the book? ___

   If not, which question before it has an answer listed?

___

F. CHAPTER ORGANIZATION

Each chapter in your text deals with a specific topic. Although each chapter is different in content, they all have similar ways of highlighting items. Some of these items are examples, definitions, properties, exercises, end of chapter review, and enrichment ideas. More than likely, your book has several of these.

1. Choose any chapter from your textbook. Write its number here. ___

2. Turn to this chapter and answer the following questions.

   (a) Which of the following ways are used to highlight the text in that chapter. Put a check mark in the box provided.

      (i) Underlining ☐

      (ii) Boxes ☐

(iii) Color ☐

(iv) Different Print ☐

(v) Graphs ☐

(vi) Diagrams ☐

(vii) Photographs ☐

(viii) Drawings ☐

(ix) Charts ☐

(x) List any others here ☐

(b) Which of the following are found in this chapter? Put a check mark in the box provided.

(i) Worked examples ☐

(ii) Definitions ☐

(iii) Formulas ☐

(iv) Properties ☐

(v) Exercises or problems ☐

(vi) End of chapter review ☐

(vii) Enrichment activities ☐

(viii) Review of exercises of material from previous chapters ☐

(ix) List any others here ☐

## How to Design Effective Questions

Levels of Questioning -

There are basically three types of questions:

1. Literal - answer is directly stated in the text.
2. Interpretive - answer requires finding information from the text, synthesizing this information, and then restating it.
3. Applied - background knowledge is necessary. Answers cannot necessarily be verified in the text because responses may vary within reasonable limits.

NOTE: It is important to vary your types of questions to suit the abilities of your students. Students of lower ability cannot usually get by the literal level unless they are guided right through to the interpretive level.

Following are examples of the three levels of questioning -- literal, interpretive, and applied.

## Grades 8 or 9 –
## Exercises for Integers and Equations

1. LITERAL LEVEL

   (a) $^{-}6 + 8 + {}^{-}12 - {}^{-}6$

   (b) $-24 \div -3$

   (c) $-1 \times -1 \times -2 + 6 + 8 - 8$

   (d) -1 squared divided by -1 cubed

   (e) find the sum of -8 and -128

   (f) find the difference of -62 and -62

   (g) $-6\ (-3 + -2 = -4)$

   (h) $-129 \div 13 \times -1 \div -13$

2. INTERPRETIVE LEVEL

   (a) $n + -6 = -12$

   (b) $2n + -6 = 0$

   (c) $-12n - 6n = 48$

   (d) $-6 + n + -4 = 2$

   (e) $2n + -3n = -6$

   (f) $-5n - -6n = -42$

   (g) $-12 - -24 = n$

   Solve for n:

   (h) negative two n plus six equals twelve

   (i) negative ten plus negative five n equals negative thirty

   (j) six n plus negative four n equals negative fifty-two

3. APPLIED

   (a) The temperature rises from -36°C to -4°C on September 25. On September 26 the temperature rose 26°. How much did the temperature rise on September 25?

   (b) The difference between 2 integers is -21. One of the numbers is -21. The second number is not -21. What is the other number?

   (c) The cube of a number is $^{-}64$. The number added to itself six times is $^{-}24$. This number subtracted from itself is zero. What is the number?

## Techniques for Reading of Content Materials

In order to familiarize the pupil with this method of reading content material, it is necessary to work through several selections in a systematic manner with the class and gradually allow the pupil to follow the steps independently with less teacher supervision. The student should acquire some degree of independence on one step before moving to the next. The aim is to give the student a completely independent method of efficiently reading content materials.

A. SQ3R - SURVEY, QUESTION, READ, RECITE, REVIEW.

*Survey:* Student glances over headings in the chapter or selection and the first sentence of each paragraph in order to get an overview of points to be noted and developed.

*Question:* Student turns heading(s) into question(s), mainly of the how, what, where, when, why variety. This, then, sets a purpose for reading.

*Read:* Student needs to answer question(s) derived from a heading(s).

*Recite:* Student goes over the questions posed and tries to answer the questions formed from what has been read.

*Review:* Student reviews for main points and the relationships. He tries to recite main points under each heading. In fact, the student then has the ideas in a logical outline (which, if this technique is applied in written form, can be a set of notes.)

Variation of SQ3R for Mathematics - SQRQCQ

*Survey:* The problem is to read rapidly to determine its nature.

*Question:* What is the problem?

*Reread:* Reread for details and interrelationships.

*Question:* What processes should be used?

*Compute:* Carry out computation.

*Question:* Is the answer correct? Check the computation against the problem facts and the arithmetic facts.

B. PQ/R-S-T - PREVIEW, QUESTION, READ, SUMMARIZE, TEST.

*Preview:* Student secures a general understanding of the selection mainly by reading carefully the topic sentences and summary paragraphs.

*Question:* While he is previewing, he should be asking himself questions related to the material being previewed.

*Read:* Student reads the whole selection carefully, trying to think about answers to some of the questions that occurred to him during the previewing he has just completed.

*Summarize:* Student tries to recall, in correct order, the main ideas of the selection he has just read.

*Test:* Answering of questions on the selection (main ideas, details, organization, between-the-lines, et cetera).

C. OARWET - OVERVIEW, ASK, READ, WRITE, EVALUATE, TEST.

A useful method for tackling a word problem in math is S Q R Q C Q -

1. Survey
2. Question
3. Read
4. Question
5. Compute
6. Question

Word problems are often called the "disaster area" of math. Most students are not aware of the slow, concentrated reading that word problems demand. A step-by-step sequence such as is used with the example below should help students to learn to attack word problems.

*Willie Mays has 31 hits for 48 times at bat. If he goes 5 for 5 in today's game, by how many points will his batting average increase?*

*1. Survey:* Read the problem thoroughly, asking "What is this all about?" Is there a word you don't know? What does "He goes 5 for 5" mean? Find the answers to these first before going ahead.

*2. Question:* Ask yourself "What have I to find here?" "I need to find Mays' present batting average and the batting average after today's game, then find the increase."

*3. Read:* Reread to find what information is already supplied. "I know the number of hits and times at bat right now, and I know that he adds 5 to each of them in the game today." Estimate what a sensible answer would be.

4. *Question:* "What processes will I use to solve the problem?"

First batting average = $\frac{31}{48}$ =

Second batting average = $\frac{31 + 5}{48 + 5}$,

then find the difference.

5. *Compute:* Carry out the above computation

$$\frac{31}{48} = 0.646$$

$$\frac{31 + 5}{48 + 5} = \frac{36}{53} = 0.679$$

$$0.679 - 0.646 = 0.033$$

6. *Question:* Is the answer correct? Compare the answer you arrived at with your estimated answer.

We will call the above sequence SQRQCQ. This may have to be modified for other types of problems.

## Suggestions for Attacking Mathematical Problems

Read and reread the problem using the following questions as guides for attacking it:

1. What am I asked to do? Try to get the general idea of the problem.
   (finding the main idea)
2. Can I visualize or diagram the problem?
   (getting meaning from the printed word or symbol)
3. What facts am I given? Take a good look at each fact, one at a time.
   (a) Important facts
   (b) Irrelevant facts
   (noting details and discriminating between relevant and irrelevant facts)
4. What "hidden" or unstated facts do I need to anticipate?
   (using critical thinking skills)
5. What step do I need to follow to compute the problem? Take each direction separately. Work with only one direction at a time.
   (a) Known facts given
   (b) Unstated or implied facts
   (c) Possible method(s) for use with given or implied facts
   (organizing materials and following directions)
6. Can I estimate the answer?
   (critical thinking)

7. Does my answer make sense?
8. Have I rechecked my computations for possible errors?

## Skills Needed for Efficient Reading of Mathematics

In order to be an efficient reader of mathematics, the student should be able to:

1. Understand specific vocabulary, word roots, symbols of mathematical operations, and spatial, temporal, quantitative relationships.
2. Solve problems which involve reading facts (details), seeing relationships (inference stated - not stated), estimating (predicting outcomes), testing results.
   (a) Understand and use rules and definitions.
   (b) Read critically - separate the relevant from the irrelevant.
   (c) Follow directions.
   (d) Understand sequence in operations.
   (e) Read compact word problems.
3. Read and understand visual materials:
   (a) diagrams
   (b) graphs
   (c) geometric forms
   (d) tables.

# Vocabulary and Symbols

*The following material was taken from a publication entitled* Reading the Language of Mathematics, *put out by the Florida Department of Education.*

## Reading Mathematical Symbols

SAMPLE ACTIVITY:

Make a set of cards with phrases of mathematical symbols. Have a student come to the front of the class and give him one of these cards. Don't allow the rest of the class to see what is on the card. The student then reads out loud (in words) what is on the card. The other students write the symbols for what they hear, and then compare their answers.

$$3 + 2$$

$$\frac{1}{7}$$

$$\frac{2}{3} + 4$$

$$\frac{2 + 1}{7} + (1 \times 4)$$

$$\frac{(X + Y)^2}{(X + Y)}$$

$$\mathrm{LOG}_3 X = 2$$

NOTE: Parentheses should not have to be silent. For greater clarification, have the student actually say, "Three minus, parenthesis two plus seven parenthesis," for "3 - (2 + 7)."

SAMPLE ACTIVITY:

Have students rewrite or read orally mathematical expressions.

Example: 7 x 2 "seven times two" or "seven multiplied by two"

$3 \times 5$

$4 \overline{)8}$

$16 \div 2$

$4 - 3$

$5 \cdot 3$

$\sqrt{25} + 2$

$\sqrt[3]{27}$

$33/11 \times 3$

$3^3$

SAMPLE ACTIVITY: "Find your Partner"

The teacher makes two sets of cards - one set with mathematical abbreviations and the other with the unabbreviated words. The class can be divided into two equal groups. One group will take the unabbreviated words and the other group will take the abbreviations. After a signal from the teacher, all the students will "scramble" to find their "partners." A person's partner is a person who has a matching card.

SAMPLE ACTIVITY:

Pictures can be drawn to interpret mathematical symbols. Along with the pictures, words can be used for a symbol. This idea can be expressed via the following exercise:

Complete:

| *Symbol* | *Word* | *Picture* |
|---|---|---|
| 3 | three | |
| $\overleftrightarrow{AB}$ | line | |
| ½ | one half | |
| 4 | four | |
| $\angle$ | angle | |

NOTE: In order to use this activity, the teacher would fill in only one of the columns. For variation, only one item from each column could be filled in, and have the students supply the missing information.

SAMPLE ACTIVITY:

Have students describe orally the situation or object expressed by each of the following:

1. 72°
2. -20°
3. +35
4. 0
5. 6'
6. 5' 3"
7. △
8. sin 30°
9. f(x)

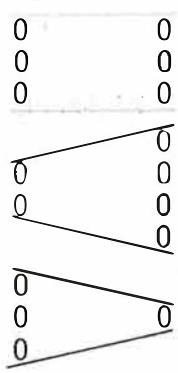
SAMPLE ACTIVITY:

Have students write a division sentence and a short story to represent the drawing:

| | | | | |
|---|---|---|---|---|
| * | * | * | * | * |
| * | * | * | * | * |
| * | * | * | * | * |

SAMPLE ACTIVITY: (For elementary-aged students.)

Have students work with a pictorial development of the meanings of =, <, and >, as well as the words.



3 = 3 "is equal to"

2 < 4 "is less than"

3 > 1 "is greater than"

SAMPLE ACTIVITY:

Have the students discuss the meaning of "_____" in each of the following:

1. $\overline{AB}$
2. 8 - 2
3. $^{-}4$
4. $3.\overline{3}$

## Directionality of Reading Mathematical Symbols

SAMPLE ACTIVITY:

Make a wall chart that contains some symbols that may create a problem in reading. Guide the students through a discussion of the directions they take in reading them. With a colored pen, draw arrows to indicate the directions taken. In addition to drawing arrows, write out, in English, the expression. Discuss this chart and leave it up for further reference.

Examples:

1. $5^2$ — "the square of five"
2. $5^2$ — "five squared"
3. $3x$ — "three times x"
4. $5 : 2$ — "five is to two"
5. $4 \overline{)12}$ — "twelve divided by four"
6. $\frac{1}{2} + 1$ — "one-half plus one"
7. $\begin{array}{r} 6 \\ -2\ 1/3 \\ \hline \boxed{\phantom{00}} \end{array}$ — "six minus two and one-third is equal to what number"
8. $7 - \boxed{\phantom{00}} = 3$ — "what number subtracted from seven leaves three"

SAMPLE ACTIVITY:

Match each word on the left with the meaning of a prefix on the right which could help with identifying the meaning of the word.

| | | | |
|---|---|---|---|
| 1. | quadrilateral | ______ | (a) five |
| 2. | exterior | ______ | (b) around |
| 3. | semicircle | ______ | (c) eight |
| 4. | pentagon | ______ | (d) two |
| 5. | circumference | ______ | (e) four |
| 6. | octagon | ______ | (f) equal |
| 7. | triangle | ______ | (g) half |
| 8. | bisect | ______ | (h) three |
| 9. | equilateral | ______ | (i) outside |

SAMPLE ACTIVITY:

Write a sentence using each of these words in a mathematical way:

| | |
|---|---|
| 1. angle | 6. square |
| 2. degree | 7. set |
| 3. plane | 8. volume |
| 4. odd | 9. solution |
| 5. base | 10. power |

## Reading Mathematical Words

SAMPLE ACTIVITY:

The following is a list of definitions taken from your reading. See if you can solve the puzzle by filling in the correct words.

*Across*

1. The result one gets when one adds.
2. The number one takes from another number.
3. A number to be added to another number or to itself.
5. The result one gets when one divides.
6. {..., $^-3$, $^-2$, $^-1$, 0, +1, +2, +3, ...}

*Down*

1. To take away.
4. $6\overline{)12}$ the number six in this problem.

SAMPLE ACTIVITY:

Allow the students to make up their own definitions, either silly or serious, for new mathematical words.

common denominator: unexciting number

parallelogram: the unit of weight for parallel lines

SAMPLE ACTIVITY:

Have the students name as many words as they can that begin with the following:

| | |
|---|---|
| tri | poly |
| mid | sub |
| in | sum |
| ex | un |
| bi | circum |

then have them identify the ones that are mathematical.

## Comprehension

SAMPLE ACTIVITY:

Students can turn to a page in the math book and answer specific questions asked by the teacher, or the teacher can write a passage and questions about it like the following:

1. What is the main idea?
2. Name several supporting details.
3. How are the main idea and details related?

SAMPLE ACTIVITY:

In addition to "walking students through a direction," a teacher can help the students learn to follow directions by having them rewrite step-by-step directions in their own words.

SAMPLE ACTIVITY:

Have the students follow the directions to add these numbers written with decimals: 1.701 + 20.03. Use this to add: 2.022 + 1.4; 13.002 + 7.0. First, rewrite them so that the decimals are lined up as shown:

| | |
|---|---|
| 1.701<br>+20.03 | This allows those digits to be added that have the same place value. The 20.03 can be rewritten with a "0" in the thousandths' place because there are no thousandths there. |
| 1.701<br>+20.030 | Then add like digits and remember to put the decimal units in proper place in the answer, keeping it lined up with the others. |

Now do the other two addition exercises.

SAMPLE ACTIVITY:

Have students make their own graphs. Small groups can graph different characteristics of the class. The following characteristics could be used:

1. Students' heights
2. Students' weights
3. Color of eyes
4. Types of books liked
5. Types of movies liked
6. Favorite school subjects

SAMPLE ACTIVITY: (For secondary-aged students.)

Some graphs use a symbol that represents a larger number. Pick your own axis for whatever labelling you want, but you must pick an axis for the names John, Pat, and Henry, in that order, either left to right or top to bottom.

1. Who has the most snowballs if names are labelled on the vertical?
2. Who has the most snowballs if names are labelled on the horizontal?
3. Who has three snowballs if names are labelled on the vertical?

0

0 0

0 0 0

Key: 0 = 3 snowballs

SAMPLE ACTIVITY:

Have students read some verbal problems and supply the information asked for in the following chart:

| | What information is needed? | What terms need defining? | What operation is needed? | What is the order of operations? | What variables are given? |
|---|---|---|---|---|---|
| *Problem 1* | | | | | |
| *Problem 2* | | | | | |
| *Problem 3* | | | | | |
| *Problem 4* | | | | | |

SAMPLE ACTIVITY:

The teacher can select a word problem that requires the students to think in sequence. Each sentence should be written on a separate cardboard strip and put in an envelope. The students can then open the envelope, take out the strips, and arrange them in order. If the teacher places numbers on the back of the strips, the students can check their own work. These strips can be made to use with a flannel board or they can be pinned on a cork board. If all students in a class are having problems in this area, the teacher can give each one an envelope, and when a student is finished, he can swap his envelope with someone else.

Suggested problem: Carmen worked as a cashier in a department store one summer. One week she worked long hours. On Monday, Tuesday, and Wednesday, she worked 9 hours per day. On Thursday, she worked 8 hours. On the other two days she worked 5 hours each. At $2.25 per hour, how much money did she earn that week?

SAMPLE ACTIVITY:

List the symbol and the word for each operation.

| | *Symbol for Operation* | *Word for Operation* |
|---|---|---|
| 1. less 7 | - | subtract |
| 2. the difference | | |
| 3. a number increased by 8 | | |
| 4. seven percent of sixty | | |
| 5. twice the amount | | |
| 6. sum of their ages | | |
| 7. decreased by | | |
| 8. 80-foot board cut in half | | |
| 9. seven hours are more than Fred worked | | |
| 10. how many dollars per day? | | |

SAMPLE ACTIVITY:

Have students take some verbal sentences and translate them into mathematical sentences. This activity shows how the translation from English into the language of mathematics involves ordering.

| *Words* | *First Translation* | *Math* |
|---|---|---|
| 1. John is two years older than Mary. | John's age is Mary's age plus two. | J = M + 2 |
| 2. For two hours, he went many miles at a particular speed. | Number of miles is found by multiplying his speed by two. | M = 2 x S |

SAMPLE ACTIVITY:

Give some word problems and ask students whether there is enough information to answer the questions.

SAMPLE ACTIVITY:

Have students indicate, by crossing out, underlining, or circling, the extraneous information in word problems.

SAMPLE ACTIVITY:

When the students have read a passage, involve them in a contest to find all the mathematics words they can in a certain time limit. Then give them a time limit to find additional information. Finally, have them close their books and see how much they remember. As the students orally recall information, the teacher writes this information on the board. Now, have the students go back to their books and check to see how much information was not remembered from the skimming activity.

SAMPLE ACTIVITY:

Assign a page in a mathematics book to be scanned by the students. After they scan it, have them summarize what they have learned. Next, have them read that same material carefully and summarize it. When this is finished, have them compare what they comprehended when they scanned to what they comprehended when they read carefully.

Since symbols hold such a quantity of information, students should be helped to see that it takes longer to read and understand them than it takes to read and understand non-symbolic terms.

# Math Assignment Cards

*The following activities have been introduced by Mrs. Margaret Wilmot of the Reading Center, University of Colorado, and Manchester, Massachusetts. Her collection includes many more subject areas besides reading and mathematics.*

These ideas for learning center activities are shared by Mrs. Margaret Wilmot. It is suggested that each idea be written on a file card. Put materials needed for each activity on the back of each card.

These cards are open-ended and can be used at many different levels. However, they are not designed for totally independent works. One of the functions of the teacher will be to help the intermediate students collect, tabulate, and interpret data so that they can apply some of their skills to situations that are outside the textbook.

My main objective for all the students is that they learn to enjoy mathematics.

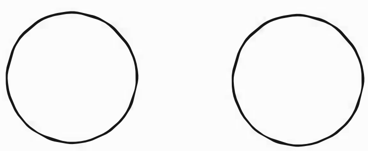
## Activity

CARD 1 - Make a collection of people. Define your collection. How many ways can you rank them? How did you measure them? How can you chart your results? (clue ... paper people)

CARD 2 - Choose six plastic containers. Estimate which holds most, which holds least. Check your estimation. How did you measure? Record your results.

You need: Plastic containers
Sand
Water
Rice
Spoon
Cup

CARD 3 - Take all the children in our class. Sort them into sets. Record your results like this:



boys girls

(clues ...
blue eyes, age, black hair, et cetera)

How many other sets can you find?

You need: Friends

CARD 4 - Take the box of shapes. Sort them into classes. How did you do it? Record your results.

You need: Box of shapes

CARD 5 - The Consumer Council hopes soon to pass a bill which will make it mandatory to send, at a consumer's request, all test results which are quoted in television (or other) advertisements.

Watch TV and the newspapers for advertisements that make extravagant claims. Write to the company concerned and ask for test results.

Try to make tests for comparison (paper towels, babies' diapers, et cetera).

Just for fun, make up some products with crazy test results and act out some commercials.

You need: Paper, envelopes, stamps
TV
Newspapers

CARD 6 - 1. Take a partner out into the school community. Observe some of the services such as collection of lunch money, movement of classes, telephone answering, making dittoes, et cetera.

2. Examine and write down the steps in each process, the personnel needed, et cetera.

3. Check whether the service is working smoothly or not. See if you can work out why.

You need: Tact
Notebook and pencil

CARD 7 - 1. Have your friend tell you the way he goes home from school.

2. Draw a map as he tells.

3. Have another friend turn the map back into words.

4. Check your map with the map of your city.

5. What more do you need to know about making and reading maps?

You need: Two friends
A map of your city

CARD 8 - You have been given $50 to pay for a party or outing for your class.

1. How would you find out what the class wants?

2. How would you spend the money so that most of the class is contented?

You need: Tact
Patience

CARD 9 - Make a school survey. During a specified interval of time, count the number of smiles seen, frowns seen, angry people, sad people, happy people, frightened people. You will need help from friends. Tabulate your data. Compare it with other data. Try to interpret it.

You need: Friends

CARD 10 - 1. Take the air temperature in as many places as you can.

2. Table the results.

3. Try to account for differences, if any.

You need: Thermometer

CARD 11 - 1. Make a collection of people. Define the collection.

2. Categorize them in many different ways, that is, height, age, and speed.

3. What measures did you use? How can you chart your results?

(clue ... paper people)

You need: Friends

CARD 12 - Statistics:

1. Form a trio with friends. Set up a traffic watch for a specific length of time. (Two people write and one spots.)

2. Keep a record of time intervals, total number of vehicles, kind of vehicles, origin of vehicles (state), and direction of vehicles.

3. Tabulate or graph your data.

4. Interpret your findings.

You need: Friends

CARD 13 - 1. Find, draw, and write about six wide and six narrow things.

2. How did you measure them?

3. Which was widest? Which was narrowest?

CARD 14 - 1. Make a collection of cereal boxes. How many ways can you order them? Shortest to tallest or largest to smallest, et cetera.

2. How did you measure?

You need: Cereal boxes<br>Sand

CARD 15 - 1. Find, draw, and write about six tall and six short things.

2. How did you measure them?

3. Which was tallest? Which was shortest?

CARD 16 - Make a collection of books. How many ways can you order them? What did you use to measure?

You need: Books

CARD 17 - 1. Make a collection of bottles. How many ways can you order them?

2. How did you measure them?

You need: Bottles
Water

CARD 18 - Choose four different plastic containers. Describe one to your partner. Use words that tell about shape, size, color, et cetera. Did your partner guess right? Take turns. Write down the best description.

CARD 19 - Estimate, then check your guess... How many children in our class? How many children in first grade? How many children in the school? How many boys in the school?

CARD 20 - 1. Form a trio with friends. Set up a traffic watch for a specific length of time. (Two people write and one spots.)

2. Keep a check list record of time intervals, total number of vehicles, kind of vehicles, state of origin, and direction of vehicles.

3. Tabulate and interpret your data.

4. Compare these results with those taken at a different time of day.

You need: Notebook and pencil
Stop watch

CARD 21 - Make a collection of boxes. How many ways can you rank them? Shortest to tallest or largest to smallest? How did you measure them?

Words you may need: Narrow
Wide
Deep
Shallow

CARD 22 - Measure the first grade play area. Plot the dimensions on squared paper. How much ground area does each first grade child have to play in?

You need: Squared paper

CARD 23 - List five staple foods. Compare the prices of these foods in as many different stores as possible. Don't forget to look at weight as well as price. If there are any differences, try to account for them.

You need: Tact
Notebook and pencil

CARD 24 - Record the length of your shadow at stated times during the day. Compare your results with a friend's. What is the relationship to your height? What is the relationship to time of day? Draw a graph to show this.

You need: String or measuring stick

CARD 25 - Find, draw, and write about six wide and six narrow things. How did you measure them?

| Words you may need: | Narrow | Narrower | Narrowest |
|---|---|---|---|
| | Wide | Wider | Widest |
| | Long | | |
| | Short | | |

CARD 26 - Which weighs most... a paper cup of sand or a paper cup of water? Use other materials to compare. Tabulate and graph your results.

You need: Paper cups
Scales
Materials to weigh

CARD 27 - What happens in one minute? Use the second hand of the clock. One person watches the time. One person performs the activity. One person counts. Tabulate your data. Suggestions... How many different numbers can you write? How many times can you touch the floor? How many times can you cross the playground?

CARD 28 - Make a marble track. Place it at the edge of a piece of carpet. Measure how far the marble rolls. Repeat 99 times. Tabulate your data. Graph the data. Interpret your results. Repeat the experiment with toy cars. Compare the results.

You need: Card for a marble track
Piece of carpet
Toy cars

CARD 29 - Measure the height and weight of twelve people. Tabulate the data. Show the data in a graph. Interpret the graph.

You need: Friends
Metre stick
Scales

CARD 30 - Take three baby food jars. Fill one with paper clips. Fill one with thumbtacks. Fill one with beans. Ask people to guess how many. Tabulate your data. Estimate the number in each jar. How can you check your estimate?

You need: Baby food jars
Friends

CARD 31 - Draw and cut out three squares, four trianges, two circles, and five rectangles. You may use the box of shapes. Make a picture with the shapes. Write about your picture.

# Part V

## Bibliography

# Bibliography

### *Compiled by*


### *Rob Cowie and Tricia Waddell*

*Rob Cowie and Tricia Waddell are graduates from the University of Lethbridge.*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