The Great Rope Robbery

by

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Problem: Two ropes hang 30 centimetres apart in a tall room, 10 metres from floor to ceiling. A rope thief with a sharp knife wants to take as much rope as possible, but while the thief can climb as high as necessary, a jump of more than 330 centimetres results in death. How much rope can the thief steal?

You will find a solution near the end of this article. Before you look at it, I want you to know that I regret posing my favorite problem in the mode of an article at all, preferring to have some control over my audience in its presentation and resolution. But the teaching of problem solving is much more important to me than any single problem, even my favorite one. So I relinquish the opportunity for direct contact with you in order to offer my ideas to a potentially wider audience. Because I believe strongly that to teach problem solving we must be problem solvers ourselves, I hope that before reading any further, you will spend some time working on it. By the way, this problem, like many, is best worked on for short periods of time, allowing the brain to rest over longer in-between periods.

I have posed this problem to young children, to young adults, and to teachers and other adults. With children and adults alike, my objective is the same: to provide an experience that is at once enriching, satisfying, stimulating, and pleasurable. With teachers I have an additional objective: to provide a model for the teaching of problem solving. I am using Bob Wirtz's concept that a problem poses a question which the solver understands, but knows neither an answer nor an algorithm for finding an answer. However, the solver does have enough information to find an answer with a small amount of effort.

Having posed the problem to a class, I permit a short time for discussion. In this way I can ascertain that it is understood and is being taken seriously. Often, on first hearing the problem, many people react by looking for some kind of gimmick or trick in the solution. (Indeed, when I first heard the problem from a friend, I experienced such a reaction. My friend told me the problem in a way that indicated that the thief dies if a jump is required of more than one-third the length of the rope. He did not mention the possible gap from the end of the rope to the floor. So I interpreted this to mean the thief could survive a jump of one-third the length of the rope plus whatever distance remained to the floor. Thus by cutting off both ropes at the ceiling, my thief could make a full legitimate jump to the floor. But my friend assured me I had misinterpreted him. It was then I set about to solve the problem as it was intended to be solved.) Once I take care of these initial responses and establish interest, I like to leave the problem and go on to another activity. I want my audience to go on to the activity as well, so I make sure it is exciting enough to take their minds away from the rope thief.

This is an important step in the problem-solving process: that is, leaving the problem, permitting the analytic side of the brain to rest, while the synthesizing side can operate on a subconscious level trying to obtain a total picture. A back-and-forth process of focusing on the problem and leaving it, returning and leaving again, should be repeated over a period of time. With fifth graders I would not consider the problem more than once or twice in a week. With adults, two or three times in a day is appropriate. As in real life, problems are not solved in a moment. Different minds work in different ways. Teachers who do not recognize the value of the subconscious in problem solving may overkill a problem and deny many students an opportunity to improve their sense of their own skills.

A word of caution, however: we have to be especially careful when we talk about a new problem at the very end of a class period. A young friend of mine was terribly frustrated when confronted with a homework problem whose solution required problem-solving skills and was due the next day. The child spent many frustrating hours with no success. When giving the assignment, my friend's teacher should have warned and urged the class not to spend more than five or ten minutes on it. In class the next day, an equal amount of time could have been spent discussing the difficulties and pitfalls encountered. Then the teacher should have requested an additional five or ten minutes consideration of the problem at home that evening. In this way the children learn more and more about the problem and about problem solving. They learn to savor both the difficulties of the problem and the nuances of the problem-solving process. And they gain an appreciation for the mind's intricate modes of operation and for their own ability to create and comprehend.

Now let's get back to our rope thief. It is the third day the problem is being discussed. (This may be the third week, but I do not recommend more than one week between discussions; as little as a day may be appropriate.) On day one, the problem was introduced and discussed only to the point of ascertaining that everyone understood it. On day two, solutions were presented and found wanting. These are the solutions that involve gimmicks like ladders, ceiling doors, windows. On day three, almost anything can happen.

With my own group of 5th-6th graders, on day three we spent only about 5 or 10 minutes of a one-hundred-minute math period with the problem. By this time everyone understood how the thief could easily obtain 1330 centimetres. (Climb one rope to the top, cut off the other rope at the top, climb down to 330 centimetres, cut, and jump to the floor.) And I said, "That's very good. Can you get any more than 1330 centimetres? No? Are you sure?"

If I had thought they needed more encouragement, I would have told them that I know a way for the thief to obtain more than 1330 centimetres, but I would not have told them how much more, nor would I have indicated any method. I prefer not to say any of this hoping that my stance of uncertainty will by itself accomplish the same thing.

Not much more happened on day three. We talked again about the difficulties of the problem; how to get 1330 centimetres, and how impossible it appeared to be to get more than 1330 centimetres unless the thief were to become a martyr for the rope (i.e. climb, cut, jump from ceiling, and die).

But the next week, a breakthrough occurred. When I relate this incident in my workshops, where I am in control of the problem-solving atmosphere, I tell my workshop audience that when I came into the classroom, Liz, one of my students, said, "If only there was a ______, I know a way for the thief to get more rope."

The word will be filled in momentarily--see HINT below. But, again, I want to give the reader a chance to stop and play with the problem some more. Already the quote above, even with a word omitted, is an additional clue. I would even like to urge you to consider posing the question to your class without knowing the solution. If you don't mind acknowledging your own uncertainties to your class, a satisfying and interesting discussion might follow.

My response to Liz was, "How much more rope could you get?"
"All of it."
"Oh? How?"

Now a discussion ensued involving a good part of the class. The student who first made the remark gave an explanation, but initial explanations are often unclear, and other students entered the discussion as their own understanding grew and in response to my remarks like, "You seem to have an idea, but I think you could express it better. Does anyone understand what Liz is trying to say?"

The discussion on this day lasted much longer than the preceding ones. Before coming out with a punch line I had in mind, I wanted to make certain that just about everyone in the class understood how Liz's thief could get the whole rope. By asking for repetition for the sake of clarifying, by asking who understood how this proposed device enabled the rope thief to obtain the whole rope, and by asking for omitted details to be filled in, I was able to determine the extent to which the class understood the proposed solution. And I was able to keep their interest as well. Even then, the entire discussion did not last more than fifteen minutes.

When it seemed to me that everyone in the class did understand how the rope thief could steal all two hundred feet of rope if there were a ______, I was ready for my punch line.

"That's a neat solution. If there were a ______, I can see how the thief could get all the rope. Too bad there isn't. We don't have any more time today to discuss this problem." (Many groans.)

By the next week, several students had solved the problem.

It is not totally clear to me what happened in the minds of those students during that last week. But certainly the discussion we had had was an important step in the problem-solving process. The effect of Liz's question was to make the problem easier. She changed the problem to a simpler, related problem. Once this easier problem has been solved, the original problem, too, is changed to a new one. Now the problem becomes: Is there some way to obtain or produce the desired device under the constraints of the original problem?

HINTS AND SOLUTION

First Hint

The question Liz asked that fourth week was, "How are the ropes attached to the ceiling?"

"With very strong nails. Why do you ask?"
"Well, if only there were a hook, I can figure out a way for the thief to get all the rope."
"Tell me."

Second Hint

"The thief climbs up one rope, grabs hold of the hook with one hand and cuts both ropes loose with the other, but does not let them drop. While holding onto the hook the thief ties the two ropes together to form a 20 metre length, and then slips the two ropes over the hook so that the knot is on one side of the hook.

Now the thief can climb down to the floor while holding onto both ropes. When the thief reaches the floor, the rope hanging on the side with the knot is pulled. The other rope is pulled up and over the hook."

"That's a very nice solution. It's too bad there is no hook."

Solution

The Fifth Week. "I know how the rope thief can get almost all of the rope. All the thief has to do is use a small part of the rope to make a hook. For example, the thief could climb up one rope to the ceiling, cut the other rope leaving ten centimetres. Use that ten centimetres of hanging rope to tie a loop. The loop serves the same purpose as a hook. Now hanging onto the looped ten centimetres of rope, the thief cuts off the first rope and ties together the two loose pieces of rope to form a single piece 1990 centimetres long. The thief slips one end through the loop until the knot reaches the loop. Now the two ropes are hanging down from the loop as from the hook and the solution proceeds as before."

In this way 1990 centimetres of rope can be obtained. Of course, the total amount that the thief can steal is $200 - x$, where x is the amount of rope it takes to form a loop.

Comments

The solution presented is not unique. It is not even the one I came up with myself, but it is the one I hear most frequently. In a large group, a few individuals usually think of making a loop right away. However, it is important to keep them from saying anything aloud, thus destroying the opportunity for the others to create for themselves. At the same time these people should be credited with their ingenuity. Both objectives can be accomplished by asking the group to whisper solutions to you or hold them until the end of the meeting.

If you do give others the opportunity to create their own solutions, you will be surprised by the many different ideas you will hear. This may help you to become more free in your own problem-solving situations and, as a result, be a better teacher of problem solving.

FROM THE EDITOR

"The Great Rope Robbery " is reminiscent of the following problem adapted from the writings of Norman R. F. Maier:

In a large room, two ropes hang from the ceiling at a considerable distance from one another. One has a small ring on its free end. The other has a small hook on its free end. In the room are a ladder, a chair, a table, a hammer, and a book. If the ropes are too far apart to simply walk from one to the other while holding the former, how might you connect the two ropes without using any unmentioned aids? Will your method always work? How is your solution affected by shortening the lengths of the ropes?

Is this the only way?

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