Issues in Mathematical Problem-Solving Research*

by

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During the seventies mathematics education researchers devoted more attention to problem solving than any other topic in the mathematics curriculum and there is every indication that this condition will exist for some time. While there is some evidence that problem-solving research is beginning to be investigated in a systematic way, it is difficult to synthesize the myriad of studies due to such factors as lack of agreement on what constitutes problem solving, how performance should be measured, what tasks should be used, and what the key variables influencing behavior are. Indeed, the nature of mathematical problem solving appears to a certain extent to be so complex and subtle as to defy description and analysis. However, there are some factors (variables) associated with problem solving that are inextricably linked together. These factors can be classified into four categories with each category involving many parts. It is immediately evident that these categories are not disjoint; in fact they are so closely related that it often is difficult to determine to which category a particular factor belongs. The four categories are:

I. Subject Factors - what the individual brings to a problem.
II. Task Factors - factors associated with the nature of the problem.
III. Process Factors - the overt and covert behavior of the individual during problem solving.
IV. Environment Factors - features of the task environment which are external to the problem and the problem solver; instructional factors comprise an important class of factors.

Categories I and III are so closely related that some further clarification is warranted. Variables within the Subject Factors category are associated with individual traits and background (e.g., previous mathematical background, age, sex, cognitive style, familiarity with certain problem types). Variables in this category serve to characterize the subject. Category III variables (Process Factors) relate directly to the behavior of the individual during problem solving. The manner in which the problem solver

Issue III. Characteristics of problem solvers greatly affect behavior and consequently severly limit generalizability of results. The kinds of subjects to use in problem-solving research is a topic of much discussion. For example, while knowledge about the processes good problem solvers use is clearly important, it is less clear that average ability problem solvers can be taught to use these processes. Should subjects used in mathematical problem-solving research be "mathematically talented" or of "average" ability?

Instruction-Related Issues.

There is every reason to believe a substantial portion of future problem-solving research will focus on instruction. For this reason it is appropriate to point out the key issues directly associated with instruction.

Issue IV. There is little agreement regarding how best to improve problem-solving performance beyond the obvious fact that attempting to solve problems is a necessary ingredient. Common points of view regarding problem-solving instruction include:

a. Having students solve many problems - no direct instruction;
b. Teaching unitary skills (tool skills);
c. Teaching heuristic strategies;
d. Modelling good problem-solving behavior and having students imitate this behavior;
e. Some combination of the above.

Issue V. In addition to a lack of consensus regarding the best ways to enhance problem solving, there is no accord about what should be the nature of problem-solving improvement. Some researchers interested in problem-solving instruction have focused on the improvement of students' abilities to use particular strategies or skills, while others have considered improvement only in terms of an increase in the number of correct solutions. Also, in many cases no attention has been given to whether newly acquired facility in solving a particular type of problem transfers to solving a different type of problem. Indeed, the extent to which various types of transfer of training should be expected is an open question.

Issue VI. The extent of instructional treatments in recent mathematical problem-solving research varies from about one week to several months with relatively short treatments being the most common. Treatments should be extensive enough to allow not only for full explication of ideas and procedures, but also to provide ample opportunity for students to practise the procedures being taught.

Research Methodology Issue.

There is a single issue related to research methodology. Typically, methodological issues become less important when a sound theoretical basis guides the conduct of inquiry. However, the present lack of adequate problem-solving theories makes issue VII a current, although possibly

short-term, concern. This issue is neither the unique domain of problem-solving researchers, nor of the same level of importance as the first six issues but it is important enough to warrant serious attention.

Issue VII. There are no generally accepted methods or instruments for measuring performance or observing behavior during problem solving which are clearly reliable and valid. Thus, the kind of instrumentation which is appropriate for a particular purpose remains an issue. The most popular instruments are of two types: paper-and-pencil tests and protocol analysis based on "thinking aloud" or retrospection. Each of these types has serious weaknesses. Paper-and-pencil tests are notoriously unreliable measures of problem-solving processes and often use only routine problems. Protocol analysis suffers equally serious limitations. Forcing the problem solver to think aloud during problem solving may have a deleterious effect on performance and the problem solver typically is unable to articulate all, or even the most important, thought processes. Retrospective analysis is often criticized for the unreliability of accounts of behavior, including all the cognitive processes used, which are reconstructed by a problem solver after an attempt to solve a problem. Should more or less emphasis be given to the development of paper-and-pencil tests? Should more or less emphasis be placed on the development of procedures for collecting and analyzing problem-solving protocols?

The individual researcher must make personal decisions regarding some, or all, of these issues before undertaking problem-solving research. At the same time the problem-solving research community as a whole should give overt attention to discussion of the controversies involved with these issues. It is only through the open exchange of ideas and points of view that progress can be made toward building a large and stable body of knowledge about the nature of mathematical problem solving.

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