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diet of questions whose solution is simple once the teacher reveals the "trick," serves to confirm these feelings.

To a math teacher or a strong math student, a question whose solution goes counter to intuition can be a delight. To a math-anxious student, mat -- in general -- tends to go against intuition. Some textbooks have made overt attempts to discourage students from trusting or using their intuitio My old text for introduction to geometry began with a series of optical illusions. The message was "don't trust your eyes because things are often not what they seem." Geometry was not supposed to make sense. Until you could establish something with a rigorous deductive proof, it was best not believe it or use it.

If we are trying to teach children that making reasonable guesses and then testing the conjectures is a legitimate problem-solving technique, the we must also teach that math is basically reasonable and intuition can be trusted.

Questions which require an "intuitive leap" can be most satisfying for students who persevere to the point where the breakthrough occurs. The majority of students, however, probably lack whatever it takes to persevere that point. The result will be one more wrong answer to a question which "tricked" them.

For example:

> A visitor arrives at a hotel seeking accommodation for a week. He has no cash but the hotel owner agrees to accept one tiny gold ring as payment for each night the visitor stays in the hotel. The problem is that the seven gold rings are linked together in a chain. The visitor doesn't trust the proprietor enough to make seven day's payments in advance and the proprietor doesn't trust the visitor to withhold payment until the end of the week. The visitor, therefore, must cut links of the chain so that payment of one link can be made on a daily basis. The question is, what is the least number of rings the visitor would have to cut so that he could make payments for his accommodation on a daily basis?

Very little thought will show that it can be achieved with six cuts. little more thinking and most students will arrive at three cuts. As this a considerable improvement over six, and there is no way of knowing whether this is the correct solution or not, most students will probably be satisfi with this. A few students may persevere to the realization that the proprietor can use rings he already has to make change and, in fact, the payments can all be made after cutting just one ring (third from one end). Will the majority who got an answer of three be amused or stimulated by thi clever solution or will they feel that they have been fooled again and be l likely to want to try another one? Questions like this, whose solution is maximum or minimum, don't provide a method for determining whether a soluti is correct or not. This lack of the possibility for verification can resul in a question being abandoned before a correct solution is found, or in a l of time being spent on an unproductive line of reasoning.

A problem which recently appeared on a Canadian math contest paper goes as follows:

> You are given one hundred coins in ten piles of ten coins each. Ninety of the coins are genuine and weigh exactly one gram each. All of the coins in one of the ten piles are counterfeit. These coins look and feel like the genuine coins but each one of them is .1 grams heavier than a genuine coin. The question is, if you are given a miniature bathroom type scale which can measure correct to .1 grams, what is the minimum number of weighings you could make to identify with certainty the counterfeit pile of coins?

The inclusion of this question on the contest paper was unfortunate for a couple of reasons. First, it was not an original problem and had appeared in problem books previously. Some test writers may have encountered the question before and could acquire full marks with almost no expenditure in time or effort. Those seeing it for the first time could spend a considerable amount of time creating any number of schemes for weighing the coins, without knowing when to quit, as they could not know whether their solution was a minimum. If this work was done in their heads or on rough paper (with no breakthrough to the correct solution) they could receive no marks for some high quality thinking.

(For those who have not seen this problem solved, the correct solution is a single weighing. This can be accomplished by placing one coin from the first pile, two from the second, etc., and noting the discrepancy between what the scale should read and what it does read. This multiple of .1 grams will reveal the counterfeit pile of coins.)

It would be a rather different question if it asked, "How is it possible to identify the couterfeit pile with a single weighing?" Then a person doing the problem would know when a correct solution had been found.

Problems like these need to be presented to those students who have the tenacity and abilities to arrive at correct solutions or learn from their failures. If handled appropriately they can be valuable for the majority of students, provided they are given credit for effort and headway in the process of solving the problem, not just for getting the correct solution. This might be achieved by presenting a problem like the coins question, limiting the time spent by students on the problem, and discussing the progress towards its solution, in class, for a few minutes each day. At first any procedures which revealed the counterfeit coins would be considered as good solutions. Then strategies for identifying the coins in fewer weighings could be discovered and discussed. Finally, with feedback and some hints, the "best" solution might be discovered. Such a procedure could reduce a sense of having failed in many students and provide insights into the problem-solving methods.

Some years ago I ran off class sets of mazes for students to work on when they had finished their work. The initial reaction from almost the entire class was "no thanks." This changed when one of the weaker students picked up a maze one day and managed to work through it in two or three minutes.

He was rather surprised and blurted out, "Hey! I got it - it's easy!" The mazes were popular from that day on and students often asked if they could take some to do at home.

The significant thing seems to be that they were afraid to try until it seemed clear that they would succeed. Many stronger students, for whom the mazes could not be considered as challenging, seemed pleased to attempt and complete the mazes.

Many problems will yield to a certain amount of perseverance on the part of almost any student. The following question, which I have used with a number of classes, is usually solved by anybody who keeps at it for a period of time. A certain amount of luck also helps and it is often one of the weaker students who arrives at a correct solution first.

A "ruler" is designed from a blank strip of wood exactly 13 centimetres long. You want to be able to measure integral values from 1 cm to 13 cm with this ruler without moving the ruler and with only four marks on the ruler at four points of your choosing. Where should you put the marks?

If we are to teach children to be effective problem solvers we will have to develop in them some confidence in their ability to solve problems and a predisposition to attempt to solve problems in the first place.

Confidence can be built through exposure to problems which are easy to solve or which require only perseverance and effort rather than the brilliant "aha" which usually eludes the majority of students.

Questions which by their nature have solutions which are not verifiable should be used in such a way that feedback can be injected at appropriate times. Similarly, questions with the possibility for a lot of work being spent chasing down blind alleys should be monitored to reduce frustrations. Trick questions should be used with discretion.

There are hundreds of problems available to teachers from a variety of sources. A judicious choice in the problems we present and the way in which we present them can help in attaining the goal of producing better problem solvers.