An Instructional System for Mathematical Problem Solving

by

Randall I. Charles
Woods County Schools

Considerable attention has been given to the topic of mathematical problem solving. Indeed, most teachers now know that one of the most important goals of mathematics education is to develop in each child the ability to solve mathematical problems. Although most teachers are aware of the important role problem-solving experiences play in mathematics education, teachers still have a variety of questions concerning how to develop and implement a mathematical problem-solving program.

Most of the questions teachers have regarding the development and implementation of a problem-solving program are concerned with two issues.

- How should one organize a mathematics program to include problem-solving experiences?
- What specifically should be done to develop students' mathematical problem-solving abilities?

The purpose of this paper is to describe an instructional system for mathematical problem solving that was designed to answer these questions. The instructional system described here was designed for and used with average and high-achieving students at the junior high school level. At the end of the paper some ideas are provided concerning ways in which the instructional system might be modified for low-achieving junior high school students.

Components of the Instructional System

There are two components to the instructional system described in this paper: organizing for instruction and teaching strategy. The organizing for instruction component provides answers to five questions.

(1) What types of problems should be used?
(2) What should be the minimum time allotment for problem solving?
(3) What grouping patterns should be used for problem-solving activities?
(4) What material is needed for problem solving?
(5) How should students be evaluated?

The teaching strategy component identifies specific behaviors one can use in the classroom to help develop students' attitudes and abilities related to mathematical problem solving.

Organizing for Instruction

Problem-solving experiences are not presently an integral part of most instructional programs. Therefore, teachers interested in providing students with problem-solving experiences must make several difficult decisions concerning the structure of a problem-solving program. This section provides one answer to each of the five questions given above related to "organizing for instruction." Although it is clear other answers could be given to each of these questions, it is also clear that each teacher must at some time provide answers to at least these questions if problem solving is to become a significant part of one's instructional program.

Question #1: What types of problems should be used?

There are many different types of mathematical problems that could be used in a problem-solving program. However, before one begins to select particular types of problems it is imperative that a position be established regarding the nature of a mathematical problem and problem solving. In this paper, a mathematical problem is considered to be a mathematical situation in which an individual or a group is called upon to perform a task for which that individual or group has no readily accessible procedure for determining a solution. Problem solving, as used in this paper, refers to the coordination of previous experience, knowledge, and intuition in an effort to determine an outcome to a situation for which a procedure for determining a solution is not known (see Lester, 1978).

Two types of mathematical problems were selected for this instructional system: process problems and translation problems. Below are examples of each:

Examples - Process Problems

> The tennis club was planning a tournament for its club with 8 members. Each member was to play every other member. How many matches need to be scheduled?
>
> It takes 1,140 pieces of type to number the pages of a book. Each piece of type is used only once. How many pages are in the book?

Examples - Translation Problems

> Fourteen bears each ate 3.4 kg of meat. After all the bears had finished eating, 7.4 kg were left over. How much meat was there in all?
>
> A record costs $5.98. How much do four records plus a sales tax of 3% total?

Process problems emphasize a three-step process: (a) understanding the problem, (b) developing and carrying out a solution strategy, and (c) evaluating the solution. Translation problems, frequently called "textbook word problems," emphasize translating from a real world situation to a mathematical sentence.

Some have argued that translation problems do not belong in a "good" problem-solving program. However, others have argued that translation problems may serve a purpose when they are used in particular ways (e.g., see Charles, 1981). Most textbook word problems are matched to the concepts and skills involved in their solution. That is, if a textbook problem is located at the end of a lesson or chapter, the concepts and skills involved in the solution process are generally developed in the same lesson or chapter. When textbook problems are presented in matched situations, there is little difficulty identifying the concepts embodied by the story situations.

Typical textbook problems are included in the system described in this paper. However, in this system these problems are only used in non-matched situations, meaning that the concepts and skills involved in the solution process are ones the students have not worked with for at least two weeks. Experience with non-matched problems may promote the types of behaviors identified in the definition of problem solving given earlier. Furthermore, experiences with non-matched problems may also promote greater understanding of the concepts embodied by story situations.

Question #2: What should be the minimum time allotment for problem solving?

Regardless of the amount of time one has available for mathematics instruction, a commitment must be made establishing problem solving as a significant part of the curriculum. Table 1 suggests possible minimum time guidelines for a junior high school program. These guidelines were selected based on a 60 minutes per day allotment for mathematics and they suggest that, as a minimum, approximately one-third of one's time teaching mathematics should be devoted to problem-solving experiences. These guidelines also suggest that experiences with process problems should dominate a problem-solving program. Furthermore, these guidelines are minimum and extensions of the time devoted to problem solving should be given to process problems, not translation problems. Finally, experience using these guidelines shows that allotting one-third of one's program to problem solving is not excessive. In fact it is quite possible that considerably more time could be given to problem-solving activities, particularly as one gets further along in the school year, without detrimental effects on the quantity and quality of other instruction.

Table 1

Possible Minimum Time Allotments for Problem Solving

Period	Content	Frequency	Length
Sept. (1 week)	translation problems	4 days/wk.	5-10 min./day

Sept. (3 wks.)	process problems	1 day/wk.	30-40 min./day
	translation problems	3 days/wk.	5-10 min./day
Oct.-June (32 wks.)	process problems	2 days/wk.	30-40 min./day
	translation problems	2 days/wk.	5-10 min./day

Question #3: What grouping patterns should be used for problem-solving activities?

An instructional program for mathematical problem solving should include individual, small-group, and whole-class experiences. Each of these grouping patterns emphasizes particular problem-solving behaviors not emphasized by the others. For example, one of the behaviors required of a student in a small group situation that is not involved in individual work is the need to comprehend, evaluate, and act upon ideas and questions raised by others. The process of dealing with the ideas and questions of other students, influences and may facilitate one's own thinking and progress toward the solution of a problem.

Following are four guidelines for selecting grouping patterns.

- use small groups (3-4) for most in-class work with process problems
- use individual work for most in-class work with translation problems
- encourage individual work with process problems via homework.

  Some teachers have found it useful to give a process problem for homework on Thursday and discuss the students' work on that problem the following Tuesday. Furthermore, the homework problem is frequently an extension of the problem attempted on Thursday or a problem whose solution involves strategies similar to those used in the problem attempted on Thursday.

- include whole-class activities as part of your teaching strategy.

Question #4: What material is needed to teach problem solving?

A collection of "good" mathematical problems is a necessary ingredient for a problem-solving program. Of course, it is not an easy task to identify "good" mathematical problems. Although experience using a problem may be the best judge of quality, there are some characteristics one should attempt to manifest in the set of problems used for instruction. For process problems there are at least four desirable characteristics. They should:

1. interest students; problems may or may not be from the real world,

2. involve relatively little formal math, that is, the mathematical content needed to solve the problem should be familiar to students,

3. not be able to be solved solely by using a computational algorithm (at least not one known to the students), and

4. be able to be solved using more than one strategy.

For both translation and process problems, characteristics like the following should be considered when organizing sets of problems.

1. content. The problem set should reflect a variety of mathematical content (e.g., geometric as well as numeric).

2. logical structure. The logical structure of problems should be varied. Logical structure refers to factors such as extraneous or insufficient data, the number of conditions in the problem, and the number of steps to solution.

3. problem setting. Problems should be presented in a variety of settings. Two factors that should be considered are real world versus "pure" mathematical settings and the existence or nonexistence of pictures accompanying problem statements.

4. reading. Reading-related factors should be varied in the problem set. Two factors to be considered are the amount of reading in problem statements and the existence of special words and symbols (e.g., "two" versus 2).

In addition to good sets of problems, the teaching strategy one selects for problem solving can establish a need for particular instructional materials. For the teaching strategy described in this paper, the problem-solving bulletin board shown in Figure 1 is required. The ways in which this bulletin board are used for instruction are explained later.

Problem-Solving Strategies

Helping Strategies	General Strategies
1. Read the problem again.	1. Look for a pattern, generalize.

2. Look for key phrases.	2. Guess and check.
3. Write what you know.	3. Work backwards.
4. Make an organized list, table, or chart.	4. Write an equation.
5. Use a picture, objects, or graph.	5. Use deduction.
6. Experiment or act-out the problem.	
7. Use simpler numbers.	
8. Solve a simpler problem.	

Figure 1: Problem-solving strategies bulletin board.

Question #5: How should students be evaluated?

This question is the most difficult one to answer. One reason it is difficult is that the realities of the classroom and the goals for teaching problem solving are not always compatible. For most students, all of their experiences in mathematics have at some time been assessed through some form of a written test. In turn, scores on a collection of written tests are transformed into a final mark for the mathematics class. Thus, for many students the importance of an activity is determined by the contribution of that activity to one's mark in mathematics.

There are two essential goals for teaching mathematical problem solving:

1. to improve a student's willingness to attempt to solve mathematical problems and to persevere in those attempts when success is not immediate, and

2. to develop a student's ability to select and utilize problem-solving strategies.

Both of these goals are not presently and perhaps may never be subject to assessment through traditional written test formats. Thus, a conflict exists between the goals of a problem-solving program and the assessment expectations of many students.

The most desirable way to deal with this conflict is to change pupils' assessment expectations. Although this is difficult the results are worth the effort. One way pupils' assessment expectations may be changed is through the use of a teaching strategy that emphasizes the goals given above. A teaching strategy that promotes the attainment of these goals is one that enables each student to find some success in most problem-solving experiences. For many students, the enjoyment provided by successful problem-solving experiences is sufficient to allay achievement expectations. The teaching strategy developed in the next section of this paper has that potential.

Another technique useful for dealing with assessment expectations is to implement some type of an accountability system. For example, teachers who assign a process problem for homework frequently verify efforts to solve the problem by collecting students' "work." The assessment in this case focuses on whether a student did or did not show evidence of attempting to solve the problem. The use of an accountability system enables one to satisfy the assessment expectations of students while focusing on the attainment of the goals for teaching problem solving.

Finally, it is important to note that the nonexistence of written assessment instruments related to the goals for teaching problem solving does not mean that one should not attempt to assess these goals. Rather, some form of a subjective but systematic process for recording observations of students' growth should be established. Individual student interviews and analyses of written work can be combined to help assess progress toward the goals for teaching problem solving.

Teaching Strategy

Fundamental to the development of a teaching strategy are the goals one has for teaching mathematical problem solving. Two essential goals for a problem-solving program were identified above. The teaching strategy described in this paper was designed to promote the attainment of these goals.

There are two related parts to the teaching strategy described here: the classroom climate and teaching actions. The classroom climate component identifies behaviors a teacher should model to develop a classroom atmosphere conducive to mathematical problem solving. The teaching actions component identifies some specific behaviors to use to help develop a student's abilities to select and utilize problem-solving strategies.

The Classroom Climate

It is absolutely essential that the classroom atmosphere be conducive to mathematical problem solving. In fact, experience suggests that the classroom climate is so important in the development of a successful problem-solving program that establishing a conducive atmosphere for problem solving should be the most important goal of all problem-solving experiences at the beginning of the school year. This is particularly true if students have not had prior experience in a mathematical problem-solving program.

Most students will have had some experience with translation problems by the time they are in junior high school. Unfortunately, by this time many students have developed a strong dislike for typical textbook problems. One reason students develop a dislike for these problems may be the result of having too few experiences with them throughout the elementary school program. Many teachers have found that providing junior high school students with frequent non-threatening experiences solving non-matched problems is a useful strategy for changing attitudes toward textbook word problems.

Process problems dominate in the instructional system developed in this paper. Process problems are quite unlike translation problems and most students have not had any exposure to process problems. For these reasons, the remainder of this section will focus on ways to develop a classroom atmosphere conducive to work with process problems. However, most of the ideas are also applicable to work with non-matched translation problems.

One must anticipate and develop ways for dealing with two probable consequences when students are introduced to process problems. First, many students will be reluctant to pursue their ideas when they are not confident these ideas will lead to a correct solution. Students sometimes reveal this situation through a comment like "I don't know what to do" or by simply not doing anything. The second consequence is that many students may not be able to obtain a correct solution or any solution to particular problems. Students frequently reveal this situation by not wanting to share their solution attempts with others, including the teacher.

Both of these consequences need not be detrimental to a student's work with process problems. There are at least two behaviors a teacher can use to deal with these consequences.

1. Encourage students to explore <u>any</u> ideas (i.e., strategies) that may help them understand and/or solve a mathematical problem and do not censor ideas generated by students.

2. Recognize and reinforce different kinds of excellence.

Regrettably, most students "new" to a problem-solving program believe there is one and only one way to solve every mathematics problem. Students must realize this is not true and process problems are an excellent vehicle for doing this. In those initial experiences with process problems, when students seemingly don't know where to start, discussions with students and questions should be used to elicit any ideas that might be explored. One idea from a group is not enough. Continued discussions and questions should be used to illustrate that different ideas are acceptable and desirable in problem-solving situations. There are, of course, situations in which students really don't know how to start work on a problem. The teaching actions discussed shortly provide one way to deal with this.

Concomitant to encouraging and eliciting ideas from students is avoiding censorship of students' ideas. When working with process problems, it is quite likely that students will generate ideas that, with great certainty, will not lead to a correct solution. At these times, it is very important that students not be stopped from pursuing their ideas. There are two reasons why censorship should be avoided. First, it frequently happens that ideas that appear "unproductive" to others may indeed be productive to the user. Another and perhaps the most important reason is that a classroom atmosphere that is conducive to problem solving is one in which students are keenly aware of the freedom as well as the desirability of exploring any strategies for understanding and/or solving mathematical problems.

Most students "new" to a problem-solving program also believe <u>the</u> goal of all problem-solving experiences is to obtain a correct solution. Because of this, students are frequently frustrated when they do not obtain a solution to a process problem. For <u>all</u> experiences with process problems and particularly for those initial experiences, the emphasis of the problem-solving activities should be on behaviors other than obtaining a correct solution. The goals for teaching problem solving suggest at least three behaviors that should be continually recognized and reinforced in problem-solving situations: (a) a student's willingness to start work on a problem, (b) a student's perseverance in attempting to solve a problem, and (c) the selecting of a strategy for solving a problem, regardless of whether that strategy did or did not lead to a correct solution.

Teaching Actions

The teaching actions selected for problem solving must be consistent with one's view of how problem solving is learned (see Bourne, Ekstrand, and Dominowski (1971). The teaching actions described here are based on an information-processing point of view. In this view of problem solving, the task of the problem solver is to select from a variety of "alternatives" those that will move him/her toward a solution. The "alternatives" confronting a problem solver lie in areas such as the different problem-solving strategies one can use (e.g., drawing a picture, working backwards, etc.), the various intuitions one generates in the process of solving a problem, and the variety of previous experiences one brings to a problem-solving situation. The primary goal of the teaching actions described here is to develop the ability to search among and evaluate alternatives when solving mathematical problems. Concomitant to this goal are the needs to make students aware of strategies useful in solving mathematical problems and to develop students' abilities to utilize these strategies.

The teaching actions developed here are for work with process problems, not translation problems. Although some of the ideas in this section may be appropriate for translation problems, experience suggests that at the junior high school level average and high-achieving students need little "teaching" to develop their abilities related to translation problems.

Learning how to solve mathematical problems is quite different from other types of learning one encounters in the study of mathematics. For example, in concept learning there is a particular kind of subject matter one is concerned with, namely, concepts. Similarly, in skill learning, a mathematical skill is the object of instruction. Problem solving, on the other hand, is not concerned with a particular kind of subject matter but instead is concerned with a process. This difference between problem solving and other types of learning in mathematics has implications for the development and evaluation of effective teaching actions.

One implication of this difference is that teaching actions for problem solving should change over time, whereas teaching actions for particular kinds of subject matter should remain fixed. For example, if a teacher develops 20 concepts over some period of time, the teaching actions used to develop the first concept should be similar to the teaching actions used to develop the last concept. On the other hand, if the teaching actions used for initial problem-solving experiences are successful at developing students' abilities related to the problem-solving process, then the teaching actions should change as students' abilities related to that process improve. In the discussion that follows, particular attention is given to the ways in which teaching actions should change as students' abilities develop.

It was suggested earlier that small groups of 3 to 4 students be used for most in-class experiences with process problems. The time allotted for work with process problems (at least 30-40 minutes per session) can be divided into three sections according to the work in small groups. The three sections are simply BEFORE students form their groups and start work on a problem, DURING the time students are in small groups working on a problem, and AFTER students have completed work on a problem (for whatever reason) and have returned to a whole-class structure. In each of these time divisions there are particular teaching actions one should use. Table 2 shows the teaching actions one should use in the "middle" of a classes' development of their problem-solving abilities. The teaching actions shown in this table are discussed first, followed by a discussion of ways in which these actions should be modified for students' initial experiences with process problems and ways in which these actions should change as students' abilities to work with process problems continue to develop.

Table 2

Teaching Actions for Process Problems

BEFORE

1. Read the problem to the class or have a student read the problem.

2. Ask if there are words or phrases they do not understand; provide explanations as needed. (Note: Be careful that one's explanations do not suggest a solution strategy.)

DURING

1. Question students and observe their work to identify where the students are in the problem-solving process.

   a. They are trying to understand the problem.
   b. They are developing or carrying out a solution strategy.
   c. They have obtained an answer.

2. If necessary, refer students to the problem-solving strategies bulletin board and encourage them to select and implement a strategy or strategies.

3. If necessary, provide hints and questions.

4. For early finishers, give an extension to the problem or have the students make up an extension to the problem.

AFTER

1. Show and discuss strategies used by the class which did and did not lead to a correct solution.

2. Name the strategies used by the students and draw their attention to the names of the strategies on the problem-solving strategies bulletin board.

3. If possible, relate the problem to previous problems and discuss possible extensions of the problem.

4. Evaluate the strategies used by the class.

5. If appropriate, discuss special features of the problem.

BEFORE. The two teaching actions at this stage should be used to illustrate the importance of carefully reading mathematical problems and of focusing on the meanings of words and phrases that may have special interpretations in mathematics. Also, at this stage, it is very important that the problem statement be visible to every student. Preferably, all students would have a copy of the problem so they can write anything on the statement which may help them understand and solve the problem.

DURING. The first three teaching actions shown in Table 2 are the most critical at this stage. Teaching action 4 is a classroom-management strategy for meeting the needs of high-achieving students. However, in the AFTER segment of the teaching actions, "problem extensions" play an important role for all students.

There are two reasons why one should identify a group's "location" in the problem-solving process. First, categorizing a group with respect to the problem-solving process enables one to diagnose a group's strengths and weaknesses related to problem solving. For example, one group of students may frequently have difficulty understanding process problems while another may frequently have difficulty generating ideas for solution strategies. Explicitly identifying a group's location in the problem-solving process enables one to provide appropriate instructional emphasis throughout the problem-solving program. The second reason it is important to categorize groups with respect to the problem-solving process is to facilitate the implementation of teaching action 3. This teaching action is discussed shortly.

The most critical moment in teaching problem solving is the time when students indicate to the teacher that they are "stuck," that is, that they have come to a blockage in their solution of the problem and they don't know what to do or try next. Teaching action 2 suggests that the first time a group encounters a blockage, they should immediately be referred to the problem-solving strategies bulletin board and encouraged to select and implement some strategy or strategies for solving the problem. Although the implementation of this teaching action is quite easy, its importance should not be underestimated. There are at least two reasons why the use of the bulletin board is important.

1. The use of the bulletin board forces a group to self-select a problem-solving strategy without relying on teacher direction. This action is consistent with the goal of developing students' abilities to search and evaluate alternatives in the problem-solving process.

2. The bulletin board provides a "crutch" needed by most students. Figure 1 shows there are at least 13 strategies from which one must select a strategy or strategies for each problem situation. Experience shows it is unrealistic to expect students to keep all of these strategies in memory unless they have had considerable experience using them.

The use of the problem-solving strategies bulletin board is often sufficient to enable a group to continue work on a problem. However, at those times when the use of the problem-solving strategies bulletin board is unproductive, teaching action 3 should be used. The purpose for using hints and questions is to facilitate, not remove, students' decision-making. To do this, however, hints and questions must be very carefully selected so as not to identify completely the direction to solution. Also, it is important to realize that hints and questions, even carefully selected ones, will usually be received differently by different students. For some students, hints and questions will be no help in moving them toward a solution. For other students, hints and questions are confusing and may even suggest to the students that their approach will be unproductive if it appears different than the approach suggested by the hint. And, of course, for many students hints and questions are indeed useful. Often, hints and questions enable students to pursue a direction previously considered unproductive, to identify the inappropriateness of a particular approach, or to identify an idea to pursue when one was not apparent.

The hints and questions one wishes to use for a particular problem should be identified prior to the problem-solving session. When hints and questions are written in advance, the teacher is forced to "think through" a problem before, rather than with, the students. As a result, the teacher is better able to identify where students are in the process of solving the problem and is better prepared to select appropriate hints and questions during the problem-solving session. The hints and questions one prepares should be categorized according to the three steps in the problem-solving process: (a) understanding the problem; (b) developing and carrying out a solution strategy; and (c) evaluating the solution.

Although hints and questions should be categorized according to the problem-solving process, the particular hint or question one provides at a given moment may not be from the category at which the students are currently working. For example, students may have reached a blockage in the process of carrying out a solution strategy and to get them past that blockage the teacher may ask a question related to understanding the problem. The teacher must also be prepared for the situation in which his pre-selected hints and questions are not appropriate, and another comment is needed to help the group along.

Finally, it is important to realize that just as the three steps in the problem-solving process are not necessarily sequential and disjoint, teaching actions 1 through 3 at this stage are also not necessarily sequential and disjoint. Frequently, these teaching actions may be used in a "cyclic" fashion and may be repeated several times in solving one problem.

AFTER. Near the end of the time for small group work, at least two students, each from a different group, should be asked to place their solution efforts on the chalkboard. If possible, one of the solution efforts should provide a correct solution and one an incorrect or no solution at all. After the students have put their work on the board, a whole-class discussion should be used for the teaching actions at this stage.

Teaching actions 1 and 2 are straightforward. Their purpose is to focus on the selection and implementation of problem-solving strategies. Teaching action 3 is intended to demonstrate to students that problem-solving strategies are not problem specific and to help students recognize different kinds of situations in which particular strategies may be useful. Although it is acceptable and desirable that different solution strategies be sought for every problem, it is important that the different approaches that led to a solution be evaluated (see teaching action 4). The criteria for evaluating solution strategies are generally dependent on the problem being solved. For example, if pattern finding is the general strategy needed for solving a problem, the approaches used should be evaluated with respect to the degree to which each facilitates identifying a generalization of the pattern.

Finally at this stage, any special features of the problem just attempted should be discussed with the class. For example, some problem statements include a picture or a diagram. In these instances, the way(s) in which the picture or diagram influenced the students' ideas should be discussed.

Adjust instruction for initial experiences. The extent to which the teaching actions in Table 2 need to be modified for experiences with process problems at the beginning of the year depends on the amount of previous experience students have had with process problems. For the guidelines below the assumption is made that students have not had any prior experience with process problems.

Before - Teaching actions 1 and 2 in Table 2 should be used at this time. Also, an additional teaching action one should use is a whole-class discussion concerning (a) understanding the problem and (b) developing and carrying out a solution strategy. The hints and questions one prepares for a problem can be used as a basis for this discussion. For these initial experiences, the first stage of the problem-solving process should be completed as a whole-class activity. In other words, students should understand what they are being asked to find in a problem before they begin work in their small groups. For the second stage of the problem-solving process, developing and carrying out a solution strategy, the class should discuss but not implement possible solution strategies. The use of a whole-class discussion for these purposes facilitates success on the students' initial experiences with process problems.

During - There are four key points at this stage.

(a) The primary goal should be to establish a classroom climate that is conducive to mathematical problem solving.

(b) Most students will not have prior experience in small-group problem-solving situations. Therefore, there is a tendency for students not to share their ideas with others and for students to work individually on their initial attempts. In these initial experiences, attention must be given to establishing a group problem-solving effort. This can be facilitated by telling a group from the start that it is fine to use more than one approach; however, everyone in the group must understand what approaches are being tried. Related to this is the fact that within a group there are usually one or two students who "see the solution" before others. To promote small-group problem solving, one should insist that everyone in the group be able to explain how a solution was obtained. This not only promotes small-group problem solving but also serves as an instructional technique with respect to the selection and implementation of problem-solving strategies.

(c) The hints and questions used during initial experiences with process problems should be more directive than those used later. That is, hints and questions should, with great certainty, enable a group to continue work on a problem.

(d) The problem-solving strategies bulletin board should not be on the wall when school starts. Rather, the problems one uses at the beginning of the program should be selected so they elicit the strategies one wants to include on the bulletin board. As each problem is solved, the strategies used in the solution should be named and the name of the strategy should be added to the bulletin board. This approach enables one to "build-up" the strategies bulletin board over time. Obviously, teaching action 2 in Table 2 cannot be used for the first process problem attempted. After that, this teaching action should play a role in the problem-solving session.

After - The concept of a "problem-solving strategy" will be new to most students. As a result, direct instruction from the teacher concerning the implementation of particular strategies is needed at the beginning of the program. For example, the first time students are exposed to "using a table," they will no doubt need help in constructing a table in a way that facilitates solving the problem. Therefore, in those initial experiences considerable time must be given to teaching actions 1 and 2 in Table 2. The only other modification in Table 2 is that teaching action 3 at this stage will not play a large role since the students have not had prior experience with process problems.

Adjusting instruction for later experiences. There are some changes that should occur in the teaching actions at this stage and, in fact, naturally occur quite often as students' problem-solving abilities develop.

Before - Both of the teaching actions shown in Table 2 may be phased out. The more experience students have with process problems the more facile they become at reading mathematical problems.

During - One modification of the actions at this stage is that the hints and questions one uses should be more general than those used earlier; that is, they should be less directive with respect to productive solution strategies. Another change in the teaching actions at this stage is related to the bulletin board. One goal of the teaching actions in Table 2 is that students will eventually commit to memory all of the strategies on the bulletin board. It may be possible at some point in the program to "tear down" the problem-solving strategies bulletin board. The first time students reach a blockage in solving a problem they should still be encouraged to select and implement a strategy. However, now the "crutch" they had been using has been removed.

After - There are two modifications in the teaching actions at this stage. First, students should assume more responsibility for naming and explaining strategies used in problem solving. Also, relating a problem to similar ones and searching for extensions of a problem should play an important role in the problem-solving session.

Modifying the Instructional System for Low-Achieving Classes

The instructional system described above was designed for and used with average and high-achieving students at the junior high school level. However, with some modification the system developed here may also be appropriate for low-achieving classes.

In the organizing-for-instruction component there are two changes that seem important.

1. Problem-solving activities should be included in the program. Some examples of problem-solving activities are: (a) identifying extra information in problem statements; (b) writing a question for a story; and (c) telling how to solve a problem that does not involve numbers.

2. Problem solving should occur daily. Work with process problems may be delayed until after students have had considerable experience with problem-solving activities.

In the teaching actions component there are seven changes that seem important.

Before

1. Teaching actions 1 and 2 in Table 2 may be needed for a longer period of time and perhaps always.

2. Whole-class discussions with respect to understanding problems and developing and carrying out solution strategies should be continued for a longer period of time.

During

3. References to the problem-solving strategies bulletin board should be somewhat specific (e.g., "Try one of these two helping strategies to get you started.").

4. Hints and questions should be more directive.

5. The problem-solving strategies bulletin board should remain on the wall for the majority of the year.

6. Considerable attention must be given to establishing a classroom atmosphere conducive to mathematical problem solving.

After

7. Do not ask students to put their solution efforts on the chalkboard initially. Copy students' work on the board for them until they develop a willingness to do so themselves.

References

Bourne, L. E. Jr., Ekstrand, B. R., & Dominowski, R. L. The psychology of thinking. Englewood Cliffs, New Jersey: Prentice-Hall, 1971.

Charles, R. I. "Get the most out of 'word problems'." Arithmetic Teacher, 29, 1981, 39-40.

Lester, F. K., Jr. "Mathematical problem solving in the elementary school: Some educational and psychological considerations." In L. Hatfield (Ed.), Mathematical problem solving. Columbus, Ohio: ERIC Center for Science and Mathematics Education, 1978.