Problem Solving: Goals and Strategies

by

Jesse A. Rudnick and Stephen Krulik
Temple University

What is problem solving? Haven't we been teaching problem solving all along? What are we doing when we go over all those word problems in the textbooks; isn't that problem solving? The answers to the latter two questions are yes, no, or maybe.

To clarify this state of confusion, let's first define what we mean by problem solving. Problem solving is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. The existence of a problem implies that the individual is confronted by an unfamiliar situation, one for which no apparent solution or path to a solution is readily seen. The key words here are unfamiliar, and for which no apparent solution or path to a solution is readily seen. You see, once a student has seen a problem and been shown a method of solution, additional exercises similar to the original (even though the names, the setting, and the numbers have been changed) are no longer problems. They are exercises, merely drill and practice. You might say that the students are engaged in solving problems, but they are not engaged in problem solving.

Problem solving is a process, a systematic search by the individual through the given data, and a synthesis of the findings into a neatly executed solution. Indeed, the solution is not the final answer, but rather the entire process from the original confrontation to the final conclusion. Thus the word problems that appear in most textbooks do not provide problem-solving experiences because most teachers do the initial thinking for the students and then provide them with a model, a method, or an algorithm for doing each of the several types of problems. The fact is that most of us and the children as well, try to identify the "type" of problem and then attempt to recall how we do that particular type. There is really nothing wrong with this procedure, if you are trying to solve specific problems, or problem types. Unfortunately, this does not serve the purposes of problem solving.

Most of the word problems that appear in a textbook are designed to support mathematical skills that have just been developed. Indeed, this is the crucial point of the recommendations of the National Council of Teachers of Mathematics (1980) that skills should be developed to give the students the power to resolve problems; the problems are not materials to support the skill. Skills, in the absence of the ability to utilize them in appropriate settings, are useless!

It is probably unrealistic at this time to believe that the school mathematics curriculum will be immediately rewritten with a central theme of problem solving. If it ever happens, it will not be for a considerable length of time. However, in the meantime, or until such time as substantial changes are made, classroom teachers can, on their own, implement the N.C.T.M. recommendations for the 1980's by making problem solving an ongoing activity in their classrooms. Some problem-solving experience can probably be worked into each and every classroom lesson, if the teacher feels it is important enough and prepares the lessons accordingly.

Before we consider problem-solving activities for the classroom, let's discuss problem solving itself. How do <u>we</u> solve problems that <u>we</u> have not seen before? What do <u>we</u> do when confronted by a perplexing situation that needs resolution? Perhaps if we can respond to these questions, we will gain some insights that will prove helpful in the school classroom.

Polya (1957) states that successful problem solving involves four steps:

1. Understanding the problem
2. Selecting a strategy
3. Solving the problem
4. Looking back at the problem.

When we are confronted by a problem, the first thing that we usually do is read the problem. This means (1) look for key words; (2) try to understand the situation; (3) visualize the situation in your mind; (4) look for the relationships that exist between the data; and (5) find out what is being sought. Then, according to Polya, a strategy should suggest itself which, if treated carefully, will result in a correct answer. (We consciously avoid using the word "solution" at this point, because we want to emphasize the fact that in problem solving the solution is the entire process, not just the answer.)

But, strategy selection is not just an automatic outcome of reading or understanding the problem. How does one select a strategy? Indeed, what are the strategies one can employ to find an answer?

We have identified eight of the most widely used strategies. They are not unique, nor is the list by any means exhaustive. We list them below:

1. Pattern recognition
2. Working backwards
3. Guess and test
4. Simulation or experimentation
5. Reduction or looking for a simpler problem
6. Exhaustive listing
7. Logical deduction

8. Data representation
    8.1 graph
    8.2 equation
    8.3 *algebraic expression*
    8.4 table
    8.5 chart
    8.6 diagram

Although it would be impossible to illustrate each of these strategies in this paper, let us take a look at some of them, with some illustrative problems for each. As an illustration of the Guess and Test strategy, consider the following problem:

> A textbook has been opened to pages 26 and 27. If we multiply these two numbers, their product is 702. Jane opened her math book and found that the product of the numbers on the two facing pages was 8,556. To what pages was her book opened?

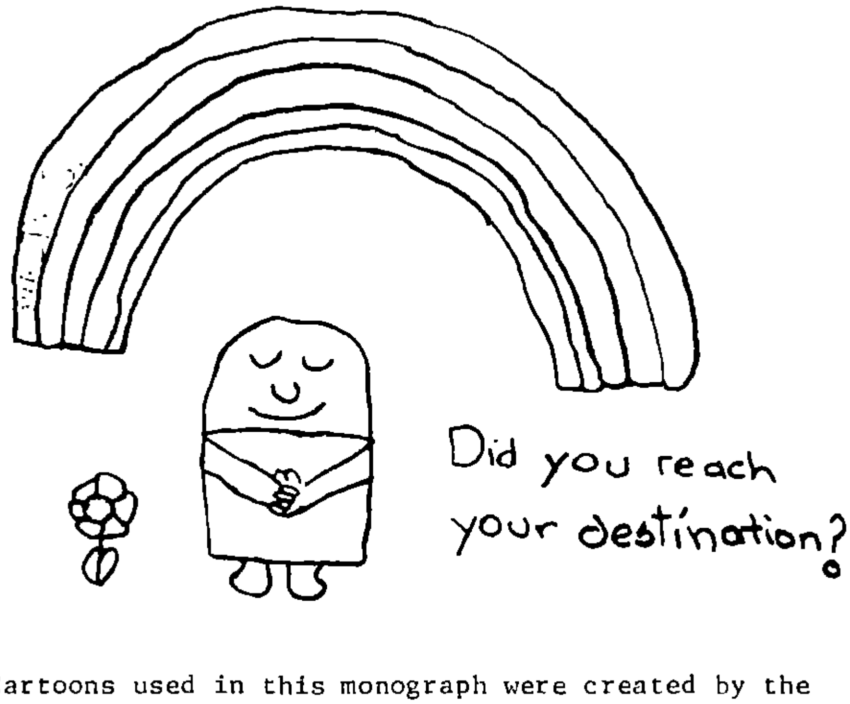
As your students read this problem and consider the information it contains, they should be brought to realize that the numbers which appear on facing pages of a textbook are always consecutive. Thus, one of them is even, one of them is odd, and their product will always be even. If the students are encouraged to try a pair of successive numbers, say 34 and 35, they find that the product of these numbers is less than 8,556. If they try a larger pair, say 110 and 111, they find that this product is too large. Thus the pair of numbers we wish to find lies somewhere between these two pairs. This is a good notion for the students to learn at this time -- the idea of approaching a limit from both sides. Now, by trying various products within this range, they should find the correct pair, 92 and 93. Notice too, that this is an excellent time for the teacher to introduce the concept of a square root. If the students take the square root of 8,556 (92.4986) they need only take the whole numbers which lie on either side of the square root, namely 92 and 93. If practice in multiplication is not needed at this time, this problem is a good one to explore with a calculator. (This is basically a consecutive integer problem, which, in Algebra, would be solved with the equation $x (x + 1) = 8,556$.)

In many cases, it requires a combination of strategies to resolve a problem. For instance, an elegant solution to the following problem employs three of the strategies on our list: simulation, recording data in a table, and pattern recognition.

> Eight members of the Harlem Globetrotters are warming up for their game with the Washington Generals. The players are in a circle. Each player passes the ball to each of the other players. How many times is the basketball tossed?

This problem could be solved by actually having eight students stand in a circle and *toss a basketball or crumpled piece of paper to each of the other* students while counting. However, you may wish to help the students prepare a table similar to the following:

Carrying Out the Plan



Looking Back

The Problem-Solving Cartoons used in this monograph were created by the students in West Block and North West Block of the University Heights Elementary School, Calgary, Alberta.