Diagnosing Reading Difficulties in Verbal Problem Solving

by

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Solving word problems has traditionally been one of the most difficult activities in the mathematics curriculum at all grade levels. This is not surprising, since word problems often require the higher levels of reasoning of analysis and synthesis. Students must not only know how to compute and work mathematical algorithms, but most importantly, they must know which algorithms to use and when.

In recent years, many models have been developed to help explain the problem-solving process. Although these models vary considerably in the number and type of stages, they all include reading and language processing in the early phases of the problem-solving process. There is, however, some disagreement as to the relative importance of reading ability in solving word problems in mathematics across age and ability levels. It seems safe to assume that the ability to read and interpret word problems with facility is a necessary, but not a sufficient condition for problem-solving success. Unfortunately, many students with poor language processing skills never really get into the analysis phase of solving word problems. Unfamiliar vocabulary and difficult syntax can distract the students' attention from the problem's structure, often resulting in confusion, frustration, and lack of confidence.

Fortunately, there is evidence that teachers can help their students become better problem solvers by devoting special attention to reading problems and language skills as they relate to mathematics. Perhaps a starting point in the development of reading instruction in mathematics is to convince both teachers and students that reading mathematical word problems and textual material is very different than reading regular English prose. These differences require the adjustment of reading habits and study skills.

In the following discussion, we shall consider a number of specific problems in the reading of mathematical word problems. For convenience, our discussion will be divided into four major areas of concern: semantics, syntax, context, and interpretation skills. Semantics refers to the <u>meanings</u> of words and phrases. Syntax refers to the <u>arrangements</u> of words and phrases, the <u>form</u> of words and symbols, and the <u>grammatical</u> structure of problem statements. Although the term "context" has several meanings, we shall restrict our attention to the setting of the problem, along the dimensions real-imaginative, concrete-abstract, and factual-hypothetical. Interpretation skills refers to the ability to understand graphs, charts, tables, etc. Within each of these areas, several problems or sources of difficulty will be identified, followed by suggestions for remediation and some follow-up activities.

Category I: Problems with Semantics

A. Problem: New vocabulary terms which a student may encounter in reading mathematical material and word problems may have no relevance to the student's everyday vocabulary. In this situation, these new words are often memorized without understanding, making recall in the appropriate problem-solving situations difficult. For example, words such as addend, secant, denominator, and hypotenuse have little use outside the context of mathematics.

Remediation: The teacher can help students understand and remember new vocabulary by writing each new word on the chalkboard or overhead, defining it, and providing examples. One effective technique is to provide several examples of the new term as well as several non-examples. Students can then be asked to try to develop their own definition, based on similarities and differences between the examples and non-examples. Students can also be asked to generate their own word problems that use the new vocabulary. These can be displayed on cards or on a bulletin board along with definitions and pictorial examples. One possible idea for a bulletin board is shown in Figure 1.

With some new vocabulary terms, teachers can use structural linguistics to help students learn and remember definitions. The meanings of prefixes, suffixes, and certain root words may already be familiar to students. Terms such as polygon, quadrilateral, isoceles, acute, obtuse, and pentagon lend themselves to structural analysis. For example, the term "obtuse" is related to the word "obese" which means fat or overweight. An "obtuse" angle is a "fat" angle; i.e., an angle whose measure is more than 90 degrees. Similarly, the term "acute" means sharp, as in an "acute pain." Therefore, an "acute" angle is a "sharp" angle or one whose measure is less than 90 degrees. With the help of a dictionary and thesaurus, many other aids to the memorization and comprehension of the terms can be found.

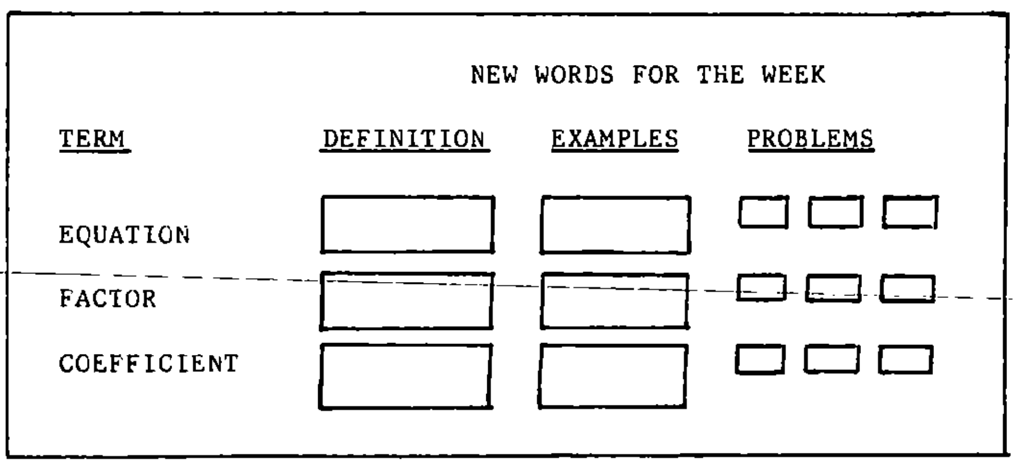


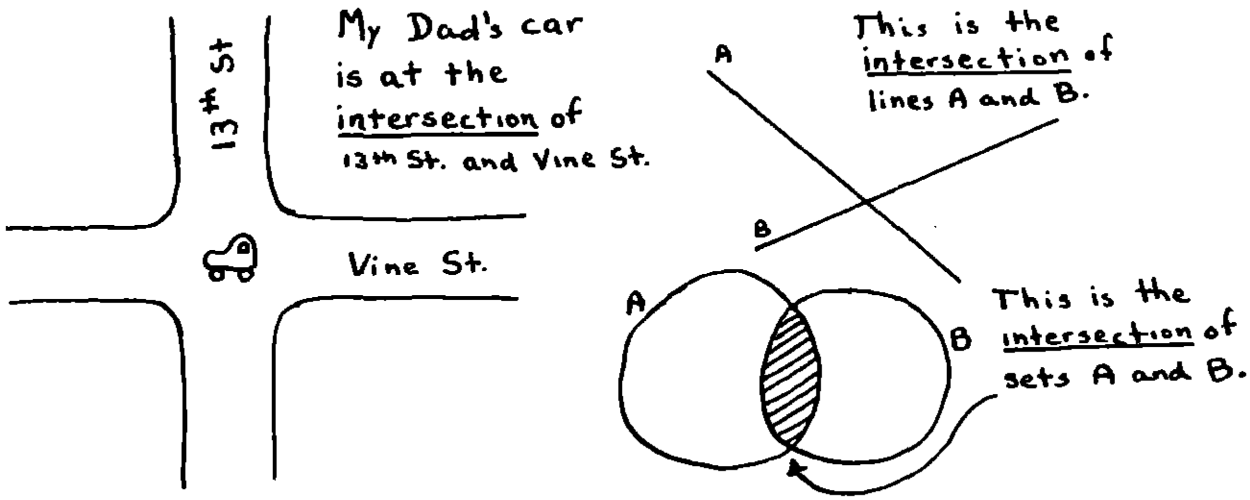
Figure 1

To ensure that new vocabulary has been learned meaningfully, continuous evaluation should be planned on quizzes, unit tests and on a day-to-day basis during class discussions. Brief, dittoed tutorials on the correct use of new vocabulary terms can be constructed by teachers to supplement classwork and the text, for those students who need additional help.

B. Problem: Some words have similar meanings in English prose and mathematical material, but students fail to perceive these similarities when the words are used in a mathematical context. Terms such as improper, union, disjoint, intersection, commutative, associative, and distributive often cause students difficulty in mathematics, even when they are familiar with these terms as part of their everyday vocabulary.

Remediation: Students will need some help from the teacher in interpreting familiar terms in a mathematical context. For example, the intersection of two streets is a familiar concept to most students. A child standing at the intersection of two streets is on both streets at the same time. Similarly, the intersection of two lines or two sets is the set of points that are in both sets or on both lines at the same time. The term "associate" means to group or to be grouped with, as in "She is associated with Girl Scouts." In the "associative" property of addition, the parentheses group or associate numbers to be added.

The parallel meanings of words such as those above should be discussed in detail when introduced for the first time. Students can be asked to write sentences which illustrate the use of the terms in both mathematical and non-mathematical contexts. As an assignment, students can draw pictures that illustrate the similarities in the use of the terms.



Since marked differences may exist between a child's familiarity with one word and a different form of the same word, special attention should be devoted to entire word families. For example, Kane, Byrne, and Hater (1974, pp. 75-90) found that 76.6 percent of seventh- and eighth-grade children were familiar with the term associative but that only 39.1 percent were familiar with the term associativity. Having children read word problems out loud is a useful technique for determining which words and phrases are causing the most difficulty.

C. Problem: Some vocabulary terms in mathematics have different meanings outside the context of mathematics.

Remediation: Special attention should be devoted to words that have the same spellings and pronunciations but different meanings in regular English prose. For example, words such as base, mean, root, times, prime, round, and right (and there are many others!) often cause confusion. In addition to pointing out these differences, teachers could have their classes make a list of words which have these different meanings and use them to make a bulletin board.

D. Problem: The teacher may not realize that some vocabulary terms are not familiar to all of the students, particularly in the beginning of the school year.

Remediation: A pretest of mathematical vocabulary can be given at the start of the year or unit of instruction, to determine which students are not familiar with the required terms. If the class members are significantly divided on their knowledge of background mathematical vocabulary, the class can be divided into groups for specific instruction on the meaning and use of the required terms.

E. Problem: Students may be unable to identify "key words" in the problem statement that provide clues as to which operation can be used to arrive at a solution.

Remediation: The problem of being able to determine how to "set up" a problem, that is, to determine which operations to use for a solution, is probably cited more often than any other difficulty that students have when solving word problems. The ability to recognize "key words" and to use them as clues to a problem's underlying structure is not easy to cultivate, but can be developed with practice over a period of time. One productive method for teaching the relationship between "key words" and problem structure is to have students underline the words they consider to be clues to the operations required. For example, in the problem below, the word "and" indicates the operation of addition, the word "of" indicates multiplication, and the word "left" indicates subtraction:

> David earned \$2.50 for cutting grass after school on Friday, <u>and</u> \$2.30 for weeding the garden on Saturday. He gave 1/3 <u>of</u> his earnings to his friend for helping. How much did he have <u>left</u>?

$$1/3 \underset{}{\overset{\text{of}}{\times}} (\$2.50 \overset{\text{and}}{+} \$2.30) = 1/3 \times \$4.80 = \$1.60$$

$$\$4.80 \overset{\text{left}}{-} \$1.60 = \$3.20$$

During the course of the year, the class can compile a list of "key words" that are associated with the four basic arithmetic operations and display them on a bulletin board along with several examples. As an activity, students could write problems using key words from the class list. Young children could be instructed to write simple problems based on given number facts in addition, subtraction, multiplication, and division. By reversing this procedure and using a calculator for the actual computations, students can be given extensive practice in translating "key words" in problem statements to mathematical operations.

Caldwell (1980) suggests the following set of problems as a basis for a class discussion on how the modification of a few "key words" can alter a problem's meaning and solution:

- John has five dollars. He earns three dollars. How much does he have now?
- John has five dollars. He saves three dollars. How much does he have now?
- John has five dollars. He spends three dollars. How much does he have now?
- John needs five dollars. He has three dollars. How much does he need now?

As a follow-up activity, Caldwell suggests writing problems with the key words deleted. Students can then experiment with varying the key words to produce problems with very different solutions. For example:

- Find the number that is the _____ of 8 and 4.
- Oscar _____ 15 cookies. He _____ three cookies. How many _____ cookies does he have _____?

F. Problem: Students may not realize that the same term may not always indicate the same operation.

Remediation: Although some words such as sum, total, difference, decrease, and, left, more, and less are often indicators of required operations, students should be cautioned to look carefully at the context, for numerous exceptions do exist. For example, in the first of the three problems that follow, the word "a" does not imply addition. In the second, the word "left" does not imply subtraction. The word "sum" in the third problem does indicate addition, but the operation has already been performed and therefore is not required in the solution process.

- What is the product of 8 and 4?
- Jane ran 6 blocks north and turned left. She then walked 7 blocks west. How far was she from her original starting place?
- Bobbi found that the sum of her 4 quiz scores was 56. What was the average score?

Perhaps the best method is to have children look for potential key words as they read through the problem. During a second reading they can use contextual clues to help determine which of the identified words are actually operational or procedural indicators. Systematic exercises of this type can help them focus on semantic hints to discover which operations or procedures are required for a solution. As a reminder, the class could compile an additional list of potentially misleading key words with an asterisk.

Category II: Problems with Syntax

A. Problem: Position of the question sentence and sequence of important data in the problem statement may cause difficulty in determining the problem's mathematical structure.

Remediation: Children need extensive practice with sets of word problems that have similar mathematical structures, but vary considerably in their wording. Teachers can provide practice sets where the position of the question sentence is varied systematically. For example:

- In how many hours can Joe and Bill paint a garage working together, if Joe can do the entire job alone in 15 hours, and Bill can do the entire job alone in 11 hours?

- Joe can paint a garage in 15 hours. In how many hours can Joe and Bill paint the garage working together, if Bill can paint it alone in 11 hours?

- Joe can paint a garage in 15 hours. Bill could paint it in 11 hours. How long would it take them to paint the garage if they worked together?

Research indicates that children tend to have more difficulty with problems when the data are presented in an order that is different from that needed to solve the problem. Students can be asked to generate sets of word problems that use the same data and require the same answer, but which vary the order of the data in the problem statements. For example, a student might come up with the following set:

- A grocer bought 17 dozen pears for $14.65. If 5 dozen spoiled, at what price per dozen must he sell the remaining pears to make a profit equal to 3/5 of the total cost?

- A grocer wished to make a profit of 3/5 of the total cost of his fruit. If 17 dozen pears cost him $14.65, and 5 dozen spoiled, for how much per dozen must he sell the remaining pears to realize the desired profit?

- In a crate containing 17 dozen pears, a grocer finds that 5 dozen have spoiled. How much per dozen should he charge for the remaining pears to make a profit of 3/5 of his total cost, if the crate costs $14.65?

As an additional activity, students can be asked to construct several more problems which use the same data, but which ask different questions. One example is the following:

- In a crate that contains 17 dozen pears, a grocer finds that 5 dozen have spoiled. He sells the remaining pears for $23.44, which will give him a profit of 3/5 of the original cost of the crate. How much per dozen did the pears in the original crate cost the grocer?

By pooling the contributed sets of problems from the members of the class, the teacher can construct activity sheets which require students to identify which problems are syntax variations of each other.

B. Problem: Students may have difficulty interpreting signs, symbols and special mathematical notation.

Remediation: When reading word problems, students should be encouraged to use the words that provide the meaning of symbols and notation, rather than the symbols and notation themselves. Word problems which use English words instead of numerals, such as "two hundred twenty-five" instead of "225", can be rewritten using the numerals instead of their English counterparts. Teachers should frequently test knowledge of symbols and notation on tests, quizzes, and during class discussions.

C. Problem: Students may center their attention on the numbers present in a word problem too early, and perform random computations without thinking through which operations are required.

Remediation: To help students avoid premature centering on numbers, the teacher could have them rewrite the problem without numerals. For example:

With numbers: A jogger runs along the edge of a field going north for 12 minutes at 10 km/hr, and then runs east for 1/2 hour at 14 km/hr. If she runs in a straight line back to where she started, at what speed must she travel to reach home in 28 minutes?

Without numbers: A jogger runs north at one rate, and then east at another rate. If she runs in a straight line back to where she started, at what rate must she travel to reach home in a given time?

This second version is one of several ways of rewriting the problem without the numerals. The numberless version could clarify the overall situation and help students see this as a rate problem involving the Pythagorean theorem.

D. Problem: Word problems which contain many pronouns can cause confusion.

Remediation: During a second reading of a problem statement, students should be encouraged to substitute nouns for pronouns if they find the action of the problem confusing.

- Henry's guinea pig has a baby which ~~he~~ Henry named Sam. ~~He~~ Sam weighed .2 kg at birth. Henry observed that ~~he~~ Sam gained .2 kg every five weeks. At that rate, how many kilograms will ~~he~~ Sam weigh after six months?

E. Problem: Students may have difficulty in distinguishing between relevant and irrelevant facts in a problem statement. Problem statements which are particularly long often cause difficulty as well, in that students can get overwhelmed with surplus information.

Remediation: Students should be encouraged to focus their attention on the action of the problem, centering on important verbs. Listing facts in their proper relationships (particularly in the form of short, mathematical sentences which can later be combined into equations) is a very useful activity. Students can be given sets of word problems and asked to try to identify all extraneous information. In the following example, the student crossed out the excess information, in an effort to simplify the problem.

- ~~Bob's uncle~~ Bill uses five ~~3 oz.~~ bottles of concentrate to make 8, ~~1/2 litre~~ bottles of root beer. How many 1/2 litre bottles of root beer can he make with twelve ~~3 oz.~~ bottles of concentrate?

Having the student read the above problem statement out loud, without the crossed-out words, can make the problem's structure more apparent. Note that the students may have to reread the problem statement several times to be able to decide which words are really not needed.

Students should also be given practice in constructing their own problems which have the same mathematical structure, but contain different amounts of extraneous information, have different sentence structures, or vary in length.

Category III: Problems with Context

A. Problem: Students may have difficulty perceiving similarities in the mathematical structure of word problems which have different contextual embodiments.

Remediation: Students need to be shown how problem settings can be modified without changing the mathematical structure of the problems. Modifications in context can be across the dimensions concrete- abstract, factual-hypothetical, or real-imaginary. The following four problems (Caldwell and Goldin, 1979) illustrate variations of context using combinations of the concrete-abstract and factual-hypothetical dimensions.

- There is a certain given number. Three more than twice this given number is equal to 15. What is the value of the given number? (Abstract-factual)

- There is a certain number. If this number were 4 more than twice as large, it would be equal to 18. What is the number? (Abstract-hypothetical)

- Susan has some dolls. Jane has 5 more than twice as many, so she has 17 dolls. How many dolls does Susan have? (Concrete-factual)

- Susan has some dolls. If she had 4 more than twice as many, she would have 14 dolls. How many does Susan really have? (Concrete-hypothetical)

Problems can be modified along the real-imaginary dimension by having students construct problem sets with given themes or subjects, such as their income and expenditures for a week (real) or problems involving fanciful characters such as dragons.

Caldwell (1980) has suggested that students could be asked to fill in missing words in problem statements, so as to change the context. For example:

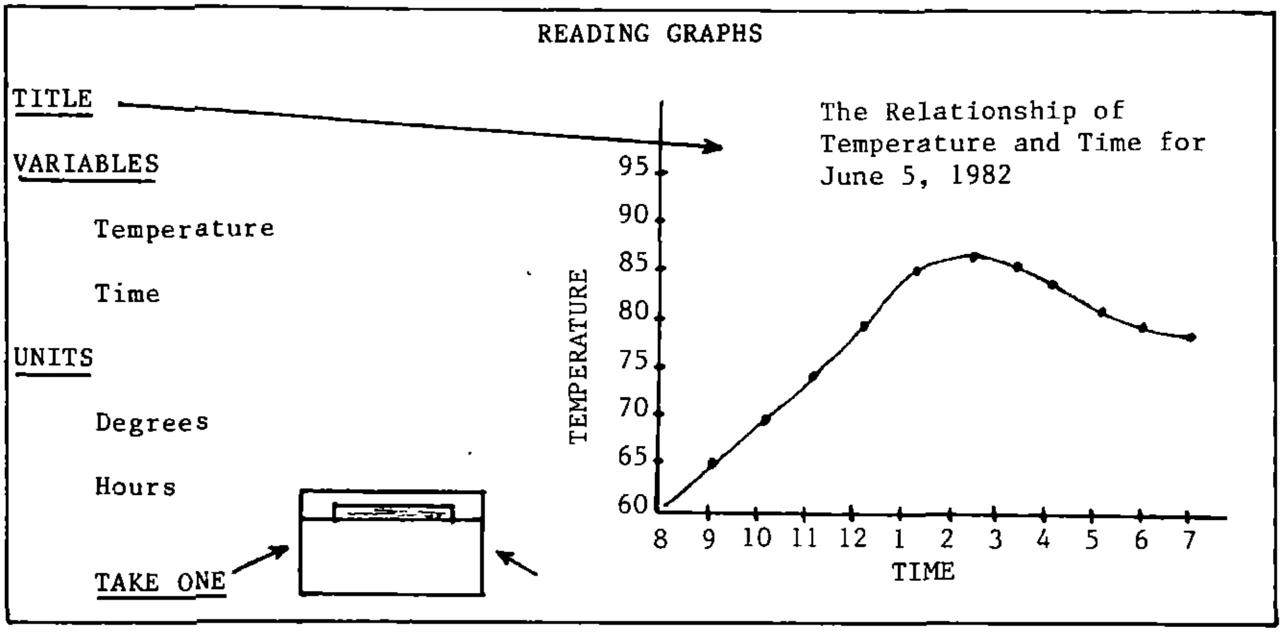
- Judy has 27 ____. She ___ 5 and then ____ 14. How many _____ does she have in all?

Activity sheets which require students to identify problems in different contexts with similar mathematical structures could provide a useful follow-up activity.

Category IV: Problems with Interpretation

A. Problem: Students may not understand processes used to read and interpret a graph.

Remediation: Students should be exposed to word problems which make use of a graph to summarize information. Conversely, students should be given practice problems which require a graph as part of the solution, or for which a graph may help conceptualize a solution. When a problem uses a graph, the student should read the title of the graph to determine the kind of information that the graph provides. The variables described on the axis or parts of the graph should be listed, with a brief description of how they are related. In some graphs, the units of measure may be spaced on different scales, so this should be checked carefully. How to read a graph is an excellent topic for a bulletin board. The diagram below makes use of a pocket of problems which make use of graphs. The teacher can change the problems weekly, or use student-generated problems.



B. Problem: Students may not know how to read tables.

Remediation: Although calculators have replaced the use of tables in many cases, there are still times when the ability to read required data from tables is essential. Students should be exposed to many types of tables and given practice finding required information. As an activity, students can be asked to write a description of how to use the table, as if their description was to be read by a younger student. The description should include information about the headings and entries, and at least one example. A bulletin board can be constructed with student-generated problems based on data available from tables collected from newspapers, magazines and old texts.

C. Problem: Students may not know where to find additional information to help them solve difficult problems.

Remediation: Students should be thoroughly familiar with the location and use of the various parts of their text, such as the index, appendix, and glossary (to look up definitions they may have forgotten). Reference texts, additional tables, study guides, etc. should also be made available.

Summary

As the above list of reading problems indicates, many students are poor problem solvers in mathematics due to the lack of language processing skills. When reading deficiencies are discovered, teachers may take steps toward remediation. However, the best plan is systematically to provide reading instruction in mathematics throughout a child's academic career. The activities suggested above are just some of the many ways that teachers can help students learn the importance of reading word problems slowly and carefully, with an attitude of aggressiveness and attention to detail. Once these adjustments in reading rate and purpose are made, students should be able to approach word problems in mathematics with increased confidence and ability.

References

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