Leading Problem Solving in an Elementary School Classroom

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In order to develop a successful problem-solving program, identifying appropriate problems for students at different ability levels is important. If a question is too easy, it is not a problem for a student and if the question is too difficult, a student will usually not even attempt to solve it. A problem can be solved only after a problem solver is willing and able to mobilize his or her resources to solve the problem.

Identifying appropriate methods of solving problems for students at different ability levels, rather than different problems, is often a challenge for a teacher. For example, the following problem could be posed for an algebra student: The sum of Susan's age and Kenneth's age is 21 and Susan is five years older than Kenneth. How old is Susan and how old is Kenneth?

Algebra students would most likely solve this problem using simultaneous equations:

$$\begin{array}{rr} x + y = 21 & x + y = 21 \\ x - y = 5 & -)\ \ x - y = \ \ 5 \\ \hline & 2y = 16 \\ & y = \ \ 8 \end{array}$$

$$x - 8 = 5$$

$$x = 13$$

Susan is 13 years old and Kenneth is eight years old. But this problem can be taught to students who are at the concrete operational stage and without the methods of algebra. A way of helping elementary school children solve the problem will be presented in the following paragraphs.

Elementary school children experience anxious feelings when they are faced with some word problems. In order to reduce their anxiety, they often

immediately write down numbers from the statement of the problem and perform an operation between the numbers "21 + 5 = 26, 26 years old"; "21 - 5 = 16 years old." One student wrote

$$\begin{array}{r} 21 \\ \underline{5} \end{array}$$

and asked her teacher, "What do I do? Do I add?"

Thus it is important in problem solving to have some rules or methods which can help one work confidently toward a solution. In How to Solve It (1957), Polya described certain rules and methods (called heuristics) which are helpful in solving problems. But Polya's list of heuristics is not in an appropriate form for elementary school children because of the number, the language, and the complexity of the heuristics. The following list is adapted for elementary school children from Polya's list of heuristics.

Heuristics for Elementary School Children

I. Understanding the problem
   (A) What is involved in the problem?
   (B) What are the relationships among the involved items?
   (C) What are the questions to be answered?

II. Making a plan
   (A) Can drawing a picture help?
   (B) Can making a chart help to solve it?
   (C) Consider special cases and look for a pattern.
   (D) Consider one condition and then add another condition.
   (E) Have you solved a similar problem?

III. Carrying out the plan
   (A) Carry out the plan
   (B) Check each step

IV. Looking back
   (A) Is your answer reasonable?
   (B) Try to find another way to solve it.
   (C) Make a similar problem.

In the early stages of teaching problem solving, teacher demonstration of problem solving by conscientious use of heuristics is essential. "At first, we must understand the problem: What are we talking about?...The problem involves the ages of two persons, Susan and Kenneth. What are the relationships between the ages of Susan and Kenneth?... Susan is five years older than Kenneth and the sum of the ages of Susan and Kenneth is 21 years. Can you give an example where Susan is five years older than Kenneth?" If there are correct responses, it is fine but if there aren't any responses, the teacher can give an example: If Susan is six years old, how old is Kenneth? At this point, some students will say, "Kenneth must be one year old." The teacher should elicit more examples from the students. This is simple enough that everyone can give an example except perhaps those who seldom pay

attention to the class. At this point, it is not wise to ask questions of the students who most likely cannot answer correctly. Rather, the teacher would ask for examples from the students who are capable of giving correct examples and would sit near the student who does not pay attention so that that student would feel that he might be the next one to be called upon. The teacher should give enough opportunities to other able students until the uninterested student could give an example. Then the teacher could ask: "You know that the problem says that Susan is five years older than Kenneth. Now, if Susan is six years old, how old should Kenneth be?" When the uninterested student answers correctly, the teacher should give a positive response which would encourage the student to be a part of the problem-solving activity.

Similarly, discuss the second condition of the problem: The sum of Susan's age and Kenneth's age is 21. This discussion will provide an opportunity for all the students to understand the problem clearly including the previously uninterested students. The students will now be ready to answer questions, regardless of whether they are sure they are correct.

"Now, what are we looking for?...Susan's age and Kenneth's age." One student remarked, "Gee, it is a hard problem. If the problem just said that Susan is five years older than Kenneth then it is easy." This remark shows that the student comprehends the problem now. The teacher can point out that "it is difficult because we must consider two conditions at the same time. So let's consider only one condition for awhile and then add the other condition later." We could start with the condition that Susan is five years older than Kenneth. The teacher then elicits examples and records them on the board.

Susan's age	Kenneth's age
6	1
5	0
10	5
18	13
12	7
11	6
13	8
15	10
17	12
.	.
.	.
.	.

Usually, the "good students" will begin to give examples, but the condition is simple enough and has been discussed in understanding the problem, so virtually everyone in the class will give examples. Hence the blackboard may be covered with examples for the one condition. Then the second condition should be considered: the sum of Susan's age and Kenneth's age is 21. The class begins to add Susan's age and Kenneth's age for each example on the board. Eventually, they will find the correct solution.

Susan's age	Kenneth's age	Sum
6	1	7
5	0	5
7	2	9
10	5	15
19	14	33
12	7	19
11	6	17
13	8	21
15	10	25
17	12	29
.	.	.
.	.	.
.	.	.

None of the students will be embarrassed because his or her predicted example is incorrect because there are so many examples on the board that no one would have noticed whose example is correct and whose example is incorrect.

After finding the solution, the teacher should point out the importance of understanding the problem which not only involves reading the statement carefully but also thinking about examples for each condition. The teacher can point out that "if you were solving this problem by yourself, it would be helpful if you would arrange your examples with one condition in an orderly way." For examples, you could write

Susan	Kenneth
.	.
.	.
.	.
9	4
10	5
11	6
12	7
13	8
14	9
15	10
16	11
.	.
.	.
.	.

and check a few of the sums of the ages

Susan	Kenneth	Sum
.	.	
.	.	
.	.	
9	4	13
10	5	15
11	6	

Susan	Kenneth	Sum
12	7	
13	8	21
14	9	23
15	10	
16	11	27
.	.	
.	.	
.	.	

Then you could find a way in which your examples would give you the correct answer without checking all the situations. However, in the author's experiences, children would often find the sums for all the cases even after they found the answer. The teacher can also present alternate ways of solving the problem so that students will recognize that there are many other ways of solving problems and they will modify learned methods to ways that are most comfortable for them. An example of a way some students solved the problem is as follows:

10	9	12	13
+5	+4	+7	+8
15 ×	13 ×	19 ×	21 ✓

These children recognized that the way to find the correct answer is by first checking two cases and then using the sums to lead to the solution.

In order to develop a successful problem-solving program, a teacher's first task is to identify problems which are not too easy and not too difficult, problems the students will be able to solve only after they mobilize their resources. Therefore the teacher must consider the students' mathematical background as well as their cognitive development levels. Second, a teacher must demonstrate, by placing himself or herself in the student's place, how conscientious use of heuristics can help to solve problems. Third, a teacher must provide many opportunities for students to engage in problem-solving activities so that each student will experience success in solving problems after hard work. The excitement of successful problem solving may be imprinted on some of the students' minds and it may help to develop a character of curiosity and inquisitiveness for their lifetime. These characteristics are common to many great thinkers.

References

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