Teaching Model Problems and the Colour Coding of Problems

by

Bruce Hedderick
John Ware Junior High School

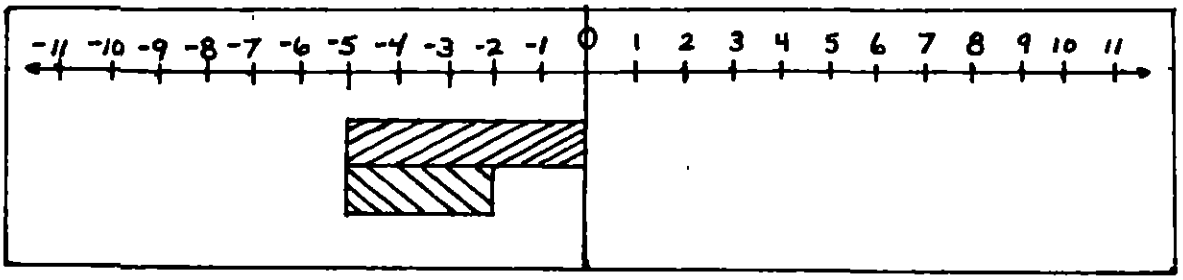
Model teaching of problems can be thought of as the excellent teaching of problems. It is hoped that my operational definition of "model problems" will encompass this idea. A model problem is a broad outline of a concept, using a particular problem, so that students may use the idea of this concept for similar questions and problems. Yaroshchuk (1969) suggests that,

> For a pupil to acquire a precise concept of a particular type of arithmetic problem, this type of problem should have a definite name. It is necessary that pupils learn to clearly isolate the mathematical structure of a model problem. In teaching model problems it is necessary to propose both numerical and subject problems, and compare them with each other. Only after they have acquired the ability to see this structure in the condition of both numerical and subject problems is it desirable to communicate the name of the given problem type to the pupils.

In other words, this material suggests that after a concept is presented using a couple of different problems, an outline for this concept would give students a general method of attack for other similar questions and problems. The outline can then be given a name to help the students remember how to attack these types of problems.

The teaching of model problems, as a concept, does not seem to be done in North American textbooks as it should. For example, the addition of integers, rational numbers, real numbers and algebra are taught, but never tied together in one large example. This example could be presented as a good review, once the pupils have been exposed to all of the ideas of integers, rational numbers, real numbers and algebra. The teaching of a model problem would incorporate these ideas: (1) a concrete example, (2) an integer example, (3) a rule, (4) a rational number example, (5) a real number example, (6) an algebraic example and (7) a problem example. The students would be asked to name this example and then some questions and problems could be given for practice.

Let us take an integer example of adding $-5 + (+3) =$ which could be made concrete by the use of a graphic diagram. If a child can get a picture in his mind of what is happening, the problems will seem much easier to him. The students draw a line down the middle of a sheet of graph paper and then using red and green pens designed for reading material, map the addition on the graph paper:



The students could then be asked to map $-5 + (-3) =$, $+5 + (-3) =$, and $+5 + (+3) =$, on the graph paper. Coloured dice could be used to generate more problems. The students could be asked to explain the rule and then solve these problems.

Integers	Rational Numbers	Real Numbers	Algebra
$-50 + (+30) =$	$-\frac{5}{7} + \frac{-3}{7} =$	$+5\sqrt{7} + (-3\sqrt{7}) =$	$+5X + (+3X) =$

The students having been given this type of understanding, can now be given both a number and a word problem. An example of a number problem is, "If Johnny adds positive $5\sqrt{11}$ to negative $3\sqrt{11}$, what is the answer?" An example of a word problem is, "A submarine is on the surface of the ocean at sea level. It dives 50 metres and then rises 30 metres. How far below sea level is the submarine now?"

Kalmukova (1975) reported on the teaching methods of a Russian elementary school teacher. The pupils of V.D. Petrova attracted attention because when difficulties in problem solving arose, the pupils returned to the text of a problem, reread it, and looked through the solution they had done. They also corrected most of the errors they made by themselves. They were, in case of failure, able to change the method of solution or find a new one. They could also outline a different path of solution for a single problem.

V.D. Petrova's classes were observed systematically while she was teaching problem solving. She emphasized reading the problem carefully with intonational expression, and emphasized that each word is important regardless of how small it is. Intonation had to be varied by the students when they saw punctuation marks. The students then separated the text of the problem into individual data and the unknown. The students completed the questions and then checked for mistakes. Homework was not considered done unless the scratch sheet was turned in as well. A pupil's mistake was analyzed by the class for errors in reading or thinking. The pupils were gradually trained in

controlling the operations they used, and in correcting their mistakes. A person learned to think with words. In addition to developing the pupil's speech, Petrova also developed their logical thought and increased the level of their analytic-synthetic activity.

One method of improving reading ability for problems, as well as improving the method of reading with intonation, is by colour coding the problems. This can be done by again using coloured reading pens. The red pen (represented by ⬭) can be used to highlight the numerical data. The green pen (represented by ▭) can be used to mark every word considered to be important in the question, and a yellow pen (represented by ☁) can be used to point out the unknown.

Problem: A submarine is on the surface of the ocean, at sea level. It dives 50 metres and then rises 30 metres. How far below sea level is the submarine now?

Solution: X = How far below the surface now?

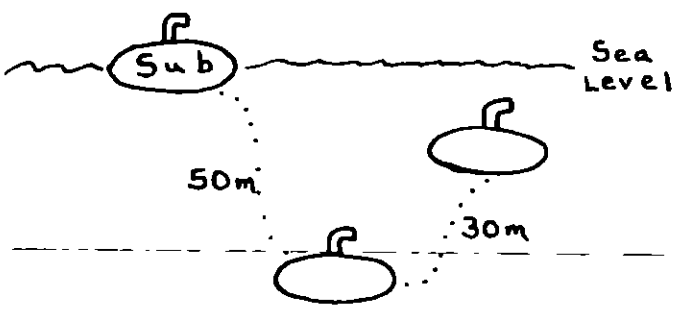
$$-50 + (+30) = X$$

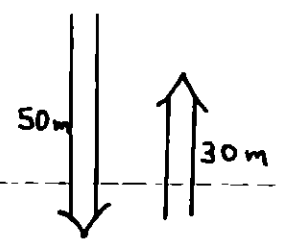
$$-20 = X$$

The submarine is 20 metres below the surface.

Try to solve the following number problem using colour coding. "If Johnny adds positive $5\sqrt{11}$ to negative $3\sqrt{11}$, what is the answer?"

M.E. Botsmanova (1972) suggests to us that the use of a graphic diagram could help students solve problems. A graphic diagram provides an abstracted and generalized expression of mathematical relationships. It starts with a subject analytic picture for the specific problem and leads to a graphic diagram for other cases. The following is a subject analytic picture and a graphic diagram for the submarine problem given above.





SUBJECT ANALYTIC PICTURE GRAPHIC DIAGRAM

Many texts give us pictures that don't help us to solve problems. These pictures should be changed to subject analytic pictures and graphic diagrams to help students find methods for attacking problems.

The combination of reading with intonation, colour coding, using a graphic diagram, and checking for errors, along with a model problem reference should help students form a wider application of their concept of problem solving. Finally, the students should name the model problem with their own words. Teachers might want to call the concept "solving model problems," while students might want to call it "solving submarine problems." Whatever the name, the idea is to make the model problems have meaning so the students remember the rule and its applications.

References

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