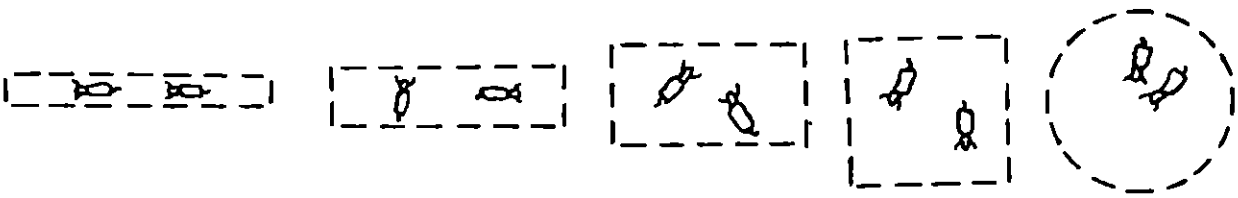
To You With Problem Solving

by

Al Anderson
Medicine Hat School System

"I want you to build a fence so that my cows will have the most possible grass to eat. That's the problem, Jim."

Jim has been given 16 five-cm rods and two plastic cows with the hidden agenda of finding a relationship between area and perimeter.



Believe it or not the above situation fits most of today's written criteria for a good problem. In short, a problem is any situation an individual faces for which no immediate solution is apparent but which holds the possibility for solution. Most descriptions would add the necessity for the person to accept the situation as a problem; otherwise, for him it is not a problem.

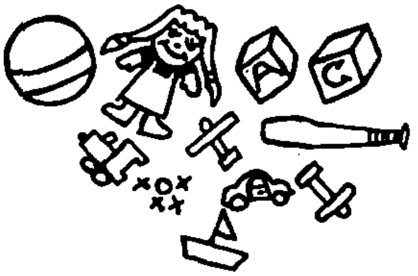
The number of students in schools who are able to experience problems like the "cow in the pasture" story is small. How many textbook problems, for example, include the following characteristics for good problems identified by Nelson and Kirkpatrick (1975)?

1. It is significant mathematically.
2. It involves a real object.
3. The child is interested in the problem.
4. The child must make modifications in the situation.
5. Several levels of solution are available.
6. The readiness with which the child attempts a solution indicates that he is convinced he can solve the problem.

Indeed we hear much about how poor students are at problem solving. What is it that they have difficulty with? Picking the equation? Getting the numerals for the sentence in the correct order? Such questions as these are indicative of a lack of experience with exciting problem-solving activities. Perhaps our students are poor problem solvers, but, given the right environment and a new mind-set toward the processes involved, both we and the students can learn. In other words, a good problem-solving experience has something for both the students and teachers. Let me illustrate through examples suitable for a range of grade levels.

Grades One-Two

Good problem-solving experiences are not limited to the upper elementary grades. The earliest pre-number development can and should be problem based. Classification tasks, for example, provide simple problem-solving experiences.



Are there more than three ways to put these toys in groups?

Even at this level pupils are required to go through a series of process actions, each one dependent on the preceding.

Eventual solution depends upon identifying minigoals and their place in the total problem.

In solving the classification task, pupils must realize that the toys have unique characteristics such as shape, color, texture, and function that relate to the "ways" to "group" the toys. They must also understand the concepts of "three" and "more than" before a realistic solution scheme can be devised. Next, there must be some type of planned actions or procedures.

The minigoals must be acted upon according to some plan toward problem solving.

Here some of the side benefits of teaching via problem solving begin to emerge. The nature of the problem will almost guarantee a pupil-teacher interaction. The setting promotes discussion. How are the toys different? Can you put some of the toys together in another way?

When the minigoals have been reached, the students must see that a solution is possible and then act on that conclusion.

The problem is to be solved.

The pupils must be able to demonstrate whether they were able to group the toys in more than three ways. A review of each setting is an important aspect of problem solving. Student and teacher should come to early and mutual agreement on the success or weakness of the strategies used.

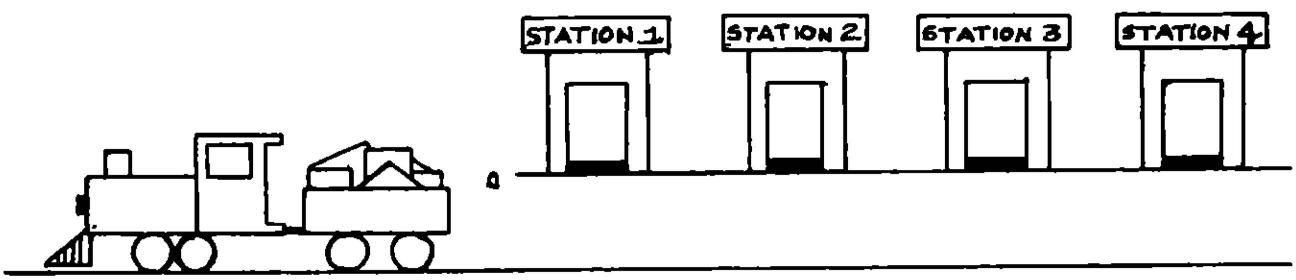
A review of the problem and solution strategy is necessary feedback.

Even though these same four steps are important at each grade level, there are obvious differences based on the prerequisite skills available to students.

Level	Prerequisite Skills	Problem-Solving Skills	Types of Problem
1-2	Sorting and classifying according to various attributes 1-1 Correspondence Equivalence Number as a class Cardinal number grouping Place value	Uses concrete materials to solve Collecting data from real experience Sequencing events in pictures and stories Discusses problems to find main parts Retells the problem Explains information in problem	Picture problem sorting, matching classification Non-number Patterns Sequences Operations concepts Numeration problems with object Calculator problems

Grades Three-Four

"You are a delivery man about to make drops at the stations but listen carefully and follow these rules:



Drop one block at Station 1 and two more blocks at Station 2 than you did at Station 1. Drop two more blocks at Station 3 than you did at Station 2, and so on. The problem is "At which station will you not have enough blocks to make a delivery (Nelson and Kirkpatrick, 1975)?"

Problems such as this can be used effectively for assessing the steps pupils take in solving problems. Do students analyze problems (identify minigoals) or do they barge ahead based on superficial hunches? What proportion of students operate as follows:

1. They do not determine the total number of blocks to be delivered (e.g., 12).
2. They tune in on problem segments and neglect the total perspective (e.g., drop one block at Station 1, two at Station 2, etc.).
3. They guess without evidence, (e.g., "Station 5").
4. They are satisfied with inappropriate solutions (no verification).

Although it is important that many problems be presented orally and that students be encouraged to talk through the problem parts and eventual solution, this oral thinking is more characteristic of young children than of children in the middle grades. Perhaps we are causing this with our abundance of written work, total class instruction and hush-hush enforcement.

Again, we can outline a distinct program of prerequisites and problem-solving skills as well as some problem types.

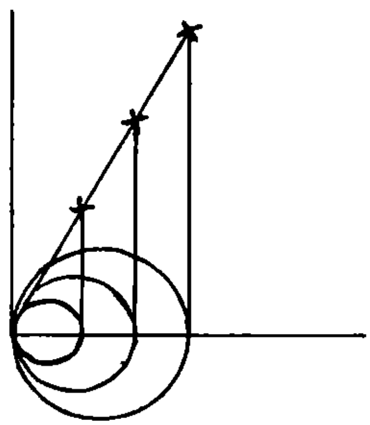
Level 1	Prerequisite Skills	Problem Solving Skills	Types of Problem
3-4	Can identify and use place value to (4-5 digits) Identifies and symbolizes operational situations Mastery of basic facts to limit of grade level Uses the algorithm to grade level limits Uses standard measuring instruments--metric linear, capacity, mass time and temperature Handles the money objectives to grade level Classifies and constructs 2-or 3-dimensional figures and objects	Uses estimation and approximation Collects data and constructs graph Interprets graphs and data charts Constructs models explores patterns Reconstructs problem Identifies relationships Makes tables for recording and interpreting data Makes projections, determines reasonableness of results Relates number sentence forms to operation situations	Real-life Can you make change for 50 cents using 6 coins, 7 coins, 8 coins? Tan-Gram puzzles Geoboard problem Number challenges If 7 cycle riders and 19 cycle wheels went by you, how many bicycles and how many tricycles passed? Calculator problems

As in our daily living, many problem breakthroughs are a result of group interaction. The following problem may best be handled in this way.

Grades Five-Six

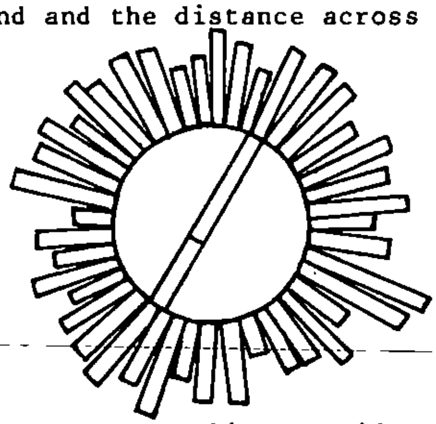
Collect sets of circular objects such as cans, cups, chips (about five different sizes). Give each grouping of two or three students one set of these circular shapes, one scissor, one metre of string, a 30 cm ruler and a large piece of paper. Students are given only one statement. "Graph the relationship of the distance around to the distance across your shapes."

Most of your groups will struggle for some time with this problem. There is a powerful temptation to rush. Don't be afraid of taking more than one class period. Some of your so-called slower pupils come through in such problems. Manipulative activities such as these often act as equalizers for these students.



Eventually, you want your pupils to cut pieces of string to fit the distance around the shapes (vertical). The horizontal distance is found by marking off the actual diameter. The result should be a straight line representation.

Another group of students can approach the same problem in a different activity. Give each group of two to three students a package of Cuisenaire rods. Ask them to illustrate or show the relationship between the distance around and the distance across a circular shape.



This method could also serve to establish the 3-1 relationship between circumference and diameter.

The next outline provides an extension of the types of problems, prerequisite skills, and problem-solving skills appropriate for grades 5 and 6.

Level	Prerequisite Skills	Problem-Solving Skills	Types of Problem
5-6	Rounding numbers Adds and subtracts whole numbers to grade limits Multiplies and divides to grade limits Ordered pairs Reads and writes coordinates Constructs and interprets graphs Uses appropriate standardized measuring units Reads distances to scale Draws diagrams to scale Knows interrelationship among units of length, capacity, and mass Uses decimals to thousand	Gains total perspective on problem Explains focus of problem Identifies required information Uses data collection and recording skills Uses diagrams and role play to solve problem Finds alternate solutions Applies equations where appropriate Checks solutions	Multi-step A Girl Guide troop sold 2000 boxes of cookies last year. This year they want to make $800. If they sell the same amount of cookies and cookies cost $1 per box, how much must they charge per box? No Solution Problem If a ship sinks one metre further in the water for every 200 people on board, how much of a ship will be under if 2000 people were aboard? Calculator Problem An average heart pumps 80 ml of blood each second. How many litres of blood has your heart pumped since birth? Fun Problem Two coins total 55 cents. One is not a nickel. What are the two coins?

It is good for students and teachers to realize that there are benefits to experiencing many ways of solving like problems. Such problems also provide opportunities for conducting informal assessment of pupils in terms of concepts, problem-solving process skills, or affective behavior. Anecdotal notes on students can be made while they are engaged in the problems. For example, how many give up before they try? Is there a reluctance to use pencil and paper for recording, drawing or graphing? Do students use the four problem-solving steps discussed earlier? Is there any evidence of their using previous knowledge (e.g., graphing, measurement, etc.)? Does their lack of response indicate a negative self-concept?

In conclusion, if success in problem solving is the primary goal of mathematics teaching and learning, why is it not more evident in our current mathematics programs? What can be done about improving the situation?

It is fairly clear that much confusion continues in regard to what problem solving is. Not only do the definitions vary, but we find different interpretations for its use in school programs. For example, some view problem solving only as an avenue for applying and practising newly acquired skills in a real-type setting. The basic purpose here is answer getting and refinement of skills. Others view problem solving as an opportunity to delve into the unknown. The essential goal here is for students to rediscover knowledge and to develop an awareness of skills needed. A growing view is that problem-solving process skills can be taught and thus applied to future problem situations be they science (discover-inquiry), social studies (value process) or language arts (comprehension).

Obviously, curriculum developers have failed in their attempts to build problem-solving skills, for whatever interpretation, into the current scope and sequence statements. Textbook publishers have typically had a narrow view of problem solving, mainly using word problems. We are also at fault for not going beyond what is handed to us.

Two final related cautions are in order as the various curriculum and instructional bodies attempt to rectify our failings on problem solving. It is important for all concerned not to impose rigidity on the teachers of problem solving. Second, let us not overemphasize the teaching of problem-solving processes to the extent that we are forced to swing back and forth to reach the necessary balance.

References

Nelson, D. and Kirkpatrick, J., "Problem solving". In J. Payne (Ed.), Mathematics learning in early childhood, NCTM's Thirty-Seventh Yearbook. Reston, VA: National Council of Teachers of Mathematics, 1975.