## Snap Number System

How would you like to develop your own number system?
Begin by selecting 2 arbitrary sets of symbols, and set up all the possible one-to-one correspondences between them.


How many combinations are there?
Did you get the combinations shown below?
Students like to make up names for each symbol, for example, Egor.


The order of the patterns and the labels (or symbols) of the elements may be chosen arbitrarily.
The patterns of the above elements can be represented on a square "snapboard" consisting of 9 nails in a pattern as shown on the diagram to the right. Rubber bands can be used to represent the patterns as shown on the diagram.


After the elements have been defined, an operation must be agreed upon. The second element may be placed on the snapboard by simply pulling the rubber bands over the third set of nails. Therefore, the position or relationship of the 2 elements " $a$ " and " $b$ " will be shown as on the diagram to the right. Students also like to make another name for "equals," for example, Gleep.


This emphasizes that our system has resulted from people's agreeing upon terminology.
Our operation between the 2 elements merely consists of "snapping" the rubber bands off the middle set of nails. The rubber bands will form a new pattern, which must be one of the 6 defined elements.

Can you think of a symbol and a name for the new operation? Let's arbitrarily call it "snap" and symbolize it as *.


In like manner, the operation of all the elements can be calculated.

What would be the pattern of elements on the following matrix as a result of the operation snap?

| $*$ | $E$ | $Z$ | $B$ | $M$ | $F$ | $T$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E$ |  |  |  |  |  |  |
| $Z$ |  |  |  |  |  |  |
| B |  |  |  |  |  |  |
| M |  |  |  |  |  |  |
| F |  |  |  |  |  |  |
| T |  |  |  |  |  |  |

Are there any similarities between this pattern and those of basic operations on whole numbers such as the identity and commutative or associative properties?

## Questions for Further Study

1. How would the matrix be affected if we begin with 2 sets, each containing 4 or more elements?
2. What is the relationship between the number of elements and the number of one-to-one correspondences?
3. Is it necessary for each set to have the same number of elements?
4. Could the operation snap be used on 3 or more elements at the same time?
5. Can you formulate any properties of the operation snap on the given elements?
6. Why should "snap" be any different from any of the basic operations?
7. Does "snap" help us to understand the properties of whole numbers? Explain.
S. Could the game "snap" have any practical applications?
8. Why has man agreed upon only the 4 basic applications?
9. What is an axiom? Cite a case in which mathematicians have used axioms. Why are axioms useful?
10. Can you create a similar game with different symbols and a different operation?
11. Could people have developed other number systems and algorithms? Explain.
