Volume of a Sphere


1. Place the sphere into the cylinder.
2. Use the cone to pour water to fill the cylinder.
3. The experiment shows that:

Volume of cylinder $=\pi r^{2} h$

$$
\begin{aligned}
& =\pi r^{2} \times 2 \mathrm{r} \\
& =2 \pi \mathrm{r}^{3}
\end{aligned}
$$

Volume of cone $\quad=\frac{1}{3} \pi r^{2} h$

$$
\begin{aligned}
& =\frac{1}{3} \pi r^{2} \times 2 r \\
& =\frac{2}{3} \pi r^{3}
\end{aligned}
$$

Therefore:
Volume of sphere $=$ Volume of cylinder-Volume of cone

$$
\begin{aligned}
& =2 \pi r^{3}-\frac{2}{3} \pi r^{3} \\
& =\left(2-\frac{2}{3}\right) \pi r^{3} \\
& =\frac{(6-2)}{3} \pi r^{3} \\
& =\frac{4}{3} \pi r^{3}
\end{aligned}
$$

## Surface Area of a Sphere



Volume of pyramid $=\frac{1}{3} a_{1} \times r$
If the sphere is made up of an infinite number of pyramids, then

$$
\begin{aligned}
& \frac{1}{3} a_{1} r+\frac{1}{3} a_{2} r+\ldots \\
& \frac{1}{3} r\left(a_{1}+a_{2}+a_{3}+\ldots\right)=\frac{4}{3} \pi r^{3} \\
& \pi r^{3}
\end{aligned}
$$

But $a_{1}+a_{2}+a_{3}+\ldots$ is the surface area of sphere.

Then $\frac{1}{3} r(A)=\frac{4}{3} \pi r^{3}$

$$
\begin{aligned}
A & =\frac{4}{3} \pi r^{3}+\frac{1}{3} r \\
A & =\frac{4}{3} \pi r^{3} \times \frac{3}{r} \\
& =4 \pi r^{2}
\end{aligned}
$$

